

# Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.3-e-x-  
 $^m-a-x^j+b-x^k-^p-c+d-x^n-^q$

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September 20, 2021

Compiled on September 20, 2021 at 6:16am

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# Chapter 1

## Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 228 ]. This is test number [ 18 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. [https://github.com/stblake/algebraic\\_integration](https://github.com/stblake/algebraic_integration). September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 228 )	0.00 ( 0 )
Mathematica	100.00 ( 228 )	0.00 ( 0 )
Fricas	100.00 ( 228 )	0.00 ( 0 )
Maple	99.12 ( 226 )	0.88 ( 2 )
Giac	93.42 ( 213 )	6.58 ( 15 )
Maxima	92.98 ( 212 )	7.02 ( 16 )
Mupad	86.40 ( 197 )	13.60 ( 31 )
IntegrateAlgebraic	62.28 ( 142 )	37.72 ( 86 )
Sympy	55.26 ( 126 )	% 44.74 ( 102 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

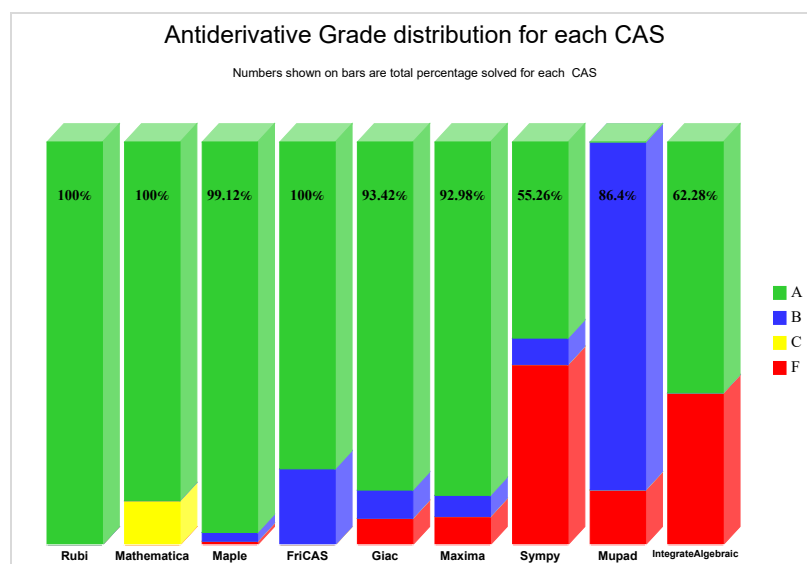
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

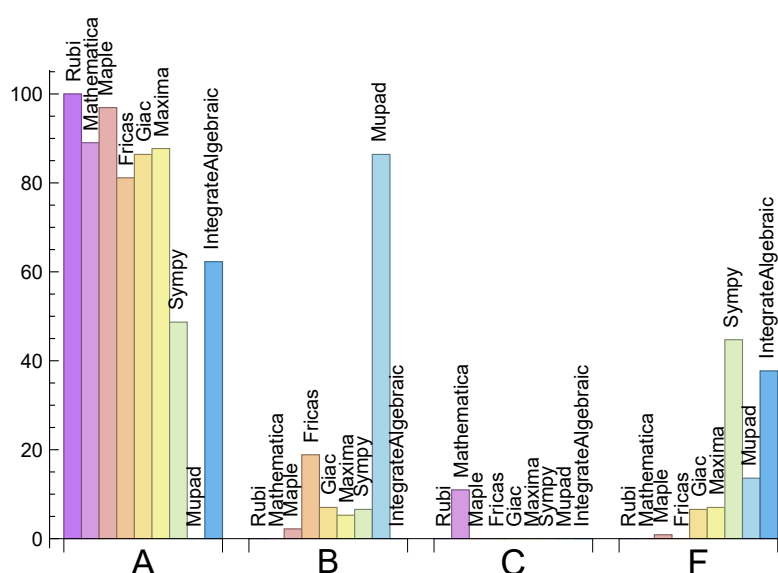
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	96.93	2.19	0.00	0.88
Mathematica	89.04	0.00	10.96	0.00
Maxima	87.72	5.26	0.00	7.02
Giac	86.40	7.02	0.00	6.58
Fricas	81.14	18.86	0.00	0.00
IntegrateAlgebraic	62.28	0.00	0.00	37.72
Sympy	48.68	6.58	0.00	44.74
Mupad	N/A	86.40	0.00	13.60

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
IntegrateAlgebraic	86	100.00 %	0.00 %	0.00 %
Giac	15	73.33 %	0.00 %	26.67 %
Maxima	16	100.00 %	0.00 %	0.00 %
Sympy	102	68.63 %	31.37 %	0.00 %
Mupad	31	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

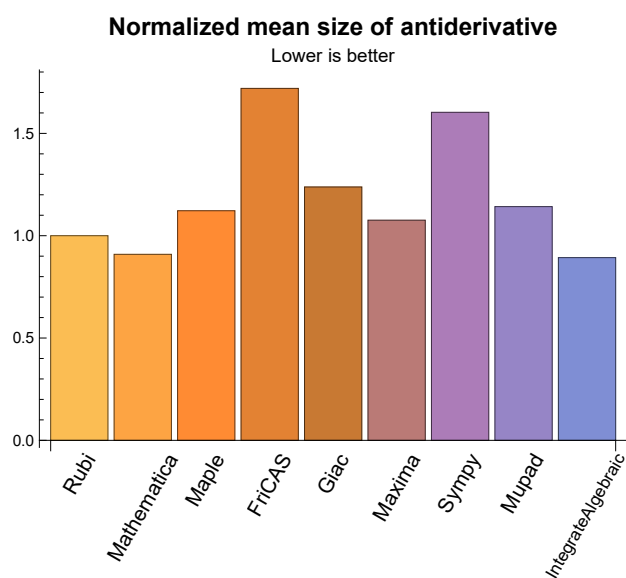
### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

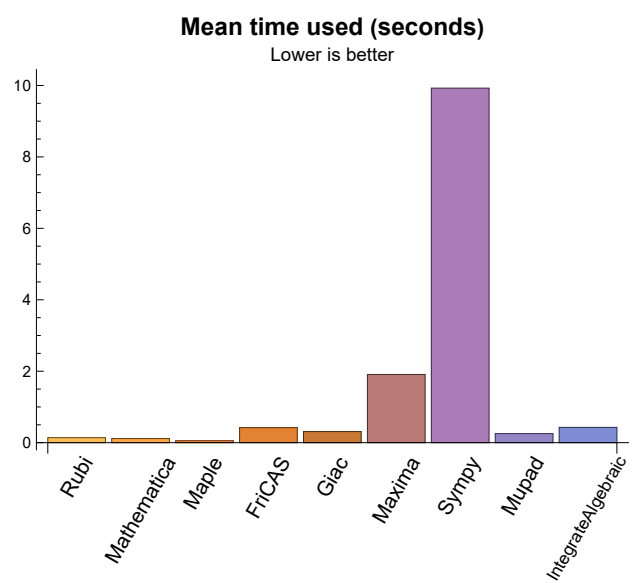
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	119.20	1.00	89.00	1.00
Mathematica	0.12	97.43	0.91	75.00	0.96
Maple	0.05	136.85	1.12	89.50	1.12
Maxima	1.91	124.24	1.08	84.00	0.99
Fricas	0.42	250.76	1.72	130.00	1.50
Sympy	9.92	130.07	1.60	80.00	1.18
Giac	0.31	144.20	1.24	97.00	1.03
Mupad	0.25	151.83	1.14	76.00	0.96
IntegrateAlgebraic	0.43	120.11	0.89	97.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.







## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

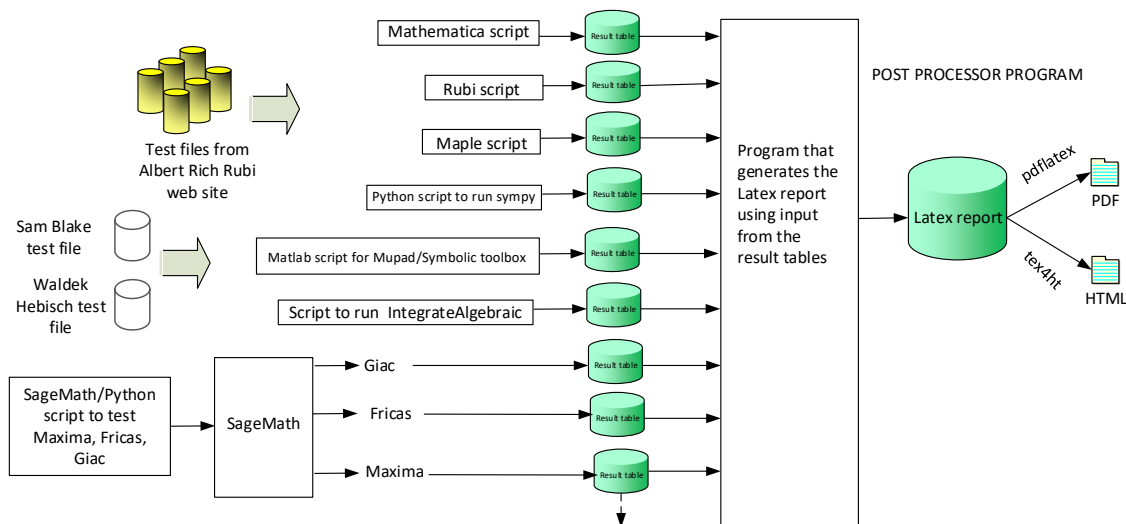
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x) \sim 2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**





# Chapter 2

## detailed summary tables of results

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## 2.1 List of integrals sorted by grade for each CAS

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### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { 112, 113, 125, 127, 128, 129, 157, 158, 191, 192, 193, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 228 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 224, 226, 227 }

B grade: { 126, 220, 221, 222, 225 }

C grade: { }

F grade: { 223, 228 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 106, 109, 110, 111, 112, 119, 120, 121, 122, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227 }

B grade: { 91, 96, 107, 108, 113, 114, 115, 116, 117, 118, 225, 228 }

C grade: { }

F grade: { 103, 104, 105, 123, 124, 125, 126, 127, 128, 129, 142, 143, 144, 156, 157, 158 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 223, 224, 225, 226, 227 }

B grade: { 84, 86, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 228 }

C grade: { }

F grade: { }

### 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 50, 53, 55, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 71, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 220, 221, 222, 224, 225, 226, 227 }

B grade: { 28, 43, 45, 49, 51, 52, 54, 56, 60, 66, 70, 72, 77, 83, 85 }

C grade: { }

F grade: { 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 183, 184, 185, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 228 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 145, 146, 147, 149, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 224, 225, 226, 227 }

B grade: { 28, 95, 96, 97, 98, 99, 113, 114, 115, 116, 117, 118, 220, 221, 222, 228 }

C grade: { }

F grade: { 139, 140, 141, 143, 144, 148, 150, 151, 152, 153, 154, 156, 157, 158, 223 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 108, 114, 115, 116, 117, 118, 119, 120, 121, 122, 133, 134, 135, 136, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227 }

C grade: { }

F grade: { 94, 95, 104, 105, 106, 107, 109, 110, 111, 112, 113, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 142, 143, 144, 145, 146, 147, 156, 157, 223, 228 }

### 2.1.9 IntegrateAlgebraic

A grade: { 4, 5, 13, 14, 15, 16, 25, 26, 27, 28, 29, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 220, 221, 222, 223, 224, 225, 226, 227, 228 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.027	0.007	0.043	1.403	0.356	0.074	0.150	0.179	0.000
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.041	0.010	0.044	1.259	0.349	0.066	0.149	0.059	0.000
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.024	0.006	0.049	1.348	0.336	0.066	0.193	0.040	0.000
Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	27	29	29	28	33
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85	1.00
time (sec)	N/A	0.038	0.008	0.043	1.307	0.374	0.067	0.148	0.059	0.019
Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	24	26	26	25	32
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89	1.14
time (sec)	N/A	0.019	0.019	0.041	1.406	0.383	0.066	0.146	0.060	0.029

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	28	28	25	27	30	26	0
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	1.03	0.90	0.00
time (sec)	N/A	0.027	0.009	0.041	1.334	0.398	0.117	0.147	0.061	0.001
Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	24	24	28	20	23	24	0
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92	0.00
time (sec)	N/A	0.023	0.009	0.059	1.325	0.361	0.115	0.158	0.073	0.001
Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	28	30	26	42	25	0
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.45	0.86	0.00
time (sec)	N/A	0.028	0.015	0.049	1.316	0.376	0.193	0.162	0.042	0.001
Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	27	25	26	29	27	28	26	0
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00	0.00
time (sec)	N/A	0.022	0.034	0.049	1.345	0.365	0.219	0.169	0.057	0.001
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	31	28	30	31	29	39	29	0
N.S.	1	1.00	1.07	0.97	1.03	1.07	1.00	1.34	1.00	0.00
time (sec)	N/A	0.027	0.027	0.049	1.396	0.389	0.374	0.153	0.074	0.001
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	33	28	29	29	32	31	29	0
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94	0.00
time (sec)	N/A	0.022	0.012	0.050	1.347	0.384	0.387	0.148	0.038	0.001

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	53	56	53	51	0
N.S.	1	1.00	1.00	0.95	0.93	0.96	1.02	0.96	0.93	0.00
time (sec)	N/A	0.048	0.010	0.043	1.333	0.339	0.076	0.145	0.085	0.000
Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	51	53	53	51	57
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.96	0.96	0.93	1.04
time (sec)	N/A	0.068	0.009	0.044	1.374	0.382	0.080	0.150	0.047	0.032
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	51	56	53	51	65
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93	1.18
time (sec)	N/A	0.039	0.009	0.044	1.322	0.369	0.080	0.167	0.049	0.035
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	51	52	51	51	53	53	51	57
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.26	1.26	1.21	1.36
time (sec)	N/A	0.070	0.016	0.039	1.363	0.368	0.082	0.162	0.045	0.031
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	48	53	50	48	60
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96	1.20
time (sec)	N/A	0.032	0.009	0.048	1.244	0.368	0.080	0.174	0.047	0.041
Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	51	51	52	49	49	53	48	0
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12	0.00
time (sec)	N/A	0.040	0.017	0.049	1.337	0.395	0.162	0.146	0.042	0.001



Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	49	48	53	48	48	48	0
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00	0.00
time (sec)	N/A	0.037	0.035	0.047	1.294	0.383	0.144	0.149	0.051	0.001
Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	49	50	52	54	48	70	48	0
N.S.	1	1.00	0.96	0.98	1.02	1.06	0.94	1.37	0.94	0.00
time (sec)	N/A	0.053	0.030	0.050	1.345	0.393	0.236	0.187	0.047	0.001
Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	50	46	50	52	51	50	50	0
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04	0.00
time (sec)	N/A	0.037	0.021	0.047	1.319	0.379	0.255	0.143	0.049	0.001
Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	50	51	54	55	51	72	51	0
N.S.	1	1.00	0.98	1.00	1.06	1.08	1.00	1.41	1.00	0.00
time (sec)	N/A	0.047	0.041	0.056	1.372	0.397	0.551	0.188	0.081	0.001
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	45	51	53	54	53	50	0
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04	0.00
time (sec)	N/A	0.037	0.027	0.055	1.389	0.366	0.609	0.155	0.068	0.001
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	53	52	55	55	56	66	51	0
N.S.	1	1.00	1.04	1.02	1.08	1.08	1.10	1.29	1.00	0.00
time (sec)	N/A	0.045	0.047	0.053	1.273	0.387	0.995	0.191	0.091	0.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	59	48	53	53	58	55	52	0
N.S.	1	1.00	1.11	0.91	1.00	1.00	1.09	1.04	0.98	0.00
time (sec)	N/A	0.036	0.021	0.049	1.370	0.384	1.074	0.150	0.038	0.001
Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	75	76	73	73	80	77	69	93
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.07	1.03	0.92	1.24
time (sec)	N/A	0.064	0.026	0.037	1.344	0.380	0.085	0.188	0.060	0.040
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	69	76	73	73	82	77	69	81
N.S.	1	1.00	1.01	1.12	1.07	1.07	1.21	1.13	1.01	1.19
time (sec)	N/A	0.134	0.031	0.039	1.317	0.381	0.086	0.212	0.030	0.039
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	75	76	73	73	82	77	69	93
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.09	1.03	0.92	1.24
time (sec)	N/A	0.055	0.014	0.045	1.367	0.371	0.086	0.153	0.030	0.040
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	69	76	73	73	80	77	69	81
N.S.	1	1.00	1.64	1.81	1.74	1.74	1.90	1.83	1.64	1.93
time (sec)	N/A	0.071	0.024	0.043	1.445	0.377	0.085	0.153	0.031	0.036
Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	73	70	70	76	73	65	85
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93	1.21
time (sec)	N/A	0.042	0.012	0.043	1.281	0.365	0.087	0.166	0.031	0.044

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	71	76	74	71	80	78	67	0
N.S.	1	1.00	1.18	1.27	1.23	1.18	1.33	1.30	1.12	0.00
time (sec)	N/A	0.053	0.022	0.045	1.315	0.382	0.180	0.152	0.035	0.001
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	71	69	75	68	70	65	0
N.S.	1	1.00	1.00	1.09	1.06	1.15	1.05	1.08	1.00	0.00
time (sec)	N/A	0.045	0.029	0.050	1.337	0.387	0.171	0.156	0.033	0.001
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	73	75	74	77	78	97	67	0
N.S.	1	1.00	1.03	1.06	1.04	1.08	1.10	1.37	0.94	0.00
time (sec)	N/A	0.079	0.044	0.051	1.301	0.386	0.265	0.173	0.039	0.001
Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	71	70	73	75	75	74	68	0
N.S.	1	1.00	1.03	1.01	1.06	1.09	1.09	1.07	0.99	0.00
time (sec)	N/A	0.050	0.035	0.049	1.331	0.371	0.291	0.151	0.057	0.001
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	73	76	76	76	75	98	76	0
N.S.	1	1.00	1.01	1.06	1.06	1.06	1.04	1.36	1.06	0.00
time (sec)	N/A	0.072	0.034	0.049	1.284	0.394	0.589	0.153	0.074	0.001
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	64	73	75	78	75	73	0
N.S.	1	1.00	1.00	0.94	1.07	1.10	1.15	1.10	1.07	0.00
time (sec)	N/A	0.050	0.036	0.047	1.326	0.384	0.676	0.164	0.057	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	75	77	77	78	99	75	0
N.S.	1	1.00	1.00	1.06	1.08	1.08	1.10	1.39	1.06	0.00
time (sec)	N/A	0.064	0.047	0.052	1.442	0.403	1.294	0.152	0.087	0.001
Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	63	73	75	80	77	71	0
N.S.	1	1.00	1.00	0.95	1.11	1.14	1.21	1.17	1.08	0.00
time (sec)	N/A	0.050	0.036	0.046	1.301	0.389	1.497	0.210	0.080	0.001
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	77	76	77	77	82	90	75	0
N.S.	1	1.00	1.22	1.21	1.22	1.22	1.30	1.43	1.19	0.00
time (sec)	N/A	0.047	0.059	0.053	1.322	0.399	2.367	0.167	0.103	0.001
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	66	75	75	83	79	74	0
N.S.	1	1.00	1.00	0.90	1.03	1.03	1.14	1.08	1.01	0.00
time (sec)	N/A	0.049	0.039	0.046	1.383	0.393	2.446	0.152	0.070	0.001
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	78	66	75	75	83	79	76	0
N.S.	1	1.00	1.59	1.35	1.53	1.53	1.69	1.61	1.55	0.00
time (sec)	N/A	0.040	0.021	0.048	1.328	0.382	3.426	0.157	0.071	0.001
Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	140	124	274	204	133	144	0
N.S.	1	1.00	1.00	1.18	1.04	2.30	1.71	1.12	1.21	0.00
time (sec)	N/A	0.087	0.085	0.048	3.005	0.408	0.444	0.170	0.171	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	92	110	97	98	94	101	100	0
N.S.	1	1.00	0.96	1.15	1.01	1.02	0.98	1.05	1.04	0.00
time (sec)	N/A	0.126	0.041	0.043	1.380	0.388	0.352	0.152	0.058	0.001
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	116	100	228	180	108	118	0
N.S.	1	1.00	1.00	1.18	1.02	2.33	1.84	1.10	1.20	0.00
time (sec)	N/A	0.076	0.068	0.049	2.940	0.405	0.408	0.182	0.043	0.001
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	71	86	74	75	70	77	76	0
N.S.	1	1.00	0.95	1.15	0.99	1.00	0.93	1.03	1.01	0.00
time (sec)	N/A	0.094	0.032	0.045	1.337	0.393	0.319	0.154	0.087	0.001
Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	92	78	178	153	85	96	0
N.S.	1	1.00	1.00	1.19	1.01	2.31	1.99	1.10	1.25	0.00
time (sec)	N/A	0.066	0.057	0.049	2.864	0.412	0.372	0.178	0.066	0.001
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	47	62	50	51	46	52	52	0
N.S.	1	1.00	0.87	1.15	0.93	0.94	0.85	0.96	0.96	0.00
time (sec)	N/A	0.069	0.021	0.051	1.326	0.391	0.286	0.152	0.074	0.001
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	68	53	129	90	57	70	0
N.S.	1	1.00	0.98	1.17	0.91	2.22	1.55	0.98	1.21	0.00
time (sec)	N/A	0.050	0.043	0.047	2.859	0.398	0.337	0.158	0.107	0.001

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	31	40	31	30	27	32	31	0
N.S.	1	1.00	0.89	1.14	0.89	0.86	0.77	0.91	0.89	0.00
time (sec)	N/A	0.043	0.013	0.044	1.299	0.386	0.250	0.232	0.056	0.001
Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	45	34	99	82	34	31	0
N.S.	1	1.00	1.00	1.12	0.85	2.48	2.05	0.85	0.78	0.00
time (sec)	N/A	0.028	0.025	0.044	2.924	0.408	0.288	0.160	0.054	0.001
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	37	35	32	26	34	32	0
N.S.	1	1.00	1.00	1.09	1.03	0.94	0.76	1.00	0.94	0.00
time (sec)	N/A	0.041	0.013	0.047	1.343	0.392	0.755	0.200	0.085	0.001
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	48	36	105	82	36	35	0
N.S.	1	1.00	1.00	1.14	0.86	2.50	1.95	0.86	0.83	0.00
time (sec)	N/A	0.032	0.049	0.048	2.944	0.397	0.423	0.165	0.092	0.001
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	39	51	103	75	38	33	0
N.S.	1	1.00	1.00	0.95	1.24	2.51	1.83	0.93	0.80	0.00
time (sec)	N/A	0.034	0.026	0.049	3.062	0.419	0.443	0.152	0.145	0.001
Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	56	48	47	41	71	46	0
N.S.	1	1.00	1.00	1.14	0.98	0.96	0.84	1.45	0.94	0.00
time (sec)	N/A	0.056	0.023	0.055	1.319	0.401	0.756	0.154	0.137	0.001

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	60	72	56	135	129	57	53	0
N.S.	1	1.00	0.98	1.18	0.92	2.21	2.11	0.93	0.87	0.00
time (sec)	N/A	0.056	0.087	0.057	2.815	0.407	0.437	0.158	0.106	0.001
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	81	70	73	61	100	70	0
N.S.	1	1.00	1.00	1.16	1.00	1.04	0.87	1.43	1.00	0.00
time (sec)	N/A	0.072	0.038	0.054	1.377	0.409	0.918	0.190	0.137	0.001
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	78	96	79	184	163	81	70	0
N.S.	1	1.00	1.00	1.23	1.01	2.36	2.09	1.04	0.90	0.00
time (sec)	N/A	0.068	0.089	0.048	3.048	0.406	0.583	0.156	0.106	0.001
Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	96	107	96	98	88	126	92	0
N.S.	1	1.00	1.04	1.16	1.04	1.07	0.96	1.37	1.00	0.00
time (sec)	N/A	0.090	0.046	0.046	1.280	0.395	1.412	0.155	0.153	0.001
Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	134	155	136	350	238	139	203	0
N.S.	1	1.00	1.01	1.17	1.02	2.63	1.79	1.05	1.53	0.00
time (sec)	N/A	0.166	0.114	0.053	2.940	0.400	1.012	0.169	0.104	0.001
Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	93	122	107	148	104	135	121	0
N.S.	1	1.00	0.89	1.16	1.02	1.41	0.99	1.29	1.15	0.00
time (sec)	N/A	0.135	0.078	0.054	1.380	0.384	0.930	0.154	0.097	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	111	132	112	298	211	115	141	0
N.S.	1	1.00	1.01	1.20	1.02	2.71	1.92	1.05	1.28	0.00
time (sec)	N/A	0.118	0.091	0.053	3.020	0.410	0.761	0.156	0.085	0.001
Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	72	98	82	121	78	106	86	0
N.S.	1	1.00	0.87	1.18	0.99	1.46	0.94	1.28	1.04	0.00
time (sec)	N/A	0.098	0.058	0.054	1.322	0.383	0.703	0.167	0.072	0.001
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	89	105	85	240	129	88	104	0
N.S.	1	1.00	1.00	1.18	0.96	2.70	1.45	0.99	1.17	0.00
time (sec)	N/A	0.089	0.080	0.064	3.026	0.409	0.689	0.157	0.110	0.001
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	50	74	60	81	56	70	62	0
N.S.	1	1.00	0.82	1.21	0.98	1.33	0.92	1.15	1.02	0.00
time (sec)	N/A	0.070	0.038	0.056	1.310	0.395	0.600	0.156	0.116	0.001
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	82	61	208	114	59	59	0
N.S.	1	1.00	1.00	1.21	0.90	3.06	1.68	0.87	0.87	0.00
time (sec)	N/A	0.061	0.056	0.053	2.999	0.422	0.553	0.191	0.126	0.001
Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	47	40	44	36	37	37	0
N.S.	1	1.00	1.00	1.15	0.98	1.07	0.88	0.90	0.90	0.00
time (sec)	N/A	0.047	0.014	0.054	1.311	0.389	0.374	0.187	0.086	0.001



Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	68	57	182	112	57	51	0
N.S.	1	1.00	1.00	1.08	0.90	2.89	1.78	0.90	0.81	0.00
time (sec)	N/A	0.033	0.051	0.056	2.965	0.395	0.417	0.182	0.120	0.001
Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	53	51	70	46	52	47	0
N.S.	1	1.00	0.90	1.04	1.00	1.37	0.90	1.02	0.92	0.00
time (sec)	N/A	0.055	0.034	0.055	1.314	0.383	0.441	0.157	0.159	0.001
Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	85	63	210	114	62	63	0
N.S.	1	1.00	1.00	1.21	0.90	3.00	1.63	0.89	0.90	0.00
time (sec)	N/A	0.074	0.036	0.056	2.977	0.409	0.506	0.213	0.129	0.001
Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	64	86	76	117	70	80	78	0
N.S.	1	1.00	0.88	1.18	1.04	1.60	0.96	1.10	1.07	0.00
time (sec)	N/A	0.080	0.060	0.070	1.332	0.379	0.880	0.159	0.147	0.001
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	90	110	93	250	184	85	83	0
N.S.	1	1.00	1.00	1.22	1.03	2.78	2.04	0.94	0.92	0.00
time (sec)	N/A	0.117	0.082	0.074	3.004	0.407	0.603	0.157	0.137	0.001
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	85	114	106	154	100	150	100	0
N.S.	1	1.00	0.88	1.18	1.09	1.59	1.03	1.55	1.03	0.00
time (sec)	N/A	0.108	0.106	0.056	1.328	0.399	1.040	0.322	0.138	0.001

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	112	136	119	308	218	112	104	0
N.S.	1	1.00	1.01	1.23	1.07	2.77	1.96	1.01	0.94	0.00
time (sec)	N/A	0.198	0.099	0.056	2.942	0.422	0.699	0.168	0.158	0.001
Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	133	174	147	416	252	138	177	0
N.S.	1	1.00	0.95	1.24	1.05	2.97	1.80	0.99	1.26	0.00
time (sec)	N/A	0.235	0.144	0.059	2.897	0.401	1.399	0.181	0.120	0.001
Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	94	134	116	179	119	132	118	0
N.S.	1	1.00	0.85	1.21	1.05	1.61	1.07	1.19	1.06	0.00
time (sec)	N/A	0.136	0.095	0.060	1.412	0.374	1.519	0.177	0.118	0.001
Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	113	147	120	358	214	111	138	0
N.S.	1	1.00	0.96	1.25	1.02	3.03	1.81	0.94	1.17	0.00
time (sec)	N/A	0.162	0.134	0.055	3.017	0.410	1.297	0.163	0.078	0.001
Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	92	109	94	142	94	93	95	0
N.S.	1	1.00	1.03	1.22	1.06	1.60	1.06	1.04	1.07	0.00
time (sec)	N/A	0.102	0.048	0.055	1.360	0.397	1.294	0.162	0.091	0.001
Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	92	122	94	328	194	80	92	0
N.S.	1	1.00	0.97	1.28	0.99	3.45	2.04	0.84	0.97	0.00
time (sec)	N/A	0.099	0.109	0.052	2.938	0.426	1.110	0.177	0.152	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	64	80	72	89	70	55	70	0
N.S.	1	1.00	0.96	1.19	1.07	1.33	1.04	0.82	1.04	0.00
time (sec)	N/A	0.076	0.026	0.054	1.335	0.388	0.932	0.189	0.108	0.001
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	83	89	92	301	155	78	82	0
N.S.	1	1.00	0.92	0.99	1.02	3.34	1.72	0.87	0.91	0.00
time (sec)	N/A	0.076	0.122	0.058	2.980	0.411	0.762	0.160	0.146	0.001
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	30	39	42	42	42	28	44	0
N.S.	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38	0.00
time (sec)	N/A	0.031	0.014	0.053	1.348	0.385	0.545	0.161	0.076	0.001
Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	84	90	92	300	150	78	82	0
N.S.	1	1.00	0.91	0.98	1.00	3.26	1.63	0.85	0.89	0.00
time (sec)	N/A	0.045	0.083	0.056	2.949	0.420	0.605	0.159	0.140	0.001
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	59	68	77	119	75	76	71	0
N.S.	1	1.00	0.87	1.00	1.13	1.75	1.10	1.12	1.04	0.00
time (sec)	N/A	0.073	0.068	0.060	1.416	0.404	0.594	0.158	0.178	0.001
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	125	96	324	194	82	113	0
N.S.	1	1.00	1.00	1.30	1.00	3.38	2.02	0.85	1.18	0.00
time (sec)	N/A	0.118	0.097	0.056	2.967	0.424	0.727	0.166	0.176	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	86	118	109	197	107	105	107	0
N.S.	1	1.00	0.89	1.22	1.12	2.03	1.10	1.08	1.10	0.00
time (sec)	N/A	0.117	0.100	0.063	1.373	0.396	1.113	0.183	0.152	0.001
Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	119	152	128	368	226	108	114	0
N.S.	1	1.00	1.02	1.30	1.09	3.15	1.93	0.92	0.97	0.00
time (sec)	N/A	0.177	0.077	0.062	3.111	0.414	0.828	0.162	0.179	0.001
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	108	150	137	229	136	132	131	0
N.S.	1	1.00	0.89	1.24	1.13	1.89	1.12	1.09	1.08	0.00
time (sec)	N/A	0.131	0.088	0.065	1.429	0.413	1.251	0.169	0.174	0.001
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	140	177	154	426	260	135	135	0
N.S.	1	1.00	1.00	1.26	1.10	3.04	1.86	0.96	0.96	0.00
time (sec)	N/A	0.332	0.085	0.072	3.024	0.427	0.920	0.162	0.195	0.001
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	135	180	170	267	165	201	155	0
N.S.	1	1.00	0.91	1.22	1.15	1.80	1.11	1.36	1.05	0.00
time (sec)	N/A	0.173	0.136	0.060	1.433	0.407	1.334	0.166	0.185	0.001
Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	193	290	321	368	0	245	289	187
N.S.	1	1.00	0.89	1.33	1.47	1.69	0.00	1.12	1.33	0.86
time (sec)	N/A	0.381	0.320	0.078	1.622	0.573	0.000	0.230	1.470	0.740

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	173	248	273	321	0	211	233	163
N.S.	1	1.00	0.96	1.37	1.51	1.77	0.00	1.17	1.29	0.90
time (sec)	N/A	0.334	0.302	0.061	1.514	0.487	0.000	0.203	0.886	0.651
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	151	206	225	272	0	177	177	139
N.S.	1	1.00	1.21	1.65	1.80	2.18	0.00	1.42	1.42	1.11
time (sec)	N/A	0.199	0.227	0.053	1.490	0.461	0.000	0.221	0.736	0.522
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	129	164	177	223	0	140	140	115
N.S.	1	1.00	1.21	1.53	1.65	2.08	0.00	1.31	1.31	1.07
time (sec)	N/A	0.154	0.199	0.053	1.424	0.427	0.000	0.193	0.752	0.587
Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	91	124	128	172	0	103	117	93
N.S.	1	1.00	0.91	1.24	1.28	1.72	0.00	1.03	1.17	0.93
time (sec)	N/A	0.197	0.155	0.054	1.451	0.437	0.000	0.185	0.609	0.497
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	78	130	105	161	0	92	-1	81
N.S.	1	1.00	0.80	1.34	1.08	1.66	0.00	0.95	-0.01	0.84
time (sec)	N/A	0.213	0.154	0.055	1.479	0.415	0.000	0.274	0.000	0.389
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	86	109	96	160	0	163	-1	86
N.S.	1	1.00	1.08	1.36	1.20	2.00	0.00	2.04	-0.01	1.08
time (sec)	N/A	0.196	0.130	0.059	1.418	0.442	0.000	0.498	0.000	0.367

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	44	48	111	59	0	250	113	66
N.S.	1	1.00	0.72	0.79	1.82	0.97	0.00	4.10	1.85	1.08
time (sec)	N/A	0.162	0.019	0.047	1.495	0.410	0.000	0.920	0.469	0.342
Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	66	70	161	85	0	310	160	90
N.S.	1	1.00	0.69	0.73	1.68	0.89	0.00	3.23	1.67	0.94
time (sec)	N/A	0.211	0.027	0.044	1.479	0.434	0.000	1.355	0.676	0.376
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	88	94	209	109	0	370	210	114
N.S.	1	1.00	0.66	0.71	1.57	0.82	0.00	2.78	1.58	0.86
time (sec)	N/A	0.245	0.030	0.053	1.494	0.469	0.000	1.970	1.039	0.404
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	94	118	257	133	0	430	260	138
N.S.	1	1.00	0.55	0.69	1.51	0.78	0.00	2.53	1.53	0.81
time (sec)	N/A	0.300	0.073	0.049	1.565	0.504	0.000	2.973	1.406	0.446
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	82	91	106	106	0	140	103	87
N.S.	1	1.00	0.63	0.69	0.81	0.81	0.00	1.07	0.79	0.66
time (sec)	N/A	0.220	0.064	0.048	1.419	0.409	0.000	0.170	0.252	0.105
Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	64	67	83	82	0	105	83	63
N.S.	1	1.00	0.68	0.71	0.88	0.87	0.00	1.12	0.88	0.67
time (sec)	N/A	0.166	0.046	0.051	1.535	0.411	0.000	0.209	0.191	0.088

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	41	45	51	57	0	72	60	41
N.S.	1	1.00	0.67	0.74	0.84	0.93	0.00	1.18	0.98	0.67
time (sec)	N/A	0.019	0.028	0.047	1.460	0.412	0.000	0.160	0.153	0.070
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	84	85	0	159	0	116	99	71
N.S.	1	1.00	1.08	1.09	0.00	2.04	0.00	1.49	1.27	0.91
time (sec)	N/A	0.146	0.086	0.050	0.000	0.411	0.000	0.196	0.498	0.142
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	94	135	0	169	0	76	-1	74
N.S.	1	1.00	0.94	1.35	0.00	1.69	0.00	0.76	-0.01	0.74
time (sec)	N/A	0.161	0.049	0.054	0.000	0.431	0.000	0.218	0.000	0.184
Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	95	174	0	198	0	132	-1	88
N.S.	1	1.00	0.92	1.69	0.00	1.92	0.00	1.28	-0.01	0.85
time (sec)	N/A	0.158	0.119	0.058	0.000	0.431	0.000	0.304	0.000	0.210
Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	215	328	363	418	0	280	-1	211
N.S.	1	1.00	0.96	1.47	1.63	1.87	0.00	1.26	-0.00	0.95
time (sec)	N/A	0.404	0.363	0.081	1.581	0.694	0.000	0.337	0.000	0.820
Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	193	286	315	369	0	246	-1	187
N.S.	1	1.00	1.16	1.71	1.89	2.21	0.00	1.47	-0.01	1.12
time (sec)	N/A	0.248	0.319	0.066	1.556	0.557	0.000	0.232	0.000	0.704

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	171	244	267	316	0	207	236	162
N.S.	1	1.00	1.16	1.65	1.80	2.14	0.00	1.40	1.59	1.09
time (sec)	N/A	0.191	0.267	0.060	1.508	0.485	0.000	0.217	1.011	0.702
Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	151	202	216	275	0	178	-1	139
N.S.	1	1.00	1.05	1.40	1.50	1.91	0.00	1.24	-0.01	0.97
time (sec)	N/A	0.266	0.243	0.057	1.493	0.471	0.000	0.202	0.000	0.711
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	130	162	168	224	0	142	-1	118
N.S.	1	1.00	0.95	1.18	1.23	1.64	0.00	1.04	-0.01	0.86
time (sec)	N/A	0.296	0.126	0.052	1.429	0.441	0.000	0.207	0.000	0.549
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	96	174	148	209	0	126	-1	100
N.S.	1	1.00	0.75	1.36	1.16	1.63	0.00	0.98	-0.01	0.78
time (sec)	N/A	0.277	0.205	0.059	1.494	0.437	0.000	0.347	0.000	0.535
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	98	219	167	189	0	225	-1	102
N.S.	1	1.00	0.72	1.61	1.23	1.39	0.00	1.65	-0.01	0.75
time (sec)	N/A	0.274	0.054	0.059	1.503	0.440	0.000	0.526	0.000	0.535
Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	94	153	177	207	0	254	-1	108
N.S.	1	1.00	0.90	1.47	1.70	1.99	0.00	2.44	-0.01	1.04
time (sec)	N/A	0.251	0.057	0.055	1.381	0.446	0.000	1.024	0.000	0.452



Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	44	48	193	82	0	370	156	90
N.S.	1	1.00	0.72	0.79	3.16	1.34	0.00	6.07	2.56	1.48
time (sec)	N/A	0.173	0.026	0.046	1.555	0.430	0.000	1.889	1.031	0.412
Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	66	70	241	109	0	430	206	114
N.S.	1	1.00	0.69	0.73	2.51	1.14	0.00	4.48	2.15	1.19
time (sec)	N/A	0.233	0.032	0.048	1.532	0.466	0.000	2.863	1.439	0.445
Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	89	94	289	134	0	490	256	138
N.S.	1	1.00	0.67	0.71	2.17	1.01	0.00	3.68	1.92	1.04
time (sec)	N/A	0.279	0.038	0.054	1.580	0.517	0.000	3.235	1.925	0.482
Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	95	118	337	157	0	550	306	162
N.S.	1	1.00	0.56	0.69	1.98	0.92	0.00	3.24	1.80	0.95
time (sec)	N/A	0.321	0.085	0.044	1.624	0.611	0.000	4.405	2.503	0.519
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	89	142	385	181	0	582	356	186
N.S.	1	1.00	0.43	0.69	1.86	0.87	0.00	2.81	1.72	0.90
time (sec)	N/A	0.352	0.077	0.046	1.650	0.749	0.000	5.435	3.210	0.556
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	113	115	150	154	0	175	143	111
N.S.	1	1.00	0.67	0.68	0.89	0.92	0.00	1.04	0.85	0.66
time (sec)	N/A	0.297	0.090	0.052	1.553	0.414	0.000	0.340	0.349	0.662

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	92	91	128	131	0	140	124	87
N.S.	1	1.00	0.70	0.69	0.98	1.00	0.00	1.07	0.95	0.66
time (sec)	N/A	0.241	0.076	0.047	1.527	0.418	0.000	0.211	0.292	0.558
Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	71	67	105	106	0	105	103	63
N.S.	1	1.00	0.74	0.70	1.09	1.10	0.00	1.09	1.07	0.66
time (sec)	N/A	0.070	0.054	0.052	1.557	0.421	0.000	0.168	0.255	0.552
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	48	45	80	80	0	72	83	41
N.S.	1	1.00	0.79	0.74	1.31	1.31	0.00	1.18	1.36	0.67
time (sec)	N/A	0.159	0.035	0.050	1.551	0.418	0.000	0.213	0.229	0.560
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	109	99	0	206	0	140	-1	95
N.S.	1	1.00	1.07	0.97	0.00	2.02	0.00	1.37	-0.01	0.93
time (sec)	N/A	0.205	0.097	0.048	0.000	0.435	0.000	0.176	0.000	0.681
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	109	172	0	195	0	115	-1	94
N.S.	1	1.00	0.82	1.29	0.00	1.47	0.00	0.86	-0.01	0.71
time (sec)	N/A	0.220	0.070	0.057	0.000	0.436	0.000	0.276	0.000	0.798
Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	63	213	0	217	0	145	-1	92
N.S.	1	1.00	0.47	1.58	0.00	1.61	0.00	1.07	-0.01	0.68
time (sec)	N/A	0.216	0.040	0.062	0.000	0.449	0.000	0.257	0.000	0.881

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	121	259	0	250	0	175	-1	112
N.S.	1	1.00	0.86	1.85	0.00	1.79	0.00	1.25	-0.01	0.80
time (sec)	N/A	0.223	0.143	0.075	0.000	0.440	0.000	0.272	0.000	1.042
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	66	302	0	299	0	214	-1	137
N.S.	1	1.00	0.37	1.71	0.00	1.69	0.00	1.21	-0.01	0.77
time (sec)	N/A	0.281	0.037	0.079	0.000	0.465	0.000	0.325	0.000	1.239
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	65	344	0	345	0	234	-1	161
N.S.	1	1.00	0.30	1.61	0.00	1.61	0.00	1.09	-0.00	0.75
time (sec)	N/A	0.339	0.036	0.090	0.000	0.510	0.000	0.309	0.000	1.496
Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	66	386	0	393	0	294	-1	185
N.S.	1	1.00	0.26	1.54	0.00	1.57	0.00	1.17	-0.00	0.74
time (sec)	N/A	0.390	0.040	0.151	0.000	0.581	0.000	0.374	0.000	1.838
Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	145	211	231	275	0	149	-1	139
N.S.	1	1.00	0.82	1.20	1.31	1.56	0.00	0.85	-0.01	0.79
time (sec)	N/A	0.328	0.262	0.058	1.379	0.464	0.000	0.223	0.000	0.566
Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	123	169	183	226	0	119	-1	115
N.S.	1	1.00	0.88	1.22	1.32	1.63	0.00	0.86	-0.01	0.83
time (sec)	N/A	0.275	0.182	0.055	1.454	0.446	0.000	0.250	0.000	0.579

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	97	127	134	177	0	91	-1	91
N.S.	1	1.00	1.17	1.53	1.61	2.13	0.00	1.10	-0.01	1.10
time (sec)	N/A	0.171	0.134	0.054	1.457	0.434	0.000	0.216	0.000	0.468
Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	81	88	88	131	0	67	89	76
N.S.	1	1.00	1.23	1.33	1.33	1.98	0.00	1.02	1.35	1.15
time (sec)	N/A	0.123	0.051	0.065	1.437	0.431	0.000	0.215	0.814	0.384
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	74	67	56	136	0	66	57	66
N.S.	1	1.00	1.30	1.18	0.98	2.39	0.00	1.16	1.00	1.16
time (sec)	N/A	0.151	0.041	0.058	1.412	0.427	0.000	0.224	0.509	0.324
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	43	47	70	38	0	88	39	44
N.S.	1	1.00	0.70	0.77	1.15	0.62	0.00	1.44	0.64	0.72
time (sec)	N/A	0.168	0.041	0.050	1.469	0.406	0.000	0.201	0.197	0.318
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	64	70	119	62	0	153	62	66
N.S.	1	1.00	0.67	0.73	1.24	0.65	0.00	1.59	0.65	0.69
time (sec)	N/A	0.209	0.029	0.051	1.489	0.412	0.000	0.209	0.251	0.343
Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	89	94	167	86	0	219	121	90
N.S.	1	1.00	0.67	0.71	1.26	0.65	0.00	1.65	0.91	0.68
time (sec)	N/A	0.253	0.051	0.054	1.488	0.421	0.000	0.308	0.301	0.366

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	94	118	215	110	0	287	156	114
N.S.	1	1.00	0.55	0.69	1.26	0.65	0.00	1.69	0.92	0.67
time (sec)	N/A	0.303	0.097	0.047	1.542	0.469	0.000	0.268	0.322	0.397
Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	85	89	106	83	0	0	87	87
N.S.	1	1.00	0.65	0.68	0.81	0.63	0.00	0.00	0.66	0.66
time (sec)	N/A	0.240	0.105	0.048	1.530	0.423	0.000	0.000	0.265	0.109
Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	63	65	83	59	0	0	64	63
N.S.	1	1.00	0.67	0.69	0.88	0.63	0.00	0.00	0.68	0.67
time (sec)	N/A	0.193	0.071	0.047	1.540	0.408	0.000	0.000	0.228	0.085
Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	40	42	50	36	0	0	41	40
N.S.	1	1.00	0.68	0.71	0.85	0.61	0.00	0.00	0.69	0.68
time (sec)	N/A	0.135	0.052	0.049	1.458	0.409	0.000	0.000	0.187	0.070
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	73	72	0	138	0	60	-1	55
N.S.	1	1.00	1.33	1.31	0.00	2.51	0.00	1.09	-0.02	1.00
time (sec)	N/A	0.020	0.039	0.056	0.000	0.433	0.000	0.326	0.000	0.108
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	87	105	0	152	0	0	-1	67
N.S.	1	1.00	1.28	1.54	0.00	2.24	0.00	0.00	-0.01	0.99
time (sec)	N/A	0.117	0.089	0.055	0.000	0.441	0.000	0.000	0.000	0.165

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	104	146	0	199	0	0	-1	89
N.S.	1	1.00	1.01	1.42	0.00	1.93	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.165	0.223	0.056	0.000	0.437	0.000	0.000	0.000	0.218
Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	136	166	237	340	0	176	-1	148
N.S.	1	1.00	0.74	0.90	1.29	1.85	0.00	0.96	-0.01	0.80
time (sec)	N/A	0.333	0.258	0.057	1.488	0.444	0.000	0.311	0.000	0.674
Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	113	140	187	289	0	147	-1	122
N.S.	1	1.00	0.77	0.95	1.27	1.97	0.00	1.00	-0.01	0.83
time (sec)	N/A	0.280	0.209	0.061	1.545	0.438	0.000	0.280	0.000	0.582
Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	91	115	138	230	0	116	-1	102
N.S.	1	1.00	0.81	1.03	1.23	2.05	0.00	1.04	-0.01	0.91
time (sec)	N/A	0.242	0.165	0.058	1.514	0.426	0.000	0.263	0.000	0.536
Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	75	75	79	188	0	0	78	85
N.S.	1	1.00	1.12	1.12	1.18	2.81	0.00	0.00	1.16	1.27
time (sec)	N/A	0.177	0.106	0.053	1.481	0.429	0.000	0.000	0.633	0.463
Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	47	65	49	0	36	53	49
N.S.	1	1.00	1.00	1.27	1.76	1.32	0.00	0.97	1.43	1.32
time (sec)	N/A	0.121	0.020	0.050	1.489	0.411	0.000	0.211	0.181	0.389

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	64	66	112	72	0	0	70	75
N.S.	1	1.00	0.97	1.00	1.70	1.09	0.00	0.00	1.06	1.14
time (sec)	N/A	0.164	0.043	0.048	1.385	0.419	0.000	0.000	0.274	0.393
Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	85	94	160	98	0	0	95	99
N.S.	1	1.00	0.84	0.93	1.58	0.97	0.00	0.00	0.94	0.98
time (sec)	N/A	0.220	0.037	0.049	1.501	0.421	0.000	0.000	0.424	0.423
Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	75	118	208	121	0	0	173	123
N.S.	1	1.00	0.54	0.86	1.51	0.88	0.00	0.00	1.25	0.89
time (sec)	N/A	0.267	0.054	0.050	1.478	0.427	0.000	0.000	0.559	0.506
Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	82	91	82	93	0	0	92	96
N.S.	1	1.00	0.59	0.65	0.59	0.67	0.00	0.00	0.66	0.69
time (sec)	N/A	0.248	0.076	0.051	1.576	0.416	0.000	0.000	0.397	1.032
Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	60	66	59	68	0	0	67	71
N.S.	1	1.00	0.58	0.63	0.57	0.65	0.00	0.00	0.64	0.68
time (sec)	N/A	0.200	0.041	0.052	1.559	0.396	0.000	0.000	0.280	0.851
Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	35	44	39	45	0	60	44	46
N.S.	1	1.00	0.51	0.64	0.57	0.65	0.00	0.87	0.64	0.67
time (sec)	N/A	0.150	0.027	0.050	1.569	0.407	0.000	0.292	0.210	0.727

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	73	79	0	199	0	0	-1	74
N.S.	1	1.00	1.14	1.23	0.00	3.11	0.00	0.00	-0.02	1.16
time (sec)	N/A	0.139	0.035	0.057	0.000	0.428	0.000	0.000	0.000	0.805
Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	61	129	0	260	0	0	-1	95
N.S.	1	1.00	0.43	0.91	0.00	1.83	0.00	0.00	-0.01	0.67
time (sec)	N/A	0.089	0.027	0.060	0.000	0.442	0.000	0.000	0.000	0.876
Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	64	157	0	315	0	0	89	120
N.S.	1	1.00	0.47	1.15	0.00	2.30	0.00	0.00	0.65	0.88
time (sec)	N/A	0.192	0.048	0.056	0.000	0.450	0.000	0.000	1.263	1.100
Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.023	0.018	0.046	1.344	0.413	20.137	0.147	0.056	0.034
Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.023	0.018	0.059	1.354	0.395	11.253	0.148	0.106	0.032
Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.023	0.017	0.049	1.330	0.397	5.592	0.151	0.044	0.031



Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	37	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	0.95	0.74	0.79	1.05
time (sec)	N/A	0.021	0.017	0.046	1.307	0.390	2.340	0.161	0.042	0.029
Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.021	0.017	0.050	1.366	0.387	2.060	0.209	0.044	0.026
Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	30	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.77	1.18	0.74	0.79	1.05
time (sec)	N/A	0.021	0.016	0.050	1.383	0.378	2.263	0.162	0.043	0.032
Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	32	27	29	44	29	31	41
N.S.	1	1.00	0.89	0.86	0.73	0.78	1.19	0.78	0.84	1.11
time (sec)	N/A	0.021	0.016	0.049	1.337	0.388	2.624	0.183	0.042	0.028
Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	32	27	29	44	29	31	35
N.S.	1	1.00	0.95	0.86	0.73	0.78	1.19	0.78	0.84	0.95
time (sec)	N/A	0.021	0.017	0.045	1.328	0.379	3.918	0.187	0.103	0.035
Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	51	56	80	53	51	69
N.S.	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.041	0.036	0.048	1.297	0.396	57.866	0.164	0.130	0.050

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.039	0.034	0.046	1.336	0.402	39.816	0.152	0.050	0.048
Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	51	56	80	53	51	69
N.S.	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.038	0.031	0.049	1.300	0.399	20.946	0.172	0.048	0.044
Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	66	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.05	0.84	0.81	1.10
time (sec)	N/A	0.038	0.032	0.049	1.383	0.402	3.729	0.178	0.049	0.048
Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.039	0.032	0.050	1.350	0.390	9.942	0.155	0.048	0.044
Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	51	56	80	53	51	69
N.S.	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.039	0.031	0.055	1.364	0.395	10.479	0.208	0.048	0.043
Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.038	0.032	0.057	1.366	0.395	12.055	0.151	0.049	0.048

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	54	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.86	1.27	0.84	0.81	1.10
time (sec)	N/A	0.038	0.034	0.054	1.332	0.395	15.463	0.173	0.051	0.045
Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.052	0.052	0.050	1.325	0.395	134.768	0.155	0.102	0.060
Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.050	0.050	0.049	1.361	0.395	92.322	0.170	0.033	0.060
Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.050	0.046	0.049	1.377	0.403	54.010	0.153	0.035	0.060
Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	95	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.12	0.91	0.81	1.14
time (sec)	N/A	0.050	0.043	0.052	1.334	0.390	5.510	0.171	0.033	0.057
Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.052	0.043	0.051	1.360	0.403	29.884	0.151	0.033	0.058

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.050	0.043	0.052	1.321	0.401	31.769	0.156	0.033	0.059
Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.052	0.040	0.053	1.403	0.398	35.910	0.163	0.036	0.057
Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.051	0.051	0.047	1.338	0.405	53.837	0.154	0.033	0.057
Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	133	336	237	920	0	298	115	180
N.S.	1	1.00	0.48	1.21	0.85	3.31	0.00	1.07	0.41	0.65
time (sec)	N/A	0.274	0.259	0.079	3.070	0.453	0.000	0.244	0.227	0.242
Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	227	330	259	714	0	298	788	181
N.S.	1	1.00	0.82	1.20	0.94	2.59	0.00	1.08	2.86	0.66
time (sec)	N/A	0.247	0.309	0.052	3.010	0.432	0.000	0.200	0.252	0.297
Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	110	308	214	899	0	264	92	160
N.S.	1	1.00	0.43	1.20	0.83	3.50	0.00	1.03	0.36	0.62
time (sec)	N/A	0.218	0.146	0.050	3.134	0.452	0.000	0.209	0.236	0.213

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	208	299	235	660	393	263	789	165
N.S.	1	1.00	0.82	1.17	0.92	2.59	1.54	1.03	3.09	0.65
time (sec)	N/A	0.209	0.224	0.062	2.990	0.429	164.461	0.196	0.275	0.217
Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	95	280	194	834	459	251	71	135
N.S.	1	1.00	0.40	1.18	0.82	3.52	1.94	1.06	0.30	0.57
time (sec)	N/A	0.188	0.080	0.053	3.019	0.463	65.599	0.227	0.153	0.206
Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	166	277	218	645	355	251	739	134
N.S.	1	1.00	0.71	1.18	0.93	2.74	1.51	1.07	3.14	0.57
time (sec)	N/A	0.184	0.146	0.056	3.159	0.440	27.828	0.183	0.292	0.204
Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	74	277	194	843	456	251	71	135
N.S.	1	1.00	0.31	1.18	0.83	3.59	1.94	1.07	0.30	0.57
time (sec)	N/A	0.189	0.097	0.062	3.011	0.454	18.455	0.194	0.222	0.201
Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	168	280	218	653	364	251	811	136
N.S.	1	1.00	0.71	1.18	0.92	2.76	1.54	1.06	3.42	0.57
time (sec)	N/A	0.188	0.154	0.060	3.067	0.440	27.101	0.228	0.301	0.215
Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	46	299	213	883	366	268	90	160
N.S.	1	1.00	0.18	1.17	0.84	3.46	1.44	1.05	0.35	0.63
time (sec)	N/A	0.215	0.022	0.061	3.080	0.440	108.599	0.241	0.226	0.226

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	47	308	247	707	405	257	555	162
N.S.	1	1.00	0.18	1.20	0.96	2.75	1.58	1.00	2.16	0.63
time (sec)	N/A	0.214	0.020	0.060	2.930	0.431	109.298	0.187	0.320	0.227
Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	47	330	237	931	0	291	107	185
N.S.	1	1.00	0.17	1.20	0.86	3.37	0.00	1.05	0.39	0.67
time (sec)	N/A	0.245	0.017	0.059	3.105	0.448	0.000	0.199	0.234	0.246
Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	47	336	276	734	0	291	563	184
N.S.	1	1.00	0.17	1.21	0.99	2.64	0.00	1.05	2.03	0.66
time (sec)	N/A	0.241	0.019	0.059	3.117	0.439	0.000	0.195	0.351	0.245
Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	332	332	417	372	298	804	0	335	857	443
N.S.	1	1.00	1.26	1.12	0.90	2.42	0.00	1.01	2.58	1.33
time (sec)	N/A	0.277	0.674	0.065	3.071	0.460	0.000	0.220	0.219	0.852
Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	154	348	247	989	0	299	127	362
N.S.	1	1.00	0.50	1.12	0.80	3.19	0.00	0.96	0.41	1.17
time (sec)	N/A	0.245	0.297	0.063	3.109	0.449	0.000	0.270	0.171	0.748
Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	385	339	271	748	0	298	823	208
N.S.	1	1.00	1.24	1.09	0.87	2.41	0.00	0.96	2.65	0.67
time (sec)	N/A	0.253	0.548	0.068	3.027	0.452	0.000	0.192	0.259	0.679

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	136	317	223	925	0	283	106	167
N.S.	1	1.00	0.47	1.10	0.77	3.20	0.00	0.98	0.37	0.58
time (sec)	N/A	0.229	0.265	0.065	3.312	0.452	0.000	0.209	0.247	0.721
Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	354	323	250	725	0	283	744	167
N.S.	1	1.00	1.22	1.12	0.87	2.51	0.00	0.98	2.57	0.58
time (sec)	N/A	0.234	0.528	0.064	3.108	0.451	0.000	0.195	0.212	0.712
Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	95	305	217	912	0	273	91	160
N.S.	1	1.00	0.36	1.17	0.83	3.49	0.00	1.05	0.35	0.61
time (sec)	N/A	0.200	0.182	0.062	3.110	0.451	0.000	0.209	0.229	0.682
Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	203	305	241	717	0	273	750	160
N.S.	1	1.00	0.78	1.17	0.92	2.75	0.00	1.05	2.87	0.61
time (sec)	N/A	0.200	0.366	0.056	3.110	0.444	0.000	0.207	0.331	0.629
Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	284	284	117	323	222	920	0	278	104	167
N.S.	1	1.00	0.41	1.14	0.78	3.24	0.00	0.98	0.37	0.59
time (sec)	N/A	0.230	0.316	0.060	3.036	0.461	0.000	0.205	0.232	0.673
Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	355	317	251	741	0	283	859	170
N.S.	1	1.00	1.23	1.10	0.87	2.56	0.00	0.98	2.97	0.59
time (sec)	N/A	0.225	0.582	0.058	3.111	0.448	0.000	0.195	0.372	0.674

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	151	339	250	974	0	303	121	200
N.S.	1	1.00	0.49	1.09	0.81	3.14	0.00	0.98	0.39	0.65
time (sec)	N/A	0.271	0.637	0.063	3.026	0.457	0.000	0.209	0.246	0.680
Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	385	348	286	795	0	292	595	200
N.S.	1	1.00	1.24	1.12	0.92	2.56	0.00	0.94	1.92	0.65
time (sec)	N/A	0.255	0.733	0.065	3.125	0.453	0.000	0.246	0.399	0.698
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	332	332	176	372	276	1024	0	328	142	224
N.S.	1	1.00	0.53	1.12	0.83	3.08	0.00	0.99	0.43	0.67
time (sec)	N/A	0.286	0.593	0.070	3.014	0.475	0.000	0.211	0.200	0.727
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	435	381	306	817	0	321	865	282
N.S.	1	1.00	1.27	1.11	0.89	2.38	0.00	0.94	2.52	0.82
time (sec)	N/A	0.278	0.628	0.076	3.080	0.449	0.000	0.234	0.249	0.895
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	176	357	256	993	0	304	138	257
N.S.	1	1.00	0.55	1.11	0.80	3.08	0.00	0.94	0.43	0.80
time (sec)	N/A	0.247	0.469	0.067	3.095	0.465	0.000	0.232	0.261	1.026
Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	403	363	283	793	0	304	760	257
N.S.	1	1.00	1.25	1.13	0.88	2.46	0.00	0.94	2.36	0.80
time (sec)	N/A	0.254	0.627	0.072	3.060	0.445	0.000	0.202	0.228	0.735



Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	137	325	251	990	0	293	122	249
N.S.	1	1.00	0.47	1.11	0.86	3.38	0.00	1.00	0.42	0.85
time (sec)	N/A	0.223	0.319	0.065	3.109	0.460	0.000	0.231	0.174	0.662
Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	389	334	280	806	0	298	799	191
N.S.	1	1.00	1.31	1.12	0.94	2.70	0.00	1.00	2.68	0.64
time (sec)	N/A	0.236	0.707	0.065	2.965	0.445	0.000	0.194	0.383	0.941
Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	62	335	253	1005	0	298	124	182
N.S.	1	1.00	0.21	1.12	0.85	3.37	0.00	1.00	0.42	0.61
time (sec)	N/A	0.229	0.087	0.070	3.014	0.457	0.000	0.233	0.165	1.025
Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	230	325	276	793	0	293	780	181
N.S.	1	1.00	0.78	1.11	0.94	2.71	0.00	1.00	2.66	0.62
time (sec)	N/A	0.231	0.378	0.064	3.054	0.444	0.000	0.195	0.392	0.826
Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	147	363	255	988	0	300	133	190
N.S.	1	1.00	0.47	1.15	0.81	3.13	0.00	0.95	0.42	0.60
time (sec)	N/A	0.256	0.294	0.069	3.074	0.469	0.000	0.212	0.261	0.794
Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	400	357	285	809	0	304	888	192
N.S.	1	1.00	1.24	1.11	0.89	2.51	0.00	0.94	2.76	0.60
time (sec)	N/A	0.251	0.496	0.068	3.089	0.460	0.000	0.205	0.446	0.643

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	189	381	285	1043	0	326	152	224
N.S.	1	1.00	0.55	1.11	0.83	3.04	0.00	0.95	0.44	0.65
time (sec)	N/A	0.275	0.517	0.070	3.038	0.465	0.000	0.221	0.194	0.663
Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	433	390	321	864	0	315	626	224
N.S.	1	1.00	1.26	1.14	0.94	2.52	0.00	0.92	1.83	0.65
time (sec)	N/A	0.278	0.571	0.068	3.145	0.463	0.000	0.219	0.449	0.735
Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	216	414	311	1093	0	351	173	248
N.S.	1	1.00	0.59	1.13	0.85	2.99	0.00	0.96	0.47	0.68
time (sec)	N/A	0.320	0.552	0.074	3.083	0.456	0.000	0.260	0.289	0.705
Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	467	420	353	894	0	351	639	248
N.S.	1	1.00	1.28	1.15	0.97	2.45	0.00	0.96	1.75	0.68
time (sec)	N/A	0.321	0.628	0.072	3.097	0.445	0.000	0.281	0.507	0.692
Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	89	474	129	381	2077	603	291	0
N.S.	1	1.00	0.93	4.94	1.34	3.97	21.64	6.28	3.03	0.00
time (sec)	N/A	0.070	0.142	0.052	1.366	0.411	9.055	0.215	0.477	0.621
Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	262	91	217	1051	340	179	0
N.S.	1	1.00	0.93	3.69	1.28	3.06	14.80	4.79	2.52	0.00
time (sec)	N/A	0.052	0.071	0.052	1.371	0.410	4.422	0.189	0.340	0.233

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	110	53	94	415	149	97	0
N.S.	1	1.00	0.93	2.44	1.18	2.09	9.22	3.31	2.16	0.00
time (sec)	N/A	0.030	0.054	0.049	1.300	0.419	1.621	0.163	0.247	0.053
Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	63	0	112	140	0	0	-1	0
N.S.	1	1.00	0.66	0.00	1.18	1.47	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.156	0.075	0.799	2.042	0.433	0.000	0.000	0.000	0.181
Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	17	17	13	0
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.89	0.68	0.00
time (sec)	N/A	0.026	0.006	0.046	2.909	0.388	0.113	0.155	0.171	0.000
Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	36	35	11	10	16	11	0
N.S.	1	1.00	1.00	2.40	2.33	0.73	0.67	1.07	0.73	0.00
time (sec)	N/A	0.022	0.006	0.050	3.010	0.390	0.121	0.152	0.153	0.000
Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	14	16	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.94	0.76	0.00
time (sec)	N/A	0.025	0.006	0.048	2.939	0.392	0.146	0.191	0.139	0.000
Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	18	10	16	14	0
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.62	1.00	0.88	0.00
time (sec)	N/A	0.014	0.004	0.053	1.623	0.389	0.102	0.183	0.129	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	116	0	37	43	0	48	-1	0
N.S.	1	1.00	6.44	0.00	2.06	2.39	0.00	2.67	-0.06	0.00
time (sec)	N/A	0.038	0.221	1.104	2.150	0.430	0.000	0.369	0.000	0.363

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [207] had the largest ratio of [.4231]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	4	3	1.00	20	0.150
3	A	3	2	1.00	19	0.105
4	A	4	3	1.00	22	0.136
5	A	3	2	1.00	22	0.091
6	A	4	3	1.00	22	0.136
7	A	3	2	1.00	22	0.091
8	A	4	3	1.00	22	0.136
9	A	3	2	1.00	22	0.091
10	A	4	3	1.00	22	0.136
11	A	3	2	1.00	22	0.091
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	24	0.125
14	A	3	2	1.00	24	0.083
15	A	4	3	1.00	24	0.125
16	A	3	2	1.00	24	0.083
17	A	5	4	1.00	24	0.167
18	A	3	2	1.00	24	0.083
19	A	4	3	1.00	24	0.125
20	A	3	2	1.00	24	0.083
21	A	4	3	1.00	24	0.125
22	A	3	2	1.00	24	0.083
23	A	4	3	1.00	24	0.125
24	A	3	2	1.00	24	0.083
25	A	3	2	1.00	24	0.083
26	A	4	3	1.00	24	0.125
27	A	3	2	1.00	24	0.083
28	A	4	3	1.00	24	0.125
29	A	3	2	1.00	24	0.083
30	A	5	4	1.00	24	0.167
31	A	3	2	1.00	24	0.083
32	A	4	3	1.00	24	0.125
33	A	3	2	1.00	24	0.083
34	A	4	3	1.00	24	0.125
35	A	3	2	1.00	24	0.083
36	A	4	3	1.00	24	0.125
37	A	3	2	1.00	24	0.083
38	A	5	4	1.00	24	0.167
39	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	4	4	1.00	24	0.167
41	A	5	4	1.00	24	0.167
42	A	4	3	1.00	24	0.125
43	A	5	4	1.00	24	0.167
44	A	4	3	1.00	24	0.125
45	A	5	4	1.00	24	0.167
46	A	4	3	1.00	24	0.125
47	A	4	4	1.00	24	0.167
48	A	4	3	1.00	24	0.125
49	A	3	3	1.00	24	0.125
50	A	4	3	1.00	22	0.136
51	A	3	3	1.00	21	0.143
52	A	3	3	1.00	22	0.136
53	A	4	3	1.00	24	0.125
54	A	4	4	1.00	24	0.167
55	A	4	3	1.00	24	0.125
56	A	5	4	1.00	24	0.167
57	A	4	3	1.00	24	0.125
58	A	5	4	1.00	24	0.167
59	A	4	3	1.00	24	0.125
60	A	5	4	1.00	24	0.167
61	A	4	3	1.00	24	0.125
62	A	5	4	1.00	24	0.167
63	A	4	3	1.00	24	0.125
64	A	4	4	1.00	24	0.167
65	A	4	3	1.00	24	0.125
66	A	3	3	1.00	24	0.125
67	A	4	3	1.00	24	0.125
68	A	4	4	1.00	24	0.167
69	A	4	3	1.00	22	0.136
70	A	5	4	1.00	21	0.190
71	A	4	3	1.00	24	0.125
72	A	5	4	1.00	24	0.167
73	A	6	5	1.00	24	0.208
74	A	4	3	1.00	24	0.125
75	A	6	5	1.00	24	0.208
76	A	4	3	1.00	24	0.125
77	A	5	5	1.00	24	0.208
78	A	4	3	1.00	24	0.125
79	A	4	4	1.00	24	0.167
80	A	3	3	1.00	24	0.125
81	A	4	4	1.00	24	0.167
82	A	4	3	1.00	24	0.125
83	A	5	4	1.00	24	0.167
84	A	4	3	1.00	24	0.125
85	A	6	5	1.00	24	0.208
86	A	4	3	1.00	22	0.136
87	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	4	3	1.00	24	0.125
89	A	8	7	1.00	26	0.269
90	A	7	7	1.00	26	0.269
91	A	5	5	1.00	26	0.192
92	A	5	5	1.00	24	0.208
93	A	5	5	1.00	26	0.192
94	A	5	5	1.00	26	0.192
95	A	5	5	1.00	26	0.192
96	A	3	3	1.00	26	0.115
97	A	4	4	1.00	26	0.154
98	A	5	4	1.00	26	0.154
99	A	6	4	1.00	26	0.154
100	A	4	3	1.00	26	0.115
101	A	3	3	1.00	26	0.115
102	A	2	2	1.00	23	0.087
103	A	4	4	1.00	26	0.154
104	A	4	4	1.00	26	0.154
105	A	4	4	1.00	26	0.154
106	A	8	7	1.00	26	0.269
107	A	6	5	1.00	26	0.192
108	A	6	5	1.00	24	0.208
109	A	6	6	1.00	26	0.231
110	A	6	6	1.00	26	0.231
111	A	6	5	1.00	26	0.192
112	A	6	6	1.00	26	0.231
113	A	6	5	1.00	26	0.192
114	A	3	3	1.00	26	0.115
115	A	4	4	1.00	26	0.154
116	A	5	4	1.00	26	0.154
117	A	6	4	1.00	26	0.154
118	A	7	4	1.00	26	0.154
119	A	5	4	1.00	26	0.154
120	A	4	4	1.00	26	0.154
121	A	3	3	1.00	23	0.130
122	A	2	2	1.00	26	0.077
123	A	5	4	1.00	26	0.154
124	A	5	4	1.00	26	0.154
125	A	5	5	1.00	26	0.192
126	A	5	4	1.00	26	0.154
127	A	6	5	1.00	26	0.192
128	A	7	5	1.00	26	0.192
129	A	8	5	1.00	26	0.192
130	A	7	6	1.00	26	0.231
131	A	6	6	1.00	26	0.231
132	A	4	4	1.00	26	0.154
133	A	4	4	1.00	24	0.167
134	A	4	4	1.00	26	0.154
135	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	4	4	1.00	26	0.154
137	A	5	4	1.00	26	0.154
138	A	6	4	1.00	26	0.154
139	A	4	3	1.00	26	0.115
140	A	3	3	1.00	26	0.115
141	A	2	2	1.00	26	0.077
142	A	3	3	1.00	23	0.130
143	A	3	3	1.00	26	0.115
144	A	4	4	1.00	26	0.154
145	A	7	6	1.00	26	0.231
146	A	6	6	1.00	26	0.231
147	A	5	5	1.00	26	0.192
148	A	4	4	1.00	26	0.154
149	A	2	2	1.00	24	0.083
150	A	3	3	1.00	26	0.115
151	A	4	4	1.00	26	0.154
152	A	5	4	1.00	26	0.154
153	A	4	3	1.00	26	0.115
154	A	3	3	1.00	26	0.115
155	A	2	2	1.00	26	0.077
156	A	3	3	1.00	26	0.115
157	A	5	5	1.00	23	0.217
158	A	5	5	1.00	26	0.192
159	A	3	2	1.00	24	0.083
160	A	3	2	1.00	24	0.083
161	A	3	2	1.00	24	0.083
162	A	3	2	1.00	24	0.083
163	A	3	2	1.00	24	0.083
164	A	3	2	1.00	24	0.083
165	A	3	2	1.00	24	0.083
166	A	3	2	1.00	24	0.083
167	A	3	2	1.00	26	0.077
168	A	3	2	1.00	26	0.077
169	A	3	2	1.00	26	0.077
170	A	3	2	1.00	26	0.077
171	A	3	2	1.00	26	0.077
172	A	3	2	1.00	26	0.077
173	A	3	2	1.00	26	0.077
174	A	3	2	1.00	26	0.077
175	A	3	2	1.00	26	0.077
176	A	3	2	1.00	26	0.077
177	A	3	2	1.00	26	0.077
178	A	3	2	1.00	26	0.077
179	A	3	2	1.00	26	0.077
180	A	3	2	1.00	26	0.077
181	A	3	2	1.00	26	0.077
182	A	3	2	1.00	26	0.077
183	A	14	10	1.00	26	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	14	10	1.00	26	0.385
185	A	13	10	1.00	26	0.385
186	A	13	10	1.00	26	0.385
187	A	12	9	1.00	26	0.346
188	A	12	9	1.00	26	0.346
189	A	12	9	1.00	26	0.346
190	A	12	9	1.00	26	0.346
191	A	13	10	1.00	26	0.385
192	A	13	10	1.00	26	0.385
193	A	14	10	1.00	26	0.385
194	A	14	10	1.00	26	0.385
195	A	15	10	1.00	26	0.385
196	A	14	10	1.00	26	0.385
197	A	14	10	1.00	26	0.385
198	A	13	10	1.00	26	0.385
199	A	13	10	1.00	26	0.385
200	A	12	9	1.00	26	0.346
201	A	12	9	1.00	26	0.346
202	A	13	10	1.00	26	0.385
203	A	13	10	1.00	26	0.385
204	A	14	10	1.00	26	0.385
205	A	14	10	1.00	26	0.385
206	A	15	10	1.00	26	0.385
207	A	15	11	1.00	26	0.423
208	A	14	11	1.00	26	0.423
209	A	14	11	1.00	26	0.423
210	A	13	10	1.00	26	0.385
211	A	13	10	1.00	26	0.385
212	A	13	10	1.00	26	0.385
213	A	13	10	1.00	26	0.385
214	A	14	11	1.00	26	0.423
215	A	14	11	1.00	26	0.423
216	A	15	11	1.00	26	0.423
217	A	15	11	1.00	26	0.423
218	A	16	11	1.00	26	0.423
219	A	16	11	1.00	26	0.423
220	A	3	2	1.00	24	0.083
221	A	3	2	1.00	24	0.083
222	A	3	2	1.00	22	0.091
223	A	2	2	1.00	32	0.062
224	A	4	3	1.00	19	0.158
225	A	4	3	1.00	15	0.200
226	A	4	3	1.00	19	0.158
227	A	3	2	1.00	17	0.118
228	A	2	2	1.00	39	0.051



# Chapter 3

## Listing of integrals

### Local contents

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3.8	$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} dx$	85
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3.14	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx$	101
3.15	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx$	104
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3.18	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx$	113
3.19	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx$	116
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3.21	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx$	122
3.22	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx$	125
3.23	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx$	128
3.24	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}} dx$	131

3.25	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx$	134
3.26	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3} dx$	137
3.27	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} dx$	140
3.28	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx$	143
3.29	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx$	146
3.30	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7} dx$	149
3.31	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} dx$	152
3.32	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx$	155
3.33	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} dx$	158
3.34	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} dx$	161
3.35	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} dx$	164
3.36	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} dx$	167
3.37	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx$	170
3.38	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} dx$	173
3.39	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} dx$	176
3.40	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}} dx$	179
3.41	$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx$	182
3.42	$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4} dx$	185
3.43	$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4} dx$	188
3.44	$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4} dx$	191
3.45	$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4} dx$	194
3.46	$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4} dx$	197
3.47	$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4} dx$	200
3.48	$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4} dx$	203
3.49	$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4} dx$	206
3.50	$\int \frac{x(A+Bx^2)}{bx^2+cx^4} dx$	209
3.51	$\int \frac{A+Bx^2}{bx^2+cx^4} dx$	212
3.52	$\int \frac{A+Bx^2}{bx^2-cx^4} dx$	215
3.53	$\int \frac{A+Bx^2}{x(bx^2+cx^4)} dx$	218
3.54	$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)} dx$	221
3.55	$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)} dx$	224
3.56	$\int \frac{A+Bx^2}{x^4(bx^2+cx^4)} dx$	227

3.57	$\int \frac{A+Bx^2}{x^5(bx^2+cx^4)} dx$	230
3.58	$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	233
3.59	$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	237
3.60	$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	240
3.61	$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx$	244
3.62	$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2} dx$	247
3.63	$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx$	250
3.64	$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2} dx$	253
3.65	$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2} dx$	256
3.66	$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2} dx$	259
3.67	$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2} dx$	262
3.68	$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2} dx$	265
3.69	$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2} dx$	268
3.70	$\int \frac{A+Bx^2}{(bx^2+cx^4)^2} dx$	271
3.71	$\int \frac{A+Bx^2}{x(bx^2+cx^4)^2} dx$	275
3.72	$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)^2} dx$	278
3.73	$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	282
3.74	$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	286
3.75	$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	289
3.76	$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	293
3.77	$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	296
3.78	$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3} dx$	300
3.79	$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3} dx$	303
3.80	$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3} dx$	306
3.81	$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3} dx$	309
3.82	$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3} dx$	312

3.83	$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3} dx$	315
3.84	$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx$	319
3.85	$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3} dx$	322
3.86	$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3} dx$	326
3.87	$\int \frac{A+Bx^2}{(bx^2+cx^4)^3} dx$	329
3.88	$\int \frac{A+Bx^2}{x(bx^2+cx^4)^3} dx$	333
3.89	$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4} dx$	336
3.90	$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4} dx$	340
3.91	$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4} dx$	344
3.92	$\int x(A+Bx^2)\sqrt{bx^2+cx^4} dx$	347
3.93	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x} dx$	350
3.94	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3} dx$	353
3.95	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5} dx$	357
3.96	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7} dx$	360
3.97	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9} dx$	363
3.98	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}} dx$	366
3.99	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}} dx$	369
3.100	$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4} dx$	373
3.101	$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4} dx$	376
3.102	$\int (A+Bx^2)\sqrt{bx^2+cx^4} dx$	379
3.103	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} dx$	382
3.104	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4} dx$	385
3.105	$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6} dx$	388
3.106	$\int x^5(A+Bx^2)(bx^2+cx^4)^{3/2} dx$	391
3.107	$\int x^3(A+Bx^2)(bx^2+cx^4)^{3/2} dx$	395
3.108	$\int x(A+Bx^2)(bx^2+cx^4)^{3/2} dx$	399
3.109	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x} dx$	403
3.110	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} dx$	407
3.111	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx$	411
3.112	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7} dx$	415
3.113	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9} dx$	419
3.114	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx$	423
3.115	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}} dx$	426

3.116	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} dx$	430
3.117	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} dx$	434
3.118	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} dx$	438
3.119	$\int x^4 (A+Bx^2)(bx^2+cx^4)^{3/2} dx$	442
3.120	$\int x^2 (A+Bx^2)(bx^2+cx^4)^{3/2} dx$	445
3.121	$\int (A+Bx^2)(bx^2+cx^4)^{3/2} dx$	448
3.122	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx$	451
3.123	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} dx$	454
3.124	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} dx$	457
3.125	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8} dx$	460
3.126	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}} dx$	463
3.127	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}} dx$	466
3.128	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx$	470
3.129	$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}} dx$	474
3.130	$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	478
3.131	$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	482
3.132	$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	486
3.133	$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	489
3.134	$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}} dx$	492
3.135	$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}} dx$	495
3.136	$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}} dx$	498
3.137	$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}} dx$	501
3.138	$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}} dx$	504
3.139	$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	507
3.140	$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	510
3.141	$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$	513
3.142	$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} dx$	516
3.143	$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}} dx$	519
3.144	$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}} dx$	522
3.145	$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	525
3.146	$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	529
3.147	$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	533

3.148	$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	537
3.149	$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	540
3.150	$\int \frac{A+Bx^2}{x(bx^2+cx^4)^{3/2}} dx$	543
3.151	$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}} dx$	546
3.152	$\int \frac{A+Bx^2}{x^5(bx^2+cx^4)^{3/2}} dx$	549
3.153	$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	552
3.154	$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	555
3.155	$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	558
3.156	$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$	561
3.157	$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}} dx$	564
3.158	$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)^{3/2}} dx$	567
3.159	$\int x^{7/2} (A + Bx^2) (bx^2 + cx^4) dx$	571
3.160	$\int x^{5/2} (A + Bx^2) (bx^2 + cx^4) dx$	574
3.161	$\int x^{3/2} (A + Bx^2) (bx^2 + cx^4) dx$	577
3.162	$\int \sqrt{x} (A + Bx^2) (bx^2 + cx^4) dx$	580
3.163	$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} dx$	582
3.164	$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} dx$	585
3.165	$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}} dx$	587
3.166	$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx$	590
3.167	$\int x^{7/2} (A + Bx^2) (bx^2 + cx^4)^2 dx$	593
3.168	$\int x^{5/2} (A + Bx^2) (bx^2 + cx^4)^2 dx$	596
3.169	$\int x^{3/2} (A + Bx^2) (bx^2 + cx^4)^2 dx$	599
3.170	$\int \sqrt{x} (A + Bx^2) (bx^2 + cx^4)^2 dx$	602
3.171	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} dx$	605
3.172	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} dx$	608
3.173	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} dx$	611
3.174	$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}} dx$	614
3.175	$\int x^{7/2} (A + Bx^2) (bx^2 + cx^4)^3 dx$	617
3.176	$\int x^{5/2} (A + Bx^2) (bx^2 + cx^4)^3 dx$	620
3.177	$\int x^{3/2} (A + Bx^2) (bx^2 + cx^4)^3 dx$	623
3.178	$\int \sqrt{x} (A + Bx^2) (bx^2 + cx^4)^3 dx$	626
3.179	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}} dx$	629



3.180	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}} dx$	632
3.181	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} dx$	635
3.182	$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}} dx$	638
3.183	$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4} dx$	641
3.184	$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4} dx$	646
3.185	$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4} dx$	651
3.186	$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4} dx$	656
3.187	$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4} dx$	662
3.188	$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4} dx$	667
3.189	$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4} dx$	672
3.190	$\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)} dx$	677
3.191	$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)} dx$	682
3.192	$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)} dx$	687
3.193	$\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)} dx$	692
3.194	$\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)} dx$	697
3.195	$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	702
3.196	$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	708
3.197	$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	713
3.198	$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	719
3.199	$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	724
3.200	$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	729
3.201	$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	734
3.202	$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	739
3.203	$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	744
3.204	$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2} dx$	750
3.205	$\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^2} dx$	755
3.206	$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2} dx$	761
3.207	$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	766
3.208	$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	772

3.209	$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	777
3.210	$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	783
3.211	$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	788
3.212	$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	794
3.213	$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	799
3.214	$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	805
3.215	$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	810
3.216	$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	816
3.217	$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	821
3.218	$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} dx$	827
3.219	$\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3} dx$	833
3.220	$\int x^m (A+Bx^2) (bx^2+cx^4)^3 dx$	839
3.221	$\int x^m (A+Bx^2) (bx^2+cx^4)^2 dx$	843
3.222	$\int x^m (A+Bx^2) (bx^2+cx^4) dx$	846
3.223	$\int x^{-1+n-jp} (c+dx^n) (ax^j+bx^{j+n})^p dx$	849
3.224	$\int \frac{4+3x^4}{5x+2x^5} dx$	852
3.225	$\int \frac{1+x^6}{x-x^7} dx$	855
3.226	$\int \frac{8+5x^{10}}{2x-x^{11}} dx$	858
3.227	$\int \frac{-3+2x}{-x^2+x^3} dx$	861
3.228	$\int x^m (a+bx^n)^p (a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}) dx$	863

### 3.1 $\int x^2 (A + Bx^2) (bx^2 + cx^4) dx$

**Optimal.** Leaf size=33

$$\frac{1}{7}x^7(Ac + bB) + \frac{1}{5}Abx^5 + \frac{1}{9}Bcx^9$$

**Rubi [A]** time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1584, 448}

$$\frac{1}{7}x^7(Ac + bB) + \frac{1}{5}Abx^5 + \frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

[In] Int[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (A\*b\*x^5)/5 + ((b\*B + A\*c)\*x^7)/7 + (B\*c\*x^9)/9

**Rule 448**

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int x^2 (A + Bx^2) (bx^2 + cx^4) dx &= \int x^4 (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^4 + (bB + Ac)x^6 + Bcx^8) dx \\ &= \frac{1}{5}Abx^5 + \frac{1}{7}(bB + Ac)x^7 + \frac{1}{9}Bcx^9 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{7}x^7(Ac + bB) + \frac{1}{5}Abx^5 + \frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (A\*b\*x^5)/5 + ((b\*B + A\*c)\*x^7)/7 + (B\*c\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (A + Bx^2) (bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] IntegrateAlgebraic[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.36, size = 29, normalized size = 0.88

$$\frac{1}{9}x^9cB + \frac{1}{7}x^7bB + \frac{1}{7}x^7cA + \frac{1}{5}x^5bA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 1/9\*x^9\*c\*B + 1/7\*x^7\*b\*B + 1/7\*x^7\*c\*A + 1/5\*x^5\*b\*A

**giac** [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{9}Bcx^9 + \frac{1}{7}Bbx^7 + \frac{1}{7}Acx^7 + \frac{1}{5}Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 1/9\*B\*c\*x^9 + 1/7\*B\*b\*x^7 + 1/7\*A\*c\*x^7 + 1/5\*A\*b\*x^5

**maple** [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bcx^9}{9} + \frac{Abx^5}{5} + \frac{(Ac + bB)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x)

[Out] 1/5\*A\*b\*x^5+1/7\*(A\*c+B\*b)\*x^7+1/9\*B\*c\*x^9

**maxima** [A] time = 1.40, size = 27, normalized size = 0.82

$$\frac{1}{9}Bcx^9 + \frac{1}{7}(Bb + Ac)x^7 + \frac{1}{5}Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 1/9\*B\*c\*x^9 + 1/7\*(B\*b + A\*c)\*x^7 + 1/5\*A\*b\*x^5

**mupad** [B] time = 0.18, size = 28, normalized size = 0.85

$$\frac{Bcx^9}{9} + \left(\frac{Ac}{7} + \frac{Bb}{7}\right)x^7 + \frac{Abx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x)

[Out] x^7\*((A\*c)/7 + (B\*b)/7) + (A\*b\*x^5)/5 + (B\*c\*x^9)/9

**sympy** [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^5}{5} + \frac{Bcx^9}{9} + x^7\left(\frac{Ac}{7} + \frac{Bb}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2),x)

[Out] A\*b\*x\*\*5/5 + B\*c\*x\*\*9/9 + x\*\*7\*(A\*c/7 + B\*b/7)

### 3.2 $\int x(A + Bx^2)(bx^2 + cx^4) dx$

**Optimal.** Leaf size=33

$$\frac{1}{6}x^6(Ac + bB) + \frac{1}{4}Abx^4 + \frac{1}{8}Bcx^8$$

**Rubi [A]** time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1584, 446, 76}

$$\frac{1}{6}x^6(Ac + bB) + \frac{1}{4}Abx^4 + \frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

[In] Int[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

[Out] (A\*b\*x^4)/4 + ((b\*B + A\*c)\*x^6)/6 + (B\*c\*x^8)/8

#### Rule 76

Int[((d\_.)\*(x\_.))^(n\_.)\*((a\_.) + (b\_.)\*(x\_.))\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int x(A + Bx^2)(bx^2 + cx^4) dx &= \int x^3(A + Bx^2)(b + cx^2) dx \\ &= \frac{1}{2} \text{Subst} \left( \int x(A + Bx)(b + cx) dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (Abx + (bB + Ac)x^2 + Bcx^3) dx, x, x^2 \right) \\ &= \frac{1}{4}Abx^4 + \frac{1}{6}(bB + Ac)x^6 + \frac{1}{8}Bcx^8 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{6}x^6(Ac + bB) + \frac{1}{4}Abx^4 + \frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

[In] Integrate[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

[Out] (A\*b\*x^4)/4 + ((b\*B + A\*c)\*x^6)/6 + (B\*c\*x^8)/8

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(A + Bx^2)(bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.35, size = 29, normalized size = 0.88

$$\frac{1}{8}x^8cB + \frac{1}{6}x^6bB + \frac{1}{6}x^6cA + \frac{1}{4}x^4bA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/8\*x^8\*c\*B + 1/6\*x^6\*b\*B + 1/6\*x^6\*c\*A + 1/4\*x^4\*b\*A

**giac** [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{8}Bcx^8 + \frac{1}{6}Bbx^6 + \frac{1}{6}Acx^6 + \frac{1}{4}Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] 1/8\*B\*c\*x^8 + 1/6\*B\*b\*x^6 + 1/6\*A\*c\*x^6 + 1/4\*A\*b\*x^4

**maple** [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bcx^8}{8} + \frac{Abx^4}{4} + \frac{(Ac + bB)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x)

[Out] 1/4\*A\*b\*x^4+1/6\*(A\*c+B\*b)\*x^6+1/8\*B\*c\*x^8

**maxima** [A] time = 1.26, size = 27, normalized size = 0.82

$$\frac{1}{8}Bcx^8 + \frac{1}{6}(Bb + Ac)x^6 + \frac{1}{4}Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x, algorithm="maxima")

[Out] 1/8\*B\*c\*x^8 + 1/6\*(B\*b + A\*c)\*x^6 + 1/4\*A\*b\*x^4

**mupad** [B] time = 0.06, size = 28, normalized size = 0.85

$$\frac{Bcx^8}{8} + \left(\frac{Ac}{6} + \frac{Bb}{6}\right)x^6 + \frac{Abx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(A + B*x^2)*(b*x^2 + c*x^4), x)`

[Out]  $x^6*((A*c)/6 + (B*b)/6) + (A*b*x^4)/4 + (B*c*x^8)/8$

**sympy** [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^4}{4} + \frac{Bcx^8}{8} + x^6 \left( \frac{Ac}{6} + \frac{Bb}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x**2+A)*(c*x**4+b*x**2), x)`

[Out]  $A*b*x**4/4 + B*c*x**8/8 + x**6*(A*c/6 + B*b/6)$

### 3.3 $\int (A + Bx^2)(bx^2 + cx^4) dx$

**Optimal.** Leaf size=33

$$\frac{1}{5}x^5(Ac + bB) + \frac{1}{3}Abx^3 + \frac{1}{7}Bcx^7$$

**Rubi [A]** time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1593, 448}

$$\frac{1}{5}x^5(Ac + bB) + \frac{1}{3}Abx^3 + \frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (A\*b\*x^3)/3 + ((b\*B + A\*c)\*x^5)/5 + (B\*c\*x^7)/7

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1593**

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int (A + Bx^2)(bx^2 + cx^4) dx &= \int x^2(A + Bx^2)(b + cx^2) dx \\ &= \int (Abx^2 + (bB + Ac)x^4 + Bcx^6) dx \\ &= \frac{1}{3}Abx^3 + \frac{1}{5}(bB + Ac)x^5 + \frac{1}{7}Bcx^7 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}x^5(Ac + bB) + \frac{1}{3}Abx^3 + \frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (A\*b\*x^3)/3 + ((b\*B + A\*c)\*x^5)/5 + (B\*c\*x^7)/7

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Bx^2)(bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]



[Out] IntegrateAlgebraic[(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.34, size = 29, normalized size = 0.88

$$\frac{1}{7}x^7cB + \frac{1}{5}x^5bB + \frac{1}{5}x^5cA + \frac{1}{3}x^3bA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 1/7\*x^7\*c\*B + 1/5\*x^5\*b\*B + 1/5\*x^5\*c\*A + 1/3\*x^3\*b\*A

**giac** [A] time = 0.19, size = 29, normalized size = 0.88

$$\frac{1}{7}Bcx^7 + \frac{1}{5}Bbx^5 + \frac{1}{5}Acx^5 + \frac{1}{3}Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 1/7\*B\*c\*x^7 + 1/5\*B\*b\*x^5 + 1/5\*A\*c\*x^5 + 1/3\*A\*b\*x^3

**maple** [A] time = 0.05, size = 28, normalized size = 0.85

$$\frac{Bcx^7}{7} + \frac{Abx^3}{3} + \frac{(Ac + bB)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2),x)

[Out] 1/3\*A\*b\*x^3+1/5\*(A\*c+B\*b)\*x^5+1/7\*B\*c\*x^7

**maxima** [A] time = 1.35, size = 27, normalized size = 0.82

$$\frac{1}{7}Bcx^7 + \frac{1}{5}(Bb + Ac)x^5 + \frac{1}{3}Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 1/7\*B\*c\*x^7 + 1/5\*(B\*b + A\*c)\*x^5 + 1/3\*A\*b\*x^3

**mupad** [B] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bcx^7}{7} + \left(\frac{Ac}{5} + \frac{Bb}{5}\right)x^5 + \frac{Abx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)\*(b\*x^2 + c\*x^4),x)

[Out] x^5\*((A\*c)/5 + (B\*b)/5) + (A\*b\*x^3)/3 + (B\*c\*x^7)/7

**sympy** [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^3}{3} + \frac{Bcx^7}{7} + x^5\left(\frac{Ac}{5} + \frac{Bb}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2),x)

[Out] A\*b\*x\*\*3/3 + B\*c\*x\*\*7/7 + x\*\*5\*(A\*c/5 + B\*b/5)

$$3.4 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x} dx$$

Optimal. Leaf size=33

$$\frac{1}{4}x^4(Ac + bB) + \frac{1}{2}Abx^2 + \frac{1}{6}Bcx^6$$

**Rubi [A]** time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 444, 43}

$$\frac{1}{4}x^4(Ac + bB) + \frac{1}{2}Abx^2 + \frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x,x]

[Out] (A\*b\*x^2)/2 + ((b\*B + A\*c)\*x^4)/4 + (B\*c\*x^6)/6

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x} dx &= \int x(A + Bx^2)(b + cx^2) dx \\ &= \frac{1}{2} \text{Subst} \left( \int (A + Bx)(b + cx) dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (Ab + (bB + Ac)x + Bcx^2) dx, x, x^2 \right) \\ &= \frac{1}{2}Abx^2 + \frac{1}{4}(bB + Ac)x^4 + \frac{1}{6}Bcx^6 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{4}x^4(Ac + bB) + \frac{1}{2}Abx^2 + \frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x,x]

[Out] (A\*b\*x^2)/2 + ((b\*B + A\*c)\*x^4)/4 + (B\*c\*x^6)/6

**IntegrateAlgebraic** [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{1}{12}x^2(6Ab + 3Acx^2 + 3bBx^2 + 2Bcx^4)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x,x]

[Out] (x^2\*(6\*A\*b + 3\*b\*B\*x^2 + 3\*A\*c\*x^2 + 2\*B\*c\*x^4))/12

**fricas** [A] time = 0.37, size = 27, normalized size = 0.82

$$\frac{1}{6}Bcx^6 + \frac{1}{4}(Bb + Ac)x^4 + \frac{1}{2}Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x,x, algorithm="fricas")

[Out] 1/6\*B\*c\*x^6 + 1/4\*(B\*b + A\*c)\*x^4 + 1/2\*A\*b\*x^2

**giac** [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{6}Bcx^6 + \frac{1}{4}Bbx^4 + \frac{1}{4}Acx^4 + \frac{1}{2}Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x,x, algorithm="giac")

[Out] 1/6\*B\*c\*x^6 + 1/4\*B\*b\*x^4 + 1/4\*A\*c\*x^4 + 1/2\*A\*b\*x^2

**maple** [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bcx^6}{6} + \frac{Abx^2}{2} + \frac{(Ac + bB)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x,x)

[Out] 1/2\*A\*b\*x^2+1/4\*(A\*c+B\*b)\*x^4+1/6\*B\*c\*x^6

**maxima** [A] time = 1.31, size = 27, normalized size = 0.82

$$\frac{1}{6}Bcx^6 + \frac{1}{4}(Bb + Ac)x^4 + \frac{1}{2}Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x,x, algorithm="maxima")

[Out] 1/6\*B\*c\*x^6 + 1/4\*(B\*b + A\*c)\*x^4 + 1/2\*A\*b\*x^2

**mupad** [B] time = 0.06, size = 28, normalized size = 0.85

$$\frac{Bcx^6}{6} + \left(\frac{Ac}{4} + \frac{Bb}{4}\right)x^4 + \frac{Abx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((A*c)/4 + (B*b)/4) + (A*b*x^2)/2 + (B*c*x^6)/6
```

**sympy [A]** time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^2}{2} + \frac{Bcx^6}{6} + x^4 \left( \frac{Ac}{4} + \frac{Bb}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x,x)
```

```
[Out] A*b*x**2/2 + B*c*x**6/6 + x**4*(A*c/4 + B*b/4)
```

$$3.5 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2} dx$$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(Ac + bB) + Abx + \frac{1}{5}Bcx^5$$

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1584, 373}

$$\frac{1}{3}x^3(Ac + bB) + Abx + \frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^2,x]

[Out] A\*b\*x + ((b\*B + A\*c)\*x^3)/3 + (B\*c\*x^5)/5

Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^2} dx &= \int (A + Bx^2)(b + cx^2) dx \\ &= \int (Ab + (bB + Ac)x^2 + Bcx^4) dx \\ &= Abx + \frac{1}{3}(bB + Ac)x^3 + \frac{1}{5}Bcx^5 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 28, normalized size = 1.00

$$\frac{1}{3}x^3(Ac + bB) + Abx + \frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^2,x]

[Out] A\*b\*x + ((b\*B + A\*c)\*x^3)/3 + (B\*c\*x^5)/5

**IntegrateAlgebraic [A]** time = 0.03, size = 32, normalized size = 1.14

$$Abx + \frac{1}{3}Acx^3 + \frac{1}{3}bBx^3 + \frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^2,x]

[Out]  $A*b*x + (b*B*x^3)/3 + (A*c*x^3)/3 + (B*c*x^5)/5$

**fricas** [A] time = 0.38, size = 24, normalized size = 0.86

$$\frac{1}{5} Bcx^5 + \frac{1}{3} (Bb + Ac)x^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="fricas")`

[Out]  $1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x$

**giac** [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{5} Bcx^5 + \frac{1}{3} Bbx^3 + \frac{1}{3} Acx^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="giac")`

[Out]  $1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + A*b*x$

**maple** [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{Bcx^5}{5} + Abx + \frac{(Ac + bB)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2)/x^2,x)`

[Out]  $A*b*x + 1/3*(A*c + B*b)*x^3 + 1/5*B*c*x^5$

**maxima** [A] time = 1.41, size = 24, normalized size = 0.86

$$\frac{1}{5} Bcx^5 + \frac{1}{3} (Bb + Ac)x^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="maxima")`

[Out]  $1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x$

**mupad** [B] time = 0.06, size = 25, normalized size = 0.89

$$\frac{Bcx^5}{5} + \left( \frac{Ac}{3} + \frac{Bb}{3} \right) x^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x)`

[Out]  $x^3*((A*c)/3 + (B*b)/3) + A*b*x + (B*c*x^5)/5$

**sympy** [A] time = 0.07, size = 26, normalized size = 0.93

$$Abx + \frac{Bcx^5}{5} + x^3 \left( \frac{Ac}{3} + \frac{Bb}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)/x**2,x)`

[Out]  $A*b*x + B*c*x**5/5 + x**3*(A*c/3 + B*b/3)$

$$3.6 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3} dx$$

Optimal. Leaf size=29

$$\frac{1}{2}x^2(Ac + bB) + Ab \log(x) + \frac{1}{4}Bcx^4$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 446, 76}

$$\frac{1}{2}x^2(Ac + bB) + Ab \log(x) + \frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^3,x]

[Out] ((b\*B + A\*c)\*x^2)/2 + (B\*c\*x^4)/4 + A\*b\*Log[x]

Rule 76

Int[((d\_.)\*(x\_))^(n\_.)\*((a\_) + (b\_.)\*(x\_))\*((e\_) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^3} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)}{x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( bB + Ac + \frac{Ab}{x} + Bcx \right) dx, x, x^2 \right) \\ &= \frac{1}{2}(bB + Ac)x^2 + \frac{1}{4}Bcx^4 + Ab \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2}x^2(Ac + bB) + Ab \log(x) + \frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^3,x]

[Out] ((b\*B + A\*c)\*x^2)/2 + (B\*c\*x^4)/4 + A\*b\*Log[x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^3,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^3, x]

**fricas** [A] time = 0.40, size = 25, normalized size = 0.86

$$\frac{1}{4} Bcx^4 + \frac{1}{2} (Bb + Ac)x^2 + Ab \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^3,x, algorithm="fricas")

[Out] 1/4\*B\*c\*x^4 + 1/2\*(B\*b + A\*c)\*x^2 + A\*b\*log(x)

**giac** [A] time = 0.15, size = 30, normalized size = 1.03

$$\frac{1}{4} Bcx^4 + \frac{1}{2} Bbx^2 + \frac{1}{2} Acx^2 + \frac{1}{2} Ab \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^3,x, algorithm="giac")

[Out] 1/4\*B\*c\*x^4 + 1/2\*B\*b\*x^2 + 1/2\*A\*c\*x^2 + 1/2\*A\*b\*log(x^2)

**maple** [A] time = 0.04, size = 28, normalized size = 0.97

$$\frac{Bc x^4}{4} + \frac{Ac x^2}{2} + \frac{Bb x^2}{2} + Ab \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^3,x)

[Out] 1/4\*B\*c\*x^4+1/2\*A\*x^2\*c+1/2\*B\*b\*x^2+A\*b\*ln(x)

**maxima** [A] time = 1.33, size = 28, normalized size = 0.97

$$\frac{1}{4} Bcx^4 + \frac{1}{2} (Bb + Ac)x^2 + \frac{1}{2} Ab \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^3,x, algorithm="maxima")

[Out] 1/4\*B\*c\*x^4 + 1/2\*(B\*b + A\*c)\*x^2 + 1/2\*A\*b\*log(x^2)

**mupad** [B] time = 0.06, size = 26, normalized size = 0.90

$$x^2 \left( \frac{Ac}{2} + \frac{Bb}{2} \right) + \frac{Bc x^4}{4} + Ab \ln(x)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x)`

[Out] `x^2*((A*c)/2 + (B*b)/2) + (B*c*x^4)/4 + A*b*log(x)`

sympy [A] time = 0.12, size = 27, normalized size = 0.93

$$Ab \log(x) + \frac{Bcx^4}{4} + x^2 \left( \frac{Ac}{2} + \frac{Bb}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)/x**3,x)`

[Out] `A*b*log(x) + B*c*x**4/4 + x**2*(A*c/2 + B*b/2)`

$$3.7 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} dx$$

**Optimal.** Leaf size=26

$$x(Ac + bB) - \frac{Ab}{x} + \frac{1}{3}Bcx^3$$

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1584, 448}

$$x(Ac + bB) - \frac{Ab}{x} + \frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^4,x]

[Out] -((A\*b)/x) + (b\*B + A\*c)\*x + (B\*c\*x^3)/3

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^4} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x^2} dx \\ &= \int \left( bB \left( 1 + \frac{Ac}{bB} \right) + \frac{Ab}{x^2} + Bcx^2 \right) dx \\ &= -\frac{Ab}{x} + (bB + Ac)x + \frac{1}{3}Bcx^3 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$x(Ac + bB) - \frac{Ab}{x} + \frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^4,x]

[Out] -((A\*b)/x) + (b\*B + A\*c)\*x + (B\*c\*x^3)/3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^4,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^4, x]

**fricas** [A] time = 0.36, size = 28, normalized size = 1.08

$$\frac{Bcx^4 + 3(Bb + Ac)x^2 - 3Ab}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^4,x, algorithm="fricas")

[Out] 1/3\*(B\*c\*x^4 + 3\*(B\*b + A\*c)\*x^2 - 3\*A\*b)/x

**giac** [A] time = 0.16, size = 23, normalized size = 0.88

$$\frac{1}{3}Bcx^3 + Bbx + Acx - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^4,x, algorithm="giac")

[Out] 1/3\*B\*c\*x^3 + B\*b\*x + A\*c\*x - A\*b/x

**maple** [A] time = 0.06, size = 24, normalized size = 0.92

$$\frac{Bcx^3}{3} + Acx + Bbx - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^4,x)

[Out] 1/3\*B\*c\*x^3+A\*c\*x+B\*b\*x-A\*b/x

**maxima** [A] time = 1.33, size = 24, normalized size = 0.92

$$\frac{1}{3}Bcx^3 + (Bb + Ac)x - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^4,x, algorithm="maxima")

[Out] 1/3\*B\*c\*x^3 + (B\*b + A\*c)\*x - A\*b/x

**mupad** [B] time = 0.07, size = 24, normalized size = 0.92

$$x(Ac + Bb) - \frac{Ab}{x} + \frac{Bcx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^4,x)

[Out] x\*(A\*c + B\*b) - (A\*b)/x + (B\*c\*x^3)/3

**sympy** [A] time = 0.11, size = 20, normalized size = 0.77

$$-\frac{Ab}{x} + \frac{Bcx^3}{3} + x(Ac + Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)/x\*\*4,x)

[Out] -A\*b/x + B\*c\*x\*\*3/3 + x\*(A\*c + B\*b)

$$3.8 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} dx$$

Optimal. Leaf size=29

$$\log(x)(Ac + bB) - \frac{Ab}{2x^2} + \frac{1}{2}Bcx^2$$

**Rubi [A]** time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 446, 76}

$$\log(x)(Ac + bB) - \frac{Ab}{2x^2} + \frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^5,x]

[Out] -(A\*b)/(2\*x^2) + (B\*c\*x^2)/2 + (b\*B + A\*c)\*Log[x]

Rule 76

Int[((d\_)\*(x\_))^(n\_)\*((a\_) + (b\_)\*(x\_))\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^5} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)}{x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( Bc + \frac{Ab}{x^2} + \frac{bB + Ac}{x} \right) dx, x, x^2 \right) \\ &= -\frac{Ab}{2x^2} + \frac{1}{2}Bcx^2 + (bB + Ac) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$\log(x)(Ac + bB) - \frac{Ab}{2x^2} + \frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^5, x]

[Out] -1/2\*(A\*b)/x^2 + (B\*c\*x^2)/2 + (b\*B + A\*c)\*Log[x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^5, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^5, x]

**fricas** [A] time = 0.38, size = 30, normalized size = 1.03

$$\frac{Bcx^4 + 2(Bb + Ac)x^2 \log(x) - Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^5, x, algorithm="fricas")

[Out] 1/2\*(B\*c\*x^4 + 2\*(B\*b + A\*c)\*x^2\*log(x) - A\*b)/x^2

**giac** [A] time = 0.16, size = 42, normalized size = 1.45

$$\frac{1}{2} Bcx^2 + \frac{1}{2} (Bb + Ac) \log(x^2) - \frac{Bbx^2 + Acx^2 + Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^5, x, algorithm="giac")

[Out] 1/2\*B\*c\*x^2 + 1/2\*(B\*b + A\*c)\*log(x^2) - 1/2\*(B\*b\*x^2 + A\*c\*x^2 + A\*b)/x^2

**maple** [A] time = 0.05, size = 26, normalized size = 0.90

$$\frac{Bcx^2}{2} + Ac \ln(x) + Bb \ln(x) - \frac{Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^5, x)

[Out] 1/2\*B\*c\*x^2-1/2\*A\*b/x^2+A\*ln(x)\*c+B\*b\*ln(x)

**maxima** [A] time = 1.32, size = 28, normalized size = 0.97

$$\frac{1}{2} Bcx^2 + \frac{1}{2} (Bb + Ac) \log(x^2) - \frac{Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^5, x, algorithm="maxima")

[Out] 1/2\*B\*c\*x^2 + 1/2\*(B\*b + A\*c)\*log(x^2) - 1/2\*A\*b/x^2

**mupad** [B] time = 0.04, size = 25, normalized size = 0.86

$$\ln(x) (Ac + Bb) - \frac{Ab}{2x^2} + \frac{Bcx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x)
```

```
[Out] log(x)*(A*c + B*b) - (A*b)/(2*x^2) + (B*c*x^2)/2
```

sympy [A] time = 0.19, size = 26, normalized size = 0.90

$$-\frac{Ab}{2x^2} + \frac{Bcx^2}{2} + (Ac + Bb)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**5,x)
```

```
[Out] -A*b/(2*x**2) + B*c*x**2/2 + (A*c + B*b)*log(x)
```

$$3.9 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} dx$$

Optimal. Leaf size=26

$$-\frac{Ac + bB}{x} - \frac{Ab}{3x^3} + Bcx$$

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1584, 448}

$$-\frac{Ac + bB}{x} - \frac{Ab}{3x^3} + Bcx$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^6, x]

[Out] -(A\*b)/(3\*x^3) - (b\*B + A\*c)/x + B\*c\*x

Rule 448

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^6} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x^4} dx \\ &= \int \left( Bc + \frac{Ab}{x^4} + \frac{bB + Ac}{x^2} \right) dx \\ &= -\frac{Ab}{3x^3} - \frac{bB + Ac}{x} + Bcx \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 27, normalized size = 1.04

$$\frac{-Ac - bB}{x} - \frac{Ab}{3x^3} + Bcx$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^6, x]

[Out] -1/3\*(A\*b)/x^3 + (-b\*B) - A\*c)/x + B\*c\*x

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^6,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^6, x]

**fricas** [A] time = 0.37, size = 29, normalized size = 1.12

$$\frac{3 B c x^4 - 3 (B b + A c) x^2 - A b}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^6,x, algorithm="fricas")

[Out] 1/3\*(3\*B\*c\*x^4 - 3\*(B\*b + A\*c)\*x^2 - A\*b)/x^3

**giac** [A] time = 0.17, size = 28, normalized size = 1.08

$$B c x - \frac{3 B b x^2 + 3 A c x^2 + A b}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^6,x, algorithm="giac")

[Out] B\*c\*x - 1/3\*(3\*B\*b\*x^2 + 3\*A\*c\*x^2 + A\*b)/x^3

**maple** [A] time = 0.05, size = 25, normalized size = 0.96

$$B c x - \frac{A b}{3 x^3} - \frac{A c + b B}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^6,x)

[Out] B\*c\*x-1/3\*A\*b/x^3-(A\*c+B\*b)/x

**maxima** [A] time = 1.34, size = 26, normalized size = 1.00

$$B c x - \frac{3 (B b + A c) x^2 + A b}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^6,x, algorithm="maxima")

[Out] B\*c\*x - 1/3\*(3\*(B\*b + A\*c)\*x^2 + A\*b)/x^3

**mupad** [B] time = 0.06, size = 26, normalized size = 1.00

$$B c x - \frac{(A c + B b) x^2 + \frac{A b}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^6,x)

[Out] B\*c\*x - ((A\*b)/3 + x^2\*(A\*c + B\*b))/x^3

**sympy** [A] time = 0.22, size = 27, normalized size = 1.04

$$B c x + \frac{-A b + x^2 (-3 A c - 3 B b)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)/x\*\*6,x)

[Out] B\*c\*x + (-A\*b + x\*\*2\*(-3\*A\*c - 3\*B\*b))/(3\*x\*\*3)



$$3.10 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx$$

**Optimal.** Leaf size=29

$$-\frac{Ac + bB}{2x^2} - \frac{Ab}{4x^4} + Bc \log(x)$$

**Rubi [A]** time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 446, 76}

$$-\frac{Ac + bB}{2x^2} - \frac{Ab}{4x^4} + Bc \log(x)$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^7, x]

[Out] -(A\*b)/(4\*x^4) - (b\*B + A\*c)/(2\*x^2) + B\*c\*Log[x]

Rule 76

Int[((d\_)\*(x\_)^(n\_))\*((a\_) + (b\_)\*(x\_)^n)\*((e\_) + (f\_)\*(x\_)^p), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^n)^p\*((c\_) + (d\_)\*(x\_)^n)^q, x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^p + (b\_)\*(x\_)^q)^n, x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^7} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x^5} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)}{x^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{Ab}{x^3} + \frac{bB + Ac}{x^2} + \frac{Bc}{x} \right) dx, x, x^2 \right) \\ &= -\frac{Ab}{4x^4} - \frac{bB + Ac}{2x^2} + Bc \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 31, normalized size = 1.07

$$-\frac{Ac - bB}{2x^2} - \frac{Ab}{4x^4} + Bc \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^7, x]

[Out] -1/4\*(A\*b)/x^4 + (- (b\*B) - A\*c)/(2\*x^2) + B\*c\*Log[x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^7, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^7, x]

**fricas** [A] time = 0.39, size = 31, normalized size = 1.07

$$\frac{4 Bcx^4 \log(x) - 2 (Bb + Ac)x^2 - Ab}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^7,x, algorithm="fricas")

[Out] 1/4\*(4\*B\*c\*x^4\*log(x) - 2\*(B\*b + A\*c)\*x^2 - A\*b)/x^4

**giac** [A] time = 0.15, size = 39, normalized size = 1.34

$$\frac{1}{2} Bc \log(x^2) - \frac{3 Bcx^4 + 2 Bbx^2 + 2 Acx^2 + Ab}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^7,x, algorithm="giac")

[Out] 1/2\*B\*c\*log(x^2) - 1/4\*(3\*B\*c\*x^4 + 2\*B\*b\*x^2 + 2\*A\*c\*x^2 + A\*b)/x^4

**maple** [A] time = 0.05, size = 28, normalized size = 0.97

$$Bc \ln(x) - \frac{Ac}{2x^2} - \frac{Bb}{2x^2} - \frac{Ab}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^7,x)

[Out] -1/4\*A\*b/x^4-1/2/x^2\*A\*c-1/2/x^2\*b\*B+B\*c\*ln(x)

**maxima** [A] time = 1.40, size = 30, normalized size = 1.03

$$\frac{1}{2} Bc \log(x^2) - \frac{2 (Bb + Ac)x^2 + Ab}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^7,x, algorithm="maxima")

[Out] 1/2\*B\*c\*log(x^2) - 1/4\*(2\*(B\*b + A\*c)\*x^2 + A\*b)/x^4

**mupad** [B] time = 0.07, size = 29, normalized size = 1.00

$$Bc \ln(x) - \frac{\left(\frac{Ac}{2} + \frac{Bb}{2}\right) x^2 + \frac{Ab}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x)`

[Out] `B*c*log(x) - ((A*b)/4 + x^2*((A*c)/2 + (B*b)/2))/x^4`

sympy [A] time = 0.37, size = 29, normalized size = 1.00

$$Bc \log(x) + \frac{-Ab + x^2(-2Ac - 2Bb)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)/x**7,x)`

[Out] `B*c*log(x) + (-A*b + x**2*(-2*A*c - 2*B*b))/(4*x**4)`

$$3.11 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} dx$$

Optimal. Leaf size=31

$$-\frac{Ac + bB}{3x^3} - \frac{Ab}{5x^5} - \frac{Bc}{x}$$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1584, 448}

$$-\frac{Ac + bB}{3x^3} - \frac{Ab}{5x^5} - \frac{Bc}{x}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^8,x]

[Out] -(A\*b)/(5\*x^5) - (b\*B + A\*c)/(3\*x^3) - (B\*c)/x

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^8} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x^6} dx \\ &= \int \left( \frac{Ab}{x^6} + \frac{bB + Ac}{x^4} + \frac{Bc}{x^2} \right) dx \\ &= -\frac{Ab}{5x^5} - \frac{bB + Ac}{3x^3} - \frac{Bc}{x} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.06

$$-\frac{Ac - bB}{3x^3} - \frac{Ab}{5x^5} - \frac{Bc}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^8,x]

[Out] -1/5\*(A\*b)/x^5 + (-b\*B) - A\*c)/(3\*x^3) - (B\*c)/x

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^8,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^8, x]

**fricas** [A] time = 0.38, size = 29, normalized size = 0.94

$$-\frac{15 B c x^4 + 5 (B b + A c) x^2 + 3 A b}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^8,x, algorithm="fricas")

[Out] -1/15\*(15\*B\*c\*x^4 + 5\*(B\*b + A\*c)\*x^2 + 3\*A\*b)/x^5

**giac** [A] time = 0.15, size = 31, normalized size = 1.00

$$-\frac{15 B c x^4 + 5 B b x^2 + 5 A c x^2 + 3 A b}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^8,x, algorithm="giac")

[Out] -1/15\*(15\*B\*c\*x^4 + 5\*B\*b\*x^2 + 5\*A\*c\*x^2 + 3\*A\*b)/x^5

**maple** [A] time = 0.05, size = 28, normalized size = 0.90

$$-\frac{B c}{x} - \frac{A b}{5 x^5} - \frac{A c + b B}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^8,x)

[Out] -1/5\*A\*b/x^5-1/3\*(A\*c+B\*b)/x^3-B\*c/x

**maxima** [A] time = 1.35, size = 29, normalized size = 0.94

$$-\frac{15 B c x^4 + 5 (B b + A c) x^2 + 3 A b}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^8,x, algorithm="maxima")

[Out] -1/15\*(15\*B\*c\*x^4 + 5\*(B\*b + A\*c)\*x^2 + 3\*A\*b)/x^5

**mupad** [B] time = 0.04, size = 29, normalized size = 0.94

$$-\frac{B c x^4 + \left(\frac{A c}{3} + \frac{B b}{3}\right) x^2 + \frac{A b}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^8,x)

[Out] -((A\*b)/5 + x^2\*((A\*c)/3 + (B\*b)/3) + B\*c\*x^4)/x^5

**sympy** [A] time = 0.39, size = 32, normalized size = 1.03

$$\frac{-3 A b - 15 B c x^4 + x^2 (-5 A c - 5 B b)}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)/x\*\*8,x)

[Out] (-3\*A\*b - 15\*B\*c\*x\*\*4 + x\*\*2\*(-5\*A\*c - 5\*B\*b))/(15\*x\*\*5)

$$3.12 \quad \int (A + Bx^2)(bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=55

$$\frac{1}{5}Ab^2x^5 + \frac{1}{9}cx^9(Ac + 2bB) + \frac{1}{7}bx^7(2Ac + bB) + \frac{1}{11}Bc^2x^{11}$$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {1593, 448}

$$\frac{1}{5}Ab^2x^5 + \frac{1}{9}cx^9(Ac + 2bB) + \frac{1}{7}bx^7(2Ac + bB) + \frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (A\*b^2\*x^5)/5 + (b\*(b\*B + 2\*A\*c)\*x^7)/7 + (c\*(2\*b\*B + A\*c)\*x^9)/9 + (B\*c^2\*x^11)/11

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int (A + Bx^2)(bx^2 + cx^4)^2 dx &= \int x^4 (A + Bx^2)(b + cx^2)^2 dx \\ &= \int (Ab^2x^4 + b(bB + 2Ac)x^6 + c(2bB + Ac)x^8 + Bc^2x^{10}) dx \\ &= \frac{1}{5}Ab^2x^5 + \frac{1}{7}b(bB + 2Ac)x^7 + \frac{1}{9}c(2bB + Ac)x^9 + \frac{1}{11}Bc^2x^{11} \end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{5}Ab^2x^5 + \frac{1}{9}cx^9(Ac + 2bB) + \frac{1}{7}bx^7(2Ac + bB) + \frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (A\*b^2\*x^5)/5 + (b\*(b\*B + 2\*A\*c)\*x^7)/7 + (c\*(2\*b\*B + A\*c)\*x^9)/9 + (B\*c^2\*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Bx^2)(bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2, x]

**fricas** [A] time = 0.34, size = 53, normalized size = 0.96

$$\frac{1}{11}x^{11}c^2B + \frac{2}{9}x^9cbB + \frac{1}{9}x^9c^2A + \frac{1}{7}x^7b^2B + \frac{2}{7}x^7cbA + \frac{1}{5}x^5b^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 1/11\*x^11\*c^2\*B + 2/9\*x^9\*c\*b\*B + 1/9\*x^9\*c^2\*A + 1/7\*x^7\*b^2\*B + 2/7\*x^7\*c\*b\*A + 1/5\*x^5\*b^2\*A

**giac** [A] time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{11}Bc^2x^{11} + \frac{2}{9}Bbcx^9 + \frac{1}{9}Ac^2x^9 + \frac{1}{7}Bb^2x^7 + \frac{2}{7}Abcx^7 + \frac{1}{5}Ab^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/11\*B\*c^2\*x^11 + 2/9\*B\*b\*c\*x^9 + 1/9\*A\*c^2\*x^9 + 1/7\*B\*b^2\*x^7 + 2/7\*A\*b\*c\*x^7 + 1/5\*A\*b^2\*x^5

**maple** [A] time = 0.04, size = 52, normalized size = 0.95

$$\frac{Bc^2x^{11}}{11} + \frac{(Ac^2 + 2bBc)x^9}{9} + \frac{Ab^2x^5}{5} + \frac{(2Abc + Bb^2)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x)

[Out] 1/11\*B\*c^2\*x^11+1/9\*(A\*c^2+2\*B\*b\*c)\*x^9+1/7\*(2\*A\*b\*c+B\*b^2)\*x^7+1/5\*A\*b^2\*x^5

**maxima** [A] time = 1.33, size = 51, normalized size = 0.93

$$\frac{1}{11}Bc^2x^{11} + \frac{1}{9}(2Bbc + Ac^2)x^9 + \frac{1}{5}Ab^2x^5 + \frac{1}{7}(Bb^2 + 2Abc)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/11\*B\*c^2\*x^11 + 1/9\*(2\*B\*b\*c + A\*c^2)\*x^9 + 1/5\*A\*b^2\*x^5 + 1/7\*(B\*b^2 + 2\*A\*b\*c)\*x^7

**mupad** [B] time = 0.09, size = 51, normalized size = 0.93

$$x^7 \left( \frac{Bb^2}{7} + \frac{2Ac b}{7} \right) + x^9 \left( \frac{Ac^2}{9} + \frac{2Bbc}{9} \right) + \frac{Ab^2x^5}{5} + \frac{Bc^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x)

[Out] x^7\*((B\*b^2)/7 + (2\*A\*b\*c)/7) + x^9\*((A\*c^2)/9 + (2\*B\*b\*c)/9) + (A\*b^2\*x^5)/5 + (B\*c^2\*x^11)/11

sympy [A] time = 0.08, size = 56, normalized size = 1.02

$$\frac{Ab^2x^5}{5} + \frac{Bc^2x^{11}}{11} + x^9 \left( \frac{Ac^2}{9} + \frac{2Bbc}{9} \right) + x^7 \left( \frac{2Abc}{7} + \frac{Bb^2}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] A\*b\*\*2\*x\*\*5/5 + B\*c\*\*2\*x\*\*11/11 + x\*\*9\*(A\*c\*\*2/9 + 2\*B\*b\*c/9) + x\*\*7\*(2\*A\*b\*c/7 + B\*b\*\*2/7)



$$3.13 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx$$

**Optimal.** Leaf size=55

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

**Rubi [A]** time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x,x]

[Out] (A\*b^2\*x^4)/4 + (b\*(b\*B + 2\*A\*c)\*x^6)/6 + (c\*(2\*b\*B + A\*c)\*x^8)/8 + (B\*c^2\*x^10)/10

#### Rule 76

Int[((d\_.)\*(x\_))^(n\_.)\*((a\_) + (b\_.)\*(x\_))\*((e\_) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx &= \int x^3 (A+Bx^2)(b+cx^2)^2 dx \\ &= \frac{1}{2} \text{Subst} \left( \int x(A+Bx)(b+cx)^2 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (Ab^2x + b(bB+2Ac)x^2 + c(2bB+Ac)x^3 + Bc^2x^4) dx, x, x^2 \right) \\ &= \frac{1}{4}Ab^2x^4 + \frac{1}{6}b(bB+2Ac)x^6 + \frac{1}{8}c(2bB+Ac)x^8 + \frac{1}{10}Bc^2x^{10} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x,x]

[Out] (A\*b^2\*x^4)/4 + (b\*(b\*B + 2\*A\*c)\*x^6)/6 + (c\*(2\*b\*B + A\*c)\*x^8)/8 + (B\*c^2\*x^10)/10

**IntegrateAlgebraic** [A] time = 0.03, size = 57, normalized size = 1.04

$$\frac{1}{120}x^4(30Ab^2 + 40Abcx^2 + 15Ac^2x^4 + 20b^2Bx^2 + 30bBcx^4 + 12Bc^2x^6)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x,x]

[Out] (x^4\*(30\*A\*b^2 + 20\*b^2\*B\*x^2 + 40\*A\*b\*c\*x^2 + 30\*b\*B\*c\*x^4 + 15\*A\*c^2\*x^4 + 12\*B\*c^2\*x^6))/120

**fricas** [A] time = 0.38, size = 51, normalized size = 0.93

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{8}(2Bbc + Ac^2)x^8 + \frac{1}{4}Ab^2x^4 + \frac{1}{6}(Bb^2 + 2Abc)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x,x, algorithm="fricas")

[Out] 1/10\*B\*c^2\*x^10 + 1/8\*(2\*B\*b\*c + A\*c^2)\*x^8 + 1/4\*A\*b^2\*x^4 + 1/6\*(B\*b^2 + 2\*A\*b\*c)\*x^6

**giac** [A] time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Abcx^6 + \frac{1}{4}Ab^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x,x, algorithm="giac")

[Out] 1/10\*B\*c^2\*x^10 + 1/4\*B\*b\*c\*x^8 + 1/8\*A\*c^2\*x^8 + 1/6\*B\*b^2\*x^6 + 1/3\*A\*b\*c\*x^6 + 1/4\*A\*b^2\*x^4

**maple** [A] time = 0.04, size = 52, normalized size = 0.95

$$\frac{Bc^2x^{10}}{10} + \frac{(Ac^2 + 2bBc)x^8}{8} + \frac{Ab^2x^4}{4} + \frac{(2Abc + Bb^2)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x,x)

[Out] 1/10\*B\*c^2\*x^10+1/8\*(A\*c^2+2\*B\*b\*c)\*x^8+1/6\*(2\*A\*b\*c+B\*b^2)\*x^6+1/4\*A\*b^2\*x^4

**maxima** [A] time = 1.37, size = 51, normalized size = 0.93

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{8}(2Bbc + Ac^2)x^8 + \frac{1}{4}Ab^2x^4 + \frac{1}{6}(Bb^2 + 2Abc)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x,x, algorithm="maxima")

[Out] 1/10\*B\*c^2\*x^10 + 1/8\*(2\*B\*b\*c + A\*c^2)\*x^8 + 1/4\*A\*b^2\*x^4 + 1/6\*(B\*b^2 + 2\*A\*b\*c)\*x^6

**mupad [B]** time = 0.05, size = 51, normalized size = 0.93

$$x^6 \left( \frac{Bb^2}{6} + \frac{Ac b}{3} \right) + x^8 \left( \frac{Ac^2}{8} + \frac{Bbc}{4} \right) + \frac{Ab^2 x^4}{4} + \frac{Bc^2 x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x,x)

[Out] x^6\*((B\*b^2)/6 + (A\*b\*c)/3) + x^8\*((A\*c^2)/8 + (B\*b\*c)/4) + (A\*b^2\*x^4)/4 + (B\*c^2\*x^10)/10

**sympy [A]** time = 0.08, size = 53, normalized size = 0.96

$$\frac{Ab^2x^4}{4} + \frac{Bc^2x^{10}}{10} + x^8 \left( \frac{Ac^2}{8} + \frac{Bbc}{4} \right) + x^6 \left( \frac{Abc}{3} + \frac{Bb^2}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x,x)

[Out] A\*b\*\*2\*x\*\*4/4 + B\*c\*\*2\*x\*\*10/10 + x\*\*8\*(A\*c\*\*2/8 + B\*b\*c/4) + x\*\*6\*(A\*b\*c/3 + B\*b\*\*2/6)

$$3.14 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx$$

**Optimal.** Leaf size=55

$$\frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac + 2bB) + \frac{1}{5}bx^5(2Ac + bB) + \frac{1}{9}Bc^2x^9$$

**Rubi [A]** time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac + 2bB) + \frac{1}{5}bx^5(2Ac + bB) + \frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^2,x]

[Out] (A\*b^2\*x^3)/3 + (b\*(b\*B + 2\*A\*c)\*x^5)/5 + (c\*(2\*b\*B + A\*c)\*x^7)/7 + (B\*c^2\*x^9)/9

**Rule 448**

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^2} dx &= \int x^2 (A + Bx^2)(b + cx^2)^2 dx \\ &= \int (Ab^2x^2 + b(bB + 2Ac)x^4 + c(2bB + Ac)x^6 + Bc^2x^8) dx \\ &= \frac{1}{3}Ab^2x^3 + \frac{1}{5}b(bB + 2Ac)x^5 + \frac{1}{7}c(2bB + Ac)x^7 + \frac{1}{9}Bc^2x^9 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac + 2bB) + \frac{1}{5}bx^5(2Ac + bB) + \frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^2,x]

[Out] (A\*b^2\*x^3)/3 + (b\*(b\*B + 2\*A\*c)\*x^5)/5 + (c\*(2\*b\*B + A\*c)\*x^7)/7 + (B\*c^2\*x^9)/9

**IntegrateAlgebraic [A]** time = 0.04, size = 65, normalized size = 1.18

$$\frac{1}{3}Ab^2x^3 + \frac{2}{5}Abcx^5 + \frac{1}{7}Ac^2x^7 + \frac{1}{5}b^2Bx^5 + \frac{2}{7}bBcx^7 + \frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^2,x]

[Out] (A\*b^2\*x^3)/3 + (b^2\*B\*x^5)/5 + (2\*A\*b\*c\*x^5)/5 + (2\*b\*B\*c\*x^7)/7 + (A\*c^2\*x^7)/7 + (B\*c^2\*x^9)/9

**fricas** [A] time = 0.37, size = 51, normalized size = 0.93

$$\frac{1}{9} Bc^2x^9 + \frac{1}{7} (2Bbc + Ac^2)x^7 + \frac{1}{3} Ab^2x^3 + \frac{1}{5} (Bb^2 + 2Abc)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^2,x, algorithm="fricas")

[Out] 1/9\*B\*c^2\*x^9 + 1/7\*(2\*B\*b\*c + A\*c^2)\*x^7 + 1/3\*A\*b^2\*x^3 + 1/5\*(B\*b^2 + 2\*A\*b\*c)\*x^5

**giac** [A] time = 0.17, size = 53, normalized size = 0.96

$$\frac{1}{9} Bc^2x^9 + \frac{2}{7} Bbcx^7 + \frac{1}{7} Ac^2x^7 + \frac{1}{5} Bb^2x^5 + \frac{2}{5} Abcx^5 + \frac{1}{3} Ab^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^2,x, algorithm="giac")

[Out] 1/9\*B\*c^2\*x^9 + 2/7\*B\*b\*c\*x^7 + 1/7\*A\*c^2\*x^7 + 1/5\*B\*b^2\*x^5 + 2/5\*A\*b\*c\*x^5 + 1/3\*A\*b^2\*x^3

**maple** [A] time = 0.04, size = 52, normalized size = 0.95

$$\frac{Bc^2x^9}{9} + \frac{(Ac^2 + 2bBc)x^7}{7} + \frac{Ab^2x^3}{3} + \frac{(2Abc + Bb^2)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^2,x)

[Out] 1/9\*B\*c^2\*x^9+1/7\*(A\*c^2+2\*B\*b\*c)\*x^7+1/5\*(2\*A\*b\*c+B\*b^2)\*x^5+1/3\*A\*b^2\*x^3

**maxima** [A] time = 1.32, size = 51, normalized size = 0.93

$$\frac{1}{9} Bc^2x^9 + \frac{1}{7} (2Bbc + Ac^2)x^7 + \frac{1}{3} Ab^2x^3 + \frac{1}{5} (Bb^2 + 2Abc)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^2,x, algorithm="maxima")

[Out] 1/9\*B\*c^2\*x^9 + 1/7\*(2\*B\*b\*c + A\*c^2)\*x^7 + 1/3\*A\*b^2\*x^3 + 1/5\*(B\*b^2 + 2\*A\*b\*c)\*x^5

**mupad** [B] time = 0.05, size = 51, normalized size = 0.93

$$x^5 \left( \frac{Bb^2}{5} + \frac{2Ac b}{5} \right) + x^7 \left( \frac{Ac^2}{7} + \frac{2Bbc}{7} \right) + \frac{Ab^2x^3}{3} + \frac{Bc^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^2,x)

[Out] x^5\*((B\*b^2)/5 + (2\*A\*b\*c)/5) + x^7\*((A\*c^2)/7 + (2\*B\*b\*c)/7) + (A\*b^2\*x^3)/3 + (B\*c^2\*x^9)/9

sympy [A] time = 0.08, size = 56, normalized size = 1.02

$$\frac{Ab^2x^3}{3} + \frac{Bc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Bbc}{7}\right) + x^5\left(\frac{2Abc}{5} + \frac{Bb^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*2,x)

[Out] A\*b\*\*2\*x\*\*3/3 + B\*c\*\*2\*x\*\*9/9 + x\*\*7\*(A\*c\*\*2/7 + 2\*B\*b\*c/7) + x\*\*5\*(2\*A\*b\*c/5 + B\*b\*\*2/5)

$$3.15 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx$$

Optimal. Leaf size=42

$$\frac{B(b+cx^2)^4}{8c^2} - \frac{(b+cx^2)^3(bB-Ac)}{6c^2}$$

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 444, 43}

$$\frac{B(b+cx^2)^4}{8c^2} - \frac{(b+cx^2)^3(bB-Ac)}{6c^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^3,x]

[Out] -((b\*B - A\*c)\*(b + c\*x^2)^3)/(6\*c^2) + (B\*(b + c\*x^2)^4)/(8\*c^2)

Rule 43

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx &= \int x(A+Bx^2)(b+cx^2)^2 dx \\ &= \frac{1}{2} \text{Subst} \left( \int (A+Bx)(b+cx)^2 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{(-bB+Ac)(b+cx)^2}{c} + \frac{B(b+cx)^3}{c} \right) dx, x, x^2 \right) \\ &= -\frac{(bB-Ac)(b+cx^2)^3}{6c^2} + \frac{B(b+cx^2)^4}{8c^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2(12Ab^2 + 4cx^4(Ac + 2bB) + 6bx^2(2Ac + bB) + 3Bc^2x^6)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^3,x]

[Out] (x^2\*(12\*A\*b^2 + 6\*b\*(b\*B + 2\*A\*c)\*x^2 + 4\*c\*(2\*b\*B + A\*c)\*x^4 + 3\*B\*c^2\*x^6))/24

**IntegrateAlgebraic [A]** time = 0.03, size = 57, normalized size = 1.36

$$\frac{1}{24}x^2(12Ab^2 + 12Abcx^2 + 4Ac^2x^4 + 6b^2Bx^2 + 8bBcx^4 + 3Bc^2x^6)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^3,x]

[Out] (x^2\*(12\*A\*b^2 + 6\*b^2\*B\*x^2 + 12\*A\*b\*c\*x^2 + 8\*b\*B\*c\*x^4 + 4\*A\*c^2\*x^4 + 3\*B\*c^2\*x^6))/24

**fricas [A]** time = 0.37, size = 51, normalized size = 1.21

$$\frac{1}{8}Bc^2x^8 + \frac{1}{6}(2Bbc + Ac^2)x^6 + \frac{1}{2}Ab^2x^2 + \frac{1}{4}(Bb^2 + 2Abc)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^3,x, algorithm="fricas")

[Out] 1/8\*B\*c^2\*x^8 + 1/6\*(2\*B\*b\*c + A\*c^2)\*x^6 + 1/2\*A\*b^2\*x^2 + 1/4\*(B\*b^2 + 2\*A\*b\*c)\*x^4

**giac [A]** time = 0.16, size = 53, normalized size = 1.26

$$\frac{1}{8}Bc^2x^8 + \frac{1}{3}Bbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{1}{4}Bb^2x^4 + \frac{1}{2}Abcx^4 + \frac{1}{2}Ab^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^3,x, algorithm="giac")

[Out] 1/8\*B\*c^2\*x^8 + 1/3\*B\*b\*c\*x^6 + 1/6\*A\*c^2\*x^6 + 1/4\*B\*b^2\*x^4 + 1/2\*A\*b\*c\*x^4 + 1/2\*A\*b^2\*x^2

**maple [A]** time = 0.04, size = 52, normalized size = 1.24

$$\frac{Bc^2x^8}{8} + \frac{(Ac^2 + 2bBc)x^6}{6} + \frac{Ab^2x^2}{2} + \frac{(2Abc + Bb^2)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^3,x)

[Out] 1/8\*B\*c^2\*x^8+1/6\*(A\*c^2+2\*B\*b\*c)\*x^6+1/4\*(2\*A\*b\*c+B\*b^2)\*x^4+1/2\*b^2\*A\*x^2

**maxima [A]** time = 1.36, size = 51, normalized size = 1.21

$$\frac{1}{8}Bc^2x^8 + \frac{1}{6}(2Bbc + Ac^2)x^6 + \frac{1}{2}Ab^2x^2 + \frac{1}{4}(Bb^2 + 2Abc)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^3,x, algorithm="maxima")

[Out] 1/8\*B\*c^2\*x^8 + 1/6\*(2\*B\*b\*c + A\*c^2)\*x^6 + 1/2\*A\*b^2\*x^2 + 1/4\*(B\*b^2 + 2\*A\*b\*c)\*x^4



**mupad [B]** time = 0.05, size = 51, normalized size = 1.21

$$x^4 \left( \frac{Bb^2}{4} + \frac{Ac b}{2} \right) + x^6 \left( \frac{Ac^2}{6} + \frac{Bbc}{3} \right) + \frac{Ab^2 x^2}{2} + \frac{Bc^2 x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^3,x)

[Out] x^4\*((B\*b^2)/4 + (A\*b\*c)/2) + x^6\*((A\*c^2)/6 + (B\*b\*c)/3) + (A\*b^2\*x^2)/2 + (B\*c^2\*x^8)/8

**sympy [A]** time = 0.08, size = 53, normalized size = 1.26

$$\frac{Ab^2x^2}{2} + \frac{Bc^2x^8}{8} + x^6 \left( \frac{Ac^2}{6} + \frac{Bbc}{3} \right) + x^4 \left( \frac{Abc}{2} + \frac{Bb^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*3,x)

[Out] A\*b\*\*2\*x\*\*2/2 + B\*c\*\*2\*x\*\*8/8 + x\*\*6\*(A\*c\*\*2/6 + B\*b\*c/3) + x\*\*4\*(A\*b\*c/2 + B\*b\*\*2/4)

$$3.16 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4} dx$$

**Optimal.** Leaf size=50

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 373}

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^4,x]

[Out] A\*b^2\*x + (b\*(b\*B + 2\*A\*c)\*x^3)/3 + (c\*(2\*b\*B + A\*c)\*x^5)/5 + (B\*c^2\*x^7)/7

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^4} dx &= \int (A + Bx^2)(b + cx^2)^2 dx \\ &= \int (Ab^2 + b(bB + 2Ac)x^2 + c(2bB + Ac)x^4 + Bc^2x^6) dx \\ &= Ab^2x + \frac{1}{3}b(bB + 2Ac)x^3 + \frac{1}{5}c(2bB + Ac)x^5 + \frac{1}{7}Bc^2x^7 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 1.00

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^4,x]

[Out] A\*b^2\*x + (b\*(b\*B + 2\*A\*c)\*x^3)/3 + (c\*(2\*b\*B + A\*c)\*x^5)/5 + (B\*c^2\*x^7)/7

**IntegrateAlgebraic [A]** time = 0.04, size = 60, normalized size = 1.20

$$Ab^2x + \frac{2}{3}Abcx^3 + \frac{1}{5}Ac^2x^5 + \frac{1}{3}b^2Bx^3 + \frac{2}{5}bBcx^5 + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^4,x]

[Out] A\*b^2\*x + (b^2\*B\*x^3)/3 + (2\*A\*b\*c\*x^3)/3 + (2\*b\*B\*c\*x^5)/5 + (A\*c^2\*x^5)/5 + (B\*c^2\*x^7)/7

**fricas** [A] time = 0.37, size = 48, normalized size = 0.96

$$\frac{1}{7}Bc^2x^7 + \frac{1}{5}(2Bbc + Ac^2)x^5 + Ab^2x + \frac{1}{3}(Bb^2 + 2Abc)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^4,x, algorithm="fricas")

[Out] 1/7\*B\*c^2\*x^7 + 1/5\*(2\*B\*b\*c + A\*c^2)\*x^5 + A\*b^2\*x + 1/3\*(B\*b^2 + 2\*A\*b\*c)\*x^3

**giac** [A] time = 0.17, size = 50, normalized size = 1.00

$$\frac{1}{7}Bc^2x^7 + \frac{2}{5}Bbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Abcx^3 + Ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^4,x, algorithm="giac")

[Out] 1/7\*B\*c^2\*x^7 + 2/5\*B\*b\*c\*x^5 + 1/5\*A\*c^2\*x^5 + 1/3\*B\*b^2\*x^3 + 2/3\*A\*b\*c\*x^3 + A\*b^2\*x

**maple** [A] time = 0.05, size = 49, normalized size = 0.98

$$\frac{Bc^2x^7}{7} + \frac{(Ac^2 + 2bBc)x^5}{5} + Ab^2x + \frac{(2Abc + Bb^2)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^4,x)

[Out] 1/7\*B\*c^2\*x^7+1/5\*(A\*c^2+2\*B\*b\*c)\*x^5+1/3\*(2\*A\*b\*c+B\*b^2)\*x^3+A\*b^2\*x

**maxima** [A] time = 1.24, size = 48, normalized size = 0.96

$$\frac{1}{7}Bc^2x^7 + \frac{1}{5}(2Bbc + Ac^2)x^5 + Ab^2x + \frac{1}{3}(Bb^2 + 2Abc)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^4,x, algorithm="maxima")

[Out] 1/7\*B\*c^2\*x^7 + 1/5\*(2\*B\*b\*c + A\*c^2)\*x^5 + A\*b^2\*x + 1/3\*(B\*b^2 + 2\*A\*b\*c)\*x^3

**mupad** [B] time = 0.05, size = 48, normalized size = 0.96

$$x^3 \left( \frac{Bb^2}{3} + \frac{2Ac b}{3} \right) + x^5 \left( \frac{Ac^2}{5} + \frac{2Bbc}{5} \right) + \frac{Bc^2x^7}{7} + Ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^4,x)

[Out] x^3\*((B\*b^2)/3 + (2\*A\*b\*c)/3) + x^5\*((A\*c^2)/5 + (2\*B\*b\*c)/5) + (B\*c^2\*x^7)/7 + A\*b^2\*x

sympy [A] time = 0.08, size = 53, normalized size = 1.06

$$Ab^2x + \frac{Bc^2x^7}{7} + x^5 \left( \frac{Ac^2}{5} + \frac{2Bbc}{5} \right) + x^3 \left( \frac{2Abc}{3} + \frac{Bb^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*4,x)

[Out] A\*b\*\*2\*x + B\*c\*\*2\*x\*\*7/7 + x\*\*5\*(A\*c\*\*2/5 + 2\*B\*b\*c/5) + x\*\*3\*(2\*A\*b\*c/3 + B\*b\*\*2/3)

$$3.17 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5} dx$$

Optimal. Leaf size=43

$$Ab^2 \log(x) + Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c}$$

**Rubi [A]** time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 446, 80, 43}

$$Ab^2 \log(x) + Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^5,x]

[Out] A\*b\*c\*x^2 + (A\*c^2\*x^4)/4 + (B\*(b + c\*x^2)^3)/(6\*c) + A\*b^2\*Log[x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 80

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^5} dx &= \int \frac{(A + Bx^2)(b + cx^2)^2}{x} dx \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)^2}{x} dx, x, x^2 \right) \\
&= \frac{B(b + cx^2)^3}{6c} + \frac{1}{2} A \text{Subst} \left( \int \frac{(b + cx)^2}{x} dx, x, x^2 \right) \\
&= \frac{B(b + cx^2)^3}{6c} + \frac{1}{2} A \text{Subst} \left( \int \left( 2bc + \frac{b^2}{x} + c^2x \right) dx, x, x^2 \right) \\
&= Abcx^2 + \frac{1}{4} Ac^2x^4 + \frac{B(b + cx^2)^3}{6c} + Ab^2 \log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 1.19

$$Ab^2 \log(x) + \frac{1}{4} cx^4 (Ac + 2bB) + \frac{1}{2} bx^2 (2Ac + bB) + \frac{1}{6} Bc^2 x^6$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^5, x]

[Out] (b\*(b\*B + 2\*A\*c)\*x^2)/2 + (c\*(2\*b\*B + A\*c)\*x^4)/4 + (B\*c^2\*x^6)/6 + A\*b^2\*log[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^5, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^5, x]

**fricas [A]** time = 0.39, size = 49, normalized size = 1.14

$$\frac{1}{6} Bc^2x^6 + \frac{1}{4} (2Bbc + Ac^2)x^4 + Ab^2 \log(x) + \frac{1}{2} (Bb^2 + 2Abc)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^5,x, algorithm="fricas")

[Out] 1/6\*B\*c^2\*x^6 + 1/4\*(2\*B\*b\*c + A\*c^2)\*x^4 + A\*b^2\*log(x) + 1/2\*(B\*b^2 + 2\*A\*b\*c)\*x^2

**giac [A]** time = 0.15, size = 53, normalized size = 1.23

$$\frac{1}{6} Bc^2x^6 + \frac{1}{2} Bbcx^4 + \frac{1}{4} Ac^2x^4 + \frac{1}{2} Bb^2x^2 + Abcx^2 + \frac{1}{2} Ab^2 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^5,x, algorithm="giac")

[Out] 1/6\*B\*c^2\*x^6 + 1/2\*B\*b\*c\*x^4 + 1/4\*A\*c^2\*x^4 + 1/2\*B\*b^2\*x^2 + A\*b\*c\*x^2 + 1/2\*A\*b^2\*log(x^2)

**maple** [A] time = 0.05, size = 51, normalized size = 1.19

$$\frac{Bc^2x^6}{6} + \frac{Ac^2x^4}{4} + \frac{Bbcx^4}{2} + Abcx^2 + \frac{Bb^2x^2}{2} + Ab^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^5,x)

[Out] 1/6\*B\*c^2\*x^6+1/4\*A\*c^2\*x^4+1/2\*B\*x^4\*b\*c+A\*b\*c\*x^2+1/2\*B\*b^2\*x^2+A\*b^2\*ln(x)

**maxima** [A] time = 1.34, size = 52, normalized size = 1.21

$$\frac{1}{6}Bc^2x^6 + \frac{1}{4}(2Bbc + Ac^2)x^4 + \frac{1}{2}Ab^2 \log(x^2) + \frac{1}{2}(Bb^2 + 2Abc)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^5,x, algorithm="maxima")

[Out] 1/6\*B\*c^2\*x^6 + 1/4\*(2\*B\*b\*c + A\*c^2)\*x^4 + 1/2\*A\*b^2\*log(x^2) + 1/2\*(B\*b^2 + 2\*A\*b\*c)\*x^2

**mupad** [B] time = 0.04, size = 48, normalized size = 1.12

$$x^2 \left( \frac{Bb^2}{2} + Acb \right) + x^4 \left( \frac{Ac^2}{4} + \frac{Bbc}{2} \right) + \frac{Bc^2x^6}{6} + Ab^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^5,x)

[Out] x^2\*((B\*b^2)/2 + A\*b\*c) + x^4\*((A\*c^2)/4 + (B\*b\*c)/2) + (B\*c^2\*x^6)/6 + A\*b^2\*log(x)

**sympy** [A] time = 0.16, size = 49, normalized size = 1.14

$$Ab^2 \log(x) + \frac{Bc^2x^6}{6} + x^4 \left( \frac{Ac^2}{4} + \frac{Bbc}{2} \right) + x^2 \left( Abc + \frac{Bb^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*5,x)

[Out] A\*b\*\*2\*log(x) + B\*c\*\*2\*x\*\*6/6 + x\*\*4\*(A\*c\*\*2/4 + B\*b\*c/2) + x\*\*2\*(A\*b\*c + B\*b\*\*2/2)

$$3.18 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx$$

Optimal. Leaf size=48

$$-\frac{Ab^2}{x} + \frac{1}{3}cx^3(Ac + 2bB) + bx(2Ac + bB) + \frac{1}{5}Bc^2x^5$$

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{Ab^2}{x} + \frac{1}{3}cx^3(Ac + 2bB) + bx(2Ac + bB) + \frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^6,x]

[Out] -((A\*b^2)/x) + b\*(b\*B + 2\*A\*c)\*x + (c\*(2\*b\*B + A\*c)\*x^3)/3 + (B\*c^2\*x^5)/5

Rule 448

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^6} dx &= \int \frac{(A + Bx^2)(b + cx^2)^2}{x^2} dx \\ &= \int \left( b(bB + 2Ac) + \frac{Ab^2}{x^2} + c(2bB + Ac)x^2 + Bc^2x^4 \right) dx \\ &= -\frac{Ab^2}{x} + b(bB + 2Ac)x + \frac{1}{3}c(2bB + Ac)x^3 + \frac{1}{5}Bc^2x^5 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{x} + \frac{1}{3}cx^3(Ac + 2bB) + bx(2Ac + bB) + \frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^6,x]

[Out] -((A\*b^2)/x) + b\*(b\*B + 2\*A\*c)\*x + (c\*(2\*b\*B + A\*c)\*x^3)/3 + (B\*c^2\*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^6} dx$$



Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^6,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^6, x]

**fricas** [A] time = 0.38, size = 53, normalized size = 1.10

$$\frac{3 B c^2 x^6 + 5 (2 B b c + A c^2) x^4 - 15 A b^2 + 15 (B b^2 + 2 A b c) x^2}{15 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^6,x, algorithm="fricas")

[Out] 1/15\*(3\*B\*c^2\*x^6 + 5\*(2\*B\*b\*c + A\*c^2)\*x^4 - 15\*A\*b^2 + 15\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x

**giac** [A] time = 0.15, size = 48, normalized size = 1.00

$$\frac{1}{5} B c^2 x^5 + \frac{2}{3} B b c x^3 + \frac{1}{3} A c^2 x^3 + B b^2 x + 2 A b c x - \frac{A b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^6,x, algorithm="giac")

[Out] 1/5\*B\*c^2\*x^5 + 2/3\*B\*b\*c\*x^3 + 1/3\*A\*c^2\*x^3 + B\*b^2\*x + 2\*A\*b\*c\*x - A\*b^2/x

**maple** [A] time = 0.05, size = 49, normalized size = 1.02

$$\frac{B c^2 x^5}{5} + \frac{A c^2 x^3}{3} + \frac{2 B b c x^3}{3} + 2 A b c x + B b^2 x - \frac{A b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^6,x)

[Out] 1/5\*B\*c^2\*x^5+1/3\*A\*x^3\*c^2+2/3\*B\*x^3\*b\*c+2\*A\*b\*c\*x+B\*b^2\*x-A\*b^2/x

**maxima** [A] time = 1.29, size = 48, normalized size = 1.00

$$\frac{1}{5} B c^2 x^5 + \frac{1}{3} (2 B b c + A c^2) x^3 - \frac{A b^2}{x} + (B b^2 + 2 A b c) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^6,x, algorithm="maxima")

[Out] 1/5\*B\*c^2\*x^5 + 1/3\*(2\*B\*b\*c + A\*c^2)\*x^3 - A\*b^2/x + (B\*b^2 + 2\*A\*b\*c)\*x

**mupad** [B] time = 0.05, size = 48, normalized size = 1.00

$$x^3 \left( \frac{A c^2}{3} + \frac{2 B b c}{3} \right) + x (B b^2 + 2 A c b) - \frac{A b^2}{x} + \frac{B c^2 x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^6,x)

[Out] x^3\*((A\*c^2)/3 + (2\*B\*b\*c)/3) + x\*(B\*b^2 + 2\*A\*b\*c) - (A\*b^2)/x + (B\*c^2\*x^5)/5

sympy [A] time = 0.14, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{x} + \frac{Bc^2x^5}{5} + x^3 \left( \frac{Ac^2}{3} + \frac{2Bbc}{3} \right) + x(2Abc + Bb^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*6,x)

[Out] -A\*b\*\*2/x + B\*c\*\*2\*x\*\*5/5 + x\*\*3\*(A\*c\*\*2/3 + 2\*B\*b\*c/3) + x\*(2\*A\*b\*c + B\*b\*\*2)

$$3.19 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx$$

**Optimal.** Leaf size=51

$$-\frac{Ab^2}{2x^2} + \frac{1}{2}cx^2(Ac + 2bB) + b \log(x)(2Ac + bB) + \frac{1}{4}Bc^2x^4$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{2x^2} + \frac{1}{2}cx^2(Ac + 2bB) + b \log(x)(2Ac + bB) + \frac{1}{4}Bc^2x^4$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^7,x]

[Out] -(A\*b^2)/(2\*x^2) + (c\*(2\*b\*B + A\*c)\*x^2)/2 + (B\*c^2\*x^4)/4 + b\*(b\*B + 2\*A\*c)\*Log[x]

#### Rule 76

Int[((d\_.)\*(x\_))^(n\_.)\*((a\_) + (b\_.)\*(x\_))\*((e\_) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A+Bx)(b+cx)^2}{x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( c(2bB+Ac) + \frac{Ab^2}{x^2} + \frac{b(bB+2Ac)}{x} + Bc^2x \right) dx, x, x^2 \right) \\ &= -\frac{Ab^2}{2x^2} + \frac{1}{2}c(2bB+Ac)x^2 + \frac{1}{4}Bc^2x^4 + b(bB+2Ac)\log(x) \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 49, normalized size = 0.96

$$\frac{1}{4} \left( -\frac{2Ab^2}{x^2} + 2cx^2(Ac + 2bB) + 4b \log(x)(2Ac + bB) + Bc^2x^4 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^7, x]

[Out] ((-2\*A\*b^2)/x^2 + 2\*c\*(2\*b\*B + A\*c)\*x^2 + B\*c^2\*x^4 + 4\*b\*(b\*B + 2\*A\*c)\*Log[x])/4

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^7, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^7, x]

**fricas** [A] time = 0.39, size = 54, normalized size = 1.06

$$\frac{Bc^2x^6 + 2(2Bbc + Ac^2)x^4 + 4(Bb^2 + 2Abc)x^2 \log(x) - 2Ab^2}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^7,x, algorithm="fricas")

[Out] 1/4\*(B\*c^2\*x^6 + 2\*(2\*B\*b\*c + A\*c^2)\*x^4 + 4\*(B\*b^2 + 2\*A\*b\*c)\*x^2\*log(x) - 2\*A\*b^2)/x^2

**giac** [A] time = 0.19, size = 70, normalized size = 1.37

$$\frac{1}{4} Bc^2x^4 + Bbcx^2 + \frac{1}{2} Ac^2x^2 + \frac{1}{2} (Bb^2 + 2Abc) \log(x^2) - \frac{Bb^2x^2 + 2Abcx^2 + Ab^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^7,x, algorithm="giac")

[Out] 1/4\*B\*c^2\*x^4 + B\*b\*c\*x^2 + 1/2\*A\*c^2\*x^2 + 1/2\*(B\*b^2 + 2\*A\*b\*c)\*log(x^2) - 1/2\*(B\*b^2\*x^2 + 2\*A\*b\*c\*x^2 + A\*b^2)/x^2

**maple** [A] time = 0.05, size = 50, normalized size = 0.98

$$\frac{Bc^2x^4}{4} + \frac{Ac^2x^2}{2} + Bbcx^2 + 2Abc \ln(x) + Bb^2 \ln(x) - \frac{Ab^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^7,x)

[Out] 1/4\*B\*c^2\*x^4+1/2\*A\*x^2\*c^2+B\*x^2\*b\*c-1/2\*A\*b^2/x^2+2\*A\*ln(x)\*b\*c+B\*ln(x)\*b^2

**maxima** [A] time = 1.35, size = 52, normalized size = 1.02

$$\frac{1}{4} Bc^2x^4 + \frac{1}{2} (2Bbc + Ac^2)x^2 + \frac{1}{2} (Bb^2 + 2Abc) \log(x^2) - \frac{Ab^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^7,x, algorithm="maxima")

[Out] 1/4\*B\*c^2\*x^4 + 1/2\*(2\*B\*b\*c + A\*c^2)\*x^2 + 1/2\*(B\*b^2 + 2\*A\*b\*c)\*log(x^2) - 1/2\*A\*b^2/x^2

mupad [B] time = 0.05, size = 48, normalized size = 0.94

$$x^2 \left( \frac{Ac^2}{2} + Bbc \right) + \ln(x) (Bb^2 + 2Ac b) - \frac{Ab^2}{2x^2} + \frac{Bc^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^7,x)

[Out] x^2\*((A\*c^2)/2 + B\*b\*c) + log(x)\*(B\*b^2 + 2\*A\*b\*c) - (A\*b^2)/(2\*x^2) + (B\*c^2\*x^4)/4

sympy [A] time = 0.24, size = 48, normalized size = 0.94

$$-\frac{Ab^2}{2x^2} + \frac{Bc^2x^4}{4} + b(2Ac + Bb) \log(x) + x^2 \left( \frac{Ac^2}{2} + Bbc \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*7,x)

[Out] -A\*b\*\*2/(2\*x\*\*2) + B\*c\*\*2\*x\*\*4/4 + b\*(2\*A\*c + B\*b)\*log(x) + x\*\*2\*(A\*c\*\*2/2 + B\*b\*c)

$$3.20 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx$$

**Optimal.** Leaf size=48

$$-\frac{Ab^2}{3x^3} + cx(Ac + 2bB) - \frac{b(2Ac + bB)}{x} + \frac{1}{3}Bc^2x^3$$

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{Ab^2}{3x^3} + cx(Ac + 2bB) - \frac{b(2Ac + bB)}{x} + \frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^8,x]

[Out] -(A\*b^2)/(3\*x^3) - (b\*(b\*B + 2\*A\*c))/x + c\*(2\*b\*B + A\*c)\*x + (B\*c^2\*x^3)/3

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^8} dx &= \int \frac{(A + Bx^2)(b + cx^2)^2}{x^4} dx \\ &= \int \left( c(2bB + Ac) + \frac{Ab^2}{x^4} + \frac{b(bB + 2Ac)}{x^2} + Bc^2x^2 \right) dx \\ &= -\frac{Ab^2}{3x^3} - \frac{b(bB + 2Ac)}{x} + c(2bB + Ac)x + \frac{1}{3}Bc^2x^3 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 1.04

$$\frac{b^2(-B) - 2Abc}{x} - \frac{Ab^2}{3x^3} + cx(Ac + 2bB) + \frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^8,x]

[Out] -1/3\*(A\*b^2)/x^3 + (-b^2\*B) - 2\*A\*b\*c)/x + c\*(2\*b\*B + A\*c)\*x + (B\*c^2\*x^3)/3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^8,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^8, x]

**fricas** [A] time = 0.38, size = 52, normalized size = 1.08

$$\frac{Bc^2x^6 + 3(2Bbc + Ac^2)x^4 - Ab^2 - 3(Bb^2 + 2Abc)x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^8,x, algorithm="fricas")

[Out] 1/3\*(B\*c^2\*x^6 + 3\*(2\*B\*b\*c + A\*c^2)\*x^4 - A\*b^2 - 3\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^3

**giac** [A] time = 0.14, size = 50, normalized size = 1.04

$$\frac{1}{3}Bc^2x^3 + 2Bbcx + Ac^2x - \frac{3Bb^2x^2 + 6Abcx^2 + Ab^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^8,x, algorithm="giac")

[Out] 1/3\*B\*c^2\*x^3 + 2\*B\*b\*c\*x + A\*c^2\*x - 1/3\*(3\*B\*b^2\*x^2 + 6\*A\*b\*c\*x^2 + A\*b^2)/x^3

**maple** [A] time = 0.05, size = 46, normalized size = 0.96

$$\frac{Bc^2x^3}{3} + Ac^2x + 2Bbcx - \frac{Ab^2}{3x^3} - \frac{(2Ac + bB)b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^8,x)

[Out] 1/3\*B\*c^2\*x^3+A\*c^2\*x+2\*b\*B\*c\*x-1/3\*A\*b^2/x^3-b\*(2\*A\*c+B\*b)/x

**maxima** [A] time = 1.32, size = 50, normalized size = 1.04

$$\frac{1}{3}Bc^2x^3 + (2Bbc + Ac^2)x - \frac{Ab^2 + 3(Bb^2 + 2Abc)x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^8,x, algorithm="maxima")

[Out] 1/3\*B\*c^2\*x^3 + (2\*B\*b\*c + A\*c^2)\*x - 1/3\*(A\*b^2 + 3\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^3

**mupad** [B] time = 0.05, size = 50, normalized size = 1.04

$$x(Ac^2 + 2Bbc) - \frac{x^2(Bb^2 + 2Ac b) + \frac{Ab^2}{3}}{x^3} + \frac{Bc^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^8,x)

[Out] x\*(A\*c^2 + 2\*B\*b\*c) - (x^2\*(B\*b^2 + 2\*A\*b\*c) + (A\*b^2)/3)/x^3 + (B\*c^2\*x^3)/3

sympy [A] time = 0.26, size = 51, normalized size = 1.06

$$\frac{Bc^2x^3}{3} + x(Ac^2 + 2Bbc) + \frac{-Ab^2 + x^2(-6Abc - 3Bb^2)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*8,x)

[Out] B\*c\*\*2\*x\*\*3/3 + x\*(A\*c\*\*2 + 2\*B\*b\*c) + (-A\*b\*\*2 + x\*\*2\*(-6\*A\*b\*c - 3\*B\*b\*\*2))/(3\*x\*\*3)



$$3.21 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx$$

**Optimal.** Leaf size=51

$$-\frac{Ab^2}{4x^4} - \frac{b(2Ac + bB)}{2x^2} + c \log(x)(Ac + 2bB) + \frac{1}{2}Bc^2x^2$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{4x^4} - \frac{b(2Ac + bB)}{2x^2} + c \log(x)(Ac + 2bB) + \frac{1}{2}Bc^2x^2$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^9,x]

[Out] -(A\*b^2)/(4\*x^4) - (b\*(b\*B + 2\*A\*c))/(2\*x^2) + (B\*c^2\*x^2)/2 + c\*(2\*b\*B + A\*c)\*Log[x]

#### Rule 76

Int[((d\_.)\*(x\_))^(n\_.)\*((a\_) + (b\_.)\*(x\_))\*((e\_) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^5} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A+Bx)(b+cx)^2}{x^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( Bc^2 + \frac{Ab^2}{x^3} + \frac{b(bB+2Ac)}{x^2} + \frac{c(2bB+Ac)}{x} \right) dx, x, x^2 \right) \\ &= -\frac{Ab^2}{4x^4} - \frac{b(bB+2Ac)}{2x^2} + \frac{1}{2}Bc^2x^2 + c(2bB+Ac) \log(x) \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 50, normalized size = 0.98

$$c \log(x)(Ac + 2bB) - \frac{Ab(b + 4cx^2) + 2Bx^2(b^2 - c^2x^4)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^9, x]

[Out] -1/4\*(A\*b\*(b + 4\*c\*x^2) + 2\*B\*x^2\*(b^2 - c^2\*x^4))/x^4 + c\*(2\*b\*B + A\*c)\*Log[x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^9} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^9, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^9, x]

**fricas** [A] time = 0.40, size = 55, normalized size = 1.08

$$\frac{2Bc^2x^6 + 4(2Bbc + Ac^2)x^4 \log(x) - Ab^2 - 2(Bb^2 + 2Abc)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^9, x, algorithm="fricas")

[Out] 1/4\*(2\*B\*c^2\*x^6 + 4\*(2\*B\*b\*c + A\*c^2)\*x^4\*log(x) - A\*b^2 - 2\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^4

**giac** [A] time = 0.19, size = 72, normalized size = 1.41

$$\frac{1}{2}Bc^2x^2 + \frac{1}{2}(2Bbc + Ac^2)\log(x^2) - \frac{6Bbcx^4 + 3Ac^2x^4 + 2Bb^2x^2 + 4Abcx^2 + Ab^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^9, x, algorithm="giac")

[Out] 1/2\*B\*c^2\*x^2 + 1/2\*(2\*B\*b\*c + A\*c^2)\*log(x^2) - 1/4\*(6\*B\*b\*c\*x^4 + 3\*A\*c^2\*x^4 + 2\*B\*b^2\*x^2 + 4\*A\*b\*c\*x^2 + A\*b^2)/x^4

**maple** [A] time = 0.06, size = 51, normalized size = 1.00

$$\frac{Bc^2x^2}{2} + Ac^2 \ln(x) + 2Bbc \ln(x) - \frac{Abc}{x^2} - \frac{Bb^2}{2x^2} - \frac{Ab^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^9, x)

[Out] 1/2\*B\*c^2\*x^2-1/4\*A\*b^2/x^4-b/x^2\*A\*c-1/2\*b^2/x^2\*B+A\*ln(x)\*c^2+2\*B\*ln(x)\*b\*c

**maxima** [A] time = 1.37, size = 54, normalized size = 1.06

$$\frac{1}{2}Bc^2x^2 + \frac{1}{2}(2Bbc + Ac^2)\log(x^2) - \frac{Ab^2 + 2(Bb^2 + 2Abc)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^9,x, algorithm="maxima")

[Out] 1/2\*B\*c^2\*x^2 + 1/2\*(2\*B\*b\*c + A\*c^2)\*log(x^2) - 1/4\*(A\*b^2 + 2\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^4

mupad [B] time = 0.08, size = 51, normalized size = 1.00

$$\ln(x) (A c^2 + 2 B b c) - \frac{x^2 \left( \frac{B b^2}{2} + A c b \right) + \frac{A b^2}{4}}{x^4} + \frac{B c^2 x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^9,x)

[Out] log(x)\*(A\*c^2 + 2\*B\*b\*c) - (x^2\*((B\*b^2)/2 + A\*b\*c) + (A\*b^2)/4)/x^4 + (B\*c^2\*x^2)/2

sympy [A] time = 0.55, size = 51, normalized size = 1.00

$$\frac{B c^2 x^2}{2} + c (A c + 2 B b) \log(x) + \frac{-A b^2 + x^2 (-4 A b c - 2 B b^2)}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*9,x)

[Out] B\*c\*\*2\*x\*\*2/2 + c\*(A\*c + 2\*B\*b)\*log(x) + (-A\*b\*\*2 + x\*\*2\*(-4\*A\*b\*c - 2\*B\*b\*\*2))/(4\*x\*\*4)

$$3.22 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx$$

**Optimal.** Leaf size=48

$$-\frac{Ab^2}{5x^5} - \frac{b(2Ac + bB)}{3x^3} - \frac{c(Ac + 2bB)}{x} + Bc^2x$$

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{Ab^2}{5x^5} - \frac{b(2Ac + bB)}{3x^3} - \frac{c(Ac + 2bB)}{x} + Bc^2x$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^10,x]

[Out] -(A\*b^2)/(5\*x^5) - (b\*(b\*B + 2\*A\*c))/(3\*x^3) - (c\*(2\*b\*B + A\*c))/x + B\*c^2\*x

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^6} dx \\ &= \int \left( Bc^2 + \frac{Ab^2}{x^6} + \frac{b(bB+2Ac)}{x^4} + \frac{c(2bB+Ac)}{x^2} \right) dx \\ &= -\frac{Ab^2}{5x^5} - \frac{b(bB+2Ac)}{3x^3} - \frac{c(2bB+Ac)}{x} + Bc^2x \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{5x^5} - \frac{b(2Ac + bB)}{3x^3} - \frac{c(Ac + 2bB)}{x} + Bc^2x$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^10,x]

[Out] -1/5\*(A\*b^2)/x^5 - (b\*(b\*B + 2\*A\*c))/(3\*x^3) - (c\*(2\*b\*B + A\*c))/x + B\*c^2\*x

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^{10}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^10,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^10, x]

**fricas** [A] time = 0.37, size = 53, normalized size = 1.10

$$\frac{15 Bc^2x^6 - 15 (2 Bbc + Ac^2)x^4 - 3 Ab^2 - 5 (Bb^2 + 2 Abc)x^2}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^10,x, algorithm="fricas")

[Out] 1/15\*(15\*B\*c^2\*x^6 - 15\*(2\*B\*b\*c + A\*c^2)\*x^4 - 3\*A\*b^2 - 5\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^5

**giac** [A] time = 0.15, size = 53, normalized size = 1.10

$$Bc^2x - \frac{30 Bbcx^4 + 15 Ac^2x^4 + 5 Bb^2x^2 + 10 Abcx^2 + 3 Ab^2}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^10,x, algorithm="giac")

[Out] B\*c^2\*x - 1/15\*(30\*B\*b\*c\*x^4 + 15\*A\*c^2\*x^4 + 5\*B\*b^2\*x^2 + 10\*A\*b\*c\*x^2 + 3\*A\*b^2)/x^5

**maple** [A] time = 0.06, size = 45, normalized size = 0.94

$$Bc^2x - \frac{(Ac + 2bB)c}{x} - \frac{Ab^2}{5x^5} - \frac{(2Ac + bB)b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^10,x)

[Out] -1/5\*A\*b^2/x^5-1/3\*b\*(2\*A\*c+B\*b)/x^3-c\*(A\*c+2\*B\*b)/x+B\*c^2\*x

**maxima** [A] time = 1.39, size = 51, normalized size = 1.06

$$Bc^2x - \frac{15 (2 Bbc + Ac^2)x^4 + 3 Ab^2 + 5 (Bb^2 + 2 Abc)x^2}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^10,x, algorithm="maxima")

[Out] B\*c^2\*x - 1/15\*(15\*(2\*B\*b\*c + A\*c^2)\*x^4 + 3\*A\*b^2 + 5\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^5

**mupad** [B] time = 0.07, size = 50, normalized size = 1.04

$$Bc^2x - \frac{x^2 \left( \frac{Bb^2}{3} + \frac{2Ac b}{3} \right) + x^4 (Ac^2 + 2Bbc) + \frac{Ab^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x)`

[Out]  $B*c^2*x - (x^2*((B*b^2)/3 + (2*A*b*c)/3) + x^4*(A*c^2 + 2*B*b*c) + (A*b^2)/5)/x^5$

sympy [A] time = 0.61, size = 54, normalized size = 1.12

$$Bc^2x + \frac{-3Ab^2 + x^4(-15Ac^2 - 30Bbc) + x^2(-10Abc - 5Bb^2)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**10,x)`

[Out]  $B*c**2*x + (-3*A*b**2 + x**4*(-15*A*c**2 - 30*B*b*c) + x**2*(-10*A*b*c - 5*B*b**2))/(15*x**5)$

$$3.23 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx$$

**Optimal.** Leaf size=51

$$-\frac{Ab^2}{6x^6} - \frac{b(2Ac + bB)}{4x^4} - \frac{c(Ac + 2bB)}{2x^2} + Bc^2 \log(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{6x^6} - \frac{b(2Ac + bB)}{4x^4} - \frac{c(Ac + 2bB)}{2x^2} + Bc^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^11,x]

[Out] -(A\*b^2)/(6\*x^6) - (b\*(b\*B + 2\*A\*c))/(4\*x^4) - (c\*(2\*b\*B + A\*c))/(2\*x^2) + B\*c^2\*Log[x]

#### Rule 76

Int[((d\_.)\*(x\_.))^(n\_.)\*((a\_.) + (b\_.)\*(x\_.))\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^7} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A+Bx)(b+cx)^2}{x^4} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{Ab^2}{x^4} + \frac{b(bB+2Ac)}{x^3} + \frac{c(2bB+Ac)}{x^2} + \frac{Bc^2}{x} \right) dx, x, x^2 \right) \\ &= -\frac{Ab^2}{6x^6} - \frac{b(bB+2Ac)}{4x^4} - \frac{c(2bB+Ac)}{2x^2} + Bc^2 \log(x) \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 53, normalized size = 1.04

$$Bc^2 \log(x) - \frac{2A(b^2 + 3bcx^2 + 3c^2x^4) + 3bBx^2(b + 4cx^2)}{12x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^11,x]

[Out] -1/12\*(3\*b\*B\*x^2\*(b + 4\*c\*x^2) + 2\*A\*(b^2 + 3\*b\*c\*x^2 + 3\*c^2\*x^4))/x^6 + B\*c^2\*Log[x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^{11}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^11,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^11, x]

**fricas** [A] time = 0.39, size = 55, normalized size = 1.08

$$\frac{12 Bc^2x^6 \log(x) - 6(2 Bbc + Ac^2)x^4 - 2 Ab^2 - 3(Bb^2 + 2 Abc)x^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^11,x, algorithm="fricas")

[Out] 1/12\*(12\*B\*c^2\*x^6\*log(x) - 6\*(2\*B\*b\*c + A\*c^2)\*x^4 - 2\*A\*b^2 - 3\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^6

**giac** [A] time = 0.19, size = 66, normalized size = 1.29

$$\frac{1}{2} Bc^2 \log(x^2) - \frac{11 Bc^2x^6 + 12 Bbcx^4 + 6 Ac^2x^4 + 3 Bb^2x^2 + 6 Abcx^2 + 2 Ab^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^11,x, algorithm="giac")

[Out] 1/2\*B\*c^2\*log(x^2) - 1/12\*(11\*B\*c^2\*x^6 + 12\*B\*b\*c\*x^4 + 6\*A\*c^2\*x^4 + 3\*B\*b^2\*x^2 + 6\*A\*b\*c\*x^2 + 2\*A\*b^2)/x^6

**maple** [A] time = 0.05, size = 52, normalized size = 1.02

$$Bc^2 \ln(x) - \frac{Ac^2}{2x^2} - \frac{Bbc}{x^2} - \frac{Abc}{2x^4} - \frac{Bb^2}{4x^4} - \frac{Ab^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^11,x)

[Out] -1/2\*b/x^4\*A\*c-1/4\*b^2/x^4\*B-1/2\*c^2/x^2\*A-c/x^2\*b\*B-1/6\*A\*b^2/x^6+B\*c^2\*ln(x)

**maxima** [A] time = 1.27, size = 55, normalized size = 1.08

$$\frac{1}{2} Bc^2 \log(x^2) - \frac{6(2 Bbc + Ac^2)x^4 + 2 Ab^2 + 3(Bb^2 + 2 Abc)x^2}{12x^6}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^11,x, algorithm="maxima")

[Out]  $1/2*B*c^2*\log(x^2) - 1/12*(6*(2*B*b*c + A*c^2)*x^4 + 2*A*b^2 + 3*(B*b^2 + 2*A*b*c)*x^2)/x^6$

mupad [B] time = 0.09, size = 51, normalized size = 1.00

$$B c^2 \ln(x) - \frac{x^2 \left( \frac{B b^2}{4} + \frac{A c b}{2} \right) + x^4 \left( \frac{A c^2}{2} + B b c \right) + \frac{A b^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^11,x)

[Out]  $B*c^2*\log(x) - (x^2*((B*b^2)/4 + (A*b*c)/2) + x^4*((A*c^2)/2 + B*b*c) + (A*b^2)/6)/x^6$

sympy [A] time = 0.99, size = 56, normalized size = 1.10

$$B c^2 \log(x) + \frac{-2 A b^2 + x^4 (-6 A c^2 - 12 B b c) + x^2 (-6 A b c - 3 B b^2)}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*11,x)

[Out]  $B*c**2*\log(x) + (-2*A*b**2 + x**4*(-6*A*c**2 - 12*B*b*c) + x**2*(-6*A*b*c - 3*B*b**2))/(12*x**6)$

$$3.24 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}} dx$$

**Optimal.** Leaf size=53

$$-\frac{Ab^2}{7x^7} - \frac{b(2Ac + bB)}{5x^5} - \frac{c(Ac + 2bB)}{3x^3} - \frac{Bc^2}{x}$$

**Rubi [A]** time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{Ab^2}{7x^7} - \frac{b(2Ac + bB)}{5x^5} - \frac{c(Ac + 2bB)}{3x^3} - \frac{Bc^2}{x}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^12,x]

[Out] -(A\*b^2)/(7\*x^7) - (b\*(b\*B + 2\*A\*c))/(5\*x^5) - (c\*(2\*b\*B + A\*c))/(3\*x^3) - (B\*c^2)/x

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^8} dx \\ &= \int \left( \frac{Ab^2}{x^8} + \frac{b(bB+2Ac)}{x^6} + \frac{c(2bB+Ac)}{x^4} + \frac{Bc^2}{x^2} \right) dx \\ &= -\frac{Ab^2}{7x^7} - \frac{b(bB+2Ac)}{5x^5} - \frac{c(2bB+Ac)}{3x^3} - \frac{Bc^2}{x} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 59, normalized size = 1.11

$$\frac{b^2(-B) - 2Abc}{5x^5} - \frac{Ab^2}{7x^7} + \frac{-Ac^2 - 2bBc}{3x^3} - \frac{Bc^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^12,x]

[Out] -1/7\*(A\*b^2)/x^7 + (-b^2\*B) - 2\*A\*b\*c)/(5\*x^5) + (-2\*b\*B\*c - A\*c^2)/(3\*x^3) - (B\*c^2)/x

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^{12}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^12,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^12, x]

**fricas** [A] time = 0.38, size = 53, normalized size = 1.00

$$\frac{105 Bc^2x^6 + 35(2Bbc + Ac^2)x^4 + 15Ab^2 + 21(Bb^2 + 2Abc)x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^12,x, algorithm="fricas")

[Out] -1/105\*(105\*B\*c^2\*x^6 + 35\*(2\*B\*b\*c + A\*c^2)\*x^4 + 15\*A\*b^2 + 21\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^7

**giac** [A] time = 0.15, size = 55, normalized size = 1.04

$$\frac{105 Bc^2x^6 + 70 Bbcx^4 + 35 Ac^2x^4 + 21 Bb^2x^2 + 42 Abcx^2 + 15 Ab^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^12,x, algorithm="giac")

[Out] -1/105\*(105\*B\*c^2\*x^6 + 70\*B\*b\*c\*x^4 + 35\*A\*c^2\*x^4 + 21\*B\*b^2\*x^2 + 42\*A\*b\*c\*x^2 + 15\*A\*b^2)/x^7

**maple** [A] time = 0.05, size = 48, normalized size = 0.91

$$\frac{Bc^2}{x} - \frac{(Ac + 2bB)c}{3x^3} - \frac{Ab^2}{7x^7} - \frac{(2Ac + bB)b}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^12,x)

[Out] -1/7\*A\*b^2/x^7-1/5\*b\*(2\*A\*c+B\*b)/x^5-1/3\*c\*(A\*c+2\*B\*b)/x^3-B\*c^2/x

**maxima** [A] time = 1.37, size = 53, normalized size = 1.00

$$\frac{105 Bc^2x^6 + 35(2Bbc + Ac^2)x^4 + 15 Ab^2 + 21(Bb^2 + 2Abc)x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^12,x, algorithm="maxima")

[Out] -1/105\*(105\*B\*c^2\*x^6 + 35\*(2\*B\*b\*c + A\*c^2)\*x^4 + 15\*A\*b^2 + 21\*(B\*b^2 + 2\*A\*b\*c)\*x^2)/x^7

**mupad** [B] time = 0.04, size = 52, normalized size = 0.98

$$\frac{x^2 \left( \frac{Bb^2}{5} + \frac{2Ac b}{5} \right) + x^4 \left( \frac{Ac^2}{3} + \frac{2Bbc}{3} \right) + \frac{Ab^2}{7} + Bc^2x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x)`

[Out]  $-(x^2*((B*b^2)/5 + (2*A*b*c)/5) + x^4*((A*c^2)/3 + (2*B*b*c)/3) + (A*b^2)/7 + B*c^2*x^6)/x^7$

sympy [A] time = 1.07, size = 58, normalized size = 1.09

$$\frac{-15Ab^2 - 105Bc^2x^6 + x^4(-35Ac^2 - 70Bbc) + x^2(-42Abc - 21Bb^2)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**12,x)`

[Out]  $(-15*A*b**2 - 105*B*c**2*x**6 + x**4*(-35*A*c**2 - 70*B*b*c) + x**2*(-42*A*b*c - 21*B*b**2))/(105*x**7)$

$$3.25 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx$$

**Optimal.** Leaf size=75

$$\frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2x^7(3Ac + bB) + \frac{1}{11}c^2x^{11}(Ac + 3bB) + \frac{1}{3}bcx^9(Ac + bB) + \frac{1}{13}Bc^3x^{13}$$

**Rubi [A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{1}{7}b^2x^7(3Ac + bB) + \frac{1}{5}Ab^3x^5 + \frac{1}{11}c^2x^{11}(Ac + 3bB) + \frac{1}{3}bcx^9(Ac + bB) + \frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^2,x]

[Out] (A\*b^3\*x^5)/5 + (b^2\*(b\*B + 3\*A\*c)\*x^7)/7 + (b\*c\*(b\*B + A\*c)\*x^9)/3 + (c^2\*(3\*b\*B + A\*c)\*x^11)/11 + (B\*c^3\*x^13)/13

**Rule 448**

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^2} dx &= \int x^4 (A + Bx^2)(b + cx^2)^3 dx \\ &= \int (Ab^3x^4 + b^2(bB + 3Ac)x^6 + 3bc(bB + Ac)x^8 + c^2(3bB + Ac)x^{10} + Bc^3x^{12}) dx \\ &= \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB + 3Ac)x^7 + \frac{1}{3}bc(bB + Ac)x^9 + \frac{1}{11}c^2(3bB + Ac)x^{11} + \frac{1}{13}Bc^3x^{13} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 75, normalized size = 1.00

$$\frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2x^7(3Ac + bB) + \frac{1}{11}c^2x^{11}(Ac + 3bB) + \frac{1}{3}bcx^9(Ac + bB) + \frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^2,x]

[Out] (A\*b^3\*x^5)/5 + (b^2\*(b\*B + 3\*A\*c)\*x^7)/7 + (b\*c\*(b\*B + A\*c)\*x^9)/3 + (c^2\*(3\*b\*B + A\*c)\*x^11)/11 + (B\*c^3\*x^13)/13

**IntegrateAlgebraic [A]** time = 0.04, size = 93, normalized size = 1.24

$$\frac{1}{5}Ab^3x^5 + \frac{3}{7}Ab^2cx^7 + \frac{1}{3}Abc^2x^9 + \frac{1}{11}Ac^3x^{11} + \frac{1}{7}b^3Bx^7 + \frac{1}{3}b^2Bcx^9 + \frac{3}{11}bBc^2x^{11} + \frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^2,x]

[Out] (A\*b^3\*x^5)/5 + (b^3\*B\*x^7)/7 + (3\*A\*b^2\*c\*x^7)/7 + (b^2\*B\*c\*x^9)/3 + (A\*b\*c^2\*x^9)/3 + (3\*b\*B\*c^2\*x^11)/11 + (A\*c^3\*x^11)/11 + (B\*c^3\*x^13)/13

**fricas** [A] time = 0.38, size = 73, normalized size = 0.97

$$\frac{1}{13} Bc^3x^{13} + \frac{1}{11} (3Bbc^2 + Ac^3)x^{11} + \frac{1}{3} (Bb^2c + Abc^2)x^9 + \frac{1}{5} Ab^3x^5 + \frac{1}{7} (Bb^3 + 3Ab^2c)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^2,x, algorithm="fricas")

[Out] 1/13\*B\*c^3\*x^13 + 1/11\*(3\*B\*b\*c^2 + A\*c^3)\*x^11 + 1/3\*(B\*b^2\*c + A\*b\*c^2)\*x^9 + 1/5\*A\*b^3\*x^5 + 1/7\*(B\*b^3 + 3\*A\*b^2\*c)\*x^7

**giac** [A] time = 0.19, size = 77, normalized size = 1.03

$$\frac{1}{13} Bc^3x^{13} + \frac{3}{11} Bbc^2x^{11} + \frac{1}{11} Ac^3x^{11} + \frac{1}{3} Bb^2cx^9 + \frac{1}{3} Abc^2x^9 + \frac{1}{7} Bb^3x^7 + \frac{3}{7} Ab^2cx^7 + \frac{1}{5} Ab^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^2,x, algorithm="giac")

[Out] 1/13\*B\*c^3\*x^13 + 3/11\*B\*b\*c^2\*x^11 + 1/11\*A\*c^3\*x^11 + 1/3\*B\*b^2\*c\*x^9 + 1/3\*A\*b\*c^2\*x^9 + 1/7\*B\*b^3\*x^7 + 3/7\*A\*b^2\*c\*x^7 + 1/5\*A\*b^3\*x^5

**maple** [A] time = 0.04, size = 76, normalized size = 1.01

$$\frac{Bc^3x^{13}}{13} + \frac{(Ac^3 + 3Bbc^2)x^{11}}{11} + \frac{Ab^3x^5}{5} + \frac{(3Abc^2 + 3Bcb^2)x^9}{9} + \frac{(3Ac b^2 + Bb^3)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^2,x)

[Out] 1/13\*B\*c^3\*x^13+1/11\*(A\*c^3+3\*B\*b\*c^2)\*x^11+1/9\*(3\*A\*b\*c^2+3\*B\*b^2\*c)\*x^9+1/7\*(3\*A\*b^2\*c+B\*b^3)\*x^7+1/5\*A\*b^3\*x^5

**maxima** [A] time = 1.34, size = 73, normalized size = 0.97

$$\frac{1}{13} Bc^3x^{13} + \frac{1}{11} (3Bbc^2 + Ac^3)x^{11} + \frac{1}{3} (Bb^2c + Abc^2)x^9 + \frac{1}{5} Ab^3x^5 + \frac{1}{7} (Bb^3 + 3Ab^2c)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^2,x, algorithm="maxima")

[Out] 1/13\*B\*c^3\*x^13 + 1/11\*(3\*B\*b\*c^2 + A\*c^3)\*x^11 + 1/3\*(B\*b^2\*c + A\*b\*c^2)\*x^9 + 1/5\*A\*b^3\*x^5 + 1/7\*(B\*b^3 + 3\*A\*b^2\*c)\*x^7

**mupad** [B] time = 0.06, size = 69, normalized size = 0.92

$$x^7 \left( \frac{Bb^3}{7} + \frac{3Ac b^2}{7} \right) + x^{11} \left( \frac{Ac^3}{11} + \frac{3Bbc^2}{11} \right) + \frac{Ab^3x^5}{5} + \frac{Bc^3x^{13}}{13} + \frac{bcx^9(Ac + Bb)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^2,x)

[Out]  $x^7 \cdot ((B \cdot b^3)/7 + (3 \cdot A \cdot b^2 \cdot c)/7) + x^{11} \cdot ((A \cdot c^3)/11 + (3 \cdot B \cdot b \cdot c^2)/11) + (A \cdot b^3 \cdot x^5)/5 + (B \cdot c^3 \cdot x^{13})/13 + (b \cdot c \cdot x^9 \cdot (A \cdot c + B \cdot b))/3$

**sympy** [A] time = 0.08, size = 80, normalized size = 1.07

$$\frac{Ab^3x^5}{5} + \frac{Bc^3x^{13}}{13} + x^{11} \left( \frac{Ac^3}{11} + \frac{3Bbc^2}{11} \right) + x^9 \left( \frac{Abc^2}{3} + \frac{Bb^2c}{3} \right) + x^7 \left( \frac{3Ab^2c}{7} + \frac{Bb^3}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*2,x)

[Out]  $A \cdot b^3 \cdot x^5/5 + B \cdot c^3 \cdot x^{13}/13 + x^{11} \cdot (A \cdot c^3/11 + 3 \cdot B \cdot b \cdot c^2/11) + x^9 \cdot (A \cdot b \cdot c^2/3 + B \cdot b^2 \cdot c/3) + x^7 \cdot (3 \cdot A \cdot b^2 \cdot c/7 + B \cdot b^3/7)$

$$3.26 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3} dx$$

**Optimal.** Leaf size=68

$$-\frac{(b+cx^2)^5(2bB-Ac)}{10c^3} + \frac{b(b+cx^2)^4(bB-Ac)}{8c^3} + \frac{B(b+cx^2)^6}{12c^3}$$

**Rubi [A]** time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$-\frac{(b+cx^2)^5(2bB-Ac)}{10c^3} + \frac{b(b+cx^2)^4(bB-Ac)}{8c^3} + \frac{B(b+cx^2)^6}{12c^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^3,x]

[Out] (b\*(b\*B - A\*c)\*(b + c\*x^2)^4)/(8\*c^3) - ((2\*b\*B - A\*c)\*(b + c\*x^2)^5)/(10\*c^3) + (B\*(b + c\*x^2)^6)/(12\*c^3)

#### Rule 76

```
Int[((d_.)*(x_.))^(n_.)*((a_.) + (b_.)*(x_.))*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

#### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3} dx &= \int x^3 (A+Bx^2)(b+cx^2)^3 dx \\ &= \frac{1}{2} \text{Subst} \left( \int x(A+Bx)(b+cx)^3 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{b(bB-Ac)(b+cx)^3}{c^2} + \frac{(-2bB+Ac)(b+cx)^4}{c^2} + \frac{B(b+cx)^5}{c^2} \right) dx, x, \right. \\ &= \frac{b(bB-Ac)(b+cx^2)^4}{8c^3} - \frac{(2bB-Ac)(b+cx^2)^5}{10c^3} + \frac{B(b+cx^2)^6}{12c^3} \end{aligned}$$



**Mathematica [A]** time = 0.03, size = 69, normalized size = 1.01

$$\frac{1}{120}x^4(30Ab^3 + 20b^2x^2(3Ac + bB) + 12c^2x^6(Ac + 3bB) + 45bcx^4(Ac + bB) + 10Bc^3x^8)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^3,x]

[Out] (x^4\*(30\*A\*b^3 + 20\*b^2\*(b\*B + 3\*A\*c)\*x^2 + 45\*b\*c\*(b\*B + A\*c)\*x^4 + 12\*c^2\*(3\*b\*B + A\*c)\*x^6 + 10\*B\*c^3\*x^8))/120

**IntegrateAlgebraic [A]** time = 0.04, size = 81, normalized size = 1.19

$$\frac{1}{120}x^4(30Ab^3 + 60Ab^2cx^2 + 45Abc^2x^4 + 12Ac^3x^6 + 20b^3Bx^2 + 45b^2Bcx^4 + 36bBc^2x^6 + 10Bc^3x^8)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^3,x]

[Out] (x^4\*(30\*A\*b^3 + 20\*b^3\*B\*x^2 + 60\*A\*b^2\*c\*x^2 + 45\*b^2\*B\*c\*x^4 + 45\*A\*b\*c^2\*x^4 + 36\*b\*B\*c^2\*x^6 + 12\*A\*c^3\*x^6 + 10\*B\*c^3\*x^8))/120

**fricas [A]** time = 0.38, size = 73, normalized size = 1.07

$$\frac{1}{12}Bc^3x^{12} + \frac{1}{10}(3Bbc^2 + Ac^3)x^{10} + \frac{3}{8}(Bb^2c + Abc^2)x^8 + \frac{1}{4}Ab^3x^4 + \frac{1}{6}(Bb^3 + 3Ab^2c)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^3,x, algorithm="fricas")

[Out] 1/12\*B\*c^3\*x^12 + 1/10\*(3\*B\*b\*c^2 + A\*c^3)\*x^10 + 3/8\*(B\*b^2\*c + A\*b\*c^2)\*x^8 + 1/4\*A\*b^3\*x^4 + 1/6\*(B\*b^3 + 3\*A\*b^2\*c)\*x^6

**giac [A]** time = 0.21, size = 77, normalized size = 1.13

$$\frac{1}{12}Bc^3x^{12} + \frac{3}{10}Bbc^2x^{10} + \frac{1}{10}Ac^3x^{10} + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{6}Bb^3x^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{4}Ab^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^3,x, algorithm="giac")

[Out] 1/12\*B\*c^3\*x^12 + 3/10\*B\*b\*c^2\*x^10 + 1/10\*A\*c^3\*x^10 + 3/8\*B\*b^2\*c\*x^8 + 3/8\*A\*b\*c^2\*x^8 + 1/6\*B\*b^3\*x^6 + 1/2\*A\*b^2\*c\*x^6 + 1/4\*A\*b^3\*x^4

**maple [A]** time = 0.04, size = 76, normalized size = 1.12

$$\frac{Bc^3x^{12}}{12} + \frac{(Ac^3 + 3Bbc^2)x^{10}}{10} + \frac{Ab^3x^4}{4} + \frac{(3Abc^2 + 3Bcb^2)x^8}{8} + \frac{(3Ac b^2 + Bb^3)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^3,x)

[Out] 1/12\*B\*c^3\*x^12+1/10\*(A\*c^3+3\*B\*b\*c^2)\*x^10+1/8\*(3\*A\*b\*c^2+3\*B\*b^2\*c)\*x^8+1/6\*(3\*A\*b^2\*c+B\*b^3)\*x^6+1/4\*A\*b^3\*x^4

**maxima [A]** time = 1.32, size = 73, normalized size = 1.07

$$\frac{1}{12}Bc^3x^{12} + \frac{1}{10}(3Bbc^2 + Ac^3)x^{10} + \frac{3}{8}(Bb^2c + Abc^2)x^8 + \frac{1}{4}Ab^3x^4 + \frac{1}{6}(Bb^3 + 3Ab^2c)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^3,x, algorithm="maxima")

[Out] 1/12\*B\*c^3\*x^12 + 1/10\*(3\*B\*b\*c^2 + A\*c^3)\*x^10 + 3/8\*(B\*b^2\*c + A\*b\*c^2)\*x^8 + 1/4\*A\*b^3\*x^4 + 1/6\*(B\*b^3 + 3\*A\*b^2\*c)\*x^6

**mupad [B]** time = 0.03, size = 69, normalized size = 1.01

$$x^6 \left( \frac{B b^3}{6} + \frac{A c b^2}{2} \right) + x^{10} \left( \frac{A c^3}{10} + \frac{3 B b c^2}{10} \right) + \frac{A b^3 x^4}{4} + \frac{B c^3 x^{12}}{12} + \frac{3 b c x^8 (A c + B b)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^3,x)

[Out] x^6\*((B\*b^3)/6 + (A\*b^2\*c)/2) + x^10\*((A\*c^3)/10 + (3\*B\*b\*c^2)/10) + (A\*b^3\*x^4)/4 + (B\*c^3\*x^12)/12 + (3\*b\*c\*x^8\*(A\*c + B\*b))/8

**sympy [A]** time = 0.09, size = 82, normalized size = 1.21

$$\frac{A b^3 x^4}{4} + \frac{B c^3 x^{12}}{12} + x^{10} \left( \frac{A c^3}{10} + \frac{3 B b c^2}{10} \right) + x^8 \left( \frac{3 A b c^2}{8} + \frac{3 B b^2 c}{8} \right) + x^6 \left( \frac{A b^2 c}{2} + \frac{B b^3}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*3,x)

[Out] A\*b\*\*3\*x\*\*4/4 + B\*c\*\*3\*x\*\*12/12 + x\*\*10\*(A\*c\*\*3/10 + 3\*B\*b\*c\*\*2/10) + x\*\*8\*(3\*A\*b\*c\*\*2/8 + 3\*B\*b\*\*2\*c/8) + x\*\*6\*(A\*b\*\*2\*c/2 + B\*b\*\*3/6)

$$3.27 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} dx$$

**Optimal.** Leaf size=75

$$\frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2x^5(3Ac + bB) + \frac{1}{9}c^2x^9(Ac + 3bB) + \frac{3}{7}bcx^7(Ac + bB) + \frac{1}{11}Bc^3x^{11}$$

**Rubi [A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{1}{5}b^2x^5(3Ac + bB) + \frac{1}{3}Ab^3x^3 + \frac{1}{9}c^2x^9(Ac + 3bB) + \frac{3}{7}bcx^7(Ac + bB) + \frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^4,x]

[Out] (A\*b^3\*x^3)/3 + (b^2\*(b\*B + 3\*A\*c)\*x^5)/5 + (3\*b\*c\*(b\*B + A\*c)\*x^7)/7 + (c^2\*(3\*b\*B + A\*c)\*x^9)/9 + (B\*c^3\*x^11)/11

**Rule 448**

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} dx &= \int x^2(A+Bx^2)(b+cx^2)^3 dx \\ &= \int (Ab^3x^2 + b^2(bB+3Ac)x^4 + 3bc(bB+Ac)x^6 + c^2(3bB+Ac)x^8 + Bc^3x^{10}) dx \\ &= \frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2(bB+3Ac)x^5 + \frac{3}{7}bc(bB+Ac)x^7 + \frac{1}{9}c^2(3bB+Ac)x^9 + \frac{1}{11}Bc^3x^{11} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 75, normalized size = 1.00

$$\frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2x^5(3Ac + bB) + \frac{1}{9}c^2x^9(Ac + 3bB) + \frac{3}{7}bcx^7(Ac + bB) + \frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^4,x]

[Out] (A\*b^3\*x^3)/3 + (b^2\*(b\*B + 3\*A\*c)\*x^5)/5 + (3\*b\*c\*(b\*B + A\*c)\*x^7)/7 + (c^2\*(3\*b\*B + A\*c)\*x^9)/9 + (B\*c^3\*x^11)/11

**IntegrateAlgebraic [A]** time = 0.04, size = 93, normalized size = 1.24

$$\frac{1}{3}Ab^3x^3 + \frac{3}{5}Ab^2cx^5 + \frac{3}{7}Abc^2x^7 + \frac{1}{9}Ac^3x^9 + \frac{1}{5}b^3Bx^5 + \frac{3}{7}b^2Bcx^7 + \frac{1}{3}bBc^2x^9 + \frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^4,x]

[Out] (A\*b^3\*x^3)/3 + (b^3\*B\*x^5)/5 + (3\*A\*b^2\*c\*x^5)/5 + (3\*b^2\*B\*c\*x^7)/7 + (3\*A\*b\*c^2\*x^7)/7 + (b\*B\*c^2\*x^9)/3 + (A\*c^3\*x^9)/9 + (B\*c^3\*x^11)/11

**fricas** [A] time = 0.37, size = 73, normalized size = 0.97

$$\frac{1}{11} Bc^3x^{11} + \frac{1}{9} (3Bbc^2 + Ac^3)x^9 + \frac{3}{7} (Bb^2c + Abc^2)x^7 + \frac{1}{3} Ab^3x^3 + \frac{1}{5} (Bb^3 + 3Ab^2c)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^4,x, algorithm="fricas")

[Out] 1/11\*B\*c^3\*x^11 + 1/9\*(3\*B\*b\*c^2 + A\*c^3)\*x^9 + 3/7\*(B\*b^2\*c + A\*b\*c^2)\*x^7 + 1/3\*A\*b^3\*x^3 + 1/5\*(B\*b^3 + 3\*A\*b^2\*c)\*x^5

**giac** [A] time = 0.15, size = 77, normalized size = 1.03

$$\frac{1}{11} Bc^3x^{11} + \frac{1}{3} Bbc^2x^9 + \frac{1}{9} Ac^3x^9 + \frac{3}{7} Bb^2cx^7 + \frac{3}{7} Abc^2x^7 + \frac{1}{5} Bb^3x^5 + \frac{3}{5} Ab^2cx^5 + \frac{1}{3} Ab^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^4,x, algorithm="giac")

[Out] 1/11\*B\*c^3\*x^11 + 1/3\*B\*b\*c^2\*x^9 + 1/9\*A\*c^3\*x^9 + 3/7\*B\*b^2\*c\*x^7 + 3/7\*A\*b\*c^2\*x^7 + 1/5\*B\*b^3\*x^5 + 3/5\*A\*b^2\*c\*x^5 + 1/3\*A\*b^3\*x^3

**maple** [A] time = 0.04, size = 76, normalized size = 1.01

$$\frac{Bc^3x^{11}}{11} + \frac{(Ac^3 + 3Bbc^2)x^9}{9} + \frac{Ab^3x^3}{3} + \frac{(3Abc^2 + 3Bcb^2)x^7}{7} + \frac{(3Ac b^2 + Bb^3)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^4,x)

[Out] 1/11\*B\*c^3\*x^11+1/9\*(A\*c^3+3\*B\*b\*c^2)\*x^9+1/7\*(3\*A\*b\*c^2+3\*B\*b^2\*c)\*x^7+1/5\*(3\*A\*b^2\*c+B\*b^3)\*x^5+1/3\*A\*b^3\*x^3

**maxima** [A] time = 1.37, size = 73, normalized size = 0.97

$$\frac{1}{11} Bc^3x^{11} + \frac{1}{9} (3Bbc^2 + Ac^3)x^9 + \frac{3}{7} (Bb^2c + Abc^2)x^7 + \frac{1}{3} Ab^3x^3 + \frac{1}{5} (Bb^3 + 3Ab^2c)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^4,x, algorithm="maxima")

[Out] 1/11\*B\*c^3\*x^11 + 1/9\*(3\*B\*b\*c^2 + A\*c^3)\*x^9 + 3/7\*(B\*b^2\*c + A\*b\*c^2)\*x^7 + 1/3\*A\*b^3\*x^3 + 1/5\*(B\*b^3 + 3\*A\*b^2\*c)\*x^5

**mupad** [B] time = 0.03, size = 69, normalized size = 0.92

$$x^5 \left( \frac{Bb^3}{5} + \frac{3Ac b^2}{5} \right) + x^9 \left( \frac{Ac^3}{9} + \frac{Bb c^2}{3} \right) + \frac{Ab^3 x^3}{3} + \frac{Bc^3 x^{11}}{11} + \frac{3bcx^7 (Ac + Bb)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^4,x)

[Out]  $x^5 \cdot ((B \cdot b^3)/5 + (3 \cdot A \cdot b^2 \cdot c)/5) + x^9 \cdot ((A \cdot c^3)/9 + (B \cdot b \cdot c^2)/3) + (A \cdot b^3 \cdot x^3)/3 + (B \cdot c^3 \cdot x^{11})/11 + (3 \cdot b \cdot c \cdot x^7 \cdot (A \cdot c + B \cdot b))/7$

**sympy [A]** time = 0.09, size = 82, normalized size = 1.09

$$\frac{Ab^3x^3}{3} + \frac{Bc^3x^{11}}{11} + x^9 \left( \frac{Ac^3}{9} + \frac{Bbc^2}{3} \right) + x^7 \left( \frac{3Abc^2}{7} + \frac{3Bb^2c}{7} \right) + x^5 \left( \frac{3Ab^2c}{5} + \frac{Bb^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*4,x)

[Out]  $A \cdot b^3 \cdot x^3/3 + B \cdot c^3 \cdot x^{11}/11 + x^9 \cdot (A \cdot c^3/9 + B \cdot b \cdot c^2/3) + x^7 \cdot (3 \cdot A \cdot b \cdot c^2/7 + 3 \cdot B \cdot b^2 \cdot c/7) + x^5 \cdot (3 \cdot A \cdot b^2 \cdot c/5 + B \cdot b^3/5)$

$$3.28 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx$$

Optimal. Leaf size=42

$$\frac{B(b+cx^2)^5}{10c^2} - \frac{(b+cx^2)^4(bB-Ac)}{8c^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 444, 43}

$$\frac{B(b+cx^2)^5}{10c^2} - \frac{(b+cx^2)^4(bB-Ac)}{8c^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^5,x]

[Out] -((b\*B - A\*c)\*(b + c\*x^2)^4)/(8\*c^2) + (B\*(b + c\*x^2)^5)/(10\*c^2)

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx &= \int x(A+Bx^2)(b+cx^2)^3 dx \\ &= \frac{1}{2} \text{Subst} \left( \int (A+Bx)(b+cx)^3 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{(-bB+Ac)(b+cx)^3}{c} + \frac{B(b+cx)^4}{c} \right) dx, x, x^2 \right) \\ &= -\frac{(bB-Ac)(b+cx^2)^4}{8c^2} + \frac{B(b+cx^2)^5}{10c^2} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 69, normalized size = 1.64

$$\frac{1}{40}x^2(20Ab^3 + 10b^2x^2(3Ac + bB) + 5c^2x^6(Ac + 3bB) + 20bcx^4(Ac + bB) + 4Bc^3x^8)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^5,x]

[Out] (x^2\*(20\*A\*b^3 + 10\*b^2\*(b\*B + 3\*A\*c)\*x^2 + 20\*b\*c\*(b\*B + A\*c)\*x^4 + 5\*c^2\*(3\*b\*B + A\*c)\*x^6 + 4\*B\*c^3\*x^8))/40

**IntegrateAlgebraic** [A] time = 0.04, size = 81, normalized size = 1.93

$$\frac{1}{40}x^2(20Ab^3 + 30Ab^2cx^2 + 20Abc^2x^4 + 5Ac^3x^6 + 10b^3Bx^2 + 20b^2Bcx^4 + 15bBc^2x^6 + 4Bc^3x^8)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^5,x]

[Out] (x^2\*(20\*A\*b^3 + 10\*b^3\*B\*x^2 + 30\*A\*b^2\*c\*x^2 + 20\*b^2\*B\*c\*x^4 + 20\*A\*b\*c^2\*x^4 + 15\*b\*B\*c^2\*x^6 + 5\*A\*c^3\*x^6 + 4\*B\*c^3\*x^8))/40

**fricas** [A] time = 0.38, size = 73, normalized size = 1.74

$$\frac{1}{10}Bc^3x^{10} + \frac{1}{8}(3Bbc^2 + Ac^3)x^8 + \frac{1}{2}(Bb^2c + Abc^2)x^6 + \frac{1}{2}Ab^3x^2 + \frac{1}{4}(Bb^3 + 3Ab^2c)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^5,x, algorithm="fricas")

[Out] 1/10\*B\*c^3\*x^10 + 1/8\*(3\*B\*b\*c^2 + A\*c^3)\*x^8 + 1/2\*(B\*b^2\*c + A\*b\*c^2)\*x^6 + 1/2\*A\*b^3\*x^2 + 1/4\*(B\*b^3 + 3\*A\*b^2\*c)\*x^4

**giac** [B] time = 0.15, size = 77, normalized size = 1.83

$$\frac{1}{10}Bc^3x^{10} + \frac{3}{8}Bbc^2x^8 + \frac{1}{8}Ac^3x^8 + \frac{1}{2}Bb^2cx^6 + \frac{1}{2}Abc^2x^6 + \frac{1}{4}Bb^3x^4 + \frac{3}{4}Ab^2cx^4 + \frac{1}{2}Ab^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^5,x, algorithm="giac")

[Out] 1/10\*B\*c^3\*x^10 + 3/8\*B\*b\*c^2\*x^8 + 1/8\*A\*c^3\*x^8 + 1/2\*B\*b^2\*c\*x^6 + 1/2\*A\*b\*c^2\*x^6 + 1/4\*B\*b^3\*x^4 + 3/4\*A\*b^2\*c\*x^4 + 1/2\*A\*b^3\*x^2

**maple** [A] time = 0.04, size = 76, normalized size = 1.81

$$\frac{Bc^3x^{10}}{10} + \frac{(Ac^3 + 3Bbc^2)x^8}{8} + \frac{Ab^3x^2}{2} + \frac{(3Abc^2 + 3Bcb^2)x^6}{6} + \frac{(3Ac b^2 + Bb^3)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^5,x)

[Out] 1/10\*B\*c^3\*x^10+1/8\*(A\*c^3+3\*B\*b\*c^2)\*x^8+1/6\*(3\*A\*b\*c^2+3\*B\*b^2\*c)\*x^6+1/4\*(3\*A\*b^2\*c+B\*b^3)\*x^4+1/2\*A\*b^3\*x^2

**maxima** [A] time = 1.44, size = 73, normalized size = 1.74

$$\frac{1}{10}Bc^3x^{10} + \frac{1}{8}(3Bbc^2 + Ac^3)x^8 + \frac{1}{2}(Bb^2c + Abc^2)x^6 + \frac{1}{2}Ab^3x^2 + \frac{1}{4}(Bb^3 + 3Ab^2c)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^5,x, algorithm="maxima")

[Out] 1/10\*B\*c^3\*x^10 + 1/8\*(3\*B\*b\*c^2 + A\*c^3)\*x^8 + 1/2\*(B\*b^2\*c + A\*b\*c^2)\*x^6 + 1/2\*A\*b^3\*x^2 + 1/4\*(B\*b^3 + 3\*A\*b^2\*c)\*x^4

**mupad [B]** time = 0.03, size = 69, normalized size = 1.64

$$x^4 \left( \frac{Bb^3}{4} + \frac{3Ac b^2}{4} \right) + x^8 \left( \frac{Ac^3}{8} + \frac{3Bbc^2}{8} \right) + \frac{Ab^3x^2}{2} + \frac{Bc^3x^{10}}{10} + \frac{bcx^6(Ac+Bb)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^5,x)

[Out] x^4\*((B\*b^3)/4 + (3\*A\*b^2\*c)/4) + x^8\*((A\*c^3)/8 + (3\*B\*b\*c^2)/8) + (A\*b^3\*x^2)/2 + (B\*c^3\*x^10)/10 + (b\*c\*x^6\*(A\*c + B\*b))/2

**sympy [B]** time = 0.09, size = 80, normalized size = 1.90

$$\frac{Ab^3x^2}{2} + \frac{Bc^3x^{10}}{10} + x^8 \left( \frac{Ac^3}{8} + \frac{3Bbc^2}{8} \right) + x^6 \left( \frac{Abc^2}{2} + \frac{Bb^2c}{2} \right) + x^4 \left( \frac{3Ab^2c}{4} + \frac{Bb^3}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*5,x)

[Out] A\*b\*\*3\*x\*\*2/2 + B\*c\*\*3\*x\*\*10/10 + x\*\*8\*(A\*c\*\*3/8 + 3\*B\*b\*c\*\*2/8) + x\*\*6\*(A\*b\*c\*\*2/2 + B\*b\*\*2\*c/2) + x\*\*4\*(3\*A\*b\*\*2\*c/4 + B\*b\*\*3/4)



$$3.29 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx$$

**Optimal.** Leaf size=70

$$Ab^3x + \frac{1}{3}b^2x^3(3Ac + bB) + \frac{1}{7}c^2x^7(Ac + 3bB) + \frac{3}{5}bcx^5(Ac + bB) + \frac{1}{9}Bc^3x^9$$

**Rubi [A]** time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 373}

$$\frac{1}{3}b^2x^3(3Ac + bB) + Ab^3x + \frac{1}{7}c^2x^7(Ac + 3bB) + \frac{3}{5}bcx^5(Ac + bB) + \frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^6,x]

[Out] A\*b^3\*x + (b^2\*(b\*B + 3\*A\*c)\*x^3)/3 + (3\*b\*c\*(b\*B + A\*c)\*x^5)/5 + (c^2\*(3\*b\*B + A\*c)\*x^7)/7 + (B\*c^3\*x^9)/9

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx &= \int (A+Bx^2)(b+cx^2)^3 dx \\ &= \int (Ab^3 + b^2(bB + 3Ac)x^2 + 3bc(bB + Ac)x^4 + c^2(3bB + Ac)x^6 + Bc^3x^8) dx \\ &= Ab^3x + \frac{1}{3}b^2(bB + 3Ac)x^3 + \frac{3}{5}bc(bB + Ac)x^5 + \frac{1}{7}c^2(3bB + Ac)x^7 + \frac{1}{9}Bc^3x^9 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 70, normalized size = 1.00

$$Ab^3x + \frac{1}{3}b^2x^3(3Ac + bB) + \frac{1}{7}c^2x^7(Ac + 3bB) + \frac{3}{5}bcx^5(Ac + bB) + \frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^6,x]

[Out] A\*b^3\*x + (b^2\*(b\*B + 3\*A\*c)\*x^3)/3 + (3\*b\*c\*(b\*B + A\*c)\*x^5)/5 + (c^2\*(3\*b\*B + A\*c)\*x^7)/7 + (B\*c^3\*x^9)/9

**IntegrateAlgebraic [A]** time = 0.04, size = 85, normalized size = 1.21

$$Ab^3x + Ab^2cx^3 + \frac{3}{5}Abc^2x^5 + \frac{1}{7}Ac^3x^7 + \frac{1}{3}b^3Bx^3 + \frac{3}{5}b^2Bcx^5 + \frac{3}{7}bBc^2x^7 + \frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^6,x]

[Out] A\*b^3\*x + (b^3\*B\*x^3)/3 + A\*b^2\*c\*x^3 + (3\*b^2\*B\*c\*x^5)/5 + (3\*A\*b\*c^2\*x^5)/5 + (3\*b\*B\*c^2\*x^7)/7 + (A\*c^3\*x^7)/7 + (B\*c^3\*x^9)/9

**fricas** [A] time = 0.36, size = 70, normalized size = 1.00

$$\frac{1}{9}Bc^3x^9 + \frac{1}{7}(3Bbc^2 + Ac^3)x^7 + \frac{3}{5}(Bb^2c + Abc^2)x^5 + Ab^3x + \frac{1}{3}(Bb^3 + 3Ab^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^6,x, algorithm="fricas")

[Out] 1/9\*B\*c^3\*x^9 + 1/7\*(3\*B\*b\*c^2 + A\*c^3)\*x^7 + 3/5\*(B\*b^2\*c + A\*b\*c^2)\*x^5 + A\*b^3\*x + 1/3\*(B\*b^3 + 3\*A\*b^2\*c)\*x^3

**giac** [A] time = 0.17, size = 73, normalized size = 1.04

$$\frac{1}{9}Bc^3x^9 + \frac{3}{7}Bbc^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{3}{5}Bb^2cx^5 + \frac{3}{5}Abc^2x^5 + \frac{1}{3}Bb^3x^3 + Ab^2cx^3 + Ab^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^6,x, algorithm="giac")

[Out] 1/9\*B\*c^3\*x^9 + 3/7\*B\*b\*c^2\*x^7 + 1/7\*A\*c^3\*x^7 + 3/5\*B\*b^2\*c\*x^5 + 3/5\*A\*b\*c^2\*x^5 + 1/3\*B\*b^3\*x^3 + A\*b^2\*c\*x^3 + A\*b^3\*x

**maple** [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{Bc^3x^9}{9} + \frac{(Ac^3 + 3Bbc^2)x^7}{7} + Ab^3x + \frac{(3Abc^2 + 3Bcb^2)x^5}{5} + \frac{(3Ac b^2 + Bb^3)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^6,x)

[Out] 1/9\*B\*c^3\*x^9+1/7\*(A\*c^3+3\*B\*b\*c^2)\*x^7+1/5\*(3\*A\*b\*c^2+3\*B\*b^2\*c)\*x^5+1/3\*(3\*A\*b^2\*c+B\*b^3)\*x^3+A\*b^3\*x

**maxima** [A] time = 1.28, size = 70, normalized size = 1.00

$$\frac{1}{9}Bc^3x^9 + \frac{1}{7}(3Bbc^2 + Ac^3)x^7 + \frac{3}{5}(Bb^2c + Abc^2)x^5 + Ab^3x + \frac{1}{3}(Bb^3 + 3Ab^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^6,x, algorithm="maxima")

[Out] 1/9\*B\*c^3\*x^9 + 1/7\*(3\*B\*b\*c^2 + A\*c^3)\*x^7 + 3/5\*(B\*b^2\*c + A\*b\*c^2)\*x^5 + A\*b^3\*x + 1/3\*(B\*b^3 + 3\*A\*b^2\*c)\*x^3

**mupad** [B] time = 0.03, size = 65, normalized size = 0.93

$$x^3 \left( \frac{Bb^3}{3} + Ac b^2 \right) + x^7 \left( \frac{Ac^3}{7} + \frac{3Bbc^2}{7} \right) + \frac{Bc^3x^9}{9} + Ab^3x + \frac{3bcx^5(Ac + Bb)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^6,x)

[Out]  $x^3 \cdot ((B \cdot b^3)/3 + A \cdot b^2 \cdot c) + x^7 \cdot ((A \cdot c^3)/7 + (3 \cdot B \cdot b \cdot c^2)/7) + (B \cdot c^3 \cdot x^9)/9 + A \cdot b^3 \cdot x + (3 \cdot b \cdot c \cdot x^5 \cdot (A \cdot c + B \cdot b))/5$

**sympy** [A] time = 0.09, size = 76, normalized size = 1.09

$$Ab^3x + \frac{Bc^3x^9}{9} + x^7 \left( \frac{Ac^3}{7} + \frac{3Bbc^2}{7} \right) + x^5 \left( \frac{3Abc^2}{5} + \frac{3Bb^2c}{5} \right) + x^3 \left( Ab^2c + \frac{Bb^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*6,x)

[Out]  $A \cdot b^3 \cdot x + B \cdot c^3 \cdot x^9/9 + x^7 \cdot (A \cdot c^3/7 + 3 \cdot B \cdot b \cdot c^2/7) + x^5 \cdot (3 \cdot A \cdot b \cdot c^2/5 + 3 \cdot B \cdot b^2 \cdot c/5) + x^3 \cdot (A \cdot b^2 \cdot c + B \cdot b^3/3)$

$$3.30 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7} dx$$

**Optimal.** Leaf size=60

$$Ab^3 \log(x) + \frac{3}{2}Ab^2cx^2 + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c}$$

**Rubi [A]** time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 446, 80, 43}

$$\frac{3}{2}Ab^2cx^2 + Ab^3 \log(x) + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^7,x]

[Out] (3\*A\*b^2\*c\*x^2)/2 + (3\*A\*b\*c^2\*x^4)/4 + (A\*c^3\*x^6)/6 + (B\*(b + c\*x^2)^4)/(8\*c) + A\*b^3\*Log[x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 80

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^7} dx &= \int \frac{(A + Bx^2)(b + cx^2)^3}{x} dx \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)^3}{x} dx, x, x^2 \right) \\
&= \frac{B(b + cx^2)^4}{8c} + \frac{1}{2} A \text{Subst} \left( \int \frac{(b + cx)^3}{x} dx, x, x^2 \right) \\
&= \frac{B(b + cx^2)^4}{8c} + \frac{1}{2} A \text{Subst} \left( \int \left( 3b^2c + \frac{b^3}{x} + 3bc^2x + c^3x^2 \right) dx, x, x^2 \right) \\
&= \frac{3}{2} Ab^2cx^2 + \frac{3}{4} Abc^2x^4 + \frac{1}{6} Ac^3x^6 + \frac{B(b + cx^2)^4}{8c} + Ab^3 \log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 71, normalized size = 1.18

$$Ab^3 \log(x) + \frac{1}{2} b^2 x^2 (3Ac + bB) + \frac{1}{6} c^2 x^6 (Ac + 3bB) + \frac{3}{4} bcx^4 (Ac + bB) + \frac{1}{8} Bc^3 x^8$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^7, x]

[Out] (b^2\*(b\*B + 3\*A\*c)\*x^2)/2 + (3\*b\*c\*(b\*B + A\*c)\*x^4)/4 + (c^2\*(3\*b\*B + A\*c)\*x^6)/6 + (B\*c^3\*x^8)/8 + A\*b^3\*Log[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^7, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^7, x]

**fricas [A]** time = 0.38, size = 71, normalized size = 1.18

$$\frac{1}{8} Bc^3x^8 + \frac{1}{6} (3Bbc^2 + Ac^3)x^6 + \frac{3}{4} (Bb^2c + Abc^2)x^4 + Ab^3 \log(x) + \frac{1}{2} (Bb^3 + 3Ab^2c)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^7,x, algorithm="fricas")

[Out] 1/8\*B\*c^3\*x^8 + 1/6\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 3/4\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + A\*b^3\*log(x) + 1/2\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2

**giac [A]** time = 0.15, size = 78, normalized size = 1.30

$$\frac{1}{8} Bc^3x^8 + \frac{1}{2} Bbc^2x^6 + \frac{1}{6} Ac^3x^6 + \frac{3}{4} Bb^2cx^4 + \frac{3}{4} Abc^2x^4 + \frac{1}{2} Bb^3x^2 + \frac{3}{2} Ab^2cx^2 + \frac{1}{2} Ab^3 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^7,x, algorithm="giac")

[Out] 1/8\*B\*c^3\*x^8 + 1/2\*B\*b\*c^2\*x^6 + 1/6\*A\*c^3\*x^6 + 3/4\*B\*b^2\*c\*x^4 + 3/4\*A\*b\*c^2\*x^4 + 1/2\*B\*b^3\*x^2 + 3/2\*A\*b^2\*c\*x^2 + 1/2\*A\*b^3\*log(x^2)

**maple [A]** time = 0.04, size = 76, normalized size = 1.27

$$\frac{Bc^3x^8}{8} + \frac{Ac^3x^6}{6} + \frac{Bbc^2x^6}{2} + \frac{3Abc^2x^4}{4} + \frac{3Bb^2cx^4}{4} + \frac{3Ab^2cx^2}{2} + \frac{Bb^3x^2}{2} + Ab^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^7,x)

[Out] 1/8\*B\*c^3\*x^8+1/6\*A\*c^3\*x^6+1/2\*B\*x^6\*b\*c^2+3/4\*A\*b\*c^2\*x^4+3/4\*B\*x^4\*b^2\*c+3/2\*A\*b^2\*c\*x^2+1/2\*B\*x^2\*b^3+A\*b^3\*ln(x)

**maxima [A]** time = 1.31, size = 74, normalized size = 1.23

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}(3Bbc^2 + Ac^3)x^6 + \frac{3}{4}(Bb^2c + Abc^2)x^4 + \frac{1}{2}Ab^3 \log(x^2) + \frac{1}{2}(Bb^3 + 3Ab^2c)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^7,x, algorithm="maxima")

[Out] 1/8\*B\*c^3\*x^8 + 1/6\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 3/4\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + 1/2\*A\*b^3\*log(x^2) + 1/2\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2

**mupad [B]** time = 0.03, size = 67, normalized size = 1.12

$$x^2 \left( \frac{Bb^3}{2} + \frac{3Ac^3}{2} \right) + x^6 \left( \frac{Ac^3}{6} + \frac{Bbc^2}{2} \right) + \frac{Bc^3x^8}{8} + Ab^3 \ln(x) + \frac{3bcx^4(Ac + Bb)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^7,x)

[Out] x^2\*((B\*b^3)/2 + (3\*A\*b^2\*c)/2) + x^6\*((A\*c^3)/6 + (B\*b\*c^2)/2) + (B\*c^3\*x^8)/8 + A\*b^3\*log(x) + (3\*b\*c\*x^4\*(A\*c + B\*b))/4

**sympy [A]** time = 0.18, size = 80, normalized size = 1.33

$$Ab^3 \log(x) + \frac{Bc^3x^8}{8} + x^6 \left( \frac{Ac^3}{6} + \frac{Bbc^2}{2} \right) + x^4 \left( \frac{3Abc^2}{4} + \frac{3Bb^2c}{4} \right) + x^2 \left( \frac{3Ab^2c}{2} + \frac{Bb^3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*7,x)

[Out] A\*b\*\*3\*log(x) + B\*c\*\*3\*x\*\*8/8 + x\*\*6\*(A\*c\*\*3/6 + B\*b\*c\*\*2/2) + x\*\*4\*(3\*A\*b\*c\*\*2/4 + 3\*B\*b\*\*2\*c/4) + x\*\*2\*(3\*A\*b\*\*2\*c/2 + B\*b\*\*3/2)

$$3.31 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} dx$$

**Optimal.** Leaf size=65

$$-\frac{Ab^3}{x} + b^2x(3Ac + bB) + \frac{1}{5}c^2x^5(Ac + 3bB) + bcx^3(Ac + bB) + \frac{1}{7}Bc^3x^7$$

**Rubi [A]** time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$b^2x(3Ac + bB) - \frac{Ab^3}{x} + \frac{1}{5}c^2x^5(Ac + 3bB) + bcx^3(Ac + bB) + \frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^8,x]

[Out] -((A\*b^3)/x) + b^2\*(b\*B + 3\*A\*c)\*x + b\*c\*(b\*B + A\*c)\*x^3 + (c^2\*(3\*b\*B + A\*c)\*x^5)/5 + (B\*c^3\*x^7)/7

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^2} dx \\ &= \int \left( b^2(bB + 3Ac) + \frac{Ab^3}{x^2} + 3bc(bB + Ac)x^2 + c^2(3bB + Ac)x^4 + Bc^3x^6 \right) dx \\ &= -\frac{Ab^3}{x} + b^2(bB + 3Ac)x + bc(bB + Ac)x^3 + \frac{1}{5}c^2(3bB + Ac)x^5 + \frac{1}{7}Bc^3x^7 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 65, normalized size = 1.00

$$-\frac{Ab^3}{x} + b^2x(3Ac + bB) + \frac{1}{5}c^2x^5(Ac + 3bB) + bcx^3(Ac + bB) + \frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^8,x]

[Out] -((A\*b^3)/x) + b^2\*(b\*B + 3\*A\*c)\*x + b\*c\*(b\*B + A\*c)\*x^3 + (c^2\*(3\*b\*B + A\*c)\*x^5)/5 + (B\*c^3\*x^7)/7

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^8,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^8, x]

**fricas** [A] time = 0.39, size = 75, normalized size = 1.15

$$\frac{5 Bc^3x^8 + 7(3 Bbc^2 + Ac^3)x^6 + 35(Bb^2c + Abc^2)x^4 - 35 Ab^3 + 35(Bb^3 + 3 Ab^2c)x^2}{35x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^8,x, algorithm="fricas")

[Out] 1/35\*(5\*B\*c^3\*x^8 + 7\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 35\*(B\*b^2\*c + A\*b\*c^2)\*x^4 - 35\*A\*b^3 + 35\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x

**giac** [A] time = 0.16, size = 70, normalized size = 1.08

$$\frac{1}{7} Bc^3x^7 + \frac{3}{5} Bbc^2x^5 + \frac{1}{5} Ac^3x^5 + Bb^2cx^3 + Abc^2x^3 + Bb^3x + 3 Ab^2cx - \frac{Ab^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^8,x, algorithm="giac")

[Out] 1/7\*B\*c^3\*x^7 + 3/5\*B\*b\*c^2\*x^5 + 1/5\*A\*c^3\*x^5 + B\*b^2\*c\*x^3 + A\*b\*c^2\*x^3 + B\*b^3\*x + 3\*A\*b^2\*c\*x - A\*b^3/x

**maple** [A] time = 0.05, size = 71, normalized size = 1.09

$$\frac{Bc^3x^7}{7} + \frac{Ac^3x^5}{5} + \frac{3Bbc^2x^5}{5} + Abc^2x^3 + Bb^2cx^3 + 3Ab^2cx + Bb^3x - \frac{Ab^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^8,x)

[Out] 1/7\*B\*c^3\*x^7+1/5\*A\*x^5\*c^3+3/5\*B\*x^5\*b\*c^2+A\*x^3\*b\*c^2+B\*x^3\*b^2\*c+3\*A\*c\*b^2\*x+B\*b^3\*x-A\*b^3/x

**maxima** [A] time = 1.34, size = 69, normalized size = 1.06

$$\frac{1}{7} Bc^3x^7 + \frac{1}{5} (3 Bbc^2 + Ac^3)x^5 + (Bb^2c + Abc^2)x^3 - \frac{Ab^3}{x} + (Bb^3 + 3 Ab^2c)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^8,x, algorithm="maxima")

[Out] 1/7\*B\*c^3\*x^7 + 1/5\*(3\*B\*b\*c^2 + A\*c^3)\*x^5 + (B\*b^2\*c + A\*b\*c^2)\*x^3 - A\*b^3/x + (B\*b^3 + 3\*A\*b^2\*c)\*x

**mupad** [B] time = 0.03, size = 65, normalized size = 1.00

$$x (Bb^3 + 3 A c b^2) + x^5 \left( \frac{A c^3}{5} + \frac{3 B b c^2}{5} \right) - \frac{A b^3}{x} + \frac{B c^3 x^7}{7} + b c x^3 (A c + B b)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x)`

[Out]  $x*(B*b^3 + 3*A*b^2*c) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) - (A*b^3)/x + (B*c^3*x^7)/7 + b*c*x^3*(A*c + B*b)$

sympy [A] time = 0.17, size = 68, normalized size = 1.05

$$-\frac{Ab^3}{x} + \frac{Bc^3x^7}{7} + x^5\left(\frac{Ac^3}{5} + \frac{3Bbc^2}{5}\right) + x^3(Abc^2 + Bb^2c) + x(3Ab^2c + Bb^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**8,x)`

[Out]  $-A*b**3/x + B*c**3*x**7/7 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**3*(A*b*c**2 + B*b**2*c) + x*(3*A*b**2*c + B*b**3)$

$$3.32 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx$$

**Optimal.** Leaf size=71

$$-\frac{Ab^3}{2x^2} + b^2 \log(x)(3Ac + bB) + \frac{1}{4}c^2x^4(Ac + 3bB) + \frac{3}{2}bcx^2(Ac + bB) + \frac{1}{6}Bc^3x^6$$

**Rubi [A]** time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$b^2 \log(x)(3Ac + bB) - \frac{Ab^3}{2x^2} + \frac{1}{4}c^2x^4(Ac + 3bB) + \frac{3}{2}bcx^2(Ac + bB) + \frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^9,x]

[Out] -(A\*b^3)/(2\*x^2) + (3\*b\*c\*(b\*B + A\*c)\*x^2)/2 + (c^2\*(3\*b\*B + A\*c)\*x^4)/4 + (B\*c^3\*x^6)/6 + b^2\*(b\*B + 3\*A\*c)\*Log[x]

### Rule 76

Int[((d\_)\*(x\_)^(n\_))\*((a\_) + (b\_)\*(x\_)^n)\*((e\_) + (f\_)\*(x\_)^p), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

### Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^n)^p\*((c\_) + (d\_)\*(x\_)^n)^q, x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^n, x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A+Bx)(b+cx)^3}{x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( 3bc(bB+Ac) + \frac{Ab^3}{x^2} + \frac{b^2(bB+3Ac)}{x} + c^2(3bB+Ac)x + Bc^3x^2 \right) dx, x, x^2 \right) \\ &= -\frac{Ab^3}{2x^2} + \frac{3}{2}bc(bB+Ac)x^2 + \frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{6}Bc^3x^6 + b^2(bB+3Ac) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 73, normalized size = 1.03

$$-\frac{Ab^3}{2x^2} + \log(x) (3Ab^2c + b^3B) + \frac{1}{4}c^2x^4(Ac + 3bB) + \frac{3}{2}bcx^2(Ac + bB) + \frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^9, x]

[Out]  $-1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 + (B*c^3*x^6)/6 + (b^3*B + 3*A*b^2*c)*\text{Log}[x]$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^9} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^9, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^9, x]

**fricas [A]** time = 0.39, size = 77, normalized size = 1.08

$$\frac{2 Bc^3x^8 + 3(3Bbc^2 + Ac^3)x^6 + 18(Bb^2c + Abc^2)x^4 - 6Ab^3 + 12(Bb^3 + 3Ab^2c)x^2 \log(x)}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^9, x, algorithm="fricas")

[Out]  $1/12*(2*B*c^3*x^8 + 3*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c^2)*x^4 - 6*A*b^3 + 12*(B*b^3 + 3*A*b^2*c)*x^2*\log(x))/x^2$

**giac [A]** time = 0.17, size = 97, normalized size = 1.37

$$\frac{1}{6} Bc^3x^6 + \frac{3}{4} Bbc^2x^4 + \frac{1}{4} Ac^3x^4 + \frac{3}{2} Bb^2cx^2 + \frac{3}{2} Abc^2x^2 + \frac{1}{2} (Bb^3 + 3Ab^2c) \log(x^2) - \frac{Bb^3x^2 + 3Ab^2cx^2 + Ab^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^9, x, algorithm="giac")

[Out]  $1/6*B*c^3*x^6 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + 3/2*B*b^2*c*x^2 + 3/2*A*b*c^2*x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*\log(x^2) - 1/2*(B*b^3*x^2 + 3*A*b^2*c*x^2 + A*b^3)/x^2$

**maple [A]** time = 0.05, size = 75, normalized size = 1.06

$$\frac{Bc^3x^6}{6} + \frac{Ac^3x^4}{4} + \frac{3Bbc^2x^4}{4} + \frac{3Abc^2x^2}{2} + \frac{3Bb^2cx^2}{2} + 3Ab^2c \ln(x) + Bb^3 \ln(x) - \frac{Ab^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^9, x)

[Out]  $1/6*B*c^3*x^6 + 1/4*A*x^4*c^3 + 3/4*B*x^4*b*c^2 + 3/2*A*x^2*b*c^2 + 3/2*B*x^2*b^2*c - 1/2*A*b^3/x^2 + 3*A*\ln(x)*b^2*c + B*\ln(x)*b^3$

**maxima [A]** time = 1.30, size = 74, normalized size = 1.04

$$\frac{1}{6} Bc^3x^6 + \frac{1}{4} (3Bbc^2 + Ac^3)x^4 + \frac{3}{2} (Bb^2c + Abc^2)x^2 - \frac{Ab^3}{2x^2} + \frac{1}{2} (Bb^3 + 3Ab^2c) \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^9, x, algorithm="maxima")

[Out]  $\frac{1}{6}B^3c^3x^6 + \frac{1}{4}(3B^2bc^2 + A^3c^3)x^4 + \frac{3}{2}(B^2b^2c + A^2b^2c^2)x^2 - \frac{1}{2}A^3b^3/x^2 + \frac{1}{2}(B^3b^3 + 3A^2b^2c)\log(x^2)$

**mupad [B]** time = 0.04, size = 67, normalized size = 0.94

$$x^4 \left( \frac{Ac^3}{4} + \frac{3Bbc^2}{4} \right) + \ln(x) (Bb^3 + 3Ac^2b^2) - \frac{Ab^3}{2x^2} + \frac{Bc^3x^6}{6} + \frac{3bcx^2(Ac + Bb)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x)`

[Out]  $x^4*((Ac^3)/4 + (3*B*b*c^2)/4) + \log(x)*(B*b^3 + 3*A*b^2*c) - (A*b^3)/(2*x^2) + (B*c^3*x^6)/6 + (3*b*c*x^2*(A*c + B*b))/2$

**sympy [A]** time = 0.27, size = 78, normalized size = 1.10

$$-\frac{Ab^3}{2x^2} + \frac{Bc^3x^6}{6} + b^2(3Ac + Bb)\log(x) + x^4\left(\frac{Ac^3}{4} + \frac{3Bbc^2}{4}\right) + x^2\left(\frac{3Abc^2}{2} + \frac{3Bb^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**9,x)`

[Out]  $-A*b**3/(2*x**2) + B*c**3*x**6/6 + b**2*(3*A*c + B*b)*\log(x) + x**4*(A*c**3/4 + 3*B*b*c**2/4) + x**2*(3*A*b*c**2/2 + 3*B*b**2*c/2)$

$$3.33 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} dx$$

**Optimal.** Leaf size=69

$$-\frac{Ab^3}{3x^3} - \frac{b^2(3Ac + bB)}{x} + \frac{1}{3}c^2x^3(Ac + 3bB) + 3bcx(Ac + bB) + \frac{1}{5}Bc^3x^5$$

**Rubi [A]** time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{b^2(3Ac + bB)}{x} - \frac{Ab^3}{3x^3} + \frac{1}{3}c^2x^3(Ac + 3bB) + 3bcx(Ac + bB) + \frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^10,x]

[Out] -(A\*b^3)/(3\*x^3) - (b^2\*(b\*B + 3\*A\*c))/x + 3\*b\*c\*(b\*B + A\*c)\*x + (c^2\*(3\*b\*B + A\*c)\*x^3)/3 + (B\*c^3\*x^5)/5

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^4} dx \\ &= \int \left( 3bc(bB + Ac) + \frac{Ab^3}{x^4} + \frac{b^2(bB + 3Ac)}{x^2} + c^2(3bB + Ac)x^2 + Bc^3x^4 \right) dx \\ &= -\frac{Ab^3}{3x^3} - \frac{b^2(bB + 3Ac)}{x} + 3bc(bB + Ac)x + \frac{1}{3}c^2(3bB + Ac)x^3 + \frac{1}{5}Bc^3x^5 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 71, normalized size = 1.03

$$-\frac{Ab^3}{3x^3} + \frac{b^3(-B) - 3Ab^2c}{x} + \frac{1}{3}c^2x^3(Ac + 3bB) + 3bcx(Ac + bB) + \frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^10,x]

[Out] -1/3\*(A\*b^3)/x^3 + (-b^3\*B) - 3\*A\*b^2\*c)/x + 3\*b\*c\*(b\*B + A\*c)\*x + (c^2\*(3\*b\*B + A\*c)\*x^3)/3 + (B\*c^3\*x^5)/5

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{10}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^10,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^10, x]

**fricas** [A] time = 0.37, size = 75, normalized size = 1.09

$$\frac{3 Bc^3x^8 + 5(3 Bbc^2 + Ac^3)x^6 + 45(Bb^2c + Abc^2)x^4 - 5 Ab^3 - 15(Bb^3 + 3 Ab^2c)x^2}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^10,x, algorithm="fricas")

[Out] 1/15\*(3\*B\*c^3\*x^8 + 5\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 45\*(B\*b^2\*c + A\*b\*c^2)\*x^4 - 5\*A\*b^3 - 15\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^3

**giac** [A] time = 0.15, size = 74, normalized size = 1.07

$$\frac{1}{5} Bc^3x^5 + Bbc^2x^3 + \frac{1}{3} Ac^3x^3 + 3 Bb^2cx + 3 Abc^2x - \frac{3 Bb^3x^2 + 9 Ab^2cx^2 + Ab^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^10,x, algorithm="giac")

[Out] 1/5\*B\*c^3\*x^5 + B\*b\*c^2\*x^3 + 1/3\*A\*c^3\*x^3 + 3\*B\*b^2\*c\*x + 3\*A\*b\*c^2\*x - 1/3\*(3\*B\*b^3\*x^2 + 9\*A\*b^2\*c\*x^2 + A\*b^3)/x^3

**maple** [A] time = 0.05, size = 70, normalized size = 1.01

$$\frac{Bc^3x^5}{5} + \frac{Ac^3x^3}{3} + Bbc^2x^3 + 3Abc^2x + 3Bb^2cx - \frac{Ab^3}{3x^3} - \frac{(3Ac + bB)b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^10,x)

[Out] 1/5\*B\*c^3\*x^5+1/3\*A\*x^3\*c^3+B\*x^3\*b\*c^2+3\*A\*b\*c^2\*x+3\*B\*c\*b^2\*x-1/3\*A\*b^3/x^3-b^2\*(3\*A\*c+B\*b)/x

**maxima** [A] time = 1.33, size = 73, normalized size = 1.06

$$\frac{1}{5} Bc^3x^5 + \frac{1}{3} (3 Bbc^2 + Ac^3)x^3 + 3 (Bb^2c + Abc^2)x - \frac{Ab^3 + 3 (Bb^3 + 3 Ab^2c)x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^10,x, algorithm="maxima")

[Out] 1/5\*B\*c^3\*x^5 + 1/3\*(3\*B\*b\*c^2 + A\*c^3)\*x^3 + 3\*(B\*b^2\*c + A\*b\*c^2)\*x - 1/3\*(A\*b^3 + 3\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^3

**mupad** [B] time = 0.06, size = 68, normalized size = 0.99

$$x^3 \left( \frac{Ac^3}{3} + Bbc^2 \right) - \frac{\frac{Ab^3}{3} + x^2 (Bb^3 + 3Ac^2b^2)}{x^3} + \frac{Bc^3x^5}{5} + 3bcx(Ac + Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x)`

[Out]  $x^3*((A*c^3)/3 + B*b*c^2) - ((A*b^3)/3 + x^2*(B*b^3 + 3*A*b^2*c))/x^3 + (B*c^3*x^5)/5 + 3*b*c*x*(A*c + B*b)$

**sympy [A]** time = 0.29, size = 75, normalized size = 1.09

$$\frac{Bc^3x^5}{5} + x^3\left(\frac{Ac^3}{3} + Bbc^2\right) + x(3Abc^2 + 3Bb^2c) + \frac{-Ab^3 + x^2(-9Ab^2c - 3Bb^3)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**10,x)`

[Out]  $B*c**3*x**5/5 + x**3*(A*c**3/3 + B*b*c**2) + x*(3*A*b*c**2 + 3*B*b**2*c) + (-A*b**3 + x**2*(-9*A*b**2*c - 3*B*b**3))/(3*x**3)$

$$3.34 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} dx$$

**Optimal.** Leaf size=72

$$-\frac{Ab^3}{4x^4} - \frac{b^2(3Ac + bB)}{2x^2} + \frac{1}{2}c^2x^2(Ac + 3bB) + 3bc \log(x)(Ac + bB) + \frac{1}{4}Bc^3x^4$$

**Rubi [A]** time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$-\frac{b^2(3Ac + bB)}{2x^2} - \frac{Ab^3}{4x^4} + \frac{1}{2}c^2x^2(Ac + 3bB) + 3bc \log(x)(Ac + bB) + \frac{1}{4}Bc^3x^4$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^11,x]

[Out] -(A\*b^3)/(4\*x^4) - (b^2\*(b\*B + 3\*A\*c))/(2\*x^2) + (c^2\*(3\*b\*B + A\*c)\*x^2)/2 + (B\*c^3\*x^4)/4 + 3\*b\*c\*(b\*B + A\*c)\*Log[x]

#### Rule 76

Int[((d\_.)\*(x\_))^(n\_.)\*((a\_) + (b\_.)\*(x\_))\*((e\_) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{11}} dx &= \int \frac{(A + Bx^2)(b + cx^2)^3}{x^5} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)^3}{x^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( c^2(3bB + Ac) + \frac{Ab^3}{x^3} + \frac{b^2(bB + 3Ac)}{x^2} + \frac{3bc(bB + Ac)}{x} + Bc^3x \right) dx, x, x^2 \right) \\ &= -\frac{Ab^3}{4x^4} - \frac{b^2(bB + 3Ac)}{2x^2} + \frac{1}{2}c^2(3bB + Ac)x^2 + \frac{1}{4}Bc^3x^4 + 3bc(bB + Ac) \log(x) \end{aligned}$$



**Mathematica [A]** time = 0.03, size = 73, normalized size = 1.01

$$\frac{Bx^2(-2b^3 + 6bc^2x^4 + c^3x^6) - A(b^3 + 6b^2cx^2 - 2c^3x^6)}{4x^4} + 3bc \log(x)(Ac + bB)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^11,x]

[Out] (-A\*(b^3 + 6\*b^2\*c\*x^2 - 2\*c^3\*x^6)) + B\*x^2\*(-2\*b^3 + 6\*b\*c^2\*x^4 + c^3\*x^6))/(4\*x^4) + 3\*b\*c\*(b\*B + A\*c)\*Log[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{11}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^11,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^11, x]

**fricas [A]** time = 0.39, size = 76, normalized size = 1.06

$$\frac{Bc^3x^8 + 2(3Bbc^2 + Ac^3)x^6 + 12(Bb^2c + Abc^2)x^4 \log(x) - Ab^3 - 2(Bb^3 + 3Ab^2c)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^11,x, algorithm="fricas")

[Out] 1/4\*(B\*c^3\*x^8 + 2\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 12\*(B\*b^2\*c + A\*b\*c^2)\*x^4\*log(x) - A\*b^3 - 2\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^4

**giac [A]** time = 0.15, size = 98, normalized size = 1.36

$$\frac{1}{4}Bc^3x^4 + \frac{3}{2}Bbc^2x^2 + \frac{1}{2}Ac^3x^2 + \frac{3}{2}(Bb^2c + Abc^2)\log(x^2) - \frac{9Bb^2cx^4 + 9Abc^2x^4 + 2Bb^3x^2 + 6Ab^2cx^2 + Ab^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^11,x, algorithm="giac")

[Out] 1/4\*B\*c^3\*x^4 + 3/2\*B\*b\*c^2\*x^2 + 1/2\*A\*c^3\*x^2 + 3/2\*(B\*b^2\*c + A\*b\*c^2)\*log(x^2) - 1/4\*(9\*B\*b^2\*c\*x^4 + 9\*A\*b\*c^2\*x^4 + 2\*B\*b^3\*x^2 + 6\*A\*b^2\*c\*x^2 + A\*b^3)/x^4

**maple [A]** time = 0.05, size = 76, normalized size = 1.06

$$\frac{Bc^3x^4}{4} + \frac{Ac^3x^2}{2} + \frac{3Bbc^2x^2}{2} + 3Abc^2 \ln(x) + 3Bb^2c \ln(x) - \frac{3Ab^2c}{2x^2} - \frac{Bb^3}{2x^2} - \frac{Ab^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^11,x)

[Out] 1/4\*B\*c^3\*x^4+1/2\*A\*x^2\*c^3+3/2\*B\*x^2\*b\*c^2-1/4\*A\*b^3/x^4-3/2\*b^2/x^2\*A\*c-1/2\*b^3/x^2\*B+3\*A\*ln(x)\*b\*c^2+3\*B\*ln(x)\*b^2\*c

**maxima [A]** time = 1.28, size = 76, normalized size = 1.06

$$\frac{1}{4}Bc^3x^4 + \frac{1}{2}(3Bbc^2 + Ac^3)x^2 + \frac{3}{2}(Bb^2c + Abc^2)\log(x^2) - \frac{Ab^3 + 2(Bb^3 + 3Ab^2c)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^11,x, algorithm="maxima")

[Out] 1/4\*B\*c^3\*x^4 + 1/2\*(3\*B\*b\*c^2 + A\*c^3)\*x^2 + 3/2\*(B\*b^2\*c + A\*b\*c^2)\*log(x^2) - 1/4\*(A\*b^3 + 2\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^4

mupad [B] time = 0.07, size = 76, normalized size = 1.06

$$\ln(x) (3 B b^2 c + 3 A b c^2) - \frac{\frac{A b^3}{4} + x^2 \left( \frac{B b^3}{2} + \frac{3 A c b^2}{2} \right)}{x^4} + x^2 \left( \frac{A c^3}{2} + \frac{3 B b c^2}{2} \right) + \frac{B c^3 x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^11,x)

[Out] log(x)\*(3\*A\*b\*c^2 + 3\*B\*b^2\*c) - ((A\*b^3)/4 + x^2\*((B\*b^3)/2 + (3\*A\*b^2\*c)/2))/x^4 + x^2\*((A\*c^3)/2 + (3\*B\*b\*c^2)/2) + (B\*c^3\*x^4)/4

sympy [A] time = 0.59, size = 75, normalized size = 1.04

$$\frac{B c^3 x^4}{4} + 3 b c (A c + B b) \log(x) + x^2 \left( \frac{A c^3}{2} + \frac{3 B b c^2}{2} \right) + \frac{-A b^3 + x^2 (-6 A b^2 c - 2 B b^3)}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*11,x)

[Out] B\*c\*\*3\*x\*\*4/4 + 3\*b\*c\*(A\*c + B\*b)\*log(x) + x\*\*2\*(A\*c\*\*3/2 + 3\*B\*b\*c\*\*2/2) + (-A\*b\*\*3 + x\*\*2\*(-6\*A\*b\*\*2\*c - 2\*B\*b\*\*3))/(4\*x\*\*4)

$$3.35 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} dx$$

**Optimal.** Leaf size=68

$$-\frac{Ab^3}{5x^5} - \frac{b^2(3Ac + bB)}{3x^3} + c^2x(Ac + 3bB) - \frac{3bc(Ac + bB)}{x} + \frac{1}{3}Bc^3x^3$$

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{b^2(3Ac + bB)}{3x^3} - \frac{Ab^3}{5x^5} + c^2x(Ac + 3bB) - \frac{3bc(Ac + bB)}{x} + \frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^12,x]

[Out] -(A\*b^3)/(5\*x^5) - (b^2\*(b\*B + 3\*A\*c))/(3\*x^3) - (3\*b\*c\*(b\*B + A\*c))/x + c^2\*(3\*b\*B + A\*c)\*x + (B\*c^3\*x^3)/3

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^6} dx \\ &= \int \left( c^2(3bB + Ac) + \frac{Ab^3}{x^6} + \frac{b^2(bB + 3Ac)}{x^4} + \frac{3bc(bB + Ac)}{x^2} + Bc^3x^2 \right) dx \\ &= -\frac{Ab^3}{5x^5} - \frac{b^2(bB + 3Ac)}{3x^3} - \frac{3bc(bB + Ac)}{x} + c^2(3bB + Ac)x + \frac{1}{3}Bc^3x^3 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 68, normalized size = 1.00

$$-\frac{Ab^3}{5x^5} - \frac{b^2(3Ac + bB)}{3x^3} + c^2x(Ac + 3bB) - \frac{3bc(Ac + bB)}{x} + \frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^12,x]

[Out] -1/5\*(A\*b^3)/x^5 - (b^2\*(b\*B + 3\*A\*c))/(3\*x^3) - (3\*b\*c\*(b\*B + A\*c))/x + c^2\*(3\*b\*B + A\*c)\*x + (B\*c^3\*x^3)/3

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{12}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^12,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^12, x]

**fricas** [A] time = 0.38, size = 75, normalized size = 1.10

$$\frac{5Bc^3x^8 + 15(3Bbc^2 + Ac^3)x^6 - 45(Bb^2c + Abc^2)x^4 - 3Ab^3 - 5(Bb^3 + 3Ab^2c)x^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^12,x, algorithm="fricas")

[Out] 1/15\*(5\*B\*c^3\*x^8 + 15\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 - 45\*(B\*b^2\*c + A\*b\*c^2)\*x^4 - 3\*A\*b^3 - 5\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^5

**giac** [A] time = 0.16, size = 75, normalized size = 1.10

$$\frac{1}{3}Bc^3x^3 + 3Bbc^2x + Ac^3x - \frac{45Bb^2cx^4 + 45Abc^2x^4 + 5Bb^3x^2 + 15Ab^2cx^2 + 3Ab^3}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^12,x, algorithm="giac")

[Out] 1/3\*B\*c^3\*x^3 + 3\*B\*b\*c^2\*x + A\*c^3\*x - 1/15\*(45\*B\*b^2\*c\*x^4 + 45\*A\*b\*c^2\*x^4 + 5\*B\*b^3\*x^2 + 15\*A\*b^2\*c\*x^2 + 3\*A\*b^3)/x^5

**maple** [A] time = 0.05, size = 64, normalized size = 0.94

$$\frac{Bc^3x^3}{3} + Ac^3x + 3Bbc^2x - \frac{3(Ac + bB)bc}{x} - \frac{Ab^3}{5x^5} - \frac{(3Ac + bB)b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^12,x)

[Out] 1/3\*B\*c^3\*x^3+A\*c^3\*x+3\*B\*b\*c^2\*x-1/5\*A\*b^3/x^5-1/3\*b^2\*(3\*A\*c+B\*b)/x^3-3\*b\*c\*(A\*c+B\*b)/x

**maxima** [A] time = 1.33, size = 73, normalized size = 1.07

$$\frac{1}{3}Bc^3x^3 + (3Bbc^2 + Ac^3)x - \frac{45(Bb^2c + Abc^2)x^4 + 3Ab^3 + 5(Bb^3 + 3Ab^2c)x^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^12,x, algorithm="maxima")

[Out] 1/3\*B\*c^3\*x^3 + (3\*B\*b\*c^2 + A\*c^3)\*x - 1/15\*(45\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + 3\*A\*b^3 + 5\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^5

**mupad** [B] time = 0.06, size = 73, normalized size = 1.07

$$x(Ac^3 + 3Bbc^2) - \frac{x^4(3Bb^2c + 3Abc^2) + \frac{Ab^3}{5} + x^2\left(\frac{Bb^3}{3} + Acb^2\right)}{x^5} + \frac{Bc^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x)`

[Out]  $x*(A*c^3 + 3*B*b*c^2) - (x^4*(3*A*b*c^2 + 3*B*b^2*c) + (A*b^3)/5 + x^2*((B*b^3)/3 + A*b^2*c))/x^5 + (B*c^3*x^3)/3$

sympy [A] time = 0.68, size = 78, normalized size = 1.15

$$\frac{Bc^3x^3}{3} + x(Ac^3 + 3Bbc^2) + \frac{-3Ab^3 + x^4(-45Abc^2 - 45Bb^2c) + x^2(-15Ab^2c - 5Bb^3)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**12,x)`

[Out]  $B*c**3*x**3/3 + x*(A*c**3 + 3*B*b*c**2) + (-3*A*b**3 + x**4*(-45*A*b*c**2 - 45*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(15*x**5)$

$$3.36 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} dx$$

**Optimal.** Leaf size=71

$$-\frac{Ab^3}{6x^6} - \frac{b^2(3Ac + bB)}{4x^4} + c^2 \log(x)(Ac + 3bB) - \frac{3bc(Ac + bB)}{2x^2} + \frac{1}{2}Bc^3x^2$$

**Rubi [A]** time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 76}

$$-\frac{b^2(3Ac + bB)}{4x^4} - \frac{Ab^3}{6x^6} + c^2 \log(x)(Ac + 3bB) - \frac{3bc(Ac + bB)}{2x^2} + \frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^13,x]

[Out] -(A\*b^3)/(6\*x^6) - (b^2\*(b\*B + 3\*A\*c))/(4\*x^4) - (3\*b\*c\*(b\*B + A\*c))/(2\*x^2) + (B\*c^3\*x^2)/2 + c^2\*(3\*b\*B + A\*c)\*Log[x]

### Rule 76

Int[((d\_.)\*(x\_))^(n\_.)\*((a\_.) + (b\_.)\*(x\_))\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b\*e + a\*f, 0] && (!IntegerQ[n] || LtQ[9\*p + 5\*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

### Rule 446

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{13}} dx &= \int \frac{(A + Bx^2)(b + cx^2)^3}{x^7} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)^3}{x^4} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( Bc^3 + \frac{Ab^3}{x^4} + \frac{b^2(bB + 3Ac)}{x^3} + \frac{3bc(bB + Ac)}{x^2} + \frac{c^2(3bB + Ac)}{x} \right) dx, \right. \\ &= -\frac{Ab^3}{6x^6} - \frac{b^2(bB + 3Ac)}{4x^4} - \frac{3bc(bB + Ac)}{2x^2} + \frac{1}{2}Bc^3x^2 + c^2(3bB + Ac) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 71, normalized size = 1.00

$$-\frac{Ab^3}{6x^6} - \frac{b^2(3Ac + bB)}{4x^4} + c^2 \log(x)(Ac + 3bB) - \frac{3bc(Ac + bB)}{2x^2} + \frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^13,x]

[Out]  $-1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*\text{Log}[x]$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{13}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^13,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^13, x]

**fricas** [A] time = 0.40, size = 77, normalized size = 1.08

$$\frac{6Bc^3x^8 + 12(3Bbc^2 + Ac^3)x^6 \log(x) - 18(Bb^2c + Abc^2)x^4 - 2Ab^3 - 3(Bb^3 + 3Ab^2c)x^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^13,x, algorithm="fricas")

[Out]  $1/12*(6*B*c^3*x^8 + 12*(3*B*b*c^2 + A*c^3)*x^6*\log(x) - 18*(B*b^2*c + A*b*c^2)*x^4 - 2*A*b^3 - 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6$

**giac** [A] time = 0.15, size = 99, normalized size = 1.39

$$\frac{1}{2}Bc^3x^2 + \frac{1}{2}(3Bbc^2 + Ac^3)\log(x^2) - \frac{33Bbc^2x^6 + 11Ac^3x^6 + 18Bb^2cx^4 + 18Abc^2x^4 + 3Bb^3x^2 + 9Ab^2cx^2 + 2Ab^3}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^13,x, algorithm="giac")

[Out]  $1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*\log(x^2) - 1/12*(33*B*b*c^2*x^6 + 11*A*c^3*x^6 + 18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 3*B*b^3*x^2 + 9*A*b^2*c*x^2 + 2*A*b^3)/x^6$

**maple** [A] time = 0.05, size = 75, normalized size = 1.06

$$\frac{Bc^3x^2}{2} + Ac^3 \ln(x) + 3Bbc^2 \ln(x) - \frac{3Abc^2}{2x^2} - \frac{3Bb^2c}{2x^2} - \frac{3Ab^2c}{4x^4} - \frac{Bb^3}{4x^4} - \frac{Ab^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^13,x)

[Out]  $1/2*B*c^3*x^2 - 3/4*b^2/x^4*A*c - 1/4*b^3/x^4*B - 3/2*b*c^2/x^2*A - 3/2*b^2*c/x^2*B - 1/6*A*b^3/x^6 + A*\ln(x)*c^3 + 3*B*\ln(x)*b*c^2$

**maxima** [A] time = 1.44, size = 77, normalized size = 1.08

$$\frac{1}{2}Bc^3x^2 + \frac{1}{2}(3Bbc^2 + Ac^3)\log(x^2) - \frac{18(Bb^2c + Abc^2)x^4 + 2Ab^3 + 3(Bb^3 + 3Ab^2c)x^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^13,x, algorithm="maxima")

[Out]  $\frac{1}{2}Bc^3x^2 + \frac{1}{2}(3Bb^2c + Ac^3)\log(x^2) - \frac{1}{12}(18(Bb^2c + Ab^2c^2)x^4 + 2Ab^3 + 3(Bb^3 + 3Ab^2c)x^2)/x^6$

mupad [B] time = 0.09, size = 75, normalized size = 1.06

$$\ln(x) (Ac^3 + 3Bbc^2) - \frac{x^4 \left( \frac{3Bb^2c}{2} + \frac{3Ab^2c^2}{2} \right) + \frac{Ab^3}{6} + x^2 \left( \frac{Bb^3}{4} + \frac{3Ac^2b^2}{4} \right)}{x^6} + \frac{Bc^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x)`

[Out]  $\log(x)(Ac^3 + 3Bb^2c) - (x^4((3Ab^2c^2)/2 + (3Bb^2c)/2) + (Ab^3)/6 + x^2((Bb^3)/4 + (3Ab^2c)/4))/x^6 + (Bc^3x^2)/2$

sympy [A] time = 1.29, size = 78, normalized size = 1.10

$$\frac{Bc^3x^2}{2} + c^2(Ac + 3Bb)\log(x) + \frac{-2Ab^3 + x^4(-18Abc^2 - 18Bb^2c) + x^2(-9Ab^2c - 3Bb^3)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**13,x)`

[Out]  $Bc^3x^2/2 + c^2(Ac + 3Bb)\log(x) + (-2Ab^3 + x^4(-18Abc^2 - 18Bb^2c) + x^2(-9Ab^2c - 3Bb^3))/(12x^6)$



$$3.37 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx$$

**Optimal.** Leaf size=66

$$-\frac{Ab^3}{7x^7} - \frac{b^2(3Ac + bB)}{5x^5} - \frac{c^2(Ac + 3bB)}{x} - \frac{bc(Ac + bB)}{x^3} + Bc^3x$$

**Rubi [A]** time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{b^2(3Ac + bB)}{5x^5} - \frac{Ab^3}{7x^7} - \frac{c^2(Ac + 3bB)}{x} - \frac{bc(Ac + bB)}{x^3} + Bc^3x$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^14,x]

[Out] -(A\*b^3)/(7\*x^7) - (b^2\*(b\*B + 3\*A\*c))/(5\*x^5) - (b\*c\*(b\*B + A\*c))/x^3 - (c^2\*(3\*b\*B + A\*c))/x + B\*c^3\*x

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^8} dx \\ &= \int \left( Bc^3 + \frac{Ab^3}{x^8} + \frac{b^2(bB+3Ac)}{x^6} + \frac{3bc(bB+Ac)}{x^4} + \frac{c^2(3bB+Ac)}{x^2} \right) dx \\ &= -\frac{Ab^3}{7x^7} - \frac{b^2(bB+3Ac)}{5x^5} - \frac{bc(bB+Ac)}{x^3} - \frac{c^2(3bB+Ac)}{x} + Bc^3x \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 66, normalized size = 1.00

$$-\frac{Ab^3}{7x^7} - \frac{b^2(3Ac + bB)}{5x^5} - \frac{c^2(Ac + 3bB)}{x} - \frac{bc(Ac + bB)}{x^3} + Bc^3x$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^14,x]

[Out] -1/7\*(A\*b^3)/x^7 - (b^2\*(b\*B + 3\*A\*c))/(5\*x^5) - (b\*c\*(b\*B + A\*c))/x^3 - (c^2\*(3\*b\*B + A\*c))/x + B\*c^3\*x

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{14}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^14,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^14, x]

**fricas** [A] time = 0.39, size = 75, normalized size = 1.14

$$\frac{35 Bc^3x^8 - 35(3Bbc^2 + Ac^3)x^6 - 35(Bb^2c + Abc^2)x^4 - 5Ab^3 - 7(Bb^3 + 3Ab^2c)x^2}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^14,x, algorithm="fricas")

[Out] 1/35\*(35\*B\*c^3\*x^8 - 35\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 - 35\*(B\*b^2\*c + A\*b\*c^2)\*x^4 - 5\*A\*b^3 - 7\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^7

**giac** [A] time = 0.21, size = 77, normalized size = 1.17

$$Bc^3x - \frac{105Bbc^2x^6 + 35Ac^3x^6 + 35Bb^2cx^4 + 35Abc^2x^4 + 7Bb^3x^2 + 21Ab^2cx^2 + 5Ab^3}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^14,x, algorithm="giac")

[Out] B\*c^3\*x - 1/35\*(105\*B\*b\*c^2\*x^6 + 35\*A\*c^3\*x^6 + 35\*B\*b^2\*c\*x^4 + 35\*A\*b\*c^2\*x^4 + 7\*B\*b^3\*x^2 + 21\*A\*b^2\*c\*x^2 + 5\*A\*b^3)/x^7

**maple** [A] time = 0.05, size = 63, normalized size = 0.95

$$Bc^3x - \frac{(Ac + 3bB)c^2}{x} - \frac{(Ac + bB)bc}{x^3} - \frac{Ab^3}{7x^7} - \frac{(3Ac + bB)b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^14,x)

[Out] -1/7\*A\*b^3/x^7-1/5\*b^2\*(3\*A\*c+B\*b)/x^5-b\*c\*(A\*c+B\*b)/x^3-c^2\*(A\*c+3\*B\*b)/x+B\*c^3\*x

**maxima** [A] time = 1.30, size = 73, normalized size = 1.11

$$Bc^3x - \frac{35(3Bbc^2 + Ac^3)x^6 + 35(Bb^2c + Abc^2)x^4 + 5Ab^3 + 7(Bb^3 + 3Ab^2c)x^2}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^14,x, algorithm="maxima")

[Out] B\*c^3\*x - 1/35\*(35\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 35\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + 5\*A\*b^3 + 7\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^7

**mupad** [B] time = 0.08, size = 71, normalized size = 1.08

$$Bc^3x - \frac{x^4(Bb^2c + Abc^2) + \frac{Ab^3}{7} + x^2\left(\frac{Bb^3}{5} + \frac{3Ac b^2}{5}\right) + x^6(Ac^3 + 3Bbc^2)}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x)`

[Out]  $B*c^3*x - (x^4*(A*b*c^2 + B*b^2*c) + (A*b^3)/7 + x^2*((B*b^3)/5 + (3*A*b^2*c)/5) + x^6*(A*c^3 + 3*B*b*c^2))/x^7$

sympy [A] time = 1.50, size = 80, normalized size = 1.21

$$Bc^3x + \frac{-5Ab^3 + x^6(-35Ac^3 - 105Bbc^2) + x^4(-35Abc^2 - 35Bb^2c) + x^2(-21Ab^2c - 7Bb^3)}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**14,x)`

[Out]  $B*c**3*x + (-5*A*b**3 + x**6*(-35*A*c**3 - 105*B*b*c**2) + x**4*(-35*A*b*c**2 - 35*B*b**2*c) + x**2*(-21*A*b**2*c - 7*B*b**3))/(35*x**7)$

$$3.38 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} dx$$

**Optimal.** Leaf size=63

$$-\frac{A(b+cx^2)^4}{8bx^8} - \frac{b^3B}{6x^6} - \frac{3b^2Bc}{4x^4} - \frac{3bBc^2}{2x^2} + Bc^3 \log(x)$$

**Rubi [A]** time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 446, 78, 43}

$$-\frac{A(b+cx^2)^4}{8bx^8} - \frac{3b^2Bc}{4x^4} - \frac{b^3B}{6x^6} - \frac{3bBc^2}{2x^2} + Bc^3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^15,x]

[Out] -(b^3\*B)/(6\*x^6) - (3\*b^2\*B\*c)/(4\*x^4) - (3\*b\*B\*c^2)/(2\*x^2) - (A\*(b + c\*x^2)^4)/(8\*b\*x^8) + B\*c^3\*Log[x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{15}} dx &= \int \frac{(A + Bx^2)(b + cx^2)^3}{x^9} dx \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)^3}{x^5} dx, x, x^2 \right) \\
&= -\frac{A(b + cx^2)^4}{8bx^8} + \frac{1}{2} B \text{Subst} \left( \int \frac{(b + cx)^3}{x^4} dx, x, x^2 \right) \\
&= -\frac{A(b + cx^2)^4}{8bx^8} + \frac{1}{2} B \text{Subst} \left( \int \left( \frac{b^3}{x^4} + \frac{3b^2c}{x^3} + \frac{3bc^2}{x^2} + \frac{c^3}{x} \right) dx, x, x^2 \right) \\
&= \frac{b^3B}{6x^6} - \frac{3b^2Bc}{4x^4} - \frac{3bBc^2}{2x^2} - \frac{A(b + cx^2)^4}{8bx^8} + Bc^3 \log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 77, normalized size = 1.22

$$Bc^3 \log(x) - \frac{3A(b^3 + 4b^2cx^2 + 6bc^2x^4 + 4c^3x^6) + 2bBx^2(2b^2 + 9bcx^2 + 18c^2x^4)}{24x^8}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^15, x]

[Out] -1/24\*(2\*b\*B\*x^2\*(2\*b^2 + 9\*b\*c\*x^2 + 18\*c^2\*x^4) + 3\*A\*(b^3 + 4\*b^2\*c\*x^2 + 6\*b\*c^2\*x^4 + 4\*c^3\*x^6))/x^8 + B\*c^3\*Log[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{15}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^15, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^15, x]

**fricas [A]** time = 0.40, size = 77, normalized size = 1.22

$$\frac{24 Bc^3x^8 \log(x) - 12(3 Bbc^2 + Ac^3)x^6 - 18(Bb^2c + Abc^2)x^4 - 3Ab^3 - 4(Bb^3 + 3Ab^2c)x^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^15,x, algorithm="fricas")

[Out] 1/24\*(24\*B\*c^3\*x^8\*log(x) - 12\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 - 18\*(B\*b^2\*c + A\*b\*c^2)\*x^4 - 3\*A\*b^3 - 4\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^8

**giac [A]** time = 0.17, size = 90, normalized size = 1.43

$$\frac{1}{2} Bc^3 \log(x^2) - \frac{25 Bc^3x^8 + 36 Bbc^2x^6 + 12 Ac^3x^6 + 18 Bb^2cx^4 + 18 Abc^2x^4 + 4 Bb^3x^2 + 12 Ab^2cx^2 + 3 Ab^3}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^15,x, algorithm="giac")

[Out]  $\frac{1}{2}Bc^3 \log(x^2) - \frac{1}{24}(25Bc^3x^8 + 36B^2bc^2x^6 + 12Ac^3x^6 + 18B^2b^2cx^4 + 18Ab^2c^2x^4 + 4B^2b^3x^2 + 12Ab^2c^2x^2 + 3Ab^3)/x^8$

**maple** [A] time = 0.05, size = 76, normalized size = 1.21

$$Bc^3 \ln(x) - \frac{Ac^3}{2x^2} - \frac{3Bbc^2}{2x^2} - \frac{3Abc^2}{4x^4} - \frac{3Bb^2c}{4x^4} - \frac{Ab^2c}{2x^6} - \frac{Bb^3}{6x^6} - \frac{Ab^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x)`

[Out]  $-3/4*b*c^2/x^4*A-3/4*b^2*B*c/x^4-1/8*A*b^3/x^8-1/2*c^3/x^2*A-3/2*b*B*c^2/x^2-1/2*b^2/x^6*A*c-1/6*b^3*B/x^6+B*c^3*\ln(x)$

**maxima** [A] time = 1.32, size = 77, normalized size = 1.22

$$\frac{1}{2}Bc^3 \log(x^2) - \frac{12(3Bbc^2 + Ac^3)x^6 + 18(Bb^2c + Abc^2)x^4 + 3Ab^3 + 4(Bb^3 + 3Ab^2c)x^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")`

[Out]  $\frac{1}{2}Bc^3 \log(x^2) - \frac{1}{24}(12(3B^2bc^2 + Ac^3)x^6 + 18(B^2b^2c + Ab^2c^2)x^4 + 3Ab^3 + 4(B^2b^3 + 3Ab^2c)x^2)/x^8$

**mupad** [B] time = 0.10, size = 75, normalized size = 1.19

$$Bc^3 \ln(x) - \frac{x^4 \left( \frac{3Bb^2c}{4} + \frac{3Abc^2}{4} \right) + \frac{Ab^3}{8} + x^2 \left( \frac{Bb^3}{6} + \frac{Ac^2}{2} \right) + x^6 \left( \frac{Ac^3}{2} + \frac{3Bbc^2}{2} \right)}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x)`

[Out]  $Bc^3 \log(x) - (x^4*((3Ab^2c)/4 + (3B^2b^2c)/4) + (Ab^3)/8 + x^2*((B^2b^3)/6 + (Ab^2c)/2) + x^6*((Ac^3)/2 + (3B^2bc^2)/2))/x^8$

**sympy** [A] time = 2.37, size = 82, normalized size = 1.30

$$Bc^3 \log(x) + \frac{-3Ab^3 + x^6(-12Ac^3 - 36Bbc^2) + x^4(-18Abc^2 - 18Bb^2c) + x^2(-12Ab^2c - 4Bb^3)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**15,x)`

[Out]  $Bc**3*\log(x) + (-3Ab**3 + x**6*(-12Ac**3 - 36B^2bc**2) + x**4*(-18Ab*c**2 - 18B^2b**2*c) + x**2*(-12Ab**2*c - 4B^2b**3))/(24*x**8)$

$$3.39 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} dx$$

**Optimal.** Leaf size=73

$$-\frac{Ab^3}{9x^9} - \frac{b^2(3Ac + bB)}{7x^7} - \frac{c^2(Ac + 3bB)}{3x^3} - \frac{3bc(Ac + bB)}{5x^5} - \frac{Bc^3}{x}$$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$-\frac{b^2(3Ac + bB)}{7x^7} - \frac{Ab^3}{9x^9} - \frac{c^2(Ac + 3bB)}{3x^3} - \frac{3bc(Ac + bB)}{5x^5} - \frac{Bc^3}{x}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^16,x]

[Out] -(A\*b^3)/(9\*x^9) - (b^2\*(b\*B + 3\*A\*c))/(7\*x^7) - (3\*b\*c\*(b\*B + A\*c))/(5\*x^5) - (c^2\*(3\*b\*B + A\*c))/(3\*x^3) - (B\*c^3)/x

Rule 448

Int[((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_.)^(m\_.)\*((a\_.)\*(x\_.)^(p\_.) + (b\_.)\*(x\_.)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^{10}} dx \\ &= \int \left( \frac{Ab^3}{x^{10}} + \frac{b^2(bB+3Ac)}{x^8} + \frac{3bc(bB+Ac)}{x^6} + \frac{c^2(3bB+Ac)}{x^4} + \frac{Bc^3}{x^2} \right) dx \\ &= -\frac{Ab^3}{9x^9} - \frac{b^2(bB+3Ac)}{7x^7} - \frac{3bc(bB+Ac)}{5x^5} - \frac{c^2(3bB+Ac)}{3x^3} - \frac{Bc^3}{x} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 73, normalized size = 1.00

$$-\frac{Ab^3}{9x^9} - \frac{b^2(3Ac + bB)}{7x^7} - \frac{c^2(Ac + 3bB)}{3x^3} - \frac{3bc(Ac + bB)}{5x^5} - \frac{Bc^3}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^16,x]

[Out] -1/9\*(A\*b^3)/x^9 - (b^2\*(b\*B + 3\*A\*c))/(7\*x^7) - (3\*b\*c\*(b\*B + A\*c))/(5\*x^5) - (c^2\*(3\*b\*B + A\*c))/(3\*x^3) - (B\*c^3)/x

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{16}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^16,x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^16, x]

**fricas** [A] time = 0.39, size = 75, normalized size = 1.03

$$\frac{315 Bc^3x^8 + 105(3 Bbc^2 + Ac^3)x^6 + 189(Bb^2c + Abc^2)x^4 + 35 Ab^3 + 45(Bb^3 + 3 Ab^2c)x^2}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^16,x, algorithm="fricas")

[Out] -1/315\*(315\*B\*c^3\*x^8 + 105\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 189\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + 35\*A\*b^3 + 45\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^9

**giac** [A] time = 0.15, size = 79, normalized size = 1.08

$$\frac{315 Bc^3x^8 + 315 Bbc^2x^6 + 105 Ac^3x^6 + 189 Bb^2cx^4 + 189 Abc^2x^4 + 45 Bb^3x^2 + 135 Ab^2cx^2 + 35 Ab^3}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^16,x, algorithm="giac")

[Out] -1/315\*(315\*B\*c^3\*x^8 + 315\*B\*b\*c^2\*x^6 + 105\*A\*c^3\*x^6 + 189\*B\*b^2\*c\*x^4 + 189\*A\*b\*c^2\*x^4 + 45\*B\*b^3\*x^2 + 135\*A\*b^2\*c\*x^2 + 35\*A\*b^3)/x^9

**maple** [A] time = 0.05, size = 66, normalized size = 0.90

$$-\frac{Bc^3}{x} - \frac{(Ac + 3bB)c^2}{3x^3} - \frac{3(Ac + bB)bc}{5x^5} - \frac{Ab^3}{9x^9} - \frac{(3Ac + bB)b^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^16,x)

[Out] -1/9\*A\*b^3/x^9-1/7\*b^2\*(3\*A\*c+B\*b)/x^7-3/5\*b\*c\*(A\*c+B\*b)/x^5-1/3\*c^2\*(A\*c+3\*B\*b)/x^3-B\*c^3/x

**maxima** [A] time = 1.38, size = 75, normalized size = 1.03

$$\frac{315 Bc^3x^8 + 105(3 Bbc^2 + Ac^3)x^6 + 189(Bb^2c + Abc^2)x^4 + 35 Ab^3 + 45(Bb^3 + 3 Ab^2c)x^2}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^16,x, algorithm="maxima")

[Out] -1/315\*(315\*B\*c^3\*x^8 + 105\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 189\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + 35\*A\*b^3 + 45\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^9

**mupad** [B] time = 0.07, size = 74, normalized size = 1.01

$$\frac{x^4 \left( \frac{3Bb^2c}{5} + \frac{3Abc^2}{5} \right) + \frac{Ab^3}{9} + x^2 \left( \frac{Bb^3}{7} + \frac{3Ac b^2}{7} \right) + x^6 \left( \frac{Ac^3}{3} + Bbc^2 \right) + Bc^3x^8}{x^9}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x)`

[Out]  $-(x^4*((3A*b*c^2)/5 + (3B*b^2*c)/5) + (A*b^3)/9 + x^2*((B*b^3)/7 + (3A*b^2*c)/7) + x^6*((A*c^3)/3 + B*b*c^2) + B*c^3*x^8)/x^9$

**sympy [A]** time = 2.45, size = 83, normalized size = 1.14

$$\frac{-35Ab^3 - 315Bc^3x^8 + x^6(-105Ac^3 - 315Bbc^2) + x^4(-189Abc^2 - 189Bb^2c) + x^2(-135Ab^2c - 45Bb^3)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**16,x)`

[Out]  $(-35*A*b**3 - 315*B*c**3*x**8 + x**6*(-105*A*c**3 - 315*B*b*c**2) + x**4*(-189*A*b*c**2 - 189*B*b**2*c) + x**2*(-135*A*b**2*c - 45*B*b**3))/(315*x**9)$

$$3.40 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}} dx$$

**Optimal.** Leaf size=49

$$-\frac{(b+cx^2)^4(5bB-Ac)}{40b^2x^8} - \frac{A(b+cx^2)^4}{10bx^{10}}$$

**Rubi [A]** time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 446, 78, 37}

$$-\frac{(b+cx^2)^4(5bB-Ac)}{40b^2x^8} - \frac{A(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^17,x]

[Out] -(A\*(b + c\*x^2)^4)/(10\*b\*x^10) - ((5\*b\*B - A\*c)\*(b + c\*x^2)^4)/(40\*b^2\*x^8)

#### Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

#### Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q.
_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{17}} dx &= \int \frac{(A + Bx^2)(b + cx^2)^3}{x^{11}} dx \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(b + cx)^3}{x^6} dx, x, x^2 \right) \\
&= -\frac{A(b + cx^2)^4}{10bx^{10}} + \frac{(5bB - Ac) \text{Subst} \left( \int \frac{(b+cx)^3}{x^5} dx, x, x^2 \right)}{10b} \\
&= -\frac{A(b + cx^2)^4}{10bx^{10}} - \frac{(5bB - Ac)(b + cx^2)^4}{40b^2x^8}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 78, normalized size = 1.59

$$\frac{A(4b^3 + 15b^2cx^2 + 20bc^2x^4 + 10c^3x^6) + 5Bx^2(b^3 + 4b^2cx^2 + 6bc^2x^4 + 4c^3x^6)}{40x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^17, x]

[Out] -1/40\*(5\*B\*x^2\*(b^3 + 4\*b^2\*c\*x^2 + 6\*b\*c^2\*x^4 + 4\*c^3\*x^6) + A\*(4\*b^3 + 15\*b^2\*c\*x^2 + 20\*b\*c^2\*x^4 + 10\*c^3\*x^6))/x^10

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{17}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^17, x]

[Out] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^17, x]

**fricas [A]** time = 0.38, size = 75, normalized size = 1.53

$$\frac{20Bc^3x^8 + 10(3Bbc^2 + Ac^3)x^6 + 20(Bb^2c + Abc^2)x^4 + 4Ab^3 + 5(Bb^3 + 3Ab^2c)x^2}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^17,x, algorithm="fricas")

[Out] -1/40\*(20\*B\*c^3\*x^8 + 10\*(3\*B\*b\*c^2 + A\*c^3)\*x^6 + 20\*(B\*b^2\*c + A\*b\*c^2)\*x^4 + 4\*A\*b^3 + 5\*(B\*b^3 + 3\*A\*b^2\*c)\*x^2)/x^10

**giac [A]** time = 0.16, size = 79, normalized size = 1.61

$$\frac{20Bc^3x^8 + 30Bbc^2x^6 + 10Ac^3x^6 + 20Bb^2cx^4 + 20Abc^2x^4 + 5Bb^3x^2 + 15Ab^2cx^2 + 4Ab^3}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^17,x, algorithm="giac")

[Out] -1/40\*(20\*B\*c^3\*x^8 + 30\*B\*b\*c^2\*x^6 + 10\*A\*c^3\*x^6 + 20\*B\*b^2\*c\*x^4 + 20\*A\*b\*c^2\*x^4 + 5\*B\*b^3\*x^2 + 15\*A\*b^2\*c\*x^2 + 4\*A\*b^3)/x^10

**maple** [A] time = 0.05, size = 66, normalized size = 1.35

$$\frac{Bc^3}{2x^2} - \frac{(Ac + 3bB)c^2}{4x^4} - \frac{(Ac + bB)bc}{2x^6} - \frac{Ab^3}{10x^{10}} - \frac{(3Ac + bB)b^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^17,x)

[Out]  $-1/4*c^2*(A*c+3*B*b)/x^4-1/8*b^2*(3*A*c+B*b)/x^8-1/2*B*c^3/x^2-1/2*b*c*(A*c+B*b)/x^6-1/10*A*b^3/x^{10}$

**maxima** [A] time = 1.33, size = 75, normalized size = 1.53

$$\frac{20Bc^3x^8 + 10(3Bbc^2 + Ac^3)x^6 + 20(Bb^2c + Abc^2)x^4 + 4Ab^3 + 5(Bb^3 + 3Ab^2c)x^2}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^17,x, algorithm="maxima")

[Out]  $-1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^{10}$

**mupad** [B] time = 0.07, size = 76, normalized size = 1.55

$$\frac{x^4 \left( \frac{Bb^2c}{2} + \frac{Abc^2}{2} \right) + \frac{Ab^3}{10} + x^2 \left( \frac{Bb^3}{8} + \frac{3Ac b^2}{8} \right) + x^6 \left( \frac{Ac^3}{4} + \frac{3Bbc^2}{4} \right) + \frac{Bc^3x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^17,x)

[Out]  $-(x^4*((A*b*c^2)/2 + (B*b^2*c)/2) + (A*b^3)/10 + x^2*((B*b^3)/8 + (3*A*b^2*c)/8) + x^6*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3*x^8)/2)/x^{10}$

**sympy** [A] time = 3.43, size = 83, normalized size = 1.69

$$\frac{-4Ab^3 - 20Bc^3x^8 + x^6(-10Ac^3 - 30Bbc^2) + x^4(-20Abc^2 - 20Bb^2c) + x^2(-15Ab^2c - 5Bb^3)}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*17,x)

[Out]  $(-4*A*b**3 - 20*B*c**3*x**8 + x**6*(-10*A*c**3 - 30*B*b*c**2) + x**4*(-20*A*b*c**2 - 20*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(40*x**10)$

$$3.41 \quad \int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=119

$$-\frac{b^{7/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}} + \frac{b^3x(bB - Ac)}{c^5} - \frac{b^2x^3(bB - Ac)}{3c^4} + \frac{bx^5(bB - Ac)}{5c^3} - \frac{x^7(bB - Ac)}{7c^2} + \frac{Bx^9}{9c}$$

**Rubi [A]** time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 459, 302, 205}

$$-\frac{b^2x^3(bB - Ac)}{3c^4} + \frac{b^3x(bB - Ac)}{c^5} - \frac{b^{7/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}} - \frac{x^7(bB - Ac)}{7c^2} + \frac{bx^5(bB - Ac)}{5c^3} + \frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

[In] Int[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (b^3\*(b\*B - A\*c)\*x)/c^5 - (b^2\*(b\*B - A\*c)\*x^3)/(3\*c^4) + (b\*(b\*B - A\*c)\*x^5)/(5\*c^3) - ((b\*B - A\*c)\*x^7)/(7\*c^2) + (B\*x^9)/(9\*c) - (b^(7/2)\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(11/2)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :> Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{x^8(A+Bx^2)}{b+cx^2} dx \\
&= \frac{Bx^9}{9c} - \frac{(9bB-9Ac) \int \frac{x^8}{b+cx^2} dx}{9c} \\
&= \frac{Bx^9}{9c} - \frac{(9bB-9Ac) \int \left( -\frac{b^3}{c^4} + \frac{b^2x^2}{c^3} - \frac{bx^4}{c^2} + \frac{x^6}{c} + \frac{b^4}{c^4(b+cx^2)} \right) dx}{9c} \\
&= \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{(b^4(bB-Ac)) \int}{c^5} \\
&= \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{b^{7/2}(bB-Ac) \tan^{-1}}{c^{11/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 119, normalized size = 1.00

$$-\frac{b^{7/2}(bB-Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}} + \frac{b^3x(bB-Ac)}{c^5} - \frac{b^2x^3(bB-Ac)}{3c^4} + \frac{bx^5(bB-Ac)}{5c^3} + \frac{x^7(Ac-bB)}{7c^2} + \frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (b^3\*(b\*B - A\*c)\*x)/c^5 - (b^2\*(b\*B - A\*c)\*x^3)/(3\*c^4) + (b\*(b\*B - A\*c)\*x^5)/(5\*c^3) + ((-(b\*B) + A\*c)\*x^7)/(7\*c^2) + (B\*x^9)/(9\*c) - (b^(7/2)\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(11/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.41, size = 274, normalized size = 2.30

$$\frac{70Bc^4x^9 - 90(Bb^3c - A^2c^4)x^7 + 126(Bb^2c^2 - Ab^2c^3)x^5 - 210(Bb^3c - Ab^2c^2)x^3 - 315(Bb^4 - Ab^3c)\sqrt{-b/c} \log\left(\frac{cx^2 + 2cx\sqrt{-b/c} - b}{cx^2 + b}\right) + 630(Bb^4 - Ab^3c)x/c^5 + 35Bc^4x^9 - 45(Bb^3c - A^2c^4)x^7 + 63(Bb^2c^2 - Ab^2c^3)x^5 - 105(Bb^3c - Ab^2c^2)x^3 - 315(Bb^4 - Ab^3c)\sqrt{b/c} \arctan\left(\frac{cx\sqrt{b/c}}{b}\right) + 315(Bb^4 - Ab^3c)x/c^5}{630c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [1/630\*(70\*B\*c^4\*x^9 - 90\*(B\*b\*c^3 - A\*c^4)\*x^7 + 126\*(B\*b^2\*c^2 - A\*b\*c^3)\*x^5 - 210\*(B\*b^3\*c - A\*b^2\*c^2)\*x^3 - 315\*(B\*b^4 - A\*b^3\*c)\*sqrt(-b/c)\*log((c\*x^2 + 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) + 630\*(B\*b^4 - A\*b^3\*c)\*x)/c^5, 1/315\*(35\*B\*c^4\*x^9 - 45\*(B\*b\*c^3 - A\*c^4)\*x^7 + 63\*(B\*b^2\*c^2 - A\*b\*c^3)\*x^5 - 105\*(B\*b^3\*c - A\*b^2\*c^2)\*x^3 - 315\*(B\*b^4 - A\*b^3\*c)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) + 315\*(B\*b^4 - A\*b^3\*c)\*x)/c^5]

**giac [A]** time = 0.17, size = 133, normalized size = 1.12

$$\frac{(Bb^5 - Ab^4c) \arctan\left(\frac{cx}{\sqrt{bc}}\right) + 35Bc^8x^9 - 45Bbc^7x^7 + 45Ac^8x^7 + 63Bb^2c^6x^5 - 63Abc^7x^5 - 105Bb^3c^5x^3 + 105Ab^2c^6x^3 + 315Bb^4c^4x - 315Ab^3c^5x}{\sqrt{bc}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>10</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>),x, algorithm="giac")

[Out]  $-(B*b^5 - A*b^4*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^5) + 1/315*(35*B*c^8*x^9 - 45*B*b*c^7*x^7 + 45*A*c^8*x^7 + 63*B*b^2*c^6*x^5 - 63*A*b*c^7*x^5 - 105*B*b^3*c^5*x^3 + 105*A*b^2*c^6*x^3 + 315*B*b^4*c^4*x - 315*A*b^3*c^5*x)/c^9$

**maple [A]** time = 0.05, size = 140, normalized size = 1.18

$$\frac{Bx^9}{9c} + \frac{Ax^7}{7c} - \frac{Bbx^7}{7c^2} - \frac{Abx^5}{5c^2} + \frac{Bb^2x^5}{5c^3} + \frac{Ab^2x^3}{3c^3} - \frac{Bb^3x^3}{3c^4} + \frac{Ab^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} - \frac{Bb^5 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^5} - \frac{Ab^3x}{c^4} + \frac{Bb^4x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>10</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>),x)

[Out]  $1/9*B*x^9/c + 1/7/c*A*x^7 - 1/7/c^2*B*x^7*b - 1/5/c^2*A*x^5*b + 1/5/c^3*B*x^5*b^2 + 1/3/c^3*A*x^3*b^2 - 1/3/c^4*B*x^3*b^3 - 1/c^4*A*b^3*x + 1/c^5*B*b^4*x + b^4/c^4/(b*c)^{(1/2)}*\arctan(c*x/(b*c)^{(1/2)})*A - b^5/c^5/(b*c)^{(1/2)}*\arctan(c*x/(b*c)^{(1/2)})*B$

**maxima [A]** time = 3.00, size = 124, normalized size = 1.04

$$\frac{(Bb^5 - Ab^4c) \arctan\left(\frac{cx}{\sqrt{bc}}\right) + 35Bc^4x^9 - 45(Bbc^3 - Ac^4)x^7 + 63(Bb^2c^2 - Abc^3)x^5 - 105(Bb^3c - Ab^2c^2)x^3 + 315(Bb^4 - Ab^3c)x}{\sqrt{bc}c^5} + \frac{315c^5}{315c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>10</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>),x, algorithm="maxima")

[Out]  $-(B*b^5 - A*b^4*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^5) + 1/315*(35*B*c^4*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)*x^5 - 105*(B*b^3*c - A*b^2*c^2)*x^3 + 315*(B*b^4 - A*b^3*c)*x)/c^5$

**mupad [B]** time = 0.17, size = 144, normalized size = 1.21

$$x^7 \left( \frac{A}{7c} - \frac{Bb}{7c^2} \right) + \frac{Bx^9}{9c} + \frac{b^2x^3 \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{3c^2} - \frac{b^{7/2} \operatorname{atan}\left(\frac{b^{7/2} \sqrt{c} x (Ac - Bb)}{Bb^5 - Ab^4c}\right) (Ac - Bb)}{c^{11/2}} - \frac{bx^5 \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{5c} - \frac{b^3x \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x<sup>10</sup>\*(A + B\*x<sup>2</sup>))/(b\*x<sup>2</sup> + c\*x<sup>4</sup>),x)

[Out]  $x^7*(A/(7*c) - (B*b)/(7*c^2)) + (B*x^9)/(9*c) + (b^2*x^3*(A/c - (B*b)/c^2))/(3*c^2) - (b^{(7/2)}*atan((b^{(7/2)}*c^{(1/2)}*x*(A*c - B*b))/(B*b^5 - A*b^4*c))*(A*c - B*b))/c^{(11/2)} - (b*x^5*(A/c - (B*b)/c^2))/(5*c) - (b^3*x*(A/c - (B*b)/c^2))/c^3$

**sympy [A]** time = 0.44, size = 204, normalized size = 1.71

$$\frac{Bx^9}{9c} + x^7 \left( \frac{A}{7c} - \frac{Bb}{7c^2} \right) + x^5 \left( -\frac{Ab}{5c^2} + \frac{Bb^2}{5c^3} \right) + x^3 \left( \frac{Ab^2}{3c^3} - \frac{Bb^3}{3c^4} \right) + x \left( -\frac{Ab^3}{c^4} + \frac{Bb^4}{c^5} \right) + \frac{\sqrt{-\frac{b^7}{c^{11}}(-Ac+Bb)} \log\left(\frac{c^5 \sqrt{\frac{b^7}{c^{11}}(-Ac+Bb)}}{-Ab^3c+Bb^4} + x\right)}{2} - \frac{\sqrt{-\frac{b^7}{c^{11}}(-Ac+Bb)} \log\left(\frac{c^5 \sqrt{\frac{b^7}{c^{11}}(-Ac+Bb)}}{-Ab^3c+Bb^4} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*10\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out]  $B*x**9/(9*c) + x**7*(A/(7*c) - B*b/(7*c**2)) + x**5*(-A*b/(5*c**2) + B*b**2/(5*c**3)) + x**3*(A*b**2/(3*c**3) - B*b**3/(3*c**4)) + x*(-A*b**3/c**4 + B*b**4/c**5) + \sqrt{-b**7/c**11}*(-A*c + B*b)*\log(-c**5*\sqrt{-b**7/c**11}*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - \sqrt{-b**7/c**11}*(-A*c + B*b)*\log(c**5*\sqrt{-b**7/c**11}*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2$

$$3.42 \quad \int \frac{x^9(A+Bx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=96

$$\frac{b^3(bB - Ac) \log(b + cx^2)}{2c^5} - \frac{b^2x^2(bB - Ac)}{2c^4} + \frac{bx^4(bB - Ac)}{4c^3} - \frac{x^6(bB - Ac)}{6c^2} + \frac{Bx^8}{8c}$$

**Rubi [A]** time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{b^2x^2(bB - Ac)}{2c^4} + \frac{b^3(bB - Ac) \log(b + cx^2)}{2c^5} - \frac{x^6(bB - Ac)}{6c^2} + \frac{bx^4(bB - Ac)}{4c^3} + \frac{Bx^8}{8c}$$

Antiderivative was successfully verified.

[In] Int[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] -(b^2\*(b\*B - A\*c)\*x^2)/(2\*c^4) + (b\*(b\*B - A\*c)\*x^4)/(4\*c^3) - ((b\*B - A\*c)\*x^6)/(6\*c^2) + (B\*x^8)/(8\*c) + (b^3\*(b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^5)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^9(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{x^7(A+Bx^2)}{b+cx^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x^3(A+Bx)}{b+cx} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -\frac{b^2(bB-Ac)}{c^4} + \frac{b(bB-Ac)x}{c^3} + \frac{(-bB+Ac)x^2}{c^2} + \frac{Bx^3}{c} + \frac{b^3(bB-Ac)}{c^4(b+cx)} \right) dx, x, \right. \\ &= -\frac{b^2(bB-Ac)x^2}{2c^4} + \frac{b(bB-Ac)x^4}{4c^3} - \frac{(bB-Ac)x^6}{6c^2} + \frac{Bx^8}{8c} + \frac{b^3(bB-Ac) \log(b+cx^2)}{2c^5} \end{aligned}$$



**Mathematica [A]** time = 0.04, size = 92, normalized size = 0.96

$$\frac{12b^3(bB - Ac)\log(b + cx^2) + cx^2(6b^2c(2A + Bx^2) - 2bc^2x^2(3A + 2Bx^2) + c^3x^4(4A + 3Bx^2) - 12b^3B)}{24c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (c\*x^2\*(-12\*b^3\*B + 6\*b^2\*c\*(2\*A + B\*x^2) - 2\*b\*c^2\*x^2\*(3\*A + 2\*B\*x^2) + c^3\*x^4\*(4\*A + 3\*B\*x^2)) + 12\*b^3\*(b\*B - A\*c)\*Log[b + c\*x^2])/(24\*c^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9(A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.39, size = 98, normalized size = 1.02

$$\frac{3Bc^4x^8 - 4(Bbc^3 - Ac^4)x^6 + 6(Bb^2c^2 - Abc^3)x^4 - 12(Bb^3c - Ab^2c^2)x^2 + 12(Bb^4 - Ab^3c)\log(cx^2 + b)}{24c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/24\*(3\*B\*c^4\*x^8 - 4\*(B\*b\*c^3 - A\*c^4)\*x^6 + 6\*(B\*b^2\*c^2 - A\*b\*c^3)\*x^4 - 12\*(B\*b^3\*c - A\*b^2\*c^2)\*x^2 + 12\*(B\*b^4 - A\*b^3\*c)\*log(c\*x^2 + b))/c^5

**giac [A]** time = 0.15, size = 101, normalized size = 1.05

$$\frac{3Bc^3x^8 - 4Bbc^2x^6 + 4Ac^3x^6 + 6Bb^2cx^4 - 6Abc^2x^4 - 12Bb^3x^2 + 12Ab^2cx^2}{24c^4} + \frac{(Bb^4 - Ab^3c)\log(|cx^2 + b|)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] 1/24\*(3\*B\*c^3\*x^8 - 4\*B\*b\*c^2\*x^6 + 4\*A\*c^3\*x^6 + 6\*B\*b^2\*c\*x^4 - 6\*A\*b\*c^2\*x^4 - 12\*B\*b^3\*x^2 + 12\*A\*b^2\*c\*x^2)/c^4 + 1/2\*(B\*b^4 - A\*b^3\*c)\*log(abs(c\*x^2 + b))/c^5

**maple [A]** time = 0.04, size = 110, normalized size = 1.15

$$\frac{Bx^8}{8c} + \frac{Ax^6}{6c} - \frac{Bbx^6}{6c^2} - \frac{Abx^4}{4c^2} + \frac{Bb^2x^4}{4c^3} + \frac{Ab^2x^2}{2c^3} - \frac{Bb^3x^2}{2c^4} - \frac{Ab^3\ln(cx^2 + b)}{2c^4} + \frac{Bb^4\ln(cx^2 + b)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2), x)

[Out] 1/8\*B\*x^8/c+1/6/c\*A\*x^6-1/6/c^2\*B\*x^6\*b-1/4/c^2\*A\*x^4\*b+1/4/c^3\*B\*x^4\*b^2+1/2/c^3\*A\*x^2\*b^2-1/2/c^4\*B\*x^2\*b^3-1/2\*b^3/c^4\*ln(c\*x^2+b)\*A+1/2\*b^4/c^5\*ln(c\*x^2+b)\*B

**maxima [A]** time = 1.38, size = 97, normalized size = 1.01

$$\frac{3Bc^3x^8 - 4(Bbc^2 - Ac^3)x^6 + 6(Bb^2c - Abc^2)x^4 - 12(Bb^3 - Ab^2c)x^2}{24c^4} + \frac{(Bb^4 - Ab^3c)\log(cx^2 + b)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 1/24\*(3\*B\*c^3\*x^8 - 4\*(B\*b\*c^2 - A\*c^3)\*x^6 + 6\*(B\*b^2\*c - A\*b\*c^2)\*x^4 - 12\*(B\*b^3 - A\*b^2\*c)\*x^2)/c^4 + 1/2\*(B\*b^4 - A\*b^3\*c)\*log(c\*x^2 + b)/c^5

**mupad [B]** time = 0.06, size = 100, normalized size = 1.04

$$x^6 \left( \frac{A}{6c} - \frac{Bb}{6c^2} \right) + \frac{Bx^8}{8c} + \frac{\ln(cx^2 + b)(Bb^4 - Ab^3c)}{2c^5} + \frac{b^2x^2 \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{2c^2} - \frac{bx^4 \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out] x^6\*(A/(6\*c) - (B\*b)/(6\*c^2)) + (B\*x^8)/(8\*c) + (log(b + c\*x^2)\*(B\*b^4 - A\*b^3\*c))/(2\*c^5) + (b^2\*x^2\*(A/c - (B\*b)/c^2))/(2\*c^2) - (b\*x^4\*(A/c - (B\*b)/c^2))/(4\*c)

**sympy [A]** time = 0.35, size = 94, normalized size = 0.98

$$\frac{Bx^8}{8c} + \frac{b^3(-Ac + Bb)\log(b + cx^2)}{2c^5} + x^6 \left( \frac{A}{6c} - \frac{Bb}{6c^2} \right) + x^4 \left( -\frac{Ab}{4c^2} + \frac{Bb^2}{4c^3} \right) + x^2 \left( \frac{Ab^2}{2c^3} - \frac{Bb^3}{2c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*9\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] B\*x\*\*8/(8\*c) + b\*\*3\*(-A\*c + B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*5) + x\*\*6\*(A/(6\*c) - B\*b/(6\*c\*\*2)) + x\*\*4\*(-A\*b/(4\*c\*\*2) + B\*b\*\*2/(4\*c\*\*3)) + x\*\*2\*(A\*b\*\*2/(2\*c\*\*3) - B\*b\*\*3/(2\*c\*\*4))

$$3.43 \quad \int \frac{x^8(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=98

$$\frac{b^{5/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}} - \frac{b^2x(bB - Ac)}{c^4} + \frac{bx^3(bB - Ac)}{3c^3} - \frac{x^5(bB - Ac)}{5c^2} + \frac{Bx^7}{7c}$$

**Rubi [A]** time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 459, 302, 205}

$$-\frac{b^2x(bB - Ac)}{c^4} + \frac{b^{5/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}} - \frac{x^5(bB - Ac)}{5c^2} + \frac{bx^3(bB - Ac)}{3c^3} + \frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

[In] Int[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] -((b^2\*(b\*B - A\*c)\*x)/c^4) + (b\*(b\*B - A\*c)\*x^3)/(3\*c^3) - ((b\*B - A\*c)\*x^5)/(5\*c^2) + (B\*x^7)/(7\*c) + (b^(5/2)\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(9/2)

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{x^6 (A + Bx^2)}{b + cx^2} dx \\
&= \frac{Bx^7}{7c} - \frac{(7bB - 7Ac) \int \frac{x^6}{b+cx^2} dx}{7c} \\
&= \frac{Bx^7}{7c} - \frac{(7bB - 7Ac) \int \left( \frac{b^2}{c^3} - \frac{bx^2}{c^2} + \frac{x^4}{c} - \frac{b^3}{c^3(b+cx^2)} \right) dx}{7c} \\
&= -\frac{b^2(bB - Ac)x}{c^4} + \frac{b(bB - Ac)x^3}{3c^3} - \frac{(bB - Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{(b^3(bB - Ac)) \int \frac{1}{b+cx^2} dx}{c^4} \\
&= -\frac{b^2(bB - Ac)x}{c^4} + \frac{b(bB - Ac)x^3}{3c^3} - \frac{(bB - Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{b^{5/2}(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{c^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 98, normalized size = 1.00

$$\frac{b^{5/2}(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{c^{9/2}} - \frac{b^2x(bB - Ac)}{c^4} + \frac{bx^3(bB - Ac)}{3c^3} + \frac{x^5(Ac - bB)}{5c^2} + \frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] -((b^2\*(b\*B - A\*c)\*x)/c^4) + (b\*(b\*B - A\*c)\*x^3)/(3\*c^3) + ((-(b\*B) + A\*c)\*x^5)/(5\*c^2) + (B\*x^7)/(7\*c) + (b^(5/2)\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(9/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.40, size = 228, normalized size = 2.33

$$\left| \frac{30 Bc^3x^7 - 42 (Bbc^2 - Ac^3)x^5 + 70 (Bb^2c - Abc^2)x^3 - 105 (Bb^3 - Ab^2c)\sqrt{\frac{c}{b}} \log\left(\frac{cx^2 - 2cx\sqrt{\frac{c}{b}} - b}{cx^2 + b}\right) - 210 (Bb^3 - Ab^2c)x}{210c^4} - \frac{15 Bc^3x^7 - 21 (Bbc^2 - Ac^3)x^5 + 35 (Bb^2c - Abc^2)x^3 + 105 (Bb^3 - Ab^2c)\sqrt{\frac{c}{b}} \arctan\left(\frac{cx\sqrt{\frac{c}{b}}}{b}\right) - 105 (Bb^3 - Ab^2c)x}{105c^4} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [1/210\*(30\*B\*c^3\*x^7 - 42\*(B\*b\*c^2 - A\*c^3)\*x^5 + 70\*(B\*b^2\*c - A\*b\*c^2)\*x^3 - 105\*(B\*b^3 - A\*b^2\*c)\*sqrt(-b/c)\*log((c\*x^2 - 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) - 210\*(B\*b^3 - A\*b^2\*c)\*x)/c^4, 1/105\*(15\*B\*c^3\*x^7 - 21\*(B\*b\*c^2 - A\*c^3)\*x^5 + 35\*(B\*b^2\*c - A\*b\*c^2)\*x^3 + 105\*(B\*b^3 - A\*b^2\*c)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) - 105\*(B\*b^3 - A\*b^2\*c)\*x)/c^4]

**giac [A]** time = 0.18, size = 108, normalized size = 1.10

$$\frac{(Bb^4 - Ab^3c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15 Bc^6x^7 - 21 Bbc^5x^5 + 21 Ac^6x^5 + 35 Bb^2c^4x^3 - 35 Abc^5x^3 - 105 Bb^3c^3x + 105 Ab^2c^4x}{105c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] (B\*b^4 - A\*b^3\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^4) + 1/105\*(15\*B\*c^6\*x^7 - 21\*B\*b\*c^5\*x^5 + 21\*A\*c^6\*x^5 + 35\*B\*b^2\*c^4\*x^3 - 35\*A\*b\*c^5\*x^3 - 105\*B\*b^3\*c^3\*x + 105\*A\*b^2\*c^4\*x)/c^7

**maple** [A] time = 0.05, size = 116, normalized size = 1.18

$$\frac{Bx^7}{7c} + \frac{Ax^5}{5c} - \frac{Bbx^5}{5c^2} - \frac{Abx^3}{3c^2} + \frac{Bb^2x^3}{3c^3} - \frac{Ab^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{Bb^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{Ab^2x}{c^3} - \frac{Bb^3x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out] 1/7\*B\*x^7/c+1/5/c\*A\*x^5-1/5/c^2\*B\*x^5\*b-1/3/c^2\*A\*x^3\*b+1/3/c^3\*B\*x^3\*b^2+1/c^3\*A\*b^2\*x-1/c^4\*B\*b^3\*x-b^3/c^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A+b^4/c^4/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B

**maxima** [A] time = 2.94, size = 100, normalized size = 1.02

$$\frac{(Bb^4 - Ab^3c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15Bc^3x^7 - 21(Bbc^2 - Ac^3)x^5 + 35(Bb^2c - Abc^2)x^3 - 105(Bb^3 - Ab^2c)x}{105c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] (B\*b^4 - A\*b^3\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^4) + 1/105\*(15\*B\*c^3\*x^7 - 21\*(B\*b\*c^2 - A\*c^3)\*x^5 + 35\*(B\*b^2\*c - A\*b\*c^2)\*x^3 - 105\*(B\*b^3 - A\*b^2\*c)\*x)/c^4

**mupad** [B] time = 0.04, size = 118, normalized size = 1.20

$$x^5 \left( \frac{A}{5c} - \frac{Bb}{5c^2} \right) + \frac{Bx^7}{7c} + \frac{b^{5/2} \operatorname{atan}\left(\frac{b^{5/2} \sqrt{c} x (Ac - Bb)}{Bb^4 - Ab^3c}\right) (Ac - Bb)}{c^{9/2}} - \frac{bx^3 \left(\frac{A}{c} - \frac{Bb}{c^2}\right)}{3c} + \frac{b^2x \left(\frac{A}{c} - \frac{Bb}{c^2}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out] x^5\*(A/(5\*c) - (B\*b)/(5\*c^2)) + (B\*x^7)/(7\*c) + (b^(5/2)\*atan((b^(5/2)\*c^(1/2)\*x\*(A\*c - B\*b))/(B\*b^4 - A\*b^3\*c))\*(A\*c - B\*b))/c^(9/2) - (b\*x^3\*(A/c - (B\*b)/c^2))/(3\*c) + (b^2\*x\*(A/c - (B\*b)/c^2))/c^2

**sympy** [B] time = 0.41, size = 180, normalized size = 1.84

$$\frac{Bx^7}{7c} + x^5 \left( \frac{A}{5c} - \frac{Bb}{5c^2} \right) + x^3 \left( \frac{Ab}{3c^2} + \frac{Bb^2}{3c^3} \right) + x \left( \frac{Ab^2}{c^3} - \frac{Bb^3}{c^4} \right) - \frac{\sqrt{\frac{b^5}{c^9}} (-Ac + Bb) \log\left(\frac{c^4 \sqrt{\frac{b^5}{c^9}} (-Ac + Bb)}{-Ab^2c + Bb^3} + x\right)}{2} + \frac{\sqrt{\frac{b^5}{c^9}} (-Ac + Bb) \log\left(\frac{c^4 \sqrt{\frac{b^5}{c^9}} (-Ac + Bb)}{-Ab^2c + Bb^3} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] B\*x\*\*7/(7\*c) + x\*\*5\*(A/(5\*c) - B\*b/(5\*c\*\*2)) + x\*\*3\*(-A\*b/(3\*c\*\*2) + B\*b\*\*2/(3\*c\*\*3)) + x\*(A\*b\*\*2/c\*\*3 - B\*b\*\*3/c\*\*4) - sqrt(-b\*\*5/c\*\*9)\*(-A\*c + B\*b)\*log(-c\*\*4\*sqrt(-b\*\*5/c\*\*9)\*(-A\*c + B\*b)/(-A\*b\*\*2\*c + B\*b\*\*3) + x)/2 + sqrt(-b\*\*5/c\*\*9)\*(-A\*c + B\*b)\*log(c\*\*4\*sqrt(-b\*\*5/c\*\*9)\*(-A\*c + B\*b)/(-A\*b\*\*2\*c + B\*b\*\*3) + x)/2

$$3.44 \quad \int \frac{x^7(A+Bx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=75

$$-\frac{b^2(bB - Ac) \log(b + cx^2)}{2c^4} + \frac{bx^2(bB - Ac)}{2c^3} - \frac{x^4(bB - Ac)}{4c^2} + \frac{Bx^6}{6c}$$

**Rubi [A]** time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{b^2(bB - Ac) \log(b + cx^2)}{2c^4} - \frac{x^4(bB - Ac)}{4c^2} + \frac{bx^2(bB - Ac)}{2c^3} + \frac{Bx^6}{6c}$$

Antiderivative was successfully verified.

[In] Int[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] (b\*(b\*B - A\*c)\*x^2)/(2\*c^3) - ((b\*B - A\*c)\*x^4)/(4\*c^2) + (B\*x^6)/(6\*c) - (b^2\*(b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^4)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^7(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{x^5(A+Bx^2)}{b+cx^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2(A+Bx)}{b+cx} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{b(bB-Ac)}{c^3} + \frac{(-bB+Ac)x}{c^2} + \frac{Bx^2}{c} - \frac{b^2(bB-Ac)}{c^3(b+cx)} \right) dx, x, x^2 \right) \\ &= \frac{b(bB-Ac)x^2}{2c^3} - \frac{(bB-Ac)x^4}{4c^2} + \frac{Bx^6}{6c} - \frac{b^2(bB-Ac) \log(b+cx^2)}{2c^4} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 71, normalized size = 0.95

$$\frac{cx^2(-3bc(2A+Bx^2)+c^2x^2(3A+2Bx^2)+6b^2B)+6b^2(Ac-bB)\log(b+cx^2)}{12c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(A+B\*x^2))/(b\*x^2+c\*x^4),x]

[Out] (c\*x^2\*(6\*b^2\*B-3\*b\*c\*(2\*A+B\*x^2))+c^2\*x^2\*(3\*A+2\*B\*x^2))+6\*b^2\*(-(b\*B)+A\*c)\*Log[b+c\*x^2]/(12\*c^4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7\*(A+B\*x^2))/(b\*x^2+c\*x^4),x]

[Out] IntegrateAlgebraic[(x^7\*(A+B\*x^2))/(b\*x^2+c\*x^4),x]

**fricas [A]** time = 0.39, size = 75, normalized size = 1.00

$$\frac{2Bc^3x^6-3(Bbc^2-Ac^3)x^4+6(Bb^2c-Abc^2)x^2-6(Bb^3-Ab^2c)\log(cx^2+b)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 1/12\*(2\*B\*c^3\*x^6-3\*(B\*b\*c^2-A\*c^3)\*x^4+6\*(B\*b^2\*c-A\*b\*c^2)\*x^2-6\*(B\*b^3-A\*b^2\*c)\*log(c\*x^2+b))/c^4

**giac [A]** time = 0.15, size = 77, normalized size = 1.03

$$\frac{2Bc^2x^6-3Bbcx^4+3Ac^2x^4+6Bb^2x^2-6Abcx^2}{12c^3} - \frac{(Bb^3-Ab^2c)\log(|cx^2+b|)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 1/12\*(2\*B\*c^2\*x^6-3\*B\*b\*c\*x^4+3\*A\*c^2\*x^4+6\*B\*b^2\*x^2-6\*A\*b\*c\*x^2)/c^3-1/2\*(B\*b^3-A\*b^2\*c)\*log(abs(c\*x^2+b))/c^4

**maple [A]** time = 0.04, size = 86, normalized size = 1.15

$$\frac{Bx^6}{6c} + \frac{Ax^4}{4c} - \frac{Bbx^4}{4c^2} - \frac{Abx^2}{2c^2} + \frac{Bb^2x^2}{2c^3} + \frac{Ab^2\ln(cx^2+b)}{2c^3} - \frac{Bb^3\ln(cx^2+b)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out] 1/6\*B\*x^6/c+1/4/c\*A\*x^4-1/4/c^2\*B\*x^4\*b-1/2/c^2\*A\*x^2\*b+1/2/c^3\*B\*x^2\*b^2+1/2\*b^2/c^3\*ln(c\*x^2+b)\*A-1/2\*b^3/c^4\*ln(c\*x^2+b)\*B

**maxima [A]** time = 1.34, size = 74, normalized size = 0.99

$$\frac{2Bc^2x^6-3(Bbc-Ac^2)x^4+6(Bb^2-Abc)x^2}{12c^3} - \frac{(Bb^3-Ab^2c)\log(cx^2+b)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 1/12\*(2\*B\*c^2\*x^6 - 3\*(B\*b\*c - A\*c^2)\*x^4 + 6\*(B\*b^2 - A\*b\*c)\*x^2)/c^3 - 1/2\*(B\*b^3 - A\*b^2\*c)\*log(c\*x^2 + b)/c^4

mupad [B] time = 0.09, size = 76, normalized size = 1.01

$$x^4 \left( \frac{A}{4c} - \frac{Bb}{4c^2} \right) + \frac{Bx^6}{6c} - \frac{\ln(cx^2 + b)(Bb^3 - Ab^2c)}{2c^4} - \frac{bx^2 \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out] x^4\*(A/(4\*c) - (B\*b)/(4\*c^2)) + (B\*x^6)/(6\*c) - (log(b + c\*x^2)\*(B\*b^3 - A\*b^2\*c))/(2\*c^4) - (b\*x^2\*(A/c - (B\*b)/c^2))/(2\*c)

sympy [A] time = 0.32, size = 70, normalized size = 0.93

$$\frac{Bx^6}{6c} - \frac{b^2(-Ac + Bb)\log(b + cx^2)}{2c^4} + x^4 \left( \frac{A}{4c} - \frac{Bb}{4c^2} \right) + x^2 \left( -\frac{Ab}{2c^2} + \frac{Bb^2}{2c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] B\*x\*\*6/(6\*c) - b\*\*2\*(-A\*c + B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*4) + x\*\*4\*(A/(4\*c) - B\*b/(4\*c\*\*2)) + x\*\*2\*(-A\*b/(2\*c\*\*2) + B\*b\*\*2/(2\*c\*\*3))



$$3.45 \quad \int \frac{x^6(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=77

$$-\frac{b^{3/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}} + \frac{bx(bB - Ac)}{c^3} - \frac{x^3(bB - Ac)}{3c^2} + \frac{Bx^5}{5c}$$

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 459, 302, 205}

$$-\frac{b^{3/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}} - \frac{x^3(bB - Ac)}{3c^2} + \frac{bx(bB - Ac)}{c^3} + \frac{Bx^5}{5c}$$

Antiderivative was successfully verified.

[In] Int[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] (b\*(b\*B - A\*c)\*x)/c^3 - ((b\*B - A\*c)\*x^3)/(3\*c^2) + (B\*x^5)/(5\*c) - (b^(3/2)\* (b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(7/2)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :> Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^6 (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{x^4 (A + Bx^2)}{b + cx^2} dx \\
&= \frac{Bx^5}{5c} - \frac{(5bB - 5Ac) \int \frac{x^4}{b+cx^2} dx}{5c} \\
&= \frac{Bx^5}{5c} - \frac{(5bB - 5Ac) \int \left( -\frac{b}{c^2} + \frac{x^2}{c} + \frac{b^2}{c^2(b+cx^2)} \right) dx}{5c} \\
&= \frac{b(bB - Ac)x}{c^3} - \frac{(bB - Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{(b^2(bB - Ac)) \int \frac{1}{b+cx^2} dx}{c^3} \\
&= \frac{b(bB - Ac)x}{c^3} - \frac{(bB - Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{b^{3/2}(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{c^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 77, normalized size = 1.00

$$-\frac{b^{3/2}(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{c^{7/2}} + \frac{bx(bB - Ac)}{c^3} + \frac{x^3(Ac - bB)}{3c^2} + \frac{Bx^5}{5c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (b\*(b\*B - A\*c)\*x)/c^3 + ((-(b\*B) + A\*c)\*x^3)/(3\*c^2) + (B\*x^5)/(5\*c) - (b^(3/2)\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(7/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.41, size = 178, normalized size = 2.31

$$\left[ \frac{6Bc^2x^5 - 10(Bbc - Ac^2)x^3 - 15(Bb^2 - Abc)\sqrt{\frac{b}{c}} \log\left(\frac{cx^2 + 2cx\sqrt{\frac{b}{c}} - b}{cx^2 + b}\right) + 30(Bb^2 - Abc)x}{30c^3}, \frac{3Bc^2x^5 - 5(Bbc - Ac^2)x^3 - 15(Bb^2 - Abc)\sqrt{\frac{b}{c}} \arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) + 15(Bb^2 - Abc)x}{15c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [1/30\*(6\*B\*c^2\*x^5 - 10\*(B\*b\*c - A\*c^2)\*x^3 - 15\*(B\*b^2 - A\*b\*c)\*sqrt(-b/c)\*log((c\*x^2 + 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) + 30\*(B\*b^2 - A\*b\*c)\*x)/c^3, 1/15\*(3\*B\*c^2\*x^5 - 5\*(B\*b\*c - A\*c^2)\*x^3 - 15\*(B\*b^2 - A\*b\*c)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) + 15\*(B\*b^2 - A\*b\*c)\*x)/c^3]

**giac [A]** time = 0.18, size = 85, normalized size = 1.10

$$-\frac{(Bb^3 - Ab^2c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3Bc^4x^5 - 5Bbc^3x^3 + 5Ac^4x^3 + 15Bb^2c^2x - 15Abc^3x}{15c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out]  $-(B*b^3 - A*b^2*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^3) + 1/15*(3*B*c^4*x^5 - 5*B*b*c^3*x^3 + 5*A*c^4*x^3 + 15*B*b^2*c^2*x - 15*A*b*c^3*x)/c^5$

**maple** [A] time = 0.05, size = 92, normalized size = 1.19

$$\frac{Bx^5}{5c} + \frac{Ax^3}{3c} - \frac{Bbx^3}{3c^2} + \frac{Ab^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} - \frac{Bb^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} - \frac{Abx}{c^2} + \frac{Bb^2x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out]  $1/5*B*x^5/c + 1/3/c*A*x^3 - 1/3/c^2*B*x^3*b - 1/c^2*A*b*x + 1/c^3*B*b^2*x + b^2/c^2/(b*c)^{(1/2)*\arctan(1/(b*c)^{(1/2)*c*x})} - b^3/c^3/(b*c)^{(1/2)*\arctan(1/(b*c)^{(1/2)*c*x})} * B$

**maxima** [A] time = 2.86, size = 78, normalized size = 1.01

$$-\frac{(Bb^3 - Ab^2c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3Bc^2x^5 - 5(Bbc - Ac^2)x^3 + 15(Bb^2 - Abc)x}{15c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $-(B*b^3 - A*b^2*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^3) + 1/15*(3*B*c^2*x^5 - 5*(B*b*c - A*c^2)*x^3 + 15*(B*b^2 - A*b*c)*x)/c^3$

**mupad** [B] time = 0.07, size = 96, normalized size = 1.25

$$x^3 \left( \frac{A}{3c} - \frac{Bb}{3c^2} \right) + \frac{Bx^5}{5c} - \frac{b^{3/2} \operatorname{atan}\left(\frac{b^{3/2} \sqrt{c} x (Ac - Bb)}{Bb^3 - Ab^2c}\right) (Ac - Bb)}{c^{7/2}} - \frac{bx \left( \frac{A}{c} - \frac{Bb}{c^2} \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out]  $x^3*(A/(3*c) - (B*b)/(3*c^2)) + (B*x^5)/(5*c) - (b^{(3/2)*\operatorname{atan}((b^{(3/2)*c^{(1/2)*x*(A*c - B*b)/(B*b^3 - A*b^2*c)})*(A*c - B*b))/c^{(7/2)}} - (b*x*(A/c - (B*b)/c^2))/c$

**sympy** [B] time = 0.37, size = 153, normalized size = 1.99

$$\frac{Bx^5}{5c} + x^3 \left( \frac{A}{3c} - \frac{Bb}{3c^2} \right) + x \left( -\frac{Ab}{c^2} + \frac{Bb^2}{c^3} \right) + \frac{\sqrt{-\frac{b^3}{c^7}} (-Ac + Bb) \log\left(-\frac{c^3 \sqrt{-\frac{b^3}{c^7}} (-Ac + Bb)}{-Abc + Bb^2} + x\right)}{2} - \frac{\sqrt{-\frac{b^3}{c^7}} (-Ac + Bb) \log\left(\frac{c^3 \sqrt{-\frac{b^3}{c^7}} (-Ac + Bb)}{-Abc + Bb^2} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out]  $B*x**5/(5*c) + x**3*(A/(3*c) - B*b/(3*c**2)) + x*(-A*b/c**2 + B*b**2/c**3) + \sqrt{-b**3/c**7}*(-A*c + B*b)*\log(-c**3*\sqrt{-b**3/c**7}*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2 - \sqrt{-b**3/c**7}*(-A*c + B*b)*\log(c**3*\sqrt{-b**3/c**7}*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2$

$$3.46 \quad \int \frac{x^5(A+Bx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=54

$$\frac{b(bB - Ac) \log(b + cx^2)}{2c^3} - \frac{x^2(bB - Ac)}{2c^2} + \frac{Bx^4}{4c}$$

**Rubi [A]** time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{x^2(bB - Ac)}{2c^2} + \frac{b(bB - Ac) \log(b + cx^2)}{2c^3} + \frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] -((b\*B - A\*c)\*x^2)/(2\*c^2) + (B\*x^4)/(4\*c) + (b\*(b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^3)

#### Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{x^5(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{x^3(A+Bx^2)}{b+cx^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x(A+Bx)}{b+cx} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{-bB+Ac}{c^2} + \frac{Bx}{c} + \frac{b(bB-Ac)}{c^2(b+cx)} \right) dx, x, x^2 \right) \\ &= -\frac{(bB-Ac)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{b(bB-Ac) \log(b+cx^2)}{2c^3} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 47, normalized size = 0.87

$$\frac{cx^2(2Ac - 2bB + Bcx^2) + 2b(bB - Ac)\log(b + cx^2)}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (c\*x^2\*(-2\*b\*B + 2\*A\*c + B\*c\*x^2) + 2\*b\*(b\*B - A\*c)\*Log[b + c\*x^2])/(4\*c^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.39, size = 51, normalized size = 0.94

$$\frac{Bc^2x^4 - 2(Bbc - Ac^2)x^2 + 2(Bb^2 - Abc)\log(cx^2 + b)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/4\*(B\*c^2\*x^4 - 2\*(B\*b\*c - A\*c^2)\*x^2 + 2\*(B\*b^2 - A\*b\*c)\*log(c\*x^2 + b))/c^3

**giac [A]** time = 0.15, size = 52, normalized size = 0.96

$$\frac{Bcx^4 - 2Bbx^2 + 2Acx^2}{4c^2} + \frac{(Bb^2 - Abc)\log(|cx^2 + b|)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] 1/4\*(B\*c\*x^4 - 2\*B\*b\*x^2 + 2\*A\*c\*x^2)/c^2 + 1/2\*(B\*b^2 - A\*b\*c)\*log(abs(c\*x^2 + b))/c^3

**maple [A]** time = 0.05, size = 62, normalized size = 1.15

$$\frac{Bx^4}{4c} + \frac{Ax^2}{2c} - \frac{Bbx^2}{2c^2} - \frac{Ab\ln(cx^2 + b)}{2c^2} + \frac{Bb^2\ln(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2), x)

[Out] 1/4\*B\*x^4/c+1/2/c\*A\*x^2-1/2/c^2\*B\*x^2\*b-1/2\*b/c^2\*ln(c\*x^2+b)\*A+1/2\*b^2/c^3\*ln(c\*x^2+b)\*B

**maxima [A]** time = 1.33, size = 50, normalized size = 0.93

$$\frac{Bcx^4 - 2(Bb - Ac)x^2}{4c^2} + \frac{(Bb^2 - Abc)\log(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 1/4\*(B\*c\*x^4 - 2\*(B\*b - A\*c)\*x^2)/c^2 + 1/2\*(B\*b^2 - A\*b\*c)\*log(c\*x^2 + b)/c^3

**mupad [B]** time = 0.07, size = 52, normalized size = 0.96

$$x^2 \left( \frac{A}{2c} - \frac{Bb}{2c^2} \right) + \frac{\ln(cx^2 + b)(Bb^2 - Abc)}{2c^3} + \frac{Bx^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out] x^2\*(A/(2\*c) - (B\*b)/(2\*c^2)) + (log(b + c\*x^2)\*(B\*b^2 - A\*b\*c))/(2\*c^3) + (B\*x^4)/(4\*c)

**sympy [A]** time = 0.29, size = 46, normalized size = 0.85

$$\frac{Bx^4}{4c} + \frac{b(-Ac + Bb)\log(b + cx^2)}{2c^3} + x^2 \left( \frac{A}{2c} - \frac{Bb}{2c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] B\*x\*\*4/(4\*c) + b\*(-A\*c + B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*3) + x\*\*2\*(A/(2\*c) - B\*b/(2\*c\*\*2))

$$3.47 \quad \int \frac{x^4(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=58

$$\frac{\sqrt{b}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}} - \frac{x(bB - Ac)}{c^2} + \frac{Bx^3}{3c}$$

**Rubi [A]** time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 459, 321, 205}

$$-\frac{x(bB - Ac)}{c^2} + \frac{\sqrt{b}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}} + \frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] -(((b\*B - A\*c)\*x)/c^2) + (B\*x^3)/(3\*c) + (Sqrt[b]\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(5/2)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^4 (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{x^2 (A + Bx^2)}{b + cx^2} dx \\
&= \frac{Bx^3}{3c} - \frac{(3bB - 3Ac) \int \frac{x^2}{b+cx^2} dx}{3c} \\
&= -\frac{(bB - Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{(b(bB - Ac)) \int \frac{1}{b+cx^2} dx}{c^2} \\
&= -\frac{(bB - Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\sqrt{b} (bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 57, normalized size = 0.98

$$\frac{\sqrt{b} (bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{c^{5/2}} + \frac{x(Ac - bB)}{c^2} + \frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] ((-(b\*B) + A\*c)\*x)/c^2 + (B\*x^3)/(3\*c) + (Sqrt[b]\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/c^(5/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.40, size = 129, normalized size = 2.22

$$\left[ \frac{2 Bc x^3 - 3 (Bb - Ac) \sqrt{-\frac{b}{c}} \log \left( \frac{cx^2 - 2cx \sqrt{-\frac{b}{c}} - b}{cx^2 + b} \right) - 6 (Bb - Ac)x}{6 c^2}, \frac{Bc x^3 + 3 (Bb - Ac) \sqrt{\frac{b}{c}} \arctan \left( \frac{cx \sqrt{\frac{b}{c}}}{b} \right) - 3 (Bb - Ac)x}{3 c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] [1/6\*(2\*B\*c\*x^3 - 3\*(B\*b - A\*c)\*sqrt(-b/c)\*log((c\*x^2 - 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) - 6\*(B\*b - A\*c)\*x)/c^2, 1/3\*(B\*c\*x^3 + 3\*(B\*b - A\*c)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) - 3\*(B\*b - A\*c)\*x)/c^2]

**giac [A]** time = 0.16, size = 57, normalized size = 0.98

$$\frac{(Bb^2 - Abc) \arctan \left( \frac{cx}{\sqrt{bc}} \right)}{\sqrt{bc} c^2} + \frac{Bc^2 x^3 - 3 Bbcx + 3 Ac^2 x}{3 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")



[Out]  $(B*b^2 - A*b*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^2) + 1/3*(B*c^2*x^3 - 3*B*b*c*x + 3*A*c^2*x)/c^3$

maple [A] time = 0.05, size = 68, normalized size = 1.17

$$\frac{Bx^3}{3c} - \frac{Ab \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} c} + \frac{Bb^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} c^2} + \frac{Ax}{c} - \frac{Bbx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(B*x^2+A)/(c*x^4+b*x^2), x)`

[Out]  $1/3*B*x^3/c + 1/c*A*x - 1/c^2*b*B*x - b/c/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*A + b^2/c^2/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*B$

maxima [A] time = 2.86, size = 53, normalized size = 0.91

$$\frac{(Bb^2 - Abc) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} c^2} + \frac{Bcx^3 - 3(Bb - Ac)x}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2), x, algorithm="maxima")`

[Out]  $(B*b^2 - A*b*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^2) + 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2$

mupad [B] time = 0.11, size = 70, normalized size = 1.21

$$x \left( \frac{A}{c} - \frac{Bb}{c^2} \right) + \frac{Bx^3}{3c} + \frac{\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{b} \sqrt{c} x (Ac - Bb)}{Bb^2 - Abc}\right) (Ac - Bb)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(A + B*x^2))/(b*x^2 + c*x^4), x)`

[Out]  $x*(A/c - (B*b)/c^2) + (B*x^3)/(3*c) + (b^{(1/2)}*\operatorname{atan}((b^{(1/2)}*c^{(1/2)}*x*(A*c - B*b))/(B*b^2 - A*b*c))*(A*c - B*b)/c^{(5/2)}$

sympy [A] time = 0.34, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3c} + x \left( \frac{A}{c} - \frac{Bb}{c^2} \right) - \frac{\sqrt{-\frac{b}{c^5}} (-Ac + Bb) \log\left(-c^2 \sqrt{-\frac{b}{c^5}} + x\right)}{2} + \frac{\sqrt{-\frac{b}{c^5}} (-Ac + Bb) \log\left(c^2 \sqrt{-\frac{b}{c^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2), x)`

[Out]  $B*x**3/(3*c) + x*(A/c - B*b/c**2) - \sqrt{-b/c**5}*(-A*c + B*b)*\log(-c**2*\sqrt{-b/c**5} + x)/2 + \sqrt{-b/c**5}*(-A*c + B*b)*\log(c**2*\sqrt{-b/c**5} + x)/2$

$$3.48 \quad \int \frac{x^3(A+Bx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=35

$$\frac{Bx^2}{2c} - \frac{(bB - Ac) \log(b + cx^2)}{2c^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 444, 43}

$$\frac{Bx^2}{2c} - \frac{(bB - Ac) \log(b + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] (B\*x^2)/(2\*c) - ((b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^2)

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^3(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{x(A+Bx^2)}{b+cx^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{b+cx} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{B}{c} + \frac{-bB+Ac}{c(b+cx)} \right) dx, x, x^2 \right) \\ &= \frac{Bx^2}{2c} - \frac{(bB - Ac) \log(b + cx^2)}{2c^2} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 0.89

$$\frac{(Ac - bB) \log(b + cx^2) + Bcx^2}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (B\*c\*x^2 + (-b\*B) + A\*c)\*Log[b + c\*x^2]/(2\*c^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.39, size = 30, normalized size = 0.86

$$\frac{Bcx^2 - (Bb - Ac) \log(cx^2 + b)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/2\*(B\*c\*x^2 - (B\*b - A\*c)\*log(c\*x^2 + b))/c^2

**giac** [A] time = 0.23, size = 32, normalized size = 0.91

$$\frac{Bx^2}{2c} - \frac{(Bb - Ac) \log(|cx^2 + b|)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] 1/2\*B\*x^2/c - 1/2\*(B\*b - A\*c)\*log(abs(c\*x^2 + b))/c^2

**maple** [A] time = 0.04, size = 40, normalized size = 1.14

$$\frac{Bx^2}{2c} + \frac{A \ln(cx^2 + b)}{2c} - \frac{Bb \ln(cx^2 + b)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2), x)

[Out] 1/2\*B\*x^2/c+1/2/c\*ln(c\*x^2+b)\*A-1/2/c^2\*ln(c\*x^2+b)\*b\*B

**maxima** [A] time = 1.30, size = 31, normalized size = 0.89

$$\frac{Bx^2}{2c} - \frac{(Bb - Ac) \log(cx^2 + b)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="maxima")

[Out] 1/2\*B\*x^2/c - 1/2\*(B\*b - A\*c)\*log(c\*x^2 + b)/c^2

**mupad** [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{Bx^2}{2c} + \frac{\ln(cx^2 + b) (Ac - Bb)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(A + B*x^2))/(b*x^2 + c*x^4),x)`

[Out]  $(B*x^2)/(2*c) + (\log(b + c*x^2)*(A*c - B*b))/(2*c^2)$

sympy [A] time = 0.25, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2c} - \frac{(-Ac + Bb)\log(b + cx^2)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2),x)`

[Out]  $B*x**2/(2*c) - (-A*c + B*b)*\log(b + c*x**2)/(2*c**2)$

$$3.49 \quad \int \frac{x^2(A+Bx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=40

$$\frac{Bx}{c} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 388, 205}

$$\frac{Bx}{c} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (B\*x)/c - ((b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(Sqrt[b]\*c^(3/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^2(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{A+Bx^2}{b+cx^2} dx \\ &= \frac{Bx}{c} - \frac{(bB-Ac) \int \frac{1}{b+cx^2} dx}{c} \\ &= \frac{Bx}{c} - \frac{(bB-Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{Bx}{c} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] (B\*x)/c - ((b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(Sqrt[b]\*c^(3/2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.41, size = 99, normalized size = 2.48

$$\left[ \frac{2 Bbcx + (Bb - Ac)\sqrt{-bc} \log\left(\frac{cx^2 - 2\sqrt{-bc}x - b}{cx^2 + b}\right)}{2bc^2}, \frac{Bbcx - (Bb - Ac)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right)}{bc^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] [1/2\*(2\*B\*b\*c\*x + (B\*b - A\*c)\*sqrt(-b\*c)\*log((c\*x^2 - 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)))/(b\*c^2), (B\*b\*c\*x - (B\*b - A\*c)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b))/(b\*c^2)]

**giac** [A] time = 0.16, size = 34, normalized size = 0.85

$$\frac{Bx}{c} - \frac{(Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] B\*x/c - (B\*b - A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c)

**maple** [A] time = 0.04, size = 45, normalized size = 1.12

$$\frac{A \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}} - \frac{Bb \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} c} + \frac{Bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out] B\*x/c+1/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A-1/c/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*b\*B

**maxima** [A] time = 2.92, size = 34, normalized size = 0.85

$$\frac{Bx}{c} - \frac{(Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $Bx/c - (Bb - Ac) \cdot \arctan(cx/\sqrt{bc}) / (\sqrt{bc} \cdot c)$

**mupad [B]** time = 0.05, size = 31, normalized size = 0.78

$$\frac{Bx}{c} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(Ac - Bb)}{\sqrt{b}c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(A + B*x^2))/(b*x^2 + c*x^4), x)`

[Out]  $(Bx)/c + (\operatorname{atan}((c^{1/2})x/b^{1/2}))(Ac - Bb) / (b^{1/2}c^{3/2})$

**sympy [B]** time = 0.29, size = 82, normalized size = 2.05

$$\frac{Bx}{c} + \frac{\sqrt{-\frac{1}{bc^3}}(-Ac + Bb) \log\left(-bc\sqrt{-\frac{1}{bc^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{bc^3}}(-Ac + Bb) \log\left(bc\sqrt{-\frac{1}{bc^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2), x)`

[Out]  $Bx/c + \sqrt{-1/(bc^3)}(-Ac + Bb) \cdot \log(-bc \cdot \sqrt{-1/(bc^3)} + x) / 2 - \sqrt{-1/(bc^3)}(-Ac + Bb) \cdot \log(bc \cdot \sqrt{-1/(bc^3)} + x) / 2$

$$3.50 \quad \int \frac{x(A+Bx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=34

$$\frac{(bB - Ac) \log(b + cx^2)}{2bc} + \frac{A \log(x)}{b}$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 446, 72}

$$\frac{(bB - Ac) \log(b + cx^2)}{2bc} + \frac{A \log(x)}{b}$$

Antiderivative was successfully verified.

[In] Int[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] (A\*Log[x])/b + ((b\*B - A\*c)\*Log[b + c\*x^2])/(2\*b\*c)

#### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{x(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{A+Bx^2}{x(b+cx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x(b+cx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{bx} + \frac{bB-Ac}{b(b+cx)} \right) dx, x, x^2 \right) \\ &= \frac{A \log(x)}{b} + \frac{(bB - Ac) \log(b + cx^2)}{2bc} \end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{(bB - Ac) \log(b + cx^2)}{2bc} + \frac{A \log(x)}{b}$$



Antiderivative was successfully verified.

[In] Integrate[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (A\*Log[x])/b + ((b\*B - A\*c)\*Log[b + c\*x^2])/(2\*b\*c)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.39, size = 32, normalized size = 0.94

$$\frac{2Ac \log(x) + (Bb - Ac) \log(cx^2 + b)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/2\*(2\*A\*c\*log(x) + (B\*b - A\*c)\*log(c\*x^2 + b))/(b\*c)

**giac** [A] time = 0.20, size = 34, normalized size = 1.00

$$\frac{A \log(|x|)}{b} + \frac{(Bb - Ac) \log(|cx^2 + b|)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] A\*log(abs(x))/b + 1/2\*(B\*b - A\*c)\*log(abs(c\*x^2 + b))/(b\*c)

**maple** [A] time = 0.05, size = 37, normalized size = 1.09

$$\frac{A \ln(x)}{b} - \frac{A \ln(cx^2 + b)}{2b} + \frac{B \ln(cx^2 + b)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)/(c\*x^4+b\*x^2), x)

[Out] -1/2/b\*ln(c\*x^2+b)\*A+1/2/c\*ln(c\*x^2+b)\*B+A\*ln(x)/b

**maxima** [A] time = 1.34, size = 35, normalized size = 1.03

$$\frac{A \log(x^2)}{2b} + \frac{(Bb - Ac) \log(cx^2 + b)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="maxima")

[Out] 1/2\*A\*log(x^2)/b + 1/2\*(B\*b - A\*c)\*log(c\*x^2 + b)/(b\*c)

**mupad** [B] time = 0.08, size = 32, normalized size = 0.94

$$\frac{A \ln(x)}{b} - \frac{\ln(cx^2 + b)(Ac - Bb)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(A + B*x^2))/(b*x^2 + c*x^4),x)`

[Out] `(A*log(x))/b - (log(b + c*x^2)*(A*c - B*b))/(2*b*c)`

sympy [A] time = 0.75, size = 26, normalized size = 0.76

$$\frac{A \log(x)}{b} + \frac{(-Ac + Bb) \log\left(\frac{b}{c} + x^2\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x**2+A)/(c*x**4+b*x**2),x)`

[Out] `A*log(x)/b + (-A*c + B*b)*log(b/c + x**2)/(2*b*c)`

$$3.51 \quad \int \frac{A+Bx^2}{bx^2+cx^4} dx$$

Optimal. Leaf size=42

$$\frac{(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1593, 453, 205}

$$\frac{(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(b\*x^2 + c\*x^4), x]

[Out] -(A/(b\*x)) + ((b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(b^(3/2)\*Sqrt[c])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(n\*p)\*(a + b\*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps

$$\begin{aligned} \int \frac{A+Bx^2}{bx^2+cx^4} dx &= \int \frac{A+Bx^2}{x^2(b+cx^2)} dx \\ &= -\frac{A}{bx} - \frac{(-bB+Ac) \int \frac{1}{b+cx^2} dx}{b} \\ &= -\frac{A}{bx} + \frac{(bB-Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{b^{3/2}\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 1.00

$$\frac{(bB - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(b\*x^2 + c\*x^4), x]

[Out] -(A/(b\*x)) + ((b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(b^(3/2)\*Sqrt[c])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.40, size = 105, normalized size = 2.50

$$\left[ \frac{(Bb - Ac)\sqrt{-bc} x \log\left(\frac{cx^2 + 2\sqrt{-bc}x - b}{cx^2 + b}\right) - 2Abc}{2b^2cx}, \frac{(Bb - Ac)\sqrt{bc} x \arctan\left(\frac{\sqrt{bc}x}{b}\right) - Abc}{b^2cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [1/2\*((B\*b - A\*c)\*sqrt(-b\*c)\*x\*log((c\*x^2 + 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)) - 2\*A\*b\*c)/(b^2\*c\*x), ((B\*b - A\*c)\*sqrt(b\*c)\*x\*arctan(sqrt(b\*c)\*x/b) - A\*b\*c)/(b^2\*c\*x)]

**giac** [A] time = 0.16, size = 36, normalized size = 0.86

$$\frac{(Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] (B\*b - A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b) - A/(b\*x)

**maple** [A] time = 0.05, size = 48, normalized size = 1.14

$$-\frac{Ac \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b} + \frac{B \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(c\*x^4+b\*x^2), x)

[Out] -1/b/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A\*c+1/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B-A/b/x

**maxima** [A] time = 2.94, size = 36, normalized size = 0.86

$$\frac{(Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] (B\*b - A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b) - A/(b\*x)

**mupad [B]** time = 0.09, size = 35, normalized size = 0.83

$$-\frac{A}{bx} - \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(Ac - Bb)}{b^{3/2}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(b\*x^2 + c\*x^4),x)

[Out] - A/(b\*x) - (atan((c^(1/2)\*x)/b^(1/2))\*(A\*c - B\*b))/(b^(3/2)\*c^(1/2))

**sympy [B]** time = 0.42, size = 82, normalized size = 1.95

$$-\frac{A}{bx} - \frac{\sqrt{-\frac{1}{b^3c}}(-Ac + Bb)\log\left(-b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{b^3c}}(-Ac + Bb)\log\left(b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] -A/(b\*x) - sqrt(-1/(b\*\*3\*c))\*(-A\*c + B\*b)\*log(-b\*\*2\*sqrt(-1/(b\*\*3\*c)) + x)/2 + sqrt(-1/(b\*\*3\*c))\*(-A\*c + B\*b)\*log(b\*\*2\*sqrt(-1/(b\*\*3\*c)) + x)/2

$$3.52 \quad \int \frac{A+Bx^2}{bx^2-cx^4} dx$$

**Optimal.** Leaf size=41

$$\frac{(Ac + bB) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

**Rubi [A]** time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1593, 453, 208}

$$\frac{(Ac + bB) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(b\*x^2 - c\*x^4), x]

[Out] -(A/(b\*x)) + ((b\*B + A\*c)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[b]])/(b^(3/2)\*Sqrt[c])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{bx^2 - cx^4} dx &= \int \frac{A + Bx^2}{x^2(b - cx^2)} dx \\ &= -\frac{A}{bx} + \frac{(bB + Ac) \int \frac{1}{b - cx^2} dx}{b} \\ &= -\frac{A}{bx} + \frac{(bB + Ac) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 1.00

$$\frac{(Ac + bB) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(b\*x^2 - c\*x^4), x]

[Out] -(A/(b\*x)) + ((b\*B + A\*c)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[b]])/(b^(3/2)\*Sqrt[c])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{bx^2 - cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 - c\*x^4), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 - c\*x^4), x]

**fricas** [A] time = 0.42, size = 103, normalized size = 2.51

$$\left[ \frac{(Bb + Ac)\sqrt{bc} x \log\left(\frac{cx^2 + 2\sqrt{bc}x + b}{cx^2 - b}\right) - 2Abc}{2b^2cx}, -\frac{(Bb + Ac)\sqrt{-bc} x \arctan\left(\frac{\sqrt{-bc}x}{b}\right) + Abc}{b^2cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(-c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [1/2\*((B\*b + A\*c)\*sqrt(b\*c)\*x\*log((c\*x^2 + 2\*sqrt(b\*c)\*x + b)/(c\*x^2 - b)) - 2\*A\*b\*c)/(b^2\*c\*x), -((B\*b + A\*c)\*sqrt(-b\*c)\*x\*arctan(sqrt(-b\*c)\*x/b) + A\*b\*c)/(b^2\*c\*x)]

**giac** [A] time = 0.15, size = 38, normalized size = 0.93

$$-\frac{(Bb + Ac) \arctan\left(\frac{cx}{\sqrt{-bc}}\right)}{\sqrt{-bc} b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(-c\*x^4+b\*x^2), x, algorithm="giac")

[Out] -(B\*b + A\*c)\*arctan(c\*x/sqrt(-b\*c))/(sqrt(-b\*c)\*b) - A/(b\*x)

**maple** [A] time = 0.05, size = 39, normalized size = 0.95

$$-\frac{(-Ac - bB) \operatorname{arctanh}\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(-c\*x^4+b\*x^2), x)

[Out] -(-A\*c-B\*b)/b/(b\*c)^(1/2)\*arctanh(1/(b\*c)^(1/2)\*c\*x)-A/b/x

**maxima** [A] time = 3.06, size = 51, normalized size = 1.24

$$-\frac{(Bb + Ac) \log\left(\frac{cx - \sqrt{bc}}{cx + \sqrt{bc}}\right)}{2\sqrt{bc} b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(-c\*x^4+b\*x^2), x, algorithm="maxima")

[Out]  $-1/2*(B*b + A*c)*\log((c*x - \sqrt{b*c})/(c*x + \sqrt{b*c}))/(\sqrt{b*c}*b) - A/(b*x)$

**mupad [B]** time = 0.15, size = 33, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(Ac + Bb)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(b*x^2 - c*x^4), x)`

[Out]  $(\operatorname{atanh}((c^{1/2}*x)/b^{1/2})*(A*c + B*b))/(b^{3/2}*c^{1/2}) - A/(b*x)$

**sympy [B]** time = 0.44, size = 75, normalized size = 1.83

$$-\frac{A}{bx} - \frac{\sqrt{\frac{1}{b^3c}}(Ac + Bb)\log\left(-b^2\sqrt{\frac{1}{b^3c}} + x\right)}{2} + \frac{\sqrt{\frac{1}{b^3c}}(Ac + Bb)\log\left(b^2\sqrt{\frac{1}{b^3c}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)/(-c*x**4+b*x**2), x)`

[Out]  $-A/(b*x) - \sqrt{1/(b**3*c)}*(A*c + B*b)*\log(-b**2*\sqrt{1/(b**3*c)} + x)/2 + \sqrt{1/(b**3*c)}*(A*c + B*b)*\log(b**2*\sqrt{1/(b**3*c)} + x)/2$



$$3.53 \quad \int \frac{A+Bx^2}{x(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=49

$$-\frac{(bB - Ac) \log(b + cx^2)}{2b^2} + \frac{\log(x)(bB - Ac)}{b^2} - \frac{A}{2bx^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{(bB - Ac) \log(b + cx^2)}{2b^2} + \frac{\log(x)(bB - Ac)}{b^2} - \frac{A}{2bx^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)), x]

[Out] -A/(2\*b\*x^2) + ((b\*B - A\*c)\*Log[x])/b^2 - ((b\*B - A\*c)\*Log[b + c\*x^2])/(2\*b^2)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^3(b + cx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^2(b + cx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{bx^2} + \frac{bB - Ac}{b^2x} - \frac{c(bB - Ac)}{b^2(b + cx)} \right) dx, x, x^2 \right) \\ &= -\frac{A}{2bx^2} + \frac{(bB - Ac) \log(x)}{b^2} - \frac{(bB - Ac) \log(b + cx^2)}{2b^2} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{(Ac - bB) \log(b + cx^2)}{2b^2} + \frac{\log(x)(bB - Ac)}{b^2} - \frac{A}{2bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)), x]

[Out] -1/2\*A/(b\*x^2) + ((b\*B - A\*c)\*Log[x])/b^2 + ((-(b\*B) + A\*c)\*Log[b + c\*x^2])/(2\*b^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)), x]

**fricas** [A] time = 0.40, size = 47, normalized size = 0.96

$$\frac{(Bb - Ac)x^2 \log(cx^2 + b) - 2(Bb - Ac)x^2 \log(x) + Ab}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] -1/2\*((B\*b - A\*c)\*x^2\*log(c\*x^2 + b) - 2\*(B\*b - A\*c)\*x^2\*log(x) + A\*b)/(b^2\*x^2)

**giac** [A] time = 0.15, size = 71, normalized size = 1.45

$$\frac{(Bb - Ac) \log(x^2)}{2b^2} - \frac{(Bbc - Ac^2) \log(|cx^2 + b|)}{2b^2c} - \frac{Bbx^2 - Acx^2 + Ab}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] 1/2\*(B\*b - A\*c)\*log(x^2)/b^2 - 1/2\*(B\*b\*c - A\*c^2)\*log(abs(c\*x^2 + b))/(b^2\*c) - 1/2\*(B\*b\*x^2 - A\*c\*x^2 + A\*b)/(b^2\*x^2)

**maple** [A] time = 0.06, size = 56, normalized size = 1.14

$$-\frac{Ac \ln(x)}{b^2} + \frac{Ac \ln(cx^2 + b)}{2b^2} + \frac{B \ln(x)}{b} - \frac{B \ln(cx^2 + b)}{2b} - \frac{A}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x/(c\*x^4+b\*x^2), x)

[Out] 1/2/b^2\*ln(c\*x^2+b)\*A\*c-1/2/b\*ln(c\*x^2+b)\*B-1/2\*A/b/x^2-1/b^2\*ln(x)\*A\*c+1/b\*ln(x)\*B

**maxima** [A] time = 1.32, size = 48, normalized size = 0.98

$$-\frac{(Bb - Ac) \log(cx^2 + b)}{2b^2} + \frac{(Bb - Ac) \log(x^2)}{2b^2} - \frac{A}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $-1/2*(B*b - A*c)*\log(c*x^2 + b)/b^2 + 1/2*(B*b - A*c)*\log(x^2)/b^2 - 1/2*A/(b*x^2)$

mupad [B] time = 0.14, size = 46, normalized size = 0.94

$$\frac{\ln(c x^2 + b) (A c - B b)}{2 b^2} - \frac{A}{2 b x^2} - \frac{\ln(x) (A c - B b)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)),x)

[Out]  $(\log(b + c*x^2)*(A*c - B*b))/(2*b^2) - A/(2*b*x^2) - (\log(x)*(A*c - B*b))/b^2$

sympy [A] time = 0.76, size = 41, normalized size = 0.84

$$-\frac{A}{2bx^2} + \frac{(-Ac + Bb)\log(x)}{b^2} - \frac{(-Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x/(c\*x\*\*4+b\*x\*\*2),x)

[Out]  $-A/(2*b*x**2) + (-A*c + B*b)*\log(x)/b**2 - (-A*c + B*b)*\log(b/c + x**2)/(2*b**2)$

$$3.54 \quad \int \frac{A+Bx^2}{x^2(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=61

$$-\frac{\sqrt{c}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{b^{5/2}} - \frac{bB - Ac}{b^2x} - \frac{A}{3bx^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 453, 325, 205}

$$-\frac{bB - Ac}{b^2x} - \frac{\sqrt{c}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{b^{5/2}} - \frac{A}{3bx^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)),x]

[Out] -A/(3\*b\*x^3) - (b\*B - A\*c)/(b^2\*x) - (Sqrt[c]\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/b^(5/2)

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m+n\*p)\*(a + b\*x^(q-p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^2(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^4(b + cx^2)} dx \\
&= -\frac{A}{3bx^3} - \frac{(-3bB + 3Ac) \int \frac{1}{x^2(b+cx^2)} dx}{3b} \\
&= -\frac{A}{3bx^3} - \frac{bB - Ac}{b^2x} - \frac{(c(bB - Ac)) \int \frac{1}{b+cx^2} dx}{b^2} \\
&= -\frac{A}{3bx^3} - \frac{bB - Ac}{b^2x} - \frac{\sqrt{c}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 60, normalized size = 0.98

$$-\frac{\sqrt{c}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}} + \frac{Ac - bB}{b^2x} - \frac{A}{3bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)), x]

[Out] -1/3\*A/(b\*x^3) + (- (b\*B) + A\*c)/(b^2\*x) - (Sqrt[c]\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/b^(5/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2(bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)), x]

**fricas [A]** time = 0.41, size = 135, normalized size = 2.21

$$\left[ \frac{3(Bb - Ac)x^3 \sqrt{\frac{-c}{b}} \log\left(\frac{cx^2 + 2bx\sqrt{\frac{-c}{b}} - b}{cx^2 + b}\right) + 6(Bb - Ac)x^2 + 2Ab}{6b^2x^3}, -\frac{3(Bb - Ac)x^3 \sqrt{\frac{c}{b}} \arctan\left(x\sqrt{\frac{c}{b}}\right) + 3(Bb - Ac)x^2 + Ab}{3b^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [-1/6\*(3\*(B\*b - A\*c)\*x^3\*sqrt(-c/b)\*log((c\*x^2 + 2\*b\*x\*sqrt(-c/b) - b)/(c\*x^2 + b)) + 6\*(B\*b - A\*c)\*x^2 + 2\*A\*b)/(b^2\*x^3), -1/3\*(3\*(B\*b - A\*c)\*x^3\*sqrt(c/b)\*arctan(x\*sqrt(c/b)) + 3\*(B\*b - A\*c)\*x^2 + A\*b)/(b^2\*x^3)]

**giac [A]** time = 0.16, size = 57, normalized size = 0.93

$$\frac{(Bbc - Ac^2) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b^2} - \frac{3Bbx^2 - 3Acx^2 + Ab}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out]  $-(B*b*c - A*c^2)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b^2) - 1/3*(3*B*b*x^2 - 3*A*c*x^2 + A*b)/(b^2*x^3)$

**maple** [A] time = 0.06, size = 72, normalized size = 1.18

$$\frac{A c^2 \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c} b^2} - \frac{B c \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c} b} + \frac{A c}{b^2 x} - \frac{B}{b x} - \frac{A}{3 b x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/x^2/(c*x^4+b*x^2),x)`

[Out]  $c^2/b^2/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*A-c/b/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*B-1/3*A/b/x^3+1/b^2/x*A*c-1/b/x*B$

**maxima** [A] time = 2.82, size = 56, normalized size = 0.92

$$-\frac{(Bbc - Ac^2) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b^2} - \frac{3(Bb - Ac)x^2 + Ab}{3 b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="maxima")`

[Out]  $-(B*b*c - A*c^2)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b^2) - 1/3*(3*(B*b - A*c)*x^2 + A*b)/(b^2*x^3)$

**mupad** [B] time = 0.11, size = 53, normalized size = 0.87

$$\frac{\sqrt{c} \operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{b}}\right) (A c - B b)}{b^{5/2}} - \frac{A}{3 b} - \frac{x^2 (A c - B b)}{b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x)`

[Out]  $(c^{(1/2)}*\operatorname{atan}((c^{(1/2)}*x)/b^{(1/2)})*(A*c - B*b))/b^{(5/2)} - (A/(3*b) - (x^2*(A*c - B*b))/b^2)/x^3$

**sympy** [B] time = 0.44, size = 129, normalized size = 2.11

$$\frac{\sqrt{-\frac{c}{b^5}} (-Ac + Bb) \log\left(-\frac{b^3 \sqrt{-\frac{c}{b^5}} (-Ac+Bb)}{-Ac^2+Bbc} + x\right)}{2} - \frac{\sqrt{-\frac{c}{b^5}} (-Ac + Bb) \log\left(\frac{b^3 \sqrt{-\frac{c}{b^5}} (-Ac+Bb)}{-Ac^2+Bbc} + x\right)}{2} + \frac{-Ab + x^2 (3Ac - 3Bb)}{3b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)/x**2/(c*x**4+b*x**2),x)`

[Out]  $\sqrt{-c/b**5}*(-A*c + B*b)*\log(-b**3*\sqrt{-c/b**5}*(-A*c + B*b)/(-A*c**2 + B*b*c) + x)/2 - \sqrt{-c/b**5}*(-A*c + B*b)*\log(b**3*\sqrt{-c/b**5}*(-A*c + B*b)/(-A*c**2 + B*b*c) + x)/2 + (-A*b + x**2*(3*A*c - 3*B*b))/(3*b**2*x**3)$

$$3.55 \quad \int \frac{A+Bx^2}{x^3(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=70

$$\frac{c(bB - Ac) \log(b + cx^2)}{2b^3} - \frac{c \log(x)(bB - Ac)}{b^3} - \frac{bB - Ac}{2b^2x^2} - \frac{A}{4bx^4}$$

**Rubi [A]** time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{bB - Ac}{2b^2x^2} + \frac{c(bB - Ac) \log(b + cx^2)}{2b^3} - \frac{c \log(x)(bB - Ac)}{b^3} - \frac{A}{4bx^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)),x]

[Out] -A/(4\*b\*x^4) - (b\*B - A\*c)/(2\*b^2\*x^2) - (c\*(b\*B - A\*c)\*Log[x])/b^3 + (c\*(b\*B - A\*c)\*Log[b + c\*x^2])/(2\*b^3)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^3(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^5(b + cx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^3(b + cx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{bx^3} + \frac{bB - Ac}{b^2x^2} - \frac{c(bB - Ac)}{b^3x} + \frac{c^2(bB - Ac)}{b^3(b + cx)} \right) dx, x, x^2 \right) \\ &= -\frac{A}{4bx^4} - \frac{bB - Ac}{2b^2x^2} - \frac{c(bB - Ac) \log(x)}{b^3} + \frac{c(bB - Ac) \log(b + cx^2)}{2b^3} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 70, normalized size = 1.00

$$\frac{4cx^4 \log(x)(Ac - bB) - b(Ab - 2Acx^2 + 2bBx^2) + 2cx^4(bB - Ac) \log(b + cx^2)}{4b^3x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)), x]

[Out]  $(-(b*(A*b + 2*b*B*x^2 - 2*A*c*x^2)) + 4*c*(-(b*B) + A*c)*x^4*\text{Log}[x] + 2*c*(b*B - A*c)*x^4*\text{Log}[b + c*x^2])/(4*b^3*x^4)$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3(bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)), x]

**fricas** [A] time = 0.41, size = 73, normalized size = 1.04

$$\frac{2(Bbc - Ac^2)x^4 \log(cx^2 + b) - 4(Bbc - Ac^2)x^4 \log(x) - Ab^2 - 2(Bb^2 - Abc)x^2}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out]  $1/4*(2*(B*b*c - A*c^2)*x^4*\log(c*x^2 + b) - 4*(B*b*c - A*c^2)*x^4*\log(x) - A*b^2 - 2*(B*b^2 - A*b*c)*x^2)/(b^3*x^4)$

**giac** [A] time = 0.19, size = 100, normalized size = 1.43

$$-\frac{(Bbc - Ac^2) \log(x^2)}{2b^3} + \frac{(Bbc^2 - Ac^3) \log(|cx^2 + b|)}{2b^3c} + \frac{3Bbcx^4 - 3Ac^2x^4 - 2Bb^2x^2 + 2Abcx^2 - Ab^2}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out]  $-1/2*(B*b*c - A*c^2)*\log(x^2)/b^3 + 1/2*(B*b*c^2 - A*c^3)*\log(\text{abs}(c*x^2 + b))/(b^3*c) + 1/4*(3*B*b*c*x^4 - 3*A*c^2*x^4 - 2*B*b^2*x^2 + 2*A*b*c*x^2 - A*b^2)/(b^3*x^4)$

**maple** [A] time = 0.05, size = 81, normalized size = 1.16

$$\frac{Ac^2 \ln(x)}{b^3} - \frac{Ac^2 \ln(cx^2 + b)}{2b^3} - \frac{Bc \ln(x)}{b^2} + \frac{Bc \ln(cx^2 + b)}{2b^2} + \frac{Ac}{2b^2x^2} - \frac{B}{2bx^2} - \frac{A}{4bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^3/(c\*x^4+b\*x^2), x)

[Out]  $-1/2*c^2/b^3*\ln(c*x^2+b)*A+1/2*c/b^2*\ln(c*x^2+b)*B-1/4*A/b/x^4+1/2/b^2/x^2*A*c-1/2/b/x^2*B+c^2/b^3*\ln(x)*A-c/b^2*\ln(x)*B$

**maxima** [A] time = 1.38, size = 70, normalized size = 1.00

$$\frac{(Bbc - Ac^2) \log(cx^2 + b)}{2b^3} - \frac{(Bbc - Ac^2) \log(x^2)}{2b^3} - \frac{2(Bb - Ac)x^2 + Ab}{4b^2x^4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(B*b*c - A*c^2)*\log(c*x^2 + b)/b^3 - \frac{1}{2}*(B*b*c - A*c^2)*\log(x^2)/b^3 - \frac{1}{4}*(2*(B*b - A*c)*x^2 + A*b)/(b^2*x^4)$

**mupad [B]** time = 0.14, size = 70, normalized size = 1.00

$$\frac{\ln(x) (A c^2 - B b c)}{b^3} - \frac{\ln(c x^2 + b) (A c^2 - B b c)}{2 b^3} - \frac{\frac{A}{4 b} - \frac{x^2 (A c - B b)}{2 b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)),x)

[Out]  $(\log(x)*(A*c^2 - B*b*c))/b^3 - (\log(b + c*x^2)*(A*c^2 - B*b*c))/(2*b^3) - (A/(4*b) - (x^2*(A*c - B*b))/(2*b^2))/x^4$

**sympy [A]** time = 0.92, size = 61, normalized size = 0.87

$$\frac{-Ab + x^2 (2Ac - 2Bb)}{4b^2x^4} - \frac{c(-Ac + Bb) \log(x)}{b^3} + \frac{c(-Ac + Bb) \log\left(\frac{b}{c} + x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*3/(c\*x\*\*4+b\*x\*\*2),x)

[Out]  $(-A*b + x**2*(2*A*c - 2*B*b))/(4*b**2*x**4) - c*(-A*c + B*b)*\log(x)/b**3 + c*(-A*c + B*b)*\log(b/c + x**2)/(2*b**3)$

$$3.56 \quad \int \frac{A+Bx^2}{x^4(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=78

$$\frac{c^{3/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{b^{7/2}} + \frac{c(bB - Ac)}{b^3x} - \frac{bB - Ac}{3b^2x^3} - \frac{A}{5bx^5}$$

**Rubi [A]** time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 453, 325, 205}

$$\frac{c^{3/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{b^{7/2}} - \frac{bB - Ac}{3b^2x^3} + \frac{c(bB - Ac)}{b^3x} - \frac{A}{5bx^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)),x]

[Out] -A/(5\*b\*x^5) - (b\*B - A\*c)/(3\*b^2\*x^3) + (c\*(b\*B - A\*c))/(b^3\*x) + (c^(3/2)\*  
\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/b^(7/2)

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 325

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^4(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^6(b + cx^2)} dx \\
&= -\frac{A}{5bx^5} - \frac{(-5bB + 5Ac) \int \frac{1}{x^4(b+cx^2)} dx}{5b} \\
&= -\frac{A}{5bx^5} - \frac{bB - Ac}{3b^2x^3} - \frac{(c(bB - Ac)) \int \frac{1}{x^2(b+cx^2)} dx}{b^2} \\
&= -\frac{A}{5bx^5} - \frac{bB - Ac}{3b^2x^3} + \frac{c(bB - Ac)}{b^3x} + \frac{(c^2(bB - Ac)) \int \frac{1}{b+cx^2} dx}{b^3} \\
&= -\frac{A}{5bx^5} - \frac{bB - Ac}{3b^2x^3} + \frac{c(bB - Ac)}{b^3x} + \frac{c^{3/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 78, normalized size = 1.00

$$\frac{c^{3/2}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}} + \frac{c(bB - Ac)}{b^3x} + \frac{Ac - bB}{3b^2x^3} - \frac{A}{5bx^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)), x]

[Out] -1/5\*A/(b\*x^5) + (- (b\*B) + A\*c)/(3\*b^2\*x^3) + (c\*(b\*B - A\*c))/(b^3\*x) + (c^(3/2)\*(b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/b^(7/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^4(bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)), x]

**fricas [A]** time = 0.41, size = 184, normalized size = 2.36

$$\left[ \frac{15(Bbc - Ac^2)x^5 \sqrt{\frac{c}{b}} \log\left(\frac{cx^2 - 2bx\sqrt{\frac{c}{b}} - b}{cx^2 + b}\right) - 30(Bbc - Ac^2)x^4 + 6Ab^2 + 10(Bb^2 - Abc)x^2}{30b^3x^5}, \frac{15(Bbc - Ac^2)x^5 \sqrt{\frac{c}{b}} \arctan\left(x\sqrt{\frac{c}{b}}\right) + 15(Bbc - Ac^2)x^4 - 3Ab^2 - 5(Bb^2 - Abc)x^2}{15b^3x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^4/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] [-1/30\*(15\*(B\*b\*c - A\*c^2)\*x^5\*sqrt(-c/b)\*log((c\*x^2 - 2\*b\*x\*sqrt(-c/b) - b)/(c\*x^2 + b)) - 30\*(B\*b\*c - A\*c^2)\*x^4 + 6\*A\*b^2 + 10\*(B\*b^2 - A\*b\*c)\*x^2)/(b^3\*x^5), 1/15\*(15\*(B\*b\*c - A\*c^2)\*x^5\*sqrt(c/b)\*arctan(x\*sqrt(c/b)) + 15\*(B\*b\*c - A\*c^2)\*x^4 - 3\*A\*b^2 - 5\*(B\*b^2 - A\*b\*c)\*x^2)/(b^3\*x^5)]

**giac [A]** time = 0.16, size = 81, normalized size = 1.04

$$\frac{(Bbc^2 - Ac^3) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc} b^3} + \frac{15 Bbcx^4 - 15 Ac^2x^4 - 5 Bb^2x^2 + 5 Abcx^2 - 3 Ab^2}{15 b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^4/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] (B\*b\*c^2 - A\*c^3)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^3) + 1/15\*(15\*B\*b\*c\*x^4 - 15\*A\*c^2\*x^4 - 5\*B\*b^2\*x^2 + 5\*A\*b\*c\*x^2 - 3\*A\*b^2)/(b^3\*x^5)

**maple** [A] time = 0.05, size = 96, normalized size = 1.23

$$-\frac{A c^3 \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c} b^3} + \frac{B c^2 \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c} b^2} - \frac{A c^2}{b^3 x} + \frac{B c}{b^2 x} + \frac{A c}{3 b^2 x^3} - \frac{B}{3 b x^3} - \frac{A}{5 b x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^4/(c\*x^4+b\*x^2),x)

[Out] -c^3/b^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A+c^2/b^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B-1/5\*A/b/x^5+1/3/b^2/x^3\*A\*c-1/3/b/x^3\*B-c^2/b^3/x\*A+c/b^2/x\*B

**maxima** [A] time = 3.05, size = 79, normalized size = 1.01

$$\frac{(B b c^2 - A c^3) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c} b^3} + \frac{15 (B b c - A c^2) x^4 - 3 A b^2 - 5 (B b^2 - A b c) x^2}{15 b^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^4/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] (B\*b\*c^2 - A\*c^3)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^3) + 1/15\*(15\*(B\*b\*c - A\*c^2)\*x^4 - 3\*A\*b^2 - 5\*(B\*b^2 - A\*b\*c)\*x^2)/(b^3\*x^5)

**mupad** [B] time = 0.11, size = 70, normalized size = 0.90

$$-\frac{\frac{A}{5 b} - \frac{x^2 (A c - B b)}{3 b^2} + \frac{c x^4 (A c - B b)}{b^3}}{x^5} - \frac{c^{3/2} \operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{b}}\right) (A c - B b)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)),x)

[Out] - (A/(5\*b) - (x^2\*(A\*c - B\*b))/(3\*b^2) + (c\*x^4\*(A\*c - B\*b))/b^3)/x^5 - (c^(3/2)\*atan((c^(1/2)\*x)/b^(1/2))\*(A\*c - B\*b))/b^(7/2)

**sympy** [B] time = 0.58, size = 163, normalized size = 2.09

$$-\frac{\sqrt{-\frac{c^3}{b^7}} (-A c + B b) \log\left(-\frac{b^4 \sqrt{\frac{c^3}{b^7}} (-A c + B b)}{-A c^3 + B b c^2} + x\right)}{2} + \frac{\sqrt{-\frac{c^3}{b^7}} (-A c + B b) \log\left(\frac{b^4 \sqrt{\frac{c^3}{b^7}} (-A c + B b)}{-A c^3 + B b c^2} + x\right)}{2} + \frac{-3 A b^2 + x^4 (-15 A c^2 + 15 B b c) + x^2 (5 A b c - 5 B b^2)}{15 b^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*4/(c\*x\*\*4+b\*x\*\*2),x)

[Out] -sqrt(-c\*\*3/b\*\*7)\*(-A\*c + B\*b)\*log(-b\*\*4\*sqrt(-c\*\*3/b\*\*7)\*(-A\*c + B\*b)/(-A\*c\*\*3 + B\*b\*c\*\*2) + x)/2 + sqrt(-c\*\*3/b\*\*7)\*(-A\*c + B\*b)\*log(b\*\*4\*sqrt(-c\*\*3/b\*\*7)\*(-A\*c + B\*b)/(-A\*c\*\*3 + B\*b\*c\*\*2) + x)/2 + (-3\*A\*b\*\*2 + x\*\*4\*(-15\*A\*c\*\*2 + 15\*B\*b\*c) + x\*\*2\*(5\*A\*b\*c - 5\*B\*b\*\*2))/(15\*b\*\*3\*x\*\*5)

$$3.57 \quad \int \frac{A+Bx^2}{x^5(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=92

$$-\frac{c^2(bB - Ac) \log(b + cx^2)}{2b^4} + \frac{c^2 \log(x)(bB - Ac)}{b^4} + \frac{c(bB - Ac)}{2b^3x^2} - \frac{bB - Ac}{4b^2x^4} - \frac{A}{6bx^6}$$

**Rubi [A]** time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{c^2(bB - Ac) \log(b + cx^2)}{2b^4} + \frac{c^2 \log(x)(bB - Ac)}{b^4} + \frac{c(bB - Ac)}{2b^3x^2} - \frac{bB - Ac}{4b^2x^4} - \frac{A}{6bx^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)),x]

[Out] -A/(6\*b\*x^6) - (b\*B - A\*c)/(4\*b^2\*x^4) + (c\*(b\*B - A\*c))/(2\*b^3\*x^2) + (c^2\*(b\*B - A\*c)\*Log[x])/b^4 - (c^2\*(b\*B - A\*c)\*Log[b + c\*x^2])/(2\*b^4)

#### Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^5(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^7(b + cx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^4(b + cx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{bx^4} + \frac{bB - Ac}{b^2x^3} - \frac{c(bB - Ac)}{b^3x^2} + \frac{c^2(bB - Ac)}{b^4x} - \frac{c^3(bB - Ac)}{b^4(b + cx)} \right) dx, x, x^2 \right) \\ &= -\frac{A}{6bx^6} - \frac{bB - Ac}{4b^2x^4} + \frac{c(bB - Ac)}{2b^3x^2} + \frac{c^2(bB - Ac) \log(x)}{b^4} - \frac{c^2(bB - Ac) \log(b + cx^2)}{2b^4} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 96, normalized size = 1.04

$$\frac{(Ac^3 - bBc^2) \log(b + cx^2)}{2b^4} + \frac{\log(x)(bBc^2 - Ac^3)}{b^4} + \frac{c(bB - Ac)}{2b^3x^2} + \frac{Ac - bB}{4b^2x^4} - \frac{A}{6bx^6}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)), x]

[Out] -1/6\*A/(b\*x^6) + (-b\*B) + A\*c)/(4\*b^2\*x^4) + (c\*(b\*B - A\*c))/(2\*b^3\*x^2) + ((b\*B\*c^2 - A\*c^3)\*Log[x])/b^4 + ((-b\*B\*c^2) + A\*c^3)\*Log[b + c\*x^2]/(2\*b^4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5(bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)), x]

**fricas [A]** time = 0.39, size = 98, normalized size = 1.07

$$\frac{6(Bbc^2 - Ac^3)x^6 \log(cx^2 + b) - 12(Bbc^2 - Ac^3)x^6 \log(x) - 6(Bb^2c - Abc^2)x^4 + 2Ab^3 + 3(Bb^3 - Ab^2c)x^2}{12b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] -1/12\*(6\*(B\*b\*c^2 - A\*c^3)\*x^6\*log(c\*x^2 + b) - 12\*(B\*b\*c^2 - A\*c^3)\*x^6\*log(x) - 6\*(B\*b^2\*c - A\*b\*c^2)\*x^4 + 2\*A\*b^3 + 3\*(B\*b^3 - A\*b^2\*c)\*x^2)/(b^4\*x^6)

**giac [A]** time = 0.16, size = 126, normalized size = 1.37

$$\frac{(Bbc^2 - Ac^3) \log(x^2)}{2b^4} - \frac{(Bbc^3 - Ac^4) \log(cx^2 + b)}{2b^4c} - \frac{11Bbc^2x^6 - 11Ac^3x^6 - 6Bb^2cx^4 + 6Abc^2x^4 + 3Bb^3x^2 - 3Ab^2cx^2 + 2Ab^3}{12b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] 1/2\*(B\*b\*c^2 - A\*c^3)\*log(x^2)/b^4 - 1/2\*(B\*b\*c^3 - A\*c^4)\*log(abs(c\*x^2 + b))/(b^4\*c) - 1/12\*(11\*B\*b\*c^2\*x^6 - 11\*A\*c^3\*x^6 - 6\*B\*b^2\*c\*x^4 + 6\*A\*b\*c^2\*x^4 + 3\*B\*b^3\*x^2 - 3\*A\*b^2\*c\*x^2 + 2\*A\*b^3)/(b^4\*x^6)

**maple [A]** time = 0.05, size = 107, normalized size = 1.16

$$-\frac{Ac^3 \ln(x)}{b^4} + \frac{Ac^3 \ln(cx^2 + b)}{2b^4} + \frac{Bc^2 \ln(x)}{b^3} - \frac{Bc^2 \ln(cx^2 + b)}{2b^3} - \frac{Ac^2}{2b^3x^2} + \frac{Bc}{2b^2x^2} + \frac{Ac}{4b^2x^4} - \frac{B}{4bx^4} - \frac{A}{6bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^5/(c\*x^4+b\*x^2), x)

[Out] 1/2\*c^3/b^4\*ln(c\*x^2+b)\*A-1/2\*c^2/b^3\*ln(c\*x^2+b)\*B-1/6\*A/b/x^6+1/4/b^2/x^4\*A\*c-1/4/b/x^4\*B-1/2\*c^2/b^3/x^2\*A+1/2\*c/b^2/x^2\*B-c^3/b^4\*ln(x)\*A+c^2/b^3\*ln(x)\*B

**maxima [A]** time = 1.28, size = 96, normalized size = 1.04

$$-\frac{(Bbc^2 - Ac^3)\log(cx^2 + b)}{2b^4} + \frac{(Bbc^2 - Ac^3)\log(x^2)}{2b^4} + \frac{6(Bbc - Ac^2)x^4 - 2Ab^2 - 3(Bb^2 - Abc)x^2}{12b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] -1/2\*(B\*b\*c^2 - A\*c^3)\*log(c\*x^2 + b)/b^4 + 1/2\*(B\*b\*c^2 - A\*c^3)\*log(x^2)/b^4 + 1/12\*(6\*(B\*b\*c - A\*c^2)\*x^4 - 2\*A\*b^2 - 3\*(B\*b^2 - A\*b\*c)\*x^2)/(b^3\*x^6)

**mupad [B]** time = 0.15, size = 92, normalized size = 1.00

$$\frac{\ln(cx^2 + b)(Ac^3 - Bbc^2)}{2b^4} - \frac{\frac{A}{6b} - \frac{x^2(Ac - Bb)}{4b^2} + \frac{cx^4(Ac - Bb)}{2b^3}}{x^6} - \frac{\ln(x)(Ac^3 - Bbc^2)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)),x)

[Out] (log(b + c\*x^2)\*(A\*c^3 - B\*b\*c^2))/(2\*b^4) - (A/(6\*b) - (x^2\*(A\*c - B\*b))/(4\*b^2) + (c\*x^4\*(A\*c - B\*b))/(2\*b^3))/x^6 - (log(x)\*(A\*c^3 - B\*b\*c^2))/b^4

**sympy [A]** time = 1.41, size = 88, normalized size = 0.96

$$\frac{-2Ab^2 + x^4(-6Ac^2 + 6Bbc) + x^2(3Abc - 3Bb^2)}{12b^3x^6} + \frac{c^2(-Ac + Bb)\log(x)}{b^4} - \frac{c^2(-Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*5/(c\*x\*\*4+b\*x\*\*2),x)

[Out] (-2\*A\*b\*\*2 + x\*\*4\*(-6\*A\*c\*\*2 + 6\*B\*b\*c) + x\*\*2\*(3\*A\*b\*c - 3\*B\*b\*\*2))/(12\*b\*\*3\*x\*\*6) + c\*\*2\*(-A\*c + B\*b)\*log(x)/b\*\*4 - c\*\*2\*(-A\*c + B\*b)\*log(b/c + x\*\*2)/(2\*b\*\*4)

$$3.58 \quad \int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=133

$$\frac{b^{5/2}(9bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}} - \frac{b^3x(bB - Ac)}{2c^5(b + cx^2)} - \frac{b^2x(4bB - 3Ac)}{c^5} + \frac{bx^3(3bB - 2Ac)}{3c^4} - \frac{x^5(2bB - Ac)}{5c^3} + \frac{Bx^7}{7c^2}$$

**Rubi [A]** time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 455, 1810, 205}

$$-\frac{b^3x(bB - Ac)}{2c^5(b + cx^2)} - \frac{b^2x(4bB - 3Ac)}{c^5} + \frac{b^{5/2}(9bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}} - \frac{x^5(2bB - Ac)}{5c^3} + \frac{bx^3(3bB - 2Ac)}{3c^4} + \frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((b^2\*(4\*b\*B - 3\*A\*c)\*x)/c^5) + (b\*(3\*b\*B - 2\*A\*c)\*x^3)/(3\*c^4) - ((2\*b\*B - A\*c)\*x^5)/(5\*c^3) + (B\*x^7)/(7\*c^2) - (b^3\*(b\*B - A\*c)\*x)/(2\*c^5\*(b + c\*x^2)) + (b^(5/2)\*(9\*b\*B - 7\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(11/2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 455**

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1)/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rule 1810**

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**



$$\begin{aligned}
\int \frac{x^{12} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^8 (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{b^3(bB - Ac)x}{2c^5(b + cx^2)} - \frac{\int \frac{-b^3(bB - Ac) + 2b^2c(bB - Ac)x^2 - 2bc^2(bB - Ac)x^4 + 2c^3(bB - Ac)x^6 - 2Bc^4x^8}{b + cx^2} dx}{2c^5} \\
&= -\frac{b^3(bB - Ac)x}{2c^5(b + cx^2)} - \frac{\int (2b^2(4bB - 3Ac) - 2bc(3bB - 2Ac)x^2 + 2c^2(2bB - Ac)x^4 - 2Bc^3x^6}{2c^5} dx}{2c^5} \\
&= -\frac{b^2(4bB - 3Ac)x}{c^5} + \frac{b(3bB - 2Ac)x^3}{3c^4} - \frac{(2bB - Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB - Ac)x}{2c^5(b + cx^2)} + \frac{(b^3(9bB - 7Ac))}{2c^{11/2}} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right) \\
&= -\frac{b^2(4bB - 3Ac)x}{c^5} + \frac{b(3bB - 2Ac)x^3}{3c^4} - \frac{(2bB - Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB - Ac)x}{2c^5(b + cx^2)} + \frac{b^{5/2}(9bB - 7Ac)}{2c^{11/2}} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 134, normalized size = 1.01

$$\frac{b^{5/2}(9bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2c^{11/2}} - \frac{b^2x(4bB - 3Ac)}{c^5} + \frac{x(Ab^3c - b^4B)}{2c^5(b + cx^2)} + \frac{bx^3(3bB - 2Ac)}{3c^4} + \frac{x^5(Ac - 2bB)}{5c^3} + \frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((b^2\*(4\*b\*B - 3\*A\*c)\*x)/c^5) + (b\*(3\*b\*B - 2\*A\*c)\*x^3)/(3\*c^4) + ((-2\*b\*B + A\*c)\*x^5)/(5\*c^3) + (B\*x^7)/(7\*c^2) + (((-b^4\*B) + A\*b^3\*c)\*x)/(2\*c^5\*(b + c\*x^2)) + (b^(5/2)\*(9\*b\*B - 7\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(11/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12} (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.40, size = 350, normalized size = 2.63

$$\frac{60Bc^{12} - 12(9Bb^2 - 7Ac^2)x^2 + 28(9Bb^2 - 7Ac^2)x^4 - 140(9Bb^2 - 7Ac^2)x^6 - 105(9Bb^2 - 7Ac^2)x^8 - 105(9Bb^2 - 7Ac^2)x^{10} - 105(9Bb^2 - 7Ac^2)x^{12}}{420(c^2 + b^2)} \sqrt{-b/c} \log\left(\frac{x^2 - 2cx\sqrt{-b/c}}{c^2 + b^2}\right) - 210(9Bb^2 - 7Ac^2)x^3 + 30Bc^{12} - 6(9Bb^2 - 7Ac^2)x^5 + 14(9Bb^2 - 7Ac^2)x^7 - 70(9Bb^2 - 7Ac^2)x^9 + 105(9Bb^2 - 7Ac^2)x^{11} - 105(9Bb^2 - 7Ac^2)x^{13}}{210(c^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [1/420\*(60\*B\*c^4\*x^9 - 12\*(9\*B\*b\*c^3 - 7\*A\*c^4)\*x^7 + 28\*(9\*B\*b^2\*c^2 - 7\*A\*b\*c^3)\*x^5 - 140\*(9\*B\*b^2\*c - 7\*A\*b^2\*c^2)\*x^3 - 105\*(9\*B\*b^4 - 7\*A\*b^3\*c + (9\*B\*b^3\*c - 7\*A\*b^2\*c^2)\*x^2)\*sqrt(-b/c)\*log((c\*x^2 - 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) - 210\*(9\*B\*b^4 - 7\*A\*b^3\*c)\*x)/(c^6\*x^2 + b\*c^5), 1/210\*(30\*B\*c^4\*x^9 - 6\*(9\*B\*b\*c^3 - 7\*A\*c^4)\*x^7 + 14\*(9\*B\*b^2\*c^2 - 7\*A\*b\*c^3)\*x^5 - 70\*(9\*B\*b^3\*c - 7\*A\*b^2\*c^2)\*x^3 + 105\*(9\*B\*b^4 - 7\*A\*b^3\*c + (9\*B\*b^3\*c

$c - 7A^2b^2c^2)x^2) \sqrt{b/c} \arctan(cx \sqrt{b/c}/b) - 105(9B^2b^4 - 7A^2b^3c)x)/(c^6x^2 + b^2c^5)]$

**giac** [A] time = 0.17, size = 139, normalized size = 1.05

$$\frac{(9Bb^4 - 7Ab^3c) \arctan\left(\frac{cx}{\sqrt{bc}}\right) - \frac{Bb^4x - Ab^3cx}{2(cx^2 + b)c^5} + \frac{15Bc^{12}x^7 - 42Bbc^{11}x^5 + 21Ac^{12}x^5 + 105Bb^2c^{10}x^3 - 70Abc^{11}x^3 - 420Bb^3c^9x + 315Ab^2c^{10}x}{105c^{14}}}{2\sqrt{bc}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $\frac{1}{2}(9B^2b^4 - 7A^2b^3c) \arctan(cx/\sqrt{bc})/(\sqrt{bc}c^5) - \frac{1}{2}(B^2b^4x - A^2b^3cx)/((cx^2 + b)c^5) + \frac{1}{105}(15B^2c^{12}x^7 - 42B^2bc^{11}x^5 + 21A^2c^{12}x^5 + 105B^2b^2c^{10}x^3 - 70A^2bc^{11}x^3 - 420B^2b^3c^9x + 315A^2b^2c^{10}x)/c^{14}$

**maple** [A] time = 0.05, size = 155, normalized size = 1.17

$$\frac{Bx^7}{7c^2} + \frac{Ax^5}{5c^2} - \frac{2Bbx^5}{5c^3} - \frac{2Abx^3}{3c^3} + \frac{Bb^2x^3}{c^4} + \frac{Ab^3x}{2(cx^2 + b)c^4} - \frac{7Ab^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^4} - \frac{Bb^4x}{2(cx^2 + b)c^5} + \frac{9Bb^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^5} + \frac{3Ab^2x}{c^4} - \frac{4Bb^3x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $\frac{1}{7}B^2x^7/c^2 + \frac{1}{5}c^2Ax^5 - \frac{2}{5}c^3B^2x^5b - \frac{2}{3}c^3Ax^3b + \frac{1}{c^4}B^2x^3b^2 + \frac{3}{c^4}A^2b^2x - \frac{4}{c^5}B^2b^3x + \frac{1}{2}b^3/c^4x/(cx^2 + b) - \frac{1}{2}b^4/c^5x/(cx^2 + b) - \frac{7}{2}b^3/c^4/(bc)^{(1/2)} \arctan(1/(bc)^{(1/2)}cx) + \frac{9}{2}b^4/c^5/(bc)^{(1/2)} \arctan(1/(bc)^{(1/2)}cx) + B$

**maxima** [A] time = 2.94, size = 136, normalized size = 1.02

$$\frac{(Bb^4 - Ab^3c)x}{2(c^6x^2 + bc^5)} + \frac{(9Bb^4 - 7Ab^3c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^5} + \frac{15Bc^3x^7 - 21(2Bbc^2 - Ac^3)x^5 + 35(3Bb^2c - 2Abc^2)x^3 - 105(4Bb^3 - 3Ab^2c)x}{105c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-\frac{1}{2}(B^2b^4 - A^2b^3c)x/(c^6x^2 + b^2c^5) + \frac{1}{2}(9B^2b^4 - 7A^2b^3c) \arctan(cx/\sqrt{bc})/(\sqrt{bc}c^5) + \frac{1}{105}(15B^2c^3x^7 - 21(2B^2bc^2 - A^2c^3)x^5 + 35(3B^2b^2c - 2A^2bc^2)x^3 - 105(4B^2b^3 - 3A^2b^2c)x)/c^5$

**mupad** [B] time = 0.10, size = 203, normalized size = 1.53

$$x \left( \frac{2b \left( \frac{A}{2} - \frac{2Bb}{c^3} \right) + \frac{Bb^2}{c^4}}{c} - \frac{b^2 \left( \frac{A}{2} - \frac{2Bb}{c^3} \right)}{c^2} \right) + x^5 \left( \frac{A}{5c^2} - \frac{2Bb}{5c^3} \right) - x^3 \left( \frac{2b \left( \frac{A}{c^2} - \frac{2Bb}{c^3} \right) + \frac{Bb^2}{3c^4}}{3c} + \frac{Bb^2}{3c^4} \right) + \frac{Bx^7}{7c^2} - \frac{x \left( \frac{Bb^4}{2} - \frac{Ab^3c}{2} \right)}{c^6x^2 + bc^5} + \frac{b^{5/2} \operatorname{atan}\left(\frac{b^{5/2} \sqrt{c} x (7Ac - 9Bb)}{9Bb^4 - 7Ab^3c}\right) (7Ac - 9Bb)}{2c^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^2 \left( \frac{2b \left( \frac{2b(A/c^2 - (2Bb)/c^3)}{c} + \frac{(Bb^2)/c^4}{c} \right)}{c} - \frac{b^2(A/c^2 - (2Bb)/c^3)}{c^2} \right) + x^5 \left( \frac{A}{5c^2} - \frac{(2Bb)/c^3}{5c^3} \right) - x^3 \left( \frac{2b \left( \frac{2b(A/c^2 - (2Bb)/c^3)}{3c} + \frac{(Bb^2)/c^4}{3c^4} \right)}{(3c) + \frac{(Bb^2)/c^4}{(3c^4)}} + \frac{(Bx^7)/(7c^2) - (x \left( \frac{(Bb^4)/2 - (A^2b^3c)/2}{(bc^5 + c^6x^2)} + \frac{b^{5/2} \operatorname{atan}\left(\frac{b^{5/2} c^{1/2} x (7Ac - 9Bb)}{9Bb^4 - 7Ab^3c}\right) (7Ac - 9Bb)}{(9Bb^4 - 7A^2b^3c) * (7Ac - 9Bb))}{2c^{11/2}} \right)}{2c^{11/2}} \right)$

sympy [A] time = 1.01, size = 238, normalized size = 1.79

$$\frac{Bx^7}{7c^2} + x^5 \left( \frac{A}{5c^2} - \frac{2Bb}{5c^3} \right) + x^3 \left( -\frac{2Ab}{3c^3} + \frac{Bb^2}{c^4} \right) + x \left( \frac{3Ab^2}{c^4} - \frac{4Bb^3}{c^5} \right) + \frac{x(Ab^3c - Bb^4)}{2bc^5 + 2c^6x^2} - \frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac + 9Bb) \log\left(-\frac{c^5 \sqrt{-\frac{b^5}{c^{11}}}(-7Ac + 9Bb)}{-7Ab^2c + 9Bb^3} + x\right)}{4} + \frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac + 9Bb) \log\left(\frac{c^5 \sqrt{-\frac{b^5}{c^{11}}}(-7Ac + 9Bb)}{-7Ab^2c + 9Bb^3} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*12\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*x\*\*7/(7\*c\*\*2) + x\*\*5\*(A/(5\*c\*\*2) - 2\*B\*b/(5\*c\*\*3)) + x\*\*3\*(-2\*A\*b/(3\*c\*\*3) + B\*b\*\*2/c\*\*4) + x\*(3\*A\*b\*\*2/c\*\*4 - 4\*B\*b\*\*3/c\*\*5) + x\*(A\*b\*\*3\*c - B\*b\*\*4)/(2\*b\*c\*\*5 + 2\*c\*\*6\*x\*\*2) - sqrt(-b\*\*5/c\*\*11)\*(-7\*A\*c + 9\*B\*b)\*log(-c\*\*5\*sqrt(-b\*\*5/c\*\*11)\*(-7\*A\*c + 9\*B\*b)/(-7\*A\*b\*\*2\*c + 9\*B\*b\*\*3) + x)/4 + sqrt(-b\*\*5/c\*\*11)\*(-7\*A\*c + 9\*B\*b)\*log(c\*\*5\*sqrt(-b\*\*5/c\*\*11)\*(-7\*A\*c + 9\*B\*b)/(-7\*A\*b\*\*2\*c + 9\*B\*b\*\*3) + x)/4

$$3.59 \quad \int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=105

$$-\frac{b^3(bB - Ac)}{2c^5(b + cx^2)} - \frac{b^2(4bB - 3Ac) \log(b + cx^2)}{2c^5} + \frac{bx^2(3bB - 2Ac)}{2c^4} - \frac{x^4(2bB - Ac)}{4c^3} + \frac{Bx^6}{6c^2}$$

**Rubi [A]** time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{b^3(bB - Ac)}{2c^5(b + cx^2)} - \frac{b^2(4bB - 3Ac) \log(b + cx^2)}{2c^5} - \frac{x^4(2bB - Ac)}{4c^3} + \frac{bx^2(3bB - 2Ac)}{2c^4} + \frac{Bx^6}{6c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*(3\*b\*B - 2\*A\*c)\*x^2)/(2\*c^4) - ((2\*b\*B - A\*c)\*x^4)/(4\*c^3) + (B\*x^6)/(6\*c^2) - (b^3\*(b\*B - A\*c))/(2\*c^5\*(b + c\*x^2)) - (b^2\*(4\*b\*B - 3\*A\*c)\*Log[b + c\*x^2])/(2\*c^5)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0]) || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{x^7(A+Bx^2)}{(b+cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x^3(A+Bx)}{(b+cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{b(3bB-2Ac)}{c^4} + \frac{(-2bB+Ac)x}{c^3} + \frac{Bx^2}{c^2} + \frac{b^3(bB-Ac)}{c^4(b+cx)^2} - \frac{b^2(4bB-3Ac)}{c^4(b+cx)} \right) dx, x, x^2 \right) \\ &= \frac{b(3bB-2Ac)x^2}{2c^4} - \frac{(2bB-Ac)x^4}{4c^3} + \frac{Bx^6}{6c^2} - \frac{b^3(bB-Ac)}{2c^5(b+cx^2)} - \frac{b^2(4bB-3Ac) \log(b+cx^2)}{2c^5} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 93, normalized size = 0.89

$$\frac{6b^3(Ac-bB)}{b+cx^2} + \frac{6b^2(3Ac-4bB)\log(b+cx^2) + 3c^2x^4(Ac-2bB) + 6bcx^2(3bB-2Ac) + 2Bc^3x^6}{12c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] (6\*b\*c\*(3\*b\*B - 2\*A\*c)\*x^2 + 3\*c^2\*(-2\*b\*B + A\*c)\*x^4 + 2\*B\*c^3\*x^6 + (6\*b^3\*(-(b\*B) + A\*c))/(b + c\*x^2) + 6\*b^2\*(-4\*b\*B + 3\*A\*c)\*Log[b + c\*x^2])/(12\*c^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] IntegrateAlgebraic[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.38, size = 148, normalized size = 1.41

$$\frac{2Bc^4x^8 - (4Bbc^3 - 3Ac^4)x^6 - 6Bb^4 + 6Ab^3c + 3(4Bb^2c^2 - 3Abc^3)x^4 + 6(3Bb^3c - 2Ab^2c^2)x^2 - 6(4Bb^4 - 3Ab^3c + (4Bb^3c - 3Ab^2c^2)x^2)\log(cx^2 + b)}{12(c^6x^2 + bc^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2, x, algorithm="fricas")

[Out] 1/12\*(2\*B\*c^4\*x^8 - (4\*B\*b\*c^3 - 3\*A\*c^4)\*x^6 - 6\*B\*b^4 + 6\*A\*b^3\*c + 3\*(4\*B\*b^2\*c^2 - 3\*A\*b\*c^3)\*x^4 + 6\*(3\*B\*b^3\*c - 2\*A\*b^2\*c^2)\*x^2 - 6\*(4\*B\*b^4 - 3\*A\*b^3\*c + (4\*B\*b^3\*c - 3\*A\*b^2\*c^2)\*x^2)\*log(c\*x^2 + b))/(c^6\*x^2 + b\*c^5)

**giac [A]** time = 0.15, size = 135, normalized size = 1.29

$$\frac{(4Bb^3 - 3Ab^2c)\log(|cx^2 + b|)}{2c^5} + \frac{2Bc^4x^6 - 6Bbc^3x^4 + 3Ac^4x^4 + 18Bb^2c^2x^2 - 12Abc^3x^2}{12c^6} + \frac{4Bb^3cx^2 - 3Ab^2c^2x^2 + 3Bb^4 - 2Ab^3c}{2(cx^2 + b)c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2, x, algorithm="giac")

[Out] -1/2\*(4\*B\*b^3 - 3\*A\*b^2\*c)\*log(abs(c\*x^2 + b))/c^5 + 1/12\*(2\*B\*c^4\*x^6 - 6\*B\*b\*c^3\*x^4 + 3\*A\*c^4\*x^4 + 18\*B\*b^2\*c^2\*x^2 - 12\*A\*b\*c^3\*x^2)/c^6 + 1/2\*(4\*B\*b^3\*c\*x^2 - 3\*A\*b^2\*c^2\*x^2 + 3\*B\*b^4 - 2\*A\*b^3\*c)/((c\*x^2 + b)\*c^5)

**maple [A]** time = 0.05, size = 122, normalized size = 1.16

$$\frac{Bx^6}{6c^2} + \frac{Ax^4}{4c^2} - \frac{Bbx^4}{2c^3} - \frac{Abx^2}{c^3} + \frac{3Bb^2x^2}{2c^4} + \frac{Ab^3}{2(cx^2 + b)c^4} + \frac{3Ab^2\ln(cx^2 + b)}{2c^4} - \frac{Bb^4}{2(cx^2 + b)c^5} - \frac{2Bb^3\ln(cx^2 + b)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2, x)

[Out] 1/6\*B\*x^6/c^2+1/4/c^2\*A\*x^4-1/2/c^3\*B\*x^4\*b-1/c^3\*A\*x^2\*b+3/2/c^4\*B\*x^2\*b^2+3/2\*b^2/c^4\*ln(c\*x^2+b)\*A-2\*b^3/c^5\*ln(c\*x^2+b)\*B+1/2\*b^3/c^4/(c\*x^2+b)\*A-1/2\*b^4/c^5/(c\*x^2+b)\*B

**maxima** [A] time = 1.38, size = 107, normalized size = 1.02

$$\frac{Bb^4 - Ab^3c}{2(c^6x^2 + bc^5)} + \frac{2Bc^2x^6 - 3(2Bbc - Ac^2)x^4 + 6(3Bb^2 - 2Abc)x^2}{12c^4} - \frac{(4Bb^3 - 3Ab^2c)\log(cx^2 + b)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] -1/2\*(B\*b^4 - A\*b^3\*c)/(c^6\*x^2 + b\*c^5) + 1/12\*(2\*B\*c^2\*x^6 - 3\*(2\*B\*b\*c - A\*c^2)\*x^4 + 6\*(3\*B\*b^2 - 2\*A\*b\*c)\*x^2)/c^4 - 1/2\*(4\*B\*b^3 - 3\*A\*b^2\*c)\*log(c\*x^2 + b)/c^5

**mupad** [B] time = 0.10, size = 121, normalized size = 1.15

$$x^4 \left( \frac{A}{4c^2} - \frac{Bb}{2c^3} \right) - x^2 \left( \frac{b \left( \frac{A}{c^2} - \frac{2Bb}{c^3} \right)}{c} + \frac{Bb^2}{2c^4} \right) + \frac{Bx^6}{6c^2} - \frac{\ln(cx^2 + b)(4Bb^3 - 3Ab^2c)}{2c^5} - \frac{Bb^4 - Ab^3c}{2c(c^5x^2 + bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] x^4\*(A/(4\*c^2) - (B\*b)/(2\*c^3)) - x^2\*((b\*(A/c^2 - (2\*B\*b)/c^3))/c + (B\*b^2)/(2\*c^4)) + (B\*x^6)/(6\*c^2) - (log(b + c\*x^2)\*(4\*B\*b^3 - 3\*A\*b^2\*c))/(2\*c^5) - (B\*b^4 - A\*b^3\*c)/(2\*c\*(b\*c^4 + c^5\*x^2))

**sympy** [A] time = 0.93, size = 104, normalized size = 0.99

$$\frac{Bx^6}{6c^2} - \frac{b^2(-3Ac + 4Bb)\log(b + cx^2)}{2c^5} + x^4 \left( \frac{A}{4c^2} - \frac{Bb}{2c^3} \right) + x^2 \left( -\frac{Ab}{c^3} + \frac{3Bb^2}{2c^4} \right) + \frac{Ab^3c - Bb^4}{2bc^5 + 2c^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*11\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*x\*\*6/(6\*c\*\*2) - b\*\*2\*(-3\*A\*c + 4\*B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*5) + x\*\*4\*(A/(4\*c\*\*2) - B\*b/(2\*c\*\*3)) + x\*\*2\*(-A\*b/c\*\*3 + 3\*B\*b\*\*2/(2\*c\*\*4)) + (A\*b\*\*3\*c - B\*b\*\*4)/(2\*b\*c\*\*5 + 2\*c\*\*6\*x\*\*2)

$$3.60 \quad \int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=110

$$-\frac{b^{3/2}(7bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} + \frac{b^2x(bB - Ac)}{2c^4(b + cx^2)} + \frac{bx(3bB - 2Ac)}{c^4} - \frac{x^3(2bB - Ac)}{3c^3} + \frac{Bx^5}{5c^2}$$

**Rubi [A]** time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 455, 1810, 205}

$$\frac{b^2x(bB - Ac)}{2c^4(b + cx^2)} - \frac{b^{3/2}(7bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} - \frac{x^3(2bB - Ac)}{3c^3} + \frac{bx(3bB - 2Ac)}{c^4} + \frac{Bx^5}{5c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*(3\*b\*B - 2\*A\*c)\*x)/c^4 - ((2\*b\*B - A\*c)\*x^3)/(3\*c^3) + (B\*x^5)/(5\*c^2) + (b^2\*(b\*B - A\*c)\*x)/(2\*c^4\*(b + c\*x^2)) - (b^(3/2)\*(7\*b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(9/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{x^6(A+Bx^2)}{(b+cx^2)^2} dx \\
&= \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \int \frac{b^2(bB-Ac)-2bc(bB-Ac)x^2+2c^2(bB-Ac)x^4-2Bc^3x^6}{b+cx^2} dx \\
&= \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{\int \left(-2b(3bB-2Ac) + 2c(2bB-Ac)x^2 - 2Bc^2x^4 + \frac{7b^3B-5Ab^2c}{b+cx^2}\right) dx}{2c^4} \\
&= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{(b^2(7bB-5Ac)) \int \frac{1}{b+cx^2} dx}{2c^4} \\
&= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 111, normalized size = 1.01

$$-\frac{b^{3/2}(7bB-5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} - \frac{x(Ab^2c-b^3B)}{2c^4(b+cx^2)} + \frac{bx(3bB-2Ac)}{c^4} + \frac{x^3(Ac-2bB)}{3c^3} + \frac{Bx^5}{5c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*(3\*b\*B - 2\*A\*c)\*x)/c^4 + ((-2\*b\*B + A\*c)\*x^3)/(3\*c^3) + (B\*x^5)/(5\*c^2) - ((-b^3\*B) + A\*b^2\*c\*x)/(2\*c^4\*(b + c\*x^2)) - (b^(3/2)\*(7\*b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.41, size = 298, normalized size = 2.71

$$\frac{12Bc^3x^7 - 4(7Bb^2 - 5Ac^2)x^5 + 20(7Bb^2 - 5Ab^2c)x^3 - 15(7Bb^2 - 5Ab^2c)^2 \sqrt{\frac{c}{b}} \log\left(\frac{c^2 + 2cx\sqrt{\frac{c}{b}} + b}{c^2 + b}\right) + 30(7Bb^2 - 5Ab^2c)x - 6Bc^3x^2 - 2(7Bb^2 - 5Ac^2)x^3 + 10(7Bb^2 - 5Ab^2c)x^4 - 15(7Bb^2 - 5Ab^2c + (7Bb^2 - 5Ab^2c)^2) \sqrt{\frac{c}{b}} \arctan\left(\frac{c\sqrt{c}x}{b}\right) + 15(7Bb^2 - 5Ab^2c)x}{60(c^2 + b^2) \sqrt{\frac{c}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [1/60\*(12\*B\*c^3\*x^7 - 4\*(7\*B\*b\*c^2 - 5\*A\*c^3)\*x^5 + 20\*(7\*B\*b^2\*c - 5\*A\*b\*c^2)\*x^3 - 15\*(7\*B\*b^3 - 5\*A\*b^2\*c + (7\*B\*b^2\*c - 5\*A\*b\*c^2)\*x^2)\*sqrt(-b/c) \*log((c\*x^2 + 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) + 30\*(7\*B\*b^3 - 5\*A\*b^2\*c)\*x)/(c^5\*x^2 + b\*c^4), 1/30\*(6\*B\*c^3\*x^7 - 2\*(7\*B\*b\*c^2 - 5\*A\*c^3)\*x^5 + 10\*(7\*B\*b^2\*c - 5\*A\*b\*c^2)\*x^3 - 15\*(7\*B\*b^3 - 5\*A\*b^2\*c + (7\*B\*b^2\*c - 5\*A\*b\*c^2)\*x^2)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) + 15\*(7\*B\*b^3 - 5\*A\*b^2\*c)\*x)/(c^5\*x^2 + b\*c^4)]



**giac** [A] time = 0.16, size = 115, normalized size = 1.05

$$\frac{(7Bb^3 - 5Ab^2c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^4} + \frac{Bb^3x - Ab^2cx}{2(cx^2 + b)c^4} + \frac{3Bc^8x^5 - 10Bbc^7x^3 + 5Ac^8x^3 + 45Bb^2c^6x - 30Abc^7x}{15c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] -1/2\*(7\*B\*b^3 - 5\*A\*b^2\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^4) + 1/2\*(B\*b^3\*x - A\*b^2\*c\*x)/((c\*x^2 + b)\*c^4) + 1/15\*(3\*B\*c^8\*x^5 - 10\*B\*b\*c^7\*x^3 + 5\*A\*c^8\*x^3 + 45\*B\*b^2\*c^6\*x - 30\*A\*b\*c^7\*x)/c^10

**maple** [A] time = 0.05, size = 132, normalized size = 1.20

$$\frac{Bx^5}{5c^2} + \frac{Ax^3}{3c^2} - \frac{2Bbx^3}{3c^3} - \frac{Ab^2x}{2(c^2x^2 + b)c^3} + \frac{5Ab^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^3} + \frac{Bb^3x}{2(c^2x^2 + b)c^4} - \frac{7Bb^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^4} - \frac{2Abx}{c^3} + \frac{3Bb^2x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 1/5\*B\*x^5/c^2+1/3/c^2\*A\*x^3-2/3/c^3\*B\*x^3\*b-2/c^3\*A\*b\*x+3/c^4\*B\*b^2\*x-1/2\*b^2/c^3\*x/(c\*x^2+b)\*A+1/2\*b^3/c^4\*x/(c\*x^2+b)\*B+5/2\*b^2/c^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A-7/2\*b^3/c^4/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B

**maxima** [A] time = 3.02, size = 112, normalized size = 1.02

$$\frac{(Bb^3 - Ab^2c)x}{2(c^5x^2 + bc^4)} - \frac{(7Bb^3 - 5Ab^2c) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^4} + \frac{3Bc^2x^5 - 5(2Bbc - Ac^2)x^3 + 15(3Bb^2 - 2Abc)x}{15c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*(B\*b^3 - A\*b^2\*c)\*x/(c^5\*x^2 + b\*c^4) - 1/2\*(7\*B\*b^3 - 5\*A\*b^2\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^4) + 1/15\*(3\*B\*c^2\*x^5 - 5\*(2\*B\*b\*c - A\*c^2)\*x^3 + 15\*(3\*B\*b^2 - 2\*A\*b\*c)\*x)/c^4

**mupad** [B] time = 0.09, size = 141, normalized size = 1.28

$$x^3 \left( \frac{A}{3c^2} - \frac{2Bb}{3c^3} \right) - x \left( \frac{2b \left( \frac{A}{c^2} - \frac{2Bb}{c^3} \right)}{c} + \frac{Bb^2}{c^4} \right) + \frac{Bx^5}{5c^2} + \frac{x \left( \frac{Bb^3}{2} - \frac{Ab^2c}{2} \right)}{c^5x^2 + bc^4} - \frac{b^{3/2} \operatorname{atan}\left(\frac{b^{3/2}\sqrt{c}x(5Ac-7Bb)}{7Bb^3-5Ab^2c}\right)}{2c^{9/2}} (5Ac-7Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] x^3\*(A/(3\*c^2) - (2\*B\*b)/(3\*c^3)) - x\*((2\*b\*(A/c^2 - (2\*B\*b)/c^3))/c + (B\*b^2)/c^4) + (B\*x^5)/(5\*c^2) + (x\*((B\*b^3)/2 - (A\*b^2\*c)/2))/(b\*c^4 + c^5\*x^2) - (b^(3/2)\*atan((b^(3/2)\*c^(1/2)\*x\*(5\*A\*c - 7\*B\*b))/(7\*B\*b^3 - 5\*A\*b^2\*c)))/(5\*A\*c - 7\*B\*b)/(2\*c^(9/2))

**sympy** [B] time = 0.76, size = 211, normalized size = 1.92

$$\frac{Bx^5}{5c^2} + x^3 \left( \frac{A}{3c^2} - \frac{2Bb}{3c^3} \right) + x \left( -\frac{2Ab}{c^3} + \frac{3Bb^2}{c^4} \right) + \frac{x(-Ab^2c + Bb^3)}{2bc^4 + 2c^5x^2} + \frac{\sqrt{-\frac{b^3}{c^9}}(-5Ac + 7Bb) \log\left(\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac + 7Bb)}{-5Abc + 7Bb^2} + x\right)}{4} - \frac{\sqrt{-\frac{b^3}{c^9}}(-5Ac + 7Bb) \log\left(\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac + 7Bb)}{-5Abc + 7Bb^2} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*10\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out]  $Bx^5/(5c^2) + x^3(A/(3c^2) - 2Bb/(3c^3)) + x(-2Ab/c^3 + 3Bb^2/c^4) + x(-Ab^2c + Bb^3)/(2bc^4 + 2c^5x^2) + \sqrt{-b^3/c^9}(-5Ac + 7Bb)\log(-c^4\sqrt{-b^3/c^9}(-5Ac + 7Bb)/(-5Abc + 7Bb^2) + x)/4 - \sqrt{-b^3/c^9}(-5Ac + 7Bb)\log(c^4\sqrt{-b^3/c^9}(-5Ac + 7Bb)/(-5Abc + 7Bb^2) + x)/4$

$$3.61 \quad \int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=83

$$\frac{b^2(bB - Ac)}{2c^4(b + cx^2)} + \frac{b(3bB - 2Ac) \log(b + cx^2)}{2c^4} - \frac{x^2(2bB - Ac)}{2c^3} + \frac{Bx^4}{4c^2}$$

**Rubi [A]** time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$\frac{b^2(bB - Ac)}{2c^4(b + cx^2)} - \frac{x^2(2bB - Ac)}{2c^3} + \frac{b(3bB - 2Ac) \log(b + cx^2)}{2c^4} + \frac{Bx^4}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((2\*b\*B - A\*c)\*x^2)/(2\*c^3) + (B\*x^4)/(4\*c^2) + (b^2\*(b\*B - A\*c))/(2\*c^4\*(b + c\*x^2)) + (b\*(3\*b\*B - 2\*A\*c)\*Log[b + c\*x^2])/(2\*c^4)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{x^5(A+Bx^2)}{(b+cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2(A+Bx)}{(b+cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{-2bB+Ac}{c^3} + \frac{Bx}{c^2} - \frac{b^2(bB-Ac)}{c^3(b+cx)^2} + \frac{b(3bB-2Ac)}{c^3(b+cx)} \right) dx, x, x^2 \right) \\ &= -\frac{(2bB-Ac)x^2}{2c^3} + \frac{Bx^4}{4c^2} + \frac{b^2(bB-Ac)}{2c^4(b+cx^2)} + \frac{b(3bB-2Ac) \log(b+cx^2)}{2c^4} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 72, normalized size = 0.87

$$\frac{\frac{2b^2(bB - Ac)}{b + cx^2} + 2cx^2(Ac - 2bB) + 2b(3bB - 2Ac) \log(b + cx^2) + Bc^2x^4}{4c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*c\*(-2\*b\*B + A\*c)\*x^2 + B\*c^2\*x^4 + (2\*b^2\*(b\*B - A\*c))/(b + c\*x^2) + 2\*b\*(3\*b\*B - 2\*A\*c)\*Log[b + c\*x^2])/(4\*c^4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.38, size = 121, normalized size = 1.46

$$\frac{Bc^3x^6 - (3Bbc^2 - 2Ac^3)x^4 + 2Bb^3 - 2Ab^2c - 2(2Bb^2c - Abc^2)x^2 + 2(3Bb^3 - 2Ab^2c + (3Bb^2c - 2Abc^2)x^2) \log(cx^2 + b)}{4(c^5x^2 + bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 1/4\*(B\*c^3\*x^6 - (3\*B\*b\*c^2 - 2\*A\*c^3)\*x^4 + 2\*B\*b^3 - 2\*A\*b^2\*c - 2\*(2\*B\*b^2\*c - A\*b\*c^2)\*x^2 + 2\*(3\*B\*b^3 - 2\*A\*b^2\*c + (3\*B\*b^2\*c - 2\*A\*b\*c^2)\*x^2) \*log(c\*x^2 + b))/(c^5\*x^2 + b\*c^4)

**giac [A]** time = 0.17, size = 106, normalized size = 1.28

$$\frac{(3Bb^2 - 2Abc) \log(|cx^2 + b|)}{2c^4} + \frac{Bc^2x^4 - 4Bbcx^2 + 2Ac^2x^2}{4c^4} - \frac{3Bb^2cx^2 - 2Abc^2x^2 + 2Bb^3 - Ab^2c}{2(cx^2 + b)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/2\*(3\*B\*b^2 - 2\*A\*b\*c)\*log(abs(c\*x^2 + b))/c^4 + 1/4\*(B\*c^2\*x^4 - 4\*B\*b\*c\*x^2 + 2\*A\*c^2\*x^2)/c^4 - 1/2\*(3\*B\*b^2\*c\*x^2 - 2\*A\*b\*c^2\*x^2 + 2\*B\*b^3 - A\*b^2\*c)/((c\*x^2 + b)\*c^4)

**maple [A]** time = 0.05, size = 98, normalized size = 1.18

$$\frac{Bx^4}{4c^2} + \frac{Ax^2}{2c^2} - \frac{Bbx^2}{c^3} - \frac{Ab^2}{2(cx^2 + b)c^3} - \frac{Ab \ln(cx^2 + b)}{c^3} + \frac{Bb^3}{2(cx^2 + b)c^4} + \frac{3Bb^2 \ln(cx^2 + b)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 1/4\*B\*x^4/c^2+1/2/c^2\*A\*x^2-1/c^3\*B\*x^2\*b-b/c^3\*ln(c\*x^2+b)\*A+3/2\*b^2/c^4\*ln(c\*x^2+b)\*B-1/2\*b^2/c^3/(c\*x^2+b)\*A+1/2\*b^3/c^4/(c\*x^2+b)\*B

**maxima [A]** time = 1.32, size = 82, normalized size = 0.99

$$\frac{Bb^3 - Ab^2c}{2(c^5x^2 + bc^4)} + \frac{Bcx^4 - 2(2Bb - Ac)x^2}{4c^3} + \frac{(3Bb^2 - 2Abc)\log(cx^2 + b)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*(B\*b^3 - A\*b^2\*c)/(c^5\*x^2 + b\*c^4) + 1/4\*(B\*c\*x^4 - 2\*(2\*B\*b - A\*c)\*x^2)/c^3 + 1/2\*(3\*B\*b^2 - 2\*A\*b\*c)\*log(c\*x^2 + b)/c^4

**mupad [B]** time = 0.07, size = 86, normalized size = 1.04

$$x^2 \left( \frac{A}{2c^2} - \frac{Bb}{c^3} \right) + \frac{\ln(cx^2 + b)(3Bb^2 - 2Abc)}{2c^4} + \frac{Bx^4}{4c^2} + \frac{Bb^3 - Ab^2c}{2c(c^4x^2 + bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] x^2\*(A/(2\*c^2) - (B\*b)/c^3) + (log(b + c\*x^2)\*(3\*B\*b^2 - 2\*A\*b\*c))/(2\*c^4) + (B\*x^4)/(4\*c^2) + (B\*b^3 - A\*b^2\*c)/(2\*c\*(b\*c^3 + c^4\*x^2))

**sympy [A]** time = 0.70, size = 78, normalized size = 0.94

$$\frac{Bx^4}{4c^2} + \frac{b(-2Ac + 3Bb)\log(b + cx^2)}{2c^4} + x^2 \left( \frac{A}{2c^2} - \frac{Bb}{c^3} \right) + \frac{-Ab^2c + Bb^3}{2bc^4 + 2c^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*9\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*x\*\*4/(4\*c\*\*2) + b\*(-2\*A\*c + 3\*B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*4) + x\*\*2\*(A/(2\*c\*\*2) - B\*b/c\*\*3) + (-A\*b\*\*2\*c + B\*b\*\*3)/(2\*b\*c\*\*4 + 2\*c\*\*5\*x\*\*2)

$$3.62 \quad \int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=89

$$\frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2c^{7/2}} - \frac{bx(bB-Ac)}{2c^3(b+cx^2)} - \frac{x(2bB-Ac)}{c^3} + \frac{Bx^3}{3c^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 455, 1153, 205}

$$-\frac{bx(bB-Ac)}{2c^3(b+cx^2)} - \frac{x(2bB-Ac)}{c^3} + \frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2c^{7/2}} + \frac{Bx^3}{3c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -(((2\*b\*B - A\*c)\*x)/c^3) + (B\*x^3)/(3\*c^2) - (b\*(b\*B - A\*c)\*x)/(2\*c^3\*(b + c\*x^2)) + (Sqrt[b]\*(5\*b\*B - 3\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(7/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1)/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^4 (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{b(bB - Ac)x}{2c^3 (b + cx^2)} - \frac{\int \frac{-b(bB - Ac) + 2c(bB - Ac)x^2 - 2Bc^2x^4}{b + cx^2} dx}{2c^3} \\
&= -\frac{b(bB - Ac)x}{2c^3 (b + cx^2)} - \frac{\int \left( 2(2bB - Ac) - 2Bcx^2 + \frac{-5b^2B + 3Abc}{b + cx^2} \right) dx}{2c^3} \\
&= -\frac{(2bB - Ac)x}{c^3} + \frac{Bx^3}{3c^2} - \frac{b(bB - Ac)x}{2c^3 (b + cx^2)} + \frac{(b(5bB - 3Ac)) \int \frac{1}{b + cx^2} dx}{2c^3} \\
&= -\frac{(2bB - Ac)x}{c^3} + \frac{Bx^3}{3c^2} - \frac{b(bB - Ac)x}{2c^3 (b + cx^2)} + \frac{\sqrt{b} (5bB - 3Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{2c^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 89, normalized size = 1.00

$$\frac{x(ABC - b^2B)}{2c^3(b + cx^2)} + \frac{\sqrt{b}(5bB - 3Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{2c^{7/2}} + \frac{x(AC - 2bB)}{c^3} + \frac{Bx^3}{3c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((-2\*b\*B + A\*c)\*x)/c^3 + (B\*x^3)/(3\*c^2) + ((-(b^2\*B) + A\*b\*c)\*x)/(2\*c^3\*(b + c\*x^2)) + (Sqrt[b]\*(5\*b\*B - 3\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(7/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.41, size = 240, normalized size = 2.70

$$\frac{4Bc^2x^5 - 4(5Bbc - 3Ac^2)x^3 - 3(5Bb^2 - 3Abc + (5Bbc - 3Ac^2)x^2)\sqrt{-b} \log\left(\frac{cx^2 - 2cx\sqrt{-b}}{c^2 + b}\right) - 6(5Bb^2 - 3Abc)x - 2Bc^2x^5 - 2(5Bbc - 3Ac^2)x^3 + 3(5Bb^2 - 3Abc + (5Bbc - 3Ac^2)x^2)\sqrt{b} \arctan\left(\frac{cx\sqrt{b}}{b}\right) - 3(5Bb^2 - 3Abc)x}{12(c^4x^2 + bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [1/12\*(4\*B\*c^2\*x^5 - 4\*(5\*B\*b\*c - 3\*A\*c^2)\*x^3 - 3\*(5\*B\*b^2 - 3\*A\*b\*c + (5\*B\*b\*c - 3\*A\*c^2)\*x^2)\*sqrt(-b/c)\*log((c\*x^2 - 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) - 6\*(5\*B\*b^2 - 3\*A\*b\*c)\*x)/(c^4\*x^2 + b\*c^3), 1/6\*(2\*B\*c^2\*x^5 - 2\*(5\*B\*b\*c - 3\*A\*c^2)\*x^3 + 3\*(5\*B\*b^2 - 3\*A\*b\*c + (5\*B\*b\*c - 3\*A\*c^2)\*x^2)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) - 3\*(5\*B\*b^2 - 3\*A\*b\*c)\*x)/(c^4\*x^2 + b\*c^3)]

**giac** [A] time = 0.16, size = 88, normalized size = 0.99

$$\frac{(5Bb^2 - 3Abc) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^3} - \frac{Bb^2x - Abcx}{2(cx^2 + b)c^3} + \frac{Bc^4x^3 - 6Bbc^3x + 3Ac^4x}{3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/2\*(5\*B\*b^2 - 3\*A\*b\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^3) - 1/2\*(B\*b^2\*x - A\*b\*c\*x)/((c\*x^2 + b)\*c^3) + 1/3\*(B\*c^4\*x^3 - 6\*B\*b\*c^3\*x + 3\*A\*c^4\*x)/c^6

**maple** [A] time = 0.06, size = 105, normalized size = 1.18

$$\frac{Bx^3}{3c^2} + \frac{Abx}{2(cx^2 + b)c^2} - \frac{3Ab \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^2} - \frac{Bb^2x}{2(cx^2 + b)c^3} + \frac{5Bb^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^3} + \frac{Ax}{c^2} - \frac{2Bbx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 1/3\*B\*x^3/c^2+1/c^2\*A\*x-2/c^3\*b\*B\*x+1/2\*b/c^2\*x/(c\*x^2+b)\*A-1/2\*b^2/c^3\*x/(c\*x^2+b)\*B-3/2\*b/c^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A+5/2\*b^2/c^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B

**maxima** [A] time = 3.03, size = 85, normalized size = 0.96

$$-\frac{(Bb^2 - Abc)x}{2(c^4x^2 + bc^3)} + \frac{(5Bb^2 - 3Abc) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^3} + \frac{Bcx^3 - 3(2Bb - Ac)x}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] -1/2\*(B\*b^2 - A\*b\*c)\*x/(c^4\*x^2 + b\*c^3) + 1/2\*(5\*B\*b^2 - 3\*A\*b\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^3) + 1/3\*(B\*c\*x^3 - 3\*(2\*B\*b - A\*c)\*x)/c^3

**mupad** [B] time = 0.11, size = 104, normalized size = 1.17

$$x \left( \frac{A}{c^2} - \frac{2Bb}{c^3} \right) - \frac{x \left( \frac{Bb^2}{2} - \frac{Abc}{2} \right)}{c^4x^2 + bc^3} + \frac{Bx^3}{3c^2} + \frac{\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c}x(3Ac-5Bb)}{5Bb^2-3Abc}\right) (3Ac-5Bb)}{2c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] x\*(A/c^2 - (2\*B\*b)/c^3) - (x\*((B\*b^2)/2 - (A\*b\*c)/2))/(b\*c^3 + c^4\*x^2) + (B\*x^3)/(3\*c^2) + (b^(1/2)\*atan((b^(1/2)\*c^(1/2)\*x\*(3\*A\*c - 5\*B\*b))/(5\*B\*b^2 - 3\*A\*b\*c))\*(3\*A\*c - 5\*B\*b))/(2\*c^(7/2))

**sympy** [A] time = 0.69, size = 129, normalized size = 1.45

$$\frac{Bx^3}{3c^2} + x \left( \frac{A}{c^2} - \frac{2Bb}{c^3} \right) + \frac{x(Abc - Bb^2)}{2bc^3 + 2c^4x^2} - \frac{\sqrt{-\frac{b}{c^7}} (-3Ac + 5Bb) \log\left(-c^3\sqrt{-\frac{b}{c^7}} + x\right)}{4} + \frac{\sqrt{-\frac{b}{c^7}} (-3Ac + 5Bb) \log\left(c^3\sqrt{-\frac{b}{c^7}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*x\*\*3/(3\*c\*\*2) + x\*(A/c\*\*2 - 2\*B\*b/c\*\*3) + x\*(A\*b\*c - B\*b\*\*2)/(2\*b\*c\*\*3 + 2\*c\*\*4\*x\*\*2) - sqrt(-b/c\*\*7)\*(-3\*A\*c + 5\*B\*b)\*log(-c\*\*3\*sqrt(-b/c\*\*7) + x)/4 + sqrt(-b/c\*\*7)\*(-3\*A\*c + 5\*B\*b)\*log(c\*\*3\*sqrt(-b/c\*\*7) + x)/4



$$3.63 \quad \int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=61

$$-\frac{b(bB - Ac)}{2c^3(b + cx^2)} - \frac{(2bB - Ac) \log(b + cx^2)}{2c^3} + \frac{Bx^2}{2c^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{b(bB - Ac)}{2c^3(b + cx^2)} - \frac{(2bB - Ac) \log(b + cx^2)}{2c^3} + \frac{Bx^2}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (B\*x^2)/(2\*c^2) - (b\*(b\*B - A\*c))/(2\*c^3\*(b + c\*x^2)) - ((2\*b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^3)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{x^3(A+Bx^2)}{(b+cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x(A+Bx)}{(b+cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{B}{c^2} + \frac{b(bB-Ac)}{c^2(b+cx)^2} + \frac{-2bB+Ac}{c^2(b+cx)} \right) dx, x, x^2 \right) \\ &= \frac{Bx^2}{2c^2} - \frac{b(bB-Ac)}{2c^3(b+cx^2)} - \frac{(2bB-Ac) \log(b+cx^2)}{2c^3} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 50, normalized size = 0.82

$$\frac{\frac{b(Ac-bB)}{b+cx^2} + (Ac - 2bB) \log(b + cx^2) + Bcx^2}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (B\*c\*x^2 + (b\*(-(b\*B) + A\*c))/(b + c\*x^2) + (-2\*b\*B + A\*c)\*Log[b + c\*x^2])/(2\*c^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.39, size = 81, normalized size = 1.33

$$\frac{Bc^2x^4 + Bbcx^2 - Bb^2 + Abc - (2Bb^2 - Abc + (2Bbc - Ac^2)x^2) \log(cx^2 + b)}{2(c^4x^2 + bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 1/2\*(B\*c^2\*x^4 + B\*b\*c\*x^2 - B\*b^2 + A\*b\*c - (2\*B\*b^2 - A\*b\*c + (2\*B\*b\*c - A\*c^2)\*x^2)\*log(c\*x^2 + b))/(c^4\*x^2 + b\*c^3)

**giac [A]** time = 0.16, size = 70, normalized size = 1.15

$$\frac{Bx^2}{2c^2} - \frac{(2Bb - Ac) \log(|cx^2 + b|)}{2c^3} + \frac{2Bbcx^2 - Ac^2x^2 + Bb^2}{2(cx^2 + b)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/2\*B\*x^2/c^2 - 1/2\*(2\*B\*b - A\*c)\*log(abs(c\*x^2 + b))/c^3 + 1/2\*(2\*B\*b\*c\*x^2 - A\*c^2\*x^2 + B\*b^2)/((c\*x^2 + b)\*c^3)

**maple [A]** time = 0.06, size = 74, normalized size = 1.21

$$\frac{Bx^2}{2c^2} + \frac{Ab}{2(cx^2 + b)c^2} + \frac{A \ln(cx^2 + b)}{2c^2} - \frac{Bb^2}{2(cx^2 + b)c^3} - \frac{Bb \ln(cx^2 + b)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 1/2\*B\*x^2/c^2+1/2/c^2\*ln(c\*x^2+b)\*A-1/c^3\*ln(c\*x^2+b)\*b\*B+1/2/c^2\*b/(c\*x^2+b)\*A-1/2/c^3\*b^2/(c\*x^2+b)\*B

**maxima** [A] time = 1.31, size = 60, normalized size = 0.98

$$\frac{Bx^2}{2c^2} - \frac{Bb^2 - Abc}{2(c^4x^2 + bc^3)} - \frac{(2Bb - Ac)\log(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*B\*x^2/c^2 - 1/2\*(B\*b^2 - A\*b\*c)/(c^4\*x^2 + b\*c^3) - 1/2\*(2\*B\*b - A\*c)\*log(c\*x^2 + b)/c^3

**mupad** [B] time = 0.12, size = 62, normalized size = 1.02

$$\frac{Bx^2}{2c^2} + \frac{\ln(cx^2 + b)(Ac - 2Bb)}{2c^3} - \frac{Bb^2 - Abc}{2c(c^3x^2 + bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] (B\*x^2)/(2\*c^2) + (log(b + c\*x^2)\*(A\*c - 2\*B\*b))/(2\*c^3) - (B\*b^2 - A\*b\*c)/(2\*c\*(b\*c^2 + c^3\*x^2))

**sympy** [A] time = 0.60, size = 56, normalized size = 0.92

$$\frac{Bx^2}{2c^2} + \frac{Abc - Bb^2}{2bc^3 + 2c^4x^2} - \frac{(-Ac + 2Bb)\log(b + cx^2)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*x\*\*2/(2\*c\*\*2) + (A\*b\*c - B\*b\*\*2)/(2\*b\*c\*\*3 + 2\*c\*\*4\*x\*\*2) - (-A\*c + 2\*B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*3)

$$3.64 \quad \int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=68

$$-\frac{(3bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}} + \frac{x(bB - Ac)}{2c^2(b + cx^2)} + \frac{Bx}{c^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 455, 388, 205}

$$\frac{x(bB - Ac)}{2c^2(b + cx^2)} - \frac{(3bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}} + \frac{Bx}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (B\*x)/c^2 + ((b\*B - A\*c)\*x)/(2\*c^2\*(b + c\*x^2)) - ((3\*b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*Sqrt[b]\*c^(5/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1)/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^6 (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^2 (A + Bx^2)}{(b + cx^2)^2} dx \\
&= \frac{(bB - Ac)x}{2c^2 (b + cx^2)} - \frac{\int \frac{bB - Ac - 2Bcx^2}{b + cx^2} dx}{2c^2} \\
&= \frac{Bx}{c^2} + \frac{(bB - Ac)x}{2c^2 (b + cx^2)} - \frac{(3bB - Ac) \int \frac{1}{b + cx^2} dx}{2c^2} \\
&= \frac{Bx}{c^2} + \frac{(bB - Ac)x}{2c^2 (b + cx^2)} - \frac{(3bB - Ac) \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right)}{2\sqrt{b} c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 68, normalized size = 1.00

$$-\frac{(3bB - Ac) \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right)}{2\sqrt{b} c^{5/2}} - \frac{x(Ac - bB)}{2c^2 (b + cx^2)} + \frac{Bx}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (B\*x)/c^2 - ((-(b\*B) + A\*c)\*x)/(2\*c^2\*(b + c\*x^2)) - ((3\*b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*Sqrt[b]\*c^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.42, size = 208, normalized size = 3.06

$$\left[ \frac{4Bbc^2x^3 + (3Bb^2 - Abc + (3Bbc - Ac^2)x^2)\sqrt{-bc} \log\left(\frac{cx^2 - \sqrt{-bc}x - b}{cx^2 + b}\right) + 2(3Bb^2c - Abc^2)x}{4(bc^4x^2 + b^2c^3)}, \frac{2Bbc^2x^3 - (3Bb^2 - Abc + (3Bbc - Ac^2)x^2)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right) + (3Bb^2c - Abc^2)x}{2(bc^4x^2 + b^2c^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [1/4\*(4\*B\*b\*c^2\*x^3 + (3\*B\*b^2 - A\*b\*c + (3\*B\*b\*c - A\*c^2)\*x^2)\*sqrt(-b\*c)\*log((c\*x^2 - 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)) + 2\*(3\*B\*b^2\*c - A\*b\*c^2)\*x)/(b\*c^4\*x^2 + b^2\*c^3), 1/2\*(2\*B\*b\*c^2\*x^3 - (3\*B\*b^2 - A\*b\*c + (3\*B\*b\*c - A\*c^2)\*x^2)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b) + (3\*B\*b^2\*c - A\*b\*c^2)\*x)/(b\*c^4\*x^2 + b^2\*c^3)]

**giac [A]** time = 0.19, size = 59, normalized size = 0.87

$$\frac{Bx}{c^2} - \frac{(3Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^2} + \frac{Bbx - Acx}{2(cx^2 + b)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] B\*x/c^2 - 1/2\*(3\*B\*b - A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^2) + 1/2\*(B\*b\*x - A\*c\*x)/((c\*x^2 + b)\*c^2)

**maple [A]** time = 0.05, size = 82, normalized size = 1.21

$$-\frac{Ax}{2(c^2x^2 + b)c} + \frac{A \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c} + \frac{Bbx}{2(c^2x^2 + b)c^2} - \frac{3Bb \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^2} + \frac{Bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] B\*x/c^2-1/2/c\*x/(c\*x^2+b)\*A+1/2/c^2\*x/(c\*x^2+b)\*b\*B+1/2/c/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A-3/2/c^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*b\*B

**maxima [A]** time = 3.00, size = 61, normalized size = 0.90

$$\frac{(Bb - Ac)x}{2(c^3x^2 + bc^2)} + \frac{Bx}{c^2} - \frac{(3Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*(B\*b - A\*c)\*x/(c^3\*x^2 + b\*c^2) + B\*x/c^2 - 1/2\*(3\*B\*b - A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^2)

**mupad [B]** time = 0.13, size = 59, normalized size = 0.87

$$\frac{Bx}{c^2} - \frac{x \left( \frac{Ac}{2} - \frac{Bb}{2} \right)}{c^3x^2 + bc^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) (Ac - 3Bb)}{2\sqrt{b}c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] (B\*x)/c^2 - (x\*((A\*c)/2 - (B\*b)/2))/(b\*c^2 + c^3\*x^2) + (atan((c^(1/2)\*x)/b^(1/2))\*(A\*c - 3\*B\*b))/(2\*b^(1/2)\*c^(5/2))

**sympy [A]** time = 0.55, size = 114, normalized size = 1.68

$$\frac{Bx}{c^2} + \frac{x(-Ac + Bb)}{2bc^2 + 2c^3x^2} + \frac{\sqrt{-\frac{1}{bc^5}}(-Ac + 3Bb) \log\left(-bc^2\sqrt{-\frac{1}{bc^5}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{bc^5}}(-Ac + 3Bb) \log\left(bc^2\sqrt{-\frac{1}{bc^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*x/c\*\*2 + x\*(-A\*c + B\*b)/(2\*b\*c\*\*2 + 2\*c\*\*3\*x\*\*2) + sqrt(-1/(b\*c\*\*5))\*(-A\*c + 3\*B\*b)\*log(-b\*c\*\*2\*sqrt(-1/(b\*c\*\*5)) + x)/4 - sqrt(-1/(b\*c\*\*5))\*(-A\*c + 3\*B\*b)\*log(b\*c\*\*2\*sqrt(-1/(b\*c\*\*5)) + x)/4

$$3.65 \quad \int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=41

$$\frac{bB - Ac}{2c^2(b + cx^2)} + \frac{B \log(b + cx^2)}{2c^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 444, 43}

$$\frac{bB - Ac}{2c^2(b + cx^2)} + \frac{B \log(b + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*B - A\*c)/(2\*c^2\*(b + c\*x^2)) + (B\*Log[b + c\*x^2])/(2\*c^2)

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{x(A+Bx^2)}{(b+cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{(b+cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{-bB+Ac}{c(b+cx)^2} + \frac{B}{c(b+cx)} \right) dx, x, x^2 \right) \\ &= \frac{bB - Ac}{2c^2(b + cx^2)} + \frac{B \log(b + cx^2)}{2c^2} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{bB - Ac}{2c^2(b + cx^2)} + \frac{B \log(b + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*B - A\*c)/(2\*c^2\*(b + c\*x^2)) + (B\*Log[b + c\*x^2])/(2\*c^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas** [A] time = 0.39, size = 44, normalized size = 1.07

$$\frac{Bb - Ac + (Bcx^2 + Bb) \log(cx^2 + b)}{2(c^3x^2 + bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 1/2\*(B\*b - A\*c + (B\*c\*x^2 + B\*b)\*log(c\*x^2 + b))/(c^3\*x^2 + b\*c^2)

**giac** [A] time = 0.19, size = 37, normalized size = 0.90

$$\frac{B \log(|cx^2 + b|)}{2c^2} - \frac{Bx^2 + A}{2(cx^2 + b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/2\*B\*log(abs(c\*x^2 + b))/c^2 - 1/2\*(B\*x^2 + A)/((c\*x^2 + b)\*c)

**maple** [A] time = 0.05, size = 47, normalized size = 1.15

$$-\frac{A}{2(cx^2 + b)c} + \frac{Bb}{2(cx^2 + b)c^2} + \frac{B \ln(cx^2 + b)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 1/2\*B\*ln(c\*x^2+b)/c^2-1/2/c/(c\*x^2+b)\*A+1/2/c^2/(c\*x^2+b)\*b\*B

**maxima** [A] time = 1.31, size = 40, normalized size = 0.98

$$\frac{Bb - Ac}{2(c^3x^2 + bc^2)} + \frac{B \log(cx^2 + b)}{2c^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*(B\*b - A\*c)/(c^3\*x^2 + b\*c^2) + 1/2\*B\*log(c\*x^2 + b)/c^2

**mupad** [B] time = 0.09, size = 37, normalized size = 0.90

$$\frac{B \ln(cx^2 + b)}{2c^2} - \frac{Ac - Bb}{2c^2(cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] (B\*log(b + c\*x^2))/(2\*c^2) - (A\*c - B\*b)/(2\*c^2\*(b + c\*x^2))

**sympy** [A] time = 0.37, size = 36, normalized size = 0.88

$$\frac{B \log(b + cx^2)}{2c^2} + \frac{-Ac + Bb}{2bc^2 + 2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] B\*log(b + c\*x\*\*2)/(2\*c\*\*2) + (-A\*c + B\*b)/(2\*b\*c\*\*2 + 2\*c\*\*3\*x\*\*2)

$$3.66 \quad \int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=63

$$\frac{(Ac + bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}} - \frac{x(bB - Ac)}{2bc(b + cx^2)}$$

**Rubi [A]** time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 385, 205}

$$\frac{(Ac + bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}} - \frac{x(bB - Ac)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((b\*B - A\*c)\*x)/(2\*b\*c\*(b + c\*x^2)) + ((b\*B + A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(3/2)\*c^(3/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{A+Bx^2}{(b+cx^2)^2} dx \\ &= -\frac{(bB-Ac)x}{2bc(b+cx^2)} + \frac{(bB+Ac) \int \frac{1}{b+cx^2} dx}{2bc} \\ &= -\frac{(bB-Ac)x}{2bc(b+cx^2)} + \frac{(bB+Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 63, normalized size = 1.00

$$\frac{(Ac + bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}} - \frac{x(bB - Ac)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] -1/2\*((b\*B - A\*c)\*x)/(b\*c\*(b + c\*x^2)) + ((b\*B + A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(3/2)\*c^(3/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.40, size = 182, normalized size = 2.89

$$\left[ \frac{(Bb^2 + Abc + (Bbc + Ac^2)x^2)\sqrt{-bc} \log\left(\frac{cx^2 - 2\sqrt{-bc}x - b}{cx^2 + b}\right) + 2(Bb^2c - Abc^2)x}{4(b^2c^3x^2 + b^3c^2)}, \frac{(Bb^2 + Abc + (Bbc + Ac^2)x^2)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right) - (Bb^2c - Abc^2)x}{2(b^2c^3x^2 + b^3c^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2, x, algorithm="fricas")

[Out] [-1/4\*((B\*b^2 + A\*b\*c + (B\*b\*c + A\*c^2)\*x^2)\*sqrt(-b\*c)\*log((c\*x^2 - 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)) + 2\*(B\*b^2\*c - A\*b\*c^2)\*x)/(b^2\*c^3\*x^2 + b^3\*c^2), 1/2\*((B\*b^2 + A\*b\*c + (B\*b\*c + A\*c^2)\*x^2)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b) - (B\*b^2\*c - A\*b\*c^2)\*x)/(b^2\*c^3\*x^2 + b^3\*c^2)]

**giac [A]** time = 0.18, size = 57, normalized size = 0.90

$$\frac{(Bb + Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}bc} - \frac{Bbx - Acx}{2(cx^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2, x, algorithm="giac")

[Out] 1/2\*(B\*b + A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b\*c) - 1/2\*(B\*b\*x - A\*c\*x)/((c\*x^2 + b)\*b\*c)

**maple [A]** time = 0.06, size = 68, normalized size = 1.08

$$\frac{A \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b} + \frac{B \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c} + \frac{(Ac - bB)x}{2(cx^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2, x)

[Out]  $1/2*(A*c-B*b)/b/c*x/(c*x^2+b)+1/2/b/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*A+1/2/c/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*B$

**maxima** [A] time = 2.96, size = 57, normalized size = 0.90

$$-\frac{(Bb - Ac)x}{2(bc^2x^2 + b^2c)} + \frac{(Bb + Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")`

[Out]  $-1/2*(B*b - A*c)*x/(b*c^2*x^2 + b^2*c) + 1/2*(B*b + A*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b*c)$

**mupad** [B] time = 0.12, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(Ac + Bb)}{2b^{3/2}c^{3/2}} + \frac{x(Ac - Bb)}{2bc(c x^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)`

[Out]  $(\operatorname{atan}((c^{1/2}*x)/b^{1/2})*(A*c + B*b))/(2*b^{3/2}*c^{3/2}) + (x*(A*c - B*b))/(2*b*c*(b + c*x^2))$

**sympy** [B] time = 0.42, size = 112, normalized size = 1.78

$$\frac{x(Ac - Bb)}{2b^2c + 2bc^2x^2} - \frac{\sqrt{-\frac{1}{b^3c^3}}(Ac + Bb) \log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c^3}}(Ac + Bb) \log\left(b^2c\sqrt{-\frac{1}{b^3c^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**2,x)`

[Out]  $x*(A*c - B*b)/(2*b**2*c + 2*b*c**2*x**2) - \sqrt{-1/(b**3*c**3)}*(A*c + B*b)*\log(-b**2*c*\sqrt{-1/(b**3*c**3)} + x)/4 + \sqrt{-1/(b**3*c**3)}*(A*c + B*b)*\log(b**2*c*\sqrt{-1/(b**3*c**3)} + x)/4$

$$3.67 \quad \int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=51

$$-\frac{A \log(b+cx^2)}{2b^2} + \frac{A \log(x)}{b^2} - \frac{bB-Ac}{2bc(b+cx^2)}$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{A \log(b+cx^2)}{2b^2} + \frac{A \log(x)}{b^2} - \frac{bB-Ac}{2bc(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -(b\*B - A\*c)/(2\*b\*c\*(b + c\*x^2)) + (A\*Log[x])/b^2 - (A\*Log[b + c\*x^2])/(2\*b^2)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{A+Bx^2}{x(b+cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x(b+cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^2x} + \frac{bB-Ac}{b(b+cx)^2} - \frac{Ac}{b^2(b+cx)} \right) dx, x, x^2 \right) \\ &= -\frac{bB-Ac}{2bc(b+cx^2)} + \frac{A \log(x)}{b^2} - \frac{A \log(b+cx^2)}{2b^2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{b(Ac-bB)}{c(b+cx^2)} - A \log(b+cx^2) + 2A \log(x)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((b\*(-(b\*B) + A\*c))/(c\*(b + c\*x^2)) + 2\*A\*Log[x] - A\*Log[b + c\*x^2])/(2\*b^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.38, size = 70, normalized size = 1.37

$$\frac{Bb^2 - Abc + (Ac^2x^2 + Abc) \log(cx^2 + b) - 2(Ac^2x^2 + Abc) \log(x)}{2(b^2c^2x^2 + b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] -1/2\*(B\*b^2 - A\*b\*c + (A\*c^2\*x^2 + A\*b\*c)\*log(c\*x^2 + b) - 2\*(A\*c^2\*x^2 + A\*b\*c)\*log(x))/(b^2\*c^2\*x^2 + b^3\*c)

**giac [A]** time = 0.16, size = 52, normalized size = 1.02

$$-\frac{A \log(|cx^2 + b|)}{2b^2} + \frac{A \log(|x|)}{b^2} - \frac{Bb^2 - Abc}{2(cx^2 + b)b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] -1/2\*A\*log(abs(c\*x^2 + b))/b^2 + A\*log(abs(x))/b^2 - 1/2\*(B\*b^2 - A\*b\*c)/((c\*x^2 + b)\*b^2\*c)

**maple [A]** time = 0.06, size = 53, normalized size = 1.04

$$\frac{A}{2(cx^2 + b)b} + \frac{A \ln(x)}{b^2} - \frac{A \ln(cx^2 + b)}{2b^2} - \frac{B}{2(cx^2 + b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] -1/2\*A\*ln(c\*x^2+b)/b^2+1/2/b/(c\*x^2+b)\*A-1/2/c/(c\*x^2+b)\*B+A\*ln(x)/b^2

**maxima [A]** time = 1.31, size = 51, normalized size = 1.00

$$-\frac{Bb - Ac}{2(bc^2x^2 + b^2c)} - \frac{A \log(cx^2 + b)}{2b^2} + \frac{A \log(x^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] -1/2\*(B\*b - A\*c)/(b\*c^2\*x^2 + b^2\*c) - 1/2\*A\*log(c\*x^2 + b)/b^2 + 1/2\*A\*log(x^2)/b^2

**mupad [B]** time = 0.16, size = 47, normalized size = 0.92

$$\frac{A \ln(x)}{b^2} - \frac{A \ln(cx^2 + b)}{2b^2} + \frac{Ac - Bb}{2bc(cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] (A\*log(x))/b^2 - (A\*log(b + c\*x^2))/(2\*b^2) + (A\*c - B\*b)/(2\*b\*c\*(b + c\*x^2))

**sympy [A]** time = 0.44, size = 46, normalized size = 0.90

$$\frac{A \log(x)}{b^2} - \frac{A \log\left(\frac{b}{c} + x^2\right)}{2b^2} + \frac{Ac - Bb}{2b^2c + 2bc^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] A\*log(x)/b\*\*2 - A\*log(b/c + x\*\*2)/(2\*b\*\*2) + (A\*c - B\*b)/(2\*b\*\*2\*c + 2\*b\*c\*\*2\*x\*\*2)

$$3.68 \quad \int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=70

$$\frac{(bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}} + \frac{x(bB - Ac)}{2b^2(b + cx^2)} - \frac{A}{b^2x}$$

**Rubi [A]** time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 456, 453, 205}

$$\frac{x(bB - Ac)}{2b^2(b + cx^2)} + \frac{(bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}} - \frac{A}{b^2x}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -(A/(b^2\*x)) + ((b\*B - A\*c)\*x)/(2\*b^2\*(b + c\*x^2)) + ((b\*B - 3\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(5/2)\*Sqrt[c])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e^(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

#### Rule 456

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p+1)/(2\*b^(m/2 + 1)\*(p+1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p+1)), Int[x^m\*(a + b\*x^2)^(p+1)\*ExpandToSum[2\*b\*(p+1)\*Together[(b^(m/2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)\*x^(-m+2)]/(a + b\*x^2)] - ((-a)^(m/2 - 1)\*(b\*c - a\*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m+n\*p)\*(a + b\*x^(q-p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps



$$\begin{aligned}
\int \frac{x^2 (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^2 (b + cx^2)^2} dx \\
&= \frac{(bB - Ac)x}{2b^2 (b + cx^2)} - \frac{1}{2} \int \frac{-\frac{2A}{b} - \frac{(bB - Ac)x^2}{b^2}}{x^2 (b + cx^2)} dx \\
&= -\frac{A}{b^2 x} + \frac{(bB - Ac)x}{2b^2 (b + cx^2)} + \frac{(bB - 3Ac) \int \frac{1}{b + cx^2} dx}{2b^2} \\
&= -\frac{A}{b^2 x} + \frac{(bB - Ac)x}{2b^2 (b + cx^2)} + \frac{(bB - 3Ac) \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right)}{2b^{5/2} \sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 70, normalized size = 1.00

$$\frac{(bB - 3Ac) \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right)}{2b^{5/2} \sqrt{c}} + \frac{x(bB - Ac)}{2b^2 (b + cx^2)} - \frac{A}{b^2 x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -(A/(b^2\*x)) + ((b\*B - A\*c)\*x)/(2\*b^2\*(b + c\*x^2)) + ((b\*B - 3\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(5/2)\*Sqrt[c])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.41, size = 210, normalized size = 3.00

$$\left[ \frac{4Ab^2c - 2(Bb^2c - 3Abc^2)x^2 - ((Bbc - 3Ac^2)x^3 + (Bb^2 - 3Abc)x)\sqrt{-bc} \log\left(\frac{cx^2 + 2\sqrt{-bc}x - b}{cx^2 + b}\right)}{4(b^3c^2x^3 + b^4cx)}, \frac{2Ab^2c - (Bb^2c - 3Abc^2)x^2 - ((Bbc - 3Ac^2)x^3 + (Bb^2 - 3Abc)x)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2(b^3c^2x^3 + b^4cx)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [-1/4\*(4\*A\*b^2\*c - 2\*(B\*b^2\*c - 3\*A\*b\*c^2)\*x^2 - ((B\*b\*c - 3\*A\*c^2)\*x^3 + (B\*b^2 - 3\*A\*b\*c)\*x)\*sqrt(-b\*c)\*log((c\*x^2 + 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)))/(b^3\*c^2\*x^3 + b^4\*c\*x), -1/2\*(2\*A\*b^2\*c - (B\*b^2\*c - 3\*A\*b\*c^2)\*x^2 - ((B\*b\*c - 3\*A\*c^2)\*x^3 + (B\*b^2 - 3\*A\*b\*c)\*x)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b)]/(b^3\*c^2\*x^3 + b^4\*c\*x)]

**giac [A]** time = 0.21, size = 62, normalized size = 0.89

$$\frac{(Bb - 3Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b^2} + \frac{Bbx^2 - 3Acx^2 - 2Ab}{2(cx^3 + bx)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/2\*(B\*b - 3\*A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^2) + 1/2\*(B\*b\*x^2 - 3\*A\*c\*x^2 - 2\*A\*b)/((c\*x^3 + b\*x)\*b^2)

**maple** [A] time = 0.06, size = 85, normalized size = 1.21

$$-\frac{Acx}{2(c^2x^2 + b)b^2} - \frac{3Ac \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc} b^2} + \frac{Bx}{2(c^2x^2 + b)b} + \frac{B \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc} b} - \frac{A}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] -1/2/b^2\*x/(c\*x^2+b)\*A\*c+1/2/b\*x/(c\*x^2+b)\*B-3/2/b^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A\*c+1/2/b/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B-A/b^2/x

**maxima** [A] time = 2.98, size = 63, normalized size = 0.90

$$\frac{(Bb - 3Ac)x^2 - 2Ab}{2(b^2cx^3 + b^3x)} + \frac{(Bb - 3Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*((B\*b - 3\*A\*c)\*x^2 - 2\*A\*b)/(b^2\*c\*x^3 + b^3\*x) + 1/2\*(B\*b - 3\*A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^2)

**mupad** [B] time = 0.13, size = 63, normalized size = 0.90

$$-\frac{\frac{A}{b} + \frac{x^2(3Ac - Bb)}{2b^2}}{cx^3 + bx} - \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(3Ac - Bb)}{2b^{5/2}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] - (A/b + (x^2\*(3\*A\*c - B\*b))/(2\*b^2))/(b\*x + c\*x^3) - (atan((c^(1/2)\*x)/b^(1/2))\*(3\*A\*c - B\*b))/(2\*b^(5/2)\*c^(1/2))

**sympy** [A] time = 0.51, size = 114, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c}}(-3Ac + Bb) \log\left(-b^3\sqrt{-\frac{1}{b^5c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^5c}}(-3Ac + Bb) \log\left(b^3\sqrt{-\frac{1}{b^5c}} + x\right)}{4} + \frac{-2Ab + x^2(-3Ac + Bb)}{2b^3x + 2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] -sqrt(-1/(b\*\*5\*c))\*(-3\*A\*c + B\*b)\*log(-b\*\*3\*sqrt(-1/(b\*\*5\*c)) + x)/4 + sqrt(-1/(b\*\*5\*c))\*(-3\*A\*c + B\*b)\*log(b\*\*3\*sqrt(-1/(b\*\*5\*c)) + x)/4 + (-2\*A\*b + x\*\*2\*(-3\*A\*c + B\*b))/(2\*b\*\*3\*x + 2\*b\*\*2\*c\*x\*\*3)

$$3.69 \quad \int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=73

$$-\frac{(bB-2Ac)\log(b+cx^2)}{2b^3} + \frac{\log(x)(bB-2Ac)}{b^3} + \frac{bB-Ac}{2b^2(b+cx^2)} - \frac{A}{2b^2x^2}$$

**Rubi [A]** time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 446, 77}

$$\frac{bB-Ac}{2b^2(b+cx^2)} - \frac{(bB-2Ac)\log(b+cx^2)}{2b^3} + \frac{\log(x)(bB-2Ac)}{b^3} - \frac{A}{2b^2x^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -A/(2\*b^2\*x^2) + (b\*B - A\*c)/(2\*b^2\*(b + c\*x^2)) + ((b\*B - 2\*A\*c)\*Log[x])/b^3 - ((b\*B - 2\*A\*c)\*Log[b + c\*x^2])/(2\*b^3)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2} dx &= \int \frac{A+Bx^2}{x^3(b+cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x^2(b+cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^2x^2} + \frac{bB-2Ac}{b^3x} - \frac{c(bB-Ac)}{b^2(b+cx)^2} - \frac{c(bB-2Ac)}{b^3(b+cx)} \right) dx, x, x^2 \right) \\ &= -\frac{A}{2b^2x^2} + \frac{bB-Ac}{2b^2(b+cx^2)} + \frac{(bB-2Ac)\log(x)}{b^3} - \frac{(bB-2Ac)\log(b+cx^2)}{2b^3} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 64, normalized size = 0.88

$$\frac{\frac{b(bB-Ac)}{b+cx^2} + (2Ac - bB) \log(b + cx^2) + 2 \log(x)(bB - 2Ac) - \frac{Ab}{x^2}}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (-((A\*b)/x^2) + (b\*(b\*B - A\*c))/(b + c\*x^2) + 2\*(b\*B - 2\*A\*c)\*Log[x] + (-b\*B + 2\*A\*c)\*Log[b + c\*x^2])/(2\*b^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.38, size = 117, normalized size = 1.60

$$\frac{Ab^2 - (Bb^2 - 2Abc)x^2 + ((Bbc - 2Ac^2)x^4 + (Bb^2 - 2Abc)x^2) \log(cx^2 + b) - 2((Bbc - 2Ac^2)x^4 + (Bb^2 - 2Abc)x^2) \log(x)}{2(b^3cx^4 + b^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] -1/2\*(A\*b^2 - (B\*b^2 - 2\*A\*b\*c)\*x^2 + ((B\*b\*c - 2\*A\*c^2)\*x^4 + (B\*b^2 - 2\*A\*b\*c)\*x^2)\*log(c\*x^2 + b) - 2\*((B\*b\*c - 2\*A\*c^2)\*x^4 + (B\*b^2 - 2\*A\*b\*c)\*x^2)\*log(x))/(b^3\*c\*x^4 + b^4\*x^2)

**giac [A]** time = 0.16, size = 80, normalized size = 1.10

$$\frac{(Bb - 2Ac) \log(|x|)}{b^3} + \frac{Bbx^2 - 2Acx^2 - Ab}{2(cx^4 + bx^2)b^2} - \frac{(Bbc - 2Ac^2) \log(|cx^2 + b|)}{2b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] (B\*b - 2\*A\*c)\*log(abs(x))/b^3 + 1/2\*(B\*b\*x^2 - 2\*A\*c\*x^2 - A\*b)/((c\*x^4 + b\*x^2)\*b^2) - 1/2\*(B\*b\*c - 2\*A\*c^2)\*log(abs(c\*x^2 + b))/(b^3\*c)

**maple [A]** time = 0.07, size = 86, normalized size = 1.18

$$-\frac{Ac}{2(cx^2 + b)b^2} - \frac{2Ac \ln(x)}{b^3} + \frac{Ac \ln(cx^2 + b)}{b^3} + \frac{B}{2(cx^2 + b)b} + \frac{B \ln(x)}{b^2} - \frac{B \ln(cx^2 + b)}{2b^2} - \frac{A}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 1/b^3\*c\*ln(c\*x^2+b)\*A-1/2/b^2\*ln(c\*x^2+b)\*B-1/2/b^2\*c/(c\*x^2+b)\*A+1/2/b/(c\*x^2+b)\*B-1/2\*A/b^2/x^2-2/b^3\*ln(x)\*A\*c+1/b^2\*ln(x)\*B

**maxima** [A] time = 1.33, size = 76, normalized size = 1.04

$$\frac{(Bb - 2Ac)x^2 - Ab}{2(b^2cx^4 + b^3x^2)} - \frac{(Bb - 2Ac)\log(cx^2 + b)}{2b^3} + \frac{(Bb - 2Ac)\log(x^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*((B\*b - 2\*A\*c)\*x^2 - A\*b)/(b^2\*c\*x^4 + b^3\*x^2) - 1/2\*(B\*b - 2\*A\*c)\*log(c\*x^2 + b)/b^3 + 1/2\*(B\*b - 2\*A\*c)\*log(x^2)/b^3

**mupad** [B] time = 0.15, size = 78, normalized size = 1.07

$$\frac{\ln(cx^2 + b)(2Ac - Bb)}{2b^3} - \frac{\frac{A}{2b} + \frac{x^2(2Ac - Bb)}{2b^2}}{cx^4 + bx^2} - \frac{\ln(x)(2Ac - Bb)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] (log(b + c\*x^2)\*(2\*A\*c - B\*b))/(2\*b^3) - (A/(2\*b) + (x^2\*(2\*A\*c - B\*b))/(2\*b^2))/(b\*x^2 + c\*x^4) - (log(x)\*(2\*A\*c - B\*b))/b^3

**sympy** [A] time = 0.88, size = 70, normalized size = 0.96

$$\frac{-Ab + x^2(-2Ac + Bb)}{2b^3x^2 + 2b^2cx^4} + \frac{(-2Ac + Bb)\log(x)}{b^3} - \frac{(-2Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] (-A\*b + x\*\*2\*(-2\*A\*c + B\*b))/(2\*b\*\*3\*x\*\*2 + 2\*b\*\*2\*c\*x\*\*4) + (-2\*A\*c + B\*b)\*log(x)/b\*\*3 - (-2\*A\*c + B\*b)\*log(b/c + x\*\*2)/(2\*b\*\*3)

$$3.70 \quad \int \frac{A+Bx^2}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=90

$$-\frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}} - \frac{cx(bB-Ac)}{2b^3(b+cx^2)} - \frac{bB-2Ac}{b^3x} - \frac{A}{3b^2x^3}$$

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {1593, 456, 1261, 205}

$$-\frac{cx(bB-Ac)}{2b^3(b+cx^2)} - \frac{bB-2Ac}{b^3x} - \frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}} - \frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(b\*x^2 + c\*x^4)^2,x]

[Out] -A/(3\*b^2\*x^3) - (b\*B - 2\*A\*c)/(b^3\*x) - (c\*(b\*B - A\*c)\*x)/(2\*b^3\*(b + c\*x^2)) - (Sqrt[c]\*(3\*b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(7/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 456

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1)/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[x^m\*(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*Together[(b^(m/2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)\*x^(-m + 2)]/(a + b\*x^2)] - ((-a)^(m/2 - 1)\*(b\*c - a\*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

Rule 1261

Int[((f\_.)\*(x\_)^(m\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(f\*x)^m\*(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^4 (b + cx^2)^2} dx \\
&= -\frac{c(bB - Ac)x}{2b^3 (b + cx^2)} - \frac{1}{2}c \int \frac{-\frac{2A}{bc} - \frac{2(bB - Ac)x^2}{b^2c} + \frac{(bB - Ac)x^4}{b^3}}{x^4 (b + cx^2)} dx \\
&= -\frac{c(bB - Ac)x}{2b^3 (b + cx^2)} - \frac{1}{2}c \int \left( -\frac{2A}{b^2cx^4} - \frac{2(bB - 2Ac)}{b^3cx^2} + \frac{3bB - 5Ac}{b^3 (b + cx^2)} \right) dx \\
&= -\frac{A}{3b^2x^3} - \frac{bB - 2Ac}{b^3x} - \frac{c(bB - Ac)x}{2b^3 (b + cx^2)} - \frac{(c(3bB - 5Ac)) \int \frac{1}{b+cx^2} dx}{2b^3} \\
&= -\frac{A}{3b^2x^3} - \frac{bB - 2Ac}{b^3x} - \frac{c(bB - Ac)x}{2b^3 (b + cx^2)} - \frac{\sqrt{c} (3bB - 5Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{2b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 90, normalized size = 1.00

$$-\frac{\sqrt{c} (3bB - 5Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{2b^{7/2}} - \frac{cx(bB - Ac)}{2b^3 (b + cx^2)} + \frac{2Ac - bB}{b^3x} - \frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(b\*x^2 + c\*x^4)^2,x]

[Out] -1/3\*A/(b^2\*x^3) + (-b\*B) + 2\*A\*c)/(b^3\*x) - (c\*(b\*B - A\*c)\*x)/(2\*b^3\*(b + c\*x^2)) - (Sqrt[c]\*(3\*b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(7/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 + c\*x^4)^2, x]

**fricas [A]** time = 0.41, size = 250, normalized size = 2.78

$$\left[ \frac{6(3Bbc - 5Ac^2)x^4 + 4Ab^2 + 4(3Bb^2 - 5Abc)x^2 + 3((3Bbc - 5Ac^2)x^3 + (3Bb^2 - 5Abc)x^3)\sqrt{\frac{c}{b}} \log\left(\frac{cx^2 + 2bx\sqrt{\frac{c}{b}} + b}{cx^2 + b}\right)}{12(b^3cx^5 + b^4x^3)}, \frac{3(3Bbc - 5Ac^2)x^4 + 2Ab^2 + 2(3Bb^2 - 5Abc)x^2 + 3((3Bbc - 5Ac^2)x^5 + (3Bb^2 - 5Abc)x^3)\sqrt{\frac{c}{b}} \arctan\left(x\sqrt{\frac{c}{b}}\right)}{6(b^3cx^5 + b^4x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [-1/12\*(6\*(3\*B\*b\*c - 5\*A\*c^2)\*x^4 + 4\*A\*b^2 + 4\*(3\*B\*b^2 - 5\*A\*b\*c)\*x^2 + 3\*((3\*B\*b\*c - 5\*A\*c^2)\*x^5 + (3\*B\*b^2 - 5\*A\*b\*c)\*x^3)\*sqrt(-c/b)\*log((c\*x^2 + 2\*b\*x\*sqrt(-c/b) - b)/(c\*x^2 + b)))/(b^3\*c\*x^5 + b^4\*x^3), -1/6\*(3\*(3\*B\*b\*c - 5\*A\*c^2)\*x^4 + 2\*A\*b^2 + 2\*(3\*B\*b^2 - 5\*A\*b\*c)\*x^2 + 3\*((3\*B\*b\*c - 5\*A\*c^2)\*x^5 + (3\*B\*b^2 - 5\*A\*b\*c)\*x^3)\*sqrt(c/b)\*arctan(x\*sqrt(c/b)))/(b^3\*c\*x^5 + b^4\*x^3)]

**giac** [A] time = 0.16, size = 85, normalized size = 0.94

$$-\frac{(3Bbc - 5Ac^2) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b^3} - \frac{Bbcx - Ac^2x}{2(cx^2 + b)b^3} - \frac{3Bbx^2 - 6Acx^2 + Ab}{3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $-1/2*(3*B*b*c - 5*A*c^2)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b^3) - 1/2*(B*b*c*x - A*c^2*x)/((c*x^2 + b)*b^3) - 1/3*(3*B*b*x^2 - 6*A*c*x^2 + A*b)/(b^3*x^3)$

**maple** [A] time = 0.07, size = 110, normalized size = 1.22

$$\frac{Ac^2x}{2(cx^2 + b)b^3} + \frac{5Ac^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b^3} - \frac{Bcx}{2(cx^2 + b)b^2} - \frac{3Bc \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b^2} + \frac{2Ac}{b^3x} - \frac{B}{b^2x} - \frac{A}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $1/2/b^3*c^2*x/(c*x^2+b)*A - 1/2/b^2*c*x/(c*x^2+b)*B + 5/2/b^3*c^2/(b*c)^{(1/2)*\arctan(1/(b*c)^{(1/2)*c*x}*A - 3/2/b^2*c/(b*c)^{(1/2)*\arctan(1/(b*c)^{(1/2)*c*x}*B - 1/3*A/b^2/x^3 + 2/b^3/x*A*c - 1/b^2/x*B}$

**maxima** [A] time = 3.00, size = 93, normalized size = 1.03

$$-\frac{3(3Bbc - 5Ac^2)x^4 + 2Ab^2 + 2(3Bb^2 - 5Abc)x^2}{6(b^3cx^5 + b^4x^3)} - \frac{(3Bbc - 5Ac^2) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-1/6*(3*(3*B*b*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2)/(b^3*c*x^5 + b^4*x^3) - 1/2*(3*B*b*c - 5*A*c^2)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b^3)$

**mupad** [B] time = 0.14, size = 83, normalized size = 0.92

$$\frac{\frac{x^2(5Ac-3Bb)}{3b^2} - \frac{A}{3b} + \frac{cx^4(5Ac-3Bb)}{2b^3}}{cx^5 + bx^3} + \frac{\sqrt{c} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(5Ac - 3Bb)}{2b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(b\*x^2 + c\*x^4)^2,x)

[Out]  $((x^2*(5*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (c*x^4*(5*A*c - 3*B*b))/(2*b^3))/(b*x^3 + c*x^5) + (c^{(1/2)*\operatorname{atan}((c^{(1/2)*x}/b^{(1/2)})*(5*A*c - 3*B*b)))/(2*b^{(7/2)})$

**sympy** [B] time = 0.60, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{c}{b^7}}(-5Ac + 3Bb) \log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac + 3Bb)}{-5Ac^2 + 3Bbc} + x\right)}{4} - \frac{\sqrt{-\frac{c}{b^7}}(-5Ac + 3Bb) \log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac + 3Bb)}{-5Ac^2 + 3Bbc} + x\right)}{4} + \frac{-2Ab^2 + x^4(15Ac^2 - 9Bbc) + x^2(10Abc - 6Bb^2)}{6b^4x^3 + 6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] sqrt(-c/b\*\*7)\*(-5\*A\*c + 3\*B\*b)\*log(-b\*\*4\*sqrt(-c/b\*\*7)\*(-5\*A\*c + 3\*B\*b)/(-5\*A\*c\*\*2 + 3\*B\*b\*c) + x)/4 - sqrt(-c/b\*\*7)\*(-5\*A\*c + 3\*B\*b)\*log(b\*\*4\*sqrt(-c/b\*\*7)\*(-5\*A\*c + 3\*B\*b)/(-5\*A\*c\*\*2 + 3\*B\*b\*c) + x)/4 + (-2\*A\*b\*\*2 + x\*\*4\*(15\*A\*c\*\*2 - 9\*B\*b\*c) + x\*\*2\*(10\*A\*b\*c - 6\*B\*b\*\*2))/(6\*b\*\*4\*x\*\*3 + 6\*b\*\*3\*c\*x\*\*5)

$$3.71 \quad \int \frac{A+Bx^2}{x(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=97

$$\frac{c(2bB - 3Ac) \log(b + cx^2)}{2b^4} - \frac{c \log(x)(2bB - 3Ac)}{b^4} - \frac{c(bB - Ac)}{2b^3(b + cx^2)} - \frac{bB - 2Ac}{2b^3x^2} - \frac{A}{4b^2x^4}$$

**Rubi [A]** time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{c(bB - Ac)}{2b^3(b + cx^2)} - \frac{bB - 2Ac}{2b^3x^2} + \frac{c(2bB - 3Ac) \log(b + cx^2)}{2b^4} - \frac{c \log(x)(2bB - 3Ac)}{b^4} - \frac{A}{4b^2x^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^2), x]

[Out] -A/(4\*b^2\*x^4) - (b\*B - 2\*A\*c)/(2\*b^3\*x^2) - (c\*(b\*B - A\*c))/(2\*b^3\*(b + c\*x^2)) - (c\*(2\*b\*B - 3\*A\*c)\*Log[x])/b^4 + (c\*(2\*b\*B - 3\*A\*c)\*Log[b + c\*x^2])/(2\*b^4)

**Rule 77**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 446**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{A + Bx^2}{x(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^5(b + cx^2)^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^3(b + cx)^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^2x^3} + \frac{bB - 2Ac}{b^3x^2} - \frac{c(2bB - 3Ac)}{b^4x} + \frac{c^2(bB - Ac)}{b^3(b + cx)^2} + \frac{c^2(2bB - 3Ac)}{b^4(b + cx)} \right) dx, x, x \right) \\ &= -\frac{A}{4b^2x^4} - \frac{bB - 2Ac}{2b^3x^2} - \frac{c(bB - Ac)}{2b^3(b + cx^2)} - \frac{c(2bB - 3Ac) \log(x)}{b^4} + \frac{c(2bB - 3Ac) \log(b + cx^2)}{2b^4} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 85, normalized size = 0.88

$$\frac{\frac{Ab^2}{x^4} + \frac{2bc(bB-Ac)}{b+cx^2} + \frac{2b(bB-2Ac)}{x^2} + 2c(3Ac - 2bB) \log(b + cx^2) - 4c \log(x)(3Ac - 2bB)}{4b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^2), x]

[Out]  $-1/4*((A*b^2)/x^4 + (2*b*(b*B - 2*A*c))/x^2 + (2*b*c*(b*B - A*c))/(b + c*x^2) - 4*c*(-2*b*B + 3*A*c)*\text{Log}[x] + 2*c*(-2*b*B + 3*A*c)*\text{Log}[b + c*x^2])/b^4$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^2), x]

**fricas [A]** time = 0.40, size = 154, normalized size = 1.59

$$\frac{2(2Bb^2c - 3Abc^2)x^4 + Ab^3 + (2Bb^3 - 3Ab^2c)x^2 - 2((2Bbc^2 - 3Ac^3)x^6 + (2Bb^2c - 3Abc^2)x^4) \log(cx^2 + b) + 4((2Bbc^2 - 3Ac^3)x^6 + (2Bb^2c - 3Abc^2)x^4) \log(x)}{4(b^4cx^6 + b^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out]  $-1/4*(2*(2*B*b^2*c - 3*A*b*c^2)*x^4 + A*b^3 + (2*B*b^3 - 3*A*b^2*c)*x^2 - 2*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*\log(c*x^2 + b) + 4*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*\log(x))/(b^4*c*x^6 + b^5*x^4)$

**giac [A]** time = 0.32, size = 150, normalized size = 1.55

$$\frac{(2Bbc - 3Ac^2) \log(x^2)}{2b^4} + \frac{(2Bbc^2 - 3Ac^3) \log(|cx^2 + b|)}{2b^4c} - \frac{2Bbc^2x^2 - 3Ac^3x^2 + 3Bb^2c - 4Abc^2}{2(cx^2 + b)b^4} + \frac{6Bbcx^4 - 9Ac^2x^4 - 2Bb^2x^2 + 4Abc^2 - Ab^2}{4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $-1/2*(2*B*b*c - 3*A*c^2)*\log(x^2)/b^4 + 1/2*(2*B*b*c^2 - 3*A*c^3)*\log(\text{abs}(c*x^2 + b))/(b^4*c) - 1/2*(2*B*b*c^2*x^2 - 3*A*c^3*x^2 + 3*B*b^2*c - 4*A*b*c^2)/((c*x^2 + b)*b^4) + 1/4*(6*B*b*c*x^4 - 9*A*c^2*x^4 - 2*B*b^2*x^2 + 4*A*b*c*x^2 - A*b^2)/(b^4*x^4)$

**maple [A]** time = 0.06, size = 114, normalized size = 1.18

$$\frac{Ac^2}{2(cx^2 + b)b^3} + \frac{3Ac^2 \ln(x)}{b^4} - \frac{3Ac^2 \ln(cx^2 + b)}{2b^4} - \frac{Bc}{2(cx^2 + b)b^2} - \frac{2Bc \ln(x)}{b^3} + \frac{Bc \ln(cx^2 + b)}{b^3} + \frac{Ac}{b^3x^2} - \frac{B}{2b^2x^2} - \frac{A}{4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x/(c\*x^4+b\*x^2)^2,x)

[Out]  $-3/2/b^4*c^2*\ln(c*x^2+b)*A+1/b^3*c*\ln(c*x^2+b)*B+1/2/b^3*c^2/(c*x^2+b)*A-1/2/b^2*c/(c*x^2+b)*B-1/4*A/b^2/x^4+1/b^3/x^2*A*c-1/2/b^2/x^2*B+3*c^2/b^4*\ln(x)*A-2*c/b^3*\ln(x)*B$

**maxima [A]** time = 1.33, size = 106, normalized size = 1.09

$$-\frac{2(2Bbc - 3Ac^2)x^4 + Ab^2 + (2Bb^2 - 3Abc)x^2}{4(b^3cx^6 + b^4x^4)} + \frac{(2Bbc - 3Ac^2)\log(cx^2 + b)}{2b^4} - \frac{(2Bbc - 3Ac^2)\log(x^2)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-1/4*(2*(2*B*b*c - 3*A*c^2)*x^4 + A*b^2 + (2*B*b^2 - 3*A*b*c)*x^2)/(b^3*c*x^6 + b^4*x^4) + 1/2*(2*B*b*c - 3*A*c^2)*\log(c*x^2 + b)/b^4 - 1/2*(2*B*b*c - 3*A*c^2)*\log(x^2)/b^4$

**mupad [B]** time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^2(3Ac-2Bb)}{4b^2} - \frac{A}{4b} + \frac{cx^4(3Ac-2Bb)}{2b^3}}{cx^6 + bx^4} - \frac{\ln(cx^2 + b)(3Ac^2 - 2Bbc)}{2b^4} + \frac{\ln(x)(3Ac^2 - 2Bbc)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^2),x)

[Out]  $((x^2*(3A*c - 2B*b))/(4*b^2) - A/(4*b) + (c*x^4*(3A*c - 2B*b))/(2*b^3))/ (b*x^4 + c*x^6) - (\log(b + c*x^2)*(3A*c^2 - 2B*b*c))/(2*b^4) + (\log(x)*(3A*c^2 - 2B*b*c))/b^4$

**sympy [A]** time = 1.04, size = 100, normalized size = 1.03

$$\frac{-Ab^2 + x^4(6Ac^2 - 4Bbc) + x^2(3Abc - 2Bb^2)}{4b^4x^4 + 4b^3cx^6} - \frac{c(-3Ac + 2Bb)\log(x)}{b^4} + \frac{c(-3Ac + 2Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out]  $(-A*b**2 + x**4*(6*A*c**2 - 4*B*b*c) + x**2*(3*A*b*c - 2*B*b**2))/(4*b**4*x**4 + 4*b**3*c*x**6) - c*(-3*A*c + 2*B*b)*\log(x)/b**4 + c*(-3*A*c + 2*B*b)*\log(b/c + x**2)/(2*b**4)$

$$3.72 \quad \int \frac{A+Bx^2}{x^2(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=111

$$\frac{c^{3/2}(5bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}} + \frac{c^2x(bB - Ac)}{2b^4(b + cx^2)} + \frac{c(2bB - 3Ac)}{b^4x} - \frac{bB - 2Ac}{3b^3x^3} - \frac{A}{5b^2x^5}$$

Rubi [A] time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 456, 1802, 205}

$$\frac{c^2x(bB - Ac)}{2b^4(b + cx^2)} + \frac{c^{3/2}(5bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}} - \frac{bB - 2Ac}{3b^3x^3} + \frac{c(2bB - 3Ac)}{b^4x} - \frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^2), x]

[Out] -A/(5\*b^2\*x^5) - (b\*B - 2\*A\*c)/(3\*b^3\*x^3) + (c\*(2\*b\*B - 3\*A\*c))/(b^4\*x) + (c^2\*(b\*B - A\*c)\*x)/(2\*b^4\*(b + c\*x^2)) + (c^(3/2)\*(5\*b\*B - 7\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(9/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 456

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[x^m\*(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*Together[(b^(m/2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)\*x^(-m + 2)]/(a + b\*x^2)] - ((-a)^(m/2 - 1)\*(b\*c - a\*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq\_)\*((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^2 (bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^6 (b + cx^2)^2} dx \\
&= \frac{c^2(bB - Ac)x}{2b^4 (b + cx^2)} - \frac{1}{2}c^2 \int \frac{-\frac{2A}{bc^2} - \frac{2(bB - Ac)x^2}{b^2c^2} + \frac{2(bB - Ac)x^4}{b^3c} - \frac{(bB - Ac)x^6}{b^4}}{x^6 (b + cx^2)} dx \\
&= \frac{c^2(bB - Ac)x}{2b^4 (b + cx^2)} - \frac{1}{2}c^2 \int \left( -\frac{2A}{b^2c^2x^6} - \frac{2(bB - 2Ac)}{b^3c^2x^4} + \frac{2(2bB - 3Ac)}{b^4cx^2} + \frac{-5bB + 7Ac}{b^4(b + cx^2)} \right) dx \\
&= -\frac{A}{5b^2x^5} - \frac{bB - 2Ac}{3b^3x^3} + \frac{c(2bB - 3Ac)}{b^4x} + \frac{c^2(bB - Ac)x}{2b^4(b + cx^2)} + \frac{(c^2(5bB - 7Ac)) \int \frac{1}{b+cx^2} dx}{2b^4} \\
&= -\frac{A}{5b^2x^5} - \frac{bB - 2Ac}{3b^3x^3} + \frac{c(2bB - 3Ac)}{b^4x} + \frac{c^2(bB - Ac)x}{2b^4(b + cx^2)} + \frac{c^{3/2}(5bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 112, normalized size = 1.01

$$\frac{c^{3/2}(5bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{2b^{9/2}} + \frac{c^2x(bB - Ac)}{2b^4(b + cx^2)} + \frac{c(2bB - 3Ac)}{b^4x} + \frac{2Ac - bB}{3b^3x^3} - \frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^2), x]

[Out] -1/5\*A/(b^2\*x^5) + (-b\*B + 2\*A\*c)/(3\*b^3\*x^3) + (c\*(2\*b\*B - 3\*A\*c))/(b^4\*x) + (c^2\*(b\*B - A\*c)\*x)/(2\*b^4\*(b + c\*x^2)) + (c^(3/2)\*(5\*b\*B - 7\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(2\*b^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2 (bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^2), x]

**fricas [A]** time = 0.42, size = 308, normalized size = 2.77

$$\left| \frac{30(5Bb^2 - 7Ac^2)x^6 + 20(5Bb^2c - 7Ab^2c^2)x^4 - 12Ab^3 - 4(5Bb^2 - 7Ab^2c^2)x^2 - 15((5Bb^2 - 7Ac^2)x^7 + (5Bb^2c - 7Ab^2c^2)x^5)\sqrt{\frac{c}{b}} \log\left(\frac{c^2 - 2bx\sqrt{\frac{c}{b}} + b}{c^2 + bx}\right)}{60(b^4cx^7 + b^5x^5)} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] [1/60\*(30\*(5\*B\*b\*c^2 - 7\*A\*c^3)\*x^6 + 20\*(5\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^4 - 12\*A\*b^3 - 4\*(5\*B\*b^3 - 7\*A\*b^2\*c)\*x^2 - 15\*((5\*B\*b\*c^2 - 7\*A\*c^3)\*x^7 + (5\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^5)\*sqrt(-c/b)\*log((c\*x^2 - 2\*b\*x\*sqrt(-c/b) - b)/(c\*x^2 + b)))/(b^4\*c\*x^7 + b^5\*x^5), 1/30\*(15\*(5\*B\*b\*c^2 - 7\*A\*c^3)\*x^6 + 10\*(5\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^4 - 6\*A\*b^3 - 2\*(5\*B\*b^3 - 7\*A\*b^2\*c)\*x^2 + 15\*((5\*B\*b\*c^2 - 7\*A\*c^3)\*x^7 + (5\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^5)\*sqrt(c/b)\*arctan(x\*sqrt(c/b)))/(b^4\*c\*x^7 + b^5\*x^5)]

**giac** [A] time = 0.17, size = 112, normalized size = 1.01

$$\frac{(5 B b c^2 - 7 A c^3) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c} b^4} + \frac{B b c^2 x - A c^3 x}{2 (c x^2 + b) b^4} + \frac{30 B b c x^4 - 45 A c^2 x^4 - 5 B b^2 x^2 + 10 A b c x^2 - 3 A b^2}{15 b^4 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/2\*(5\*B\*b\*c^2 - 7\*A\*c^3)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^4) + 1/2\*(B\*b\*c^2\*x - A\*c^3\*x)/((c\*x^2 + b)\*b^4) + 1/15\*(30\*B\*b\*c\*x^4 - 45\*A\*c^2\*x^4 - 5\*B\*b^2\*x^2 + 10\*A\*b\*c\*x^2 - 3\*A\*b^2)/(b^4\*x^5)

**maple** [A] time = 0.06, size = 136, normalized size = 1.23

$$\frac{A c^3 x}{2 (c x^2 + b) b^4} - \frac{7 A c^3 \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c} b^4} + \frac{B c^2 x}{2 (c x^2 + b) b^3} + \frac{5 B c^2 \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c} b^3} - \frac{3 A c^2}{b^4 x} + \frac{2 B c}{b^3 x} + \frac{2 A c}{3 b^3 x^3} - \frac{B}{3 b^2 x^3} - \frac{A}{5 b^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^2,x)

[Out] -1/2/b^4\*c^3\*x/(c\*x^2+b)\*A+1/2/b^3\*c^2\*x/(c\*x^2+b)\*B-7/2/b^4\*c^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A+5/2/b^3\*c^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B-1/5\*A/b^2/x^5+2/3/b^3/x^3\*A\*c-1/3/b^2/x^3\*B-3\*c^2/b^4/x\*A+2\*c/b^3/x\*B

**maxima** [A] time = 2.94, size = 119, normalized size = 1.07

$$\frac{15 (5 B b c^2 - 7 A c^3) x^6 + 10 (5 B b^2 c - 7 A b c^2) x^4 - 6 A b^3 - 2 (5 B b^3 - 7 A b^2 c) x^2}{30 (b^4 c x^7 + b^5 x^5)} + \frac{(5 B b c^2 - 7 A c^3) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c} b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/30\*(15\*(5\*B\*b\*c^2 - 7\*A\*c^3)\*x^6 + 10\*(5\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^4 - 6\*A\*b^3 - 2\*(5\*B\*b^3 - 7\*A\*b^2\*c)\*x^2)/(b^4\*c\*x^7 + b^5\*x^5) + 1/2\*(5\*B\*b\*c^2 - 7\*A\*c^3)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^4)

**mupad** [B] time = 0.16, size = 104, normalized size = 0.94

$$\frac{\frac{A}{5 b} - \frac{x^2 (7 A c - 5 B b)}{15 b^2} + \frac{c^2 x^6 (7 A c - 5 B b)}{2 b^4} + \frac{c x^4 (7 A c - 5 B b)}{3 b^3}}{c x^7 + b x^5} - \frac{c^{3/2} \operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{b}}\right) (7 A c - 5 B b)}{2 b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^2),x)

[Out] - (A/(5\*b) - (x^2\*(7\*A\*c - 5\*B\*b))/(15\*b^2) + (c^2\*x^6\*(7\*A\*c - 5\*B\*b))/(2\*b^4) + (c\*x^4\*(7\*A\*c - 5\*B\*b))/(3\*b^3))/(b\*x^5 + c\*x^7) - (c^(3/2)\*atan((c^(1/2)\*x)/b^(1/2))\*(7\*A\*c - 5\*B\*b))/(2\*b^(9/2))

**sympy** [B] time = 0.70, size = 218, normalized size = 1.96

$$\frac{\sqrt{\frac{c^3}{b^9}} (-7 A c + 5 B b) \log\left(\frac{b^5 \sqrt{\frac{c^3}{b^9}} (-7 A c + 5 B b)}{-7 A c^3 + 5 B b c^2} + x\right)}{4} + \frac{\sqrt{\frac{c^3}{b^9}} (-7 A c + 5 B b) \log\left(\frac{b^5 \sqrt{\frac{c^3}{b^9}} (-7 A c + 5 B b)}{-7 A c^3 + 5 B b c^2} - x\right)}{4} + \frac{-6 A b^3 + x^6 (-105 A c^3 + 75 B b c^2) + x^4 (-70 A b c^2 + 50 B b^2 c) + x^2 (14 A b^2 c - 10 B b^3)}{30 b^5 x^5 + 30 b^4 c x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*2/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out]  $-\sqrt{-c^{**3}/b^{**9}}*(-7*A*c + 5*B*b)*\log(-b^{**5}\sqrt{-c^{**3}/b^{**9}}*(-7*A*c + 5*B*b)/(-7*A*c^{**3} + 5*B*b*c^{**2}) + x)/4 + \sqrt{-c^{**3}/b^{**9}}*(-7*A*c + 5*B*b)*\log(b^{**5}\sqrt{-c^{**3}/b^{**9}}*(-7*A*c + 5*B*b)/(-7*A*c^{**3} + 5*B*b*c^{**2}) + x)/4 + (-6*A*b^{**3} + x^{**6}*(-105*A*c^{**3} + 75*B*b*c^{**2}) + x^{**4}*(-70*A*b*c^{**2} + 50*B*b*2*c) + x^{**2}(14*A*b^{**2}*c - 10*B*b^{**3}))/ (30*b^{**5}*x^{**5} + 30*b^{**4}*c*x^{**7})$



$$3.73 \quad \int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=140

$$-\frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}} - \frac{b^3x(bB-Ac)}{4c^5(b+cx^2)^2} + \frac{b^2x(17bB-13Ac)}{8c^5(b+cx^2)} + \frac{3bx(2bB-Ac)}{c^5} - \frac{x^3(3bB-Ac)}{3c^4} + \frac{Bx^5}{5c^3}$$

**Rubi [A]** time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1584, 455, 1814, 1810, 205}

$$\frac{b^2x(17bB-13Ac)}{8c^5(b+cx^2)} - \frac{b^3x(bB-Ac)}{4c^5(b+cx^2)^2} - \frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}} - \frac{x^3(3bB-Ac)}{3c^4} + \frac{3bx(2bB-Ac)}{c^5} + \frac{Bx^5}{5c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^14\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (3\*b\*(2\*b\*B - A\*c)\*x)/c^5 - ((3\*b\*B - A\*c)\*x^3)/(3\*c^4) + (B\*x^5)/(5\*c^3) - (b^3\*(b\*B - A\*c)\*x)/(4\*c^5\*(b + c\*x^2)^2) + (b^2\*(17\*b\*B - 13\*A\*c)\*x)/(8\*c^5\*(b + c\*x^2)) - (7\*b^(3/2)\*(9\*b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*c^(11/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^n, x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /

; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^{14} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^8 (A + Bx^2)}{(b + cx^2)^3} dx \\ &= -\frac{b^3(bB - Ac)x}{4c^5 (b + cx^2)^2} - \frac{\int \frac{-b^3(bB - Ac) + 4b^2c(bB - Ac)x^2 - 4bc^2(bB - Ac)x^4 + 4c^3(bB - Ac)x^6 - 4Bc^4x^8}{(b + cx^2)^2} dx}{4c^5} \\ &= -\frac{b^3(bB - Ac)x}{4c^5 (b + cx^2)^2} + \frac{b^2(17bB - 13Ac)x}{8c^5 (b + cx^2)} + \frac{\int \frac{-b^3(15bB - 11Ac) + 8b^2c(3bB - 2Ac)x^2 - 8bc^2(2bB - Ac)x^4 + 8bBc^3x^6}{b + cx^2} dx}{8bc^5} \\ &= -\frac{b^3(bB - Ac)x}{4c^5 (b + cx^2)^2} + \frac{b^2(17bB - 13Ac)x}{8c^5 (b + cx^2)} + \frac{\int (24b^2(2bB - Ac) - 8bc(3bB - Ac)x^2 + 8bBc^2x^4) dx}{8bc^5} \\ &= \frac{3b(2bB - Ac)x}{c^5} - \frac{(3bB - Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB - Ac)x}{4c^5 (b + cx^2)^2} + \frac{b^2(17bB - 13Ac)x}{8c^5 (b + cx^2)} - \frac{(7b^2(9bB - 11Ac))}{8c^5} \\ &= \frac{3b(2bB - Ac)x}{c^5} - \frac{(3bB - Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB - Ac)x}{4c^5 (b + cx^2)^2} + \frac{b^2(17bB - 13Ac)x}{8c^5 (b + cx^2)} - \frac{7b^{3/2}(9bB - 11Ac)}{8c^{11/2}} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 133, normalized size = 0.95

$$\frac{x(-525b^3c(A - 3Bx^2) + 7b^2c^2x^2(72Bx^2 - 125A) - 8bc^3x^4(35A + 9Bx^2) + 8c^4x^6(5A + 3Bx^2) + 945b^4B)}{120c^5(b + cx^2)^2} - \frac{7b^{3/2}(9bB - 5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^14\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (x\*(945\*b^4\*B - 525\*b^3\*c\*(A - 3\*B\*x^2) + 8\*c^4\*x^6\*(5\*A + 3\*B\*x^2) - 8\*b\*c^3\*x^4\*(35\*A + 9\*B\*x^2) + 7\*b^2\*c^2\*x^2\*(-125\*A + 72\*B\*x^2)))/(120\*c^5\*(b + c\*x^2)^2) - (7\*b^(3/2)\*(9\*b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*c^(11/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14} (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^14\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^14\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.40, size = 416, normalized size = 2.97

$$\frac{48b^4c^3 - 16(9bB^2 - 5Ac^2)c^2 + 112(9bB^2 - 5Ac^2)c^3 + 350(9bB^2 - 5Ac^2)c^4 - 118(9bB^2 - 5Ac^2)c^5 + (9bB^2 - 5Ac^2)c^6 + 2(9bB^2 - 5Ac^2)c^7\sqrt{\frac{c}{b}} \log\left(\frac{c^2 + bx^2}{2bx^2}\right) + 210(9bB^2 - 5Ac^2)c^8 + 24bBc^4 - 8(9bB^2 - 5Ac^2)c^5 + 56(9bB^2 - 5Ac^2)c^6 + 175(9bB^2 - 5Ac^2)c^7 - 118(9bB^2 - 5Ac^2)c^8 + (9bB^2 - 5Ac^2)c^9 + 2(9bB^2 - 5Ac^2)c^{10}\sqrt{\frac{c}{b}} \operatorname{arctan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) + 105(9bB^2 - 5Ac^2)c^{11}}{120(c^5 + 2bc^3 + b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>14</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>)<sup>3</sup>,x, algorithm="fricas")

[Out] [1/240\*(48\*B\*c<sup>4</sup>\*x<sup>9</sup> - 16\*(9\*B\*b\*c<sup>3</sup> - 5\*A\*c<sup>4</sup>)\*x<sup>7</sup> + 112\*(9\*B\*b<sup>2</sup>\*c<sup>2</sup> - 5\*A\*b\*c<sup>3</sup>)\*x<sup>5</sup> + 350\*(9\*B\*b<sup>3</sup>\*c - 5\*A\*b<sup>2</sup>\*c<sup>2</sup>)\*x<sup>3</sup> - 105\*(9\*B\*b<sup>4</sup> - 5\*A\*b<sup>3</sup>\*c + (9\*B\*b<sup>2</sup>\*c<sup>2</sup> - 5\*A\*b\*c<sup>3</sup>)\*x<sup>4</sup> + 2\*(9\*B\*b<sup>3</sup>\*c - 5\*A\*b<sup>2</sup>\*c<sup>2</sup>)\*x<sup>2</sup>)\*sqrt(-b/c)\*log((c\*x<sup>2</sup> + 2\*c\*x\*sqrt(-b/c) - b)/(c\*x<sup>2</sup> + b)) + 210\*(9\*B\*b<sup>4</sup> - 5\*A\*b<sup>3</sup>\*c)\*x)/(c<sup>7</sup>\*x<sup>4</sup> + 2\*b\*c<sup>6</sup>\*x<sup>2</sup> + b<sup>2</sup>\*c<sup>5</sup>), 1/120\*(24\*B\*c<sup>4</sup>\*x<sup>9</sup> - 8\*(9\*B\*b\*c<sup>3</sup> - 5\*A\*c<sup>4</sup>)\*x<sup>7</sup> + 56\*(9\*B\*b<sup>2</sup>\*c<sup>2</sup> - 5\*A\*b\*c<sup>3</sup>)\*x<sup>5</sup> + 175\*(9\*B\*b<sup>3</sup>\*c - 5\*A\*b<sup>2</sup>\*c<sup>2</sup>)\*x<sup>3</sup> - 105\*(9\*B\*b<sup>4</sup> - 5\*A\*b<sup>3</sup>\*c + (9\*B\*b<sup>2</sup>\*c<sup>2</sup> - 5\*A\*b\*c<sup>3</sup>)\*x<sup>4</sup> + 2\*(9\*B\*b<sup>3</sup>\*c - 5\*A\*b<sup>2</sup>\*c<sup>2</sup>)\*x<sup>2</sup>)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) + 105\*(9\*B\*b<sup>4</sup> - 5\*A\*b<sup>3</sup>\*c)\*x)/(c<sup>7</sup>\*x<sup>4</sup> + 2\*b\*c<sup>6</sup>\*x<sup>2</sup> + b<sup>2</sup>\*c<sup>5</sup>)]

**giac** [A] time = 0.18, size = 138, normalized size = 0.99

$$\frac{7(9Bb^3 - 5Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right) + \frac{17Bb^3cx^3 - 13Ab^2c^2x^3 + 15Bb^4x - 11Ab^3cx}{8(cx^2 + b)^2c^5} + \frac{3Bc^{12}x^5 - 15Bbc^{11}x^3 + 5Ac^{12}x^3 + 90Bb^2c^{10}x - 45Abc^{11}x}{15c^{15}}}{8\sqrt{bc}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>14</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>)<sup>3</sup>,x, algorithm="giac")

[Out] -7/8\*(9\*B\*b<sup>3</sup> - 5\*A\*b<sup>2</sup>\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c<sup>5</sup>) + 1/8\*(17\*B\*b<sup>3</sup>\*c\*x<sup>3</sup> - 13\*A\*b<sup>2</sup>\*c<sup>2</sup>\*x<sup>3</sup> + 15\*B\*b<sup>4</sup>\*x - 11\*A\*b<sup>3</sup>\*c\*x)/((c\*x<sup>2</sup> + b)<sup>2</sup>\*c<sup>5</sup>) + 1/15\*(3\*B\*c<sup>12</sup>\*x<sup>5</sup> - 15\*B\*b\*c<sup>11</sup>\*x<sup>3</sup> + 5\*A\*c<sup>12</sup>\*x<sup>3</sup> + 90\*B\*b<sup>2</sup>\*c<sup>10</sup>\*x - 45\*A\*b\*c<sup>11</sup>\*x)/c<sup>15</sup>

**maple** [A] time = 0.06, size = 174, normalized size = 1.24

$$\frac{\frac{13Ab^2x^3}{8(cx^2 + b)^2c^3} + \frac{17Bb^3x^3}{8(cx^2 + b)^2c^4} + \frac{Bx^5}{5c^3} - \frac{11Ab^3x}{8(cx^2 + b)^2c^4} + \frac{Ax^3}{3c^3} + \frac{15Bb^4x}{8(cx^2 + b)^2c^5} - \frac{Bbx^3}{c^4} + \frac{35Ab^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^4} - \frac{63Bb^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^5} - \frac{3Abx}{c^4} + \frac{6Bb^2x}{c^5}}{8\sqrt{bc}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>14</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>)<sup>3</sup>,x)

[Out] 1/5\*B\*x<sup>5</sup>/c<sup>3</sup>+1/3/c<sup>3</sup>\*A\*x<sup>3</sup>-1/c<sup>4</sup>\*B\*x<sup>3</sup>\*b-3/c<sup>4</sup>\*A\*b\*x+6/c<sup>5</sup>\*B\*b<sup>2</sup>\*x-13/8\*b<sup>2</sup>/c<sup>3</sup>/(c\*x<sup>2</sup>+b)<sup>2</sup>\*A\*x<sup>3</sup>+17/8\*b<sup>3</sup>/c<sup>4</sup>/(c\*x<sup>2</sup>+b)<sup>2</sup>\*B\*x<sup>3</sup>-11/8\*b<sup>3</sup>/c<sup>4</sup>/(c\*x<sup>2</sup>+b)<sup>2</sup>\*A\*x+15/8\*b<sup>4</sup>/c<sup>5</sup>/(c\*x<sup>2</sup>+b)<sup>2</sup>\*B\*x+35/8\*b<sup>2</sup>/c<sup>4</sup>/(b\*c)<sup>(1/2)</sup>\*arctan(1/(b\*c)<sup>(1/2)</sup>\*c\*x)\*A-63/8\*b<sup>3</sup>/c<sup>5</sup>/(b\*c)<sup>(1/2)</sup>\*arctan(1/(b\*c)<sup>(1/2)</sup>\*c\*x)\*B

**maxima** [A] time = 2.90, size = 147, normalized size = 1.05

$$\frac{(17Bb^3c - 13Ab^2c^2)x^3 + (15Bb^4 - 11Ab^3c)x}{8(c^7x^4 + 2bc^6x^2 + b^2c^5)} - \frac{7(9Bb^3 - 5Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^5} + \frac{3Bc^2x^5 - 5(3Bbc - Ac^2)x^3 + 45(2Bb^2 - Abc)x}{15c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>14</sup>\*(B\*x<sup>2</sup>+A)/(c\*x<sup>4</sup>+b\*x<sup>2</sup>)<sup>3</sup>,x, algorithm="maxima")

[Out] 1/8\*((17\*B\*b<sup>3</sup>\*c - 13\*A\*b<sup>2</sup>\*c<sup>2</sup>)\*x<sup>3</sup> + (15\*B\*b<sup>4</sup> - 11\*A\*b<sup>3</sup>\*c)\*x)/(c<sup>7</sup>\*x<sup>4</sup> + 2\*b\*c<sup>6</sup>\*x<sup>2</sup> + b<sup>2</sup>\*c<sup>5</sup>) - 7/8\*(9\*B\*b<sup>3</sup> - 5\*A\*b<sup>2</sup>\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c<sup>5</sup>) + 1/15\*(3\*B\*c<sup>12</sup>\*x<sup>5</sup> - 5\*(3\*B\*b\*c - A\*c<sup>2</sup>)\*x<sup>3</sup> + 45\*(2\*B\*b<sup>2</sup> - A\*b\*c)\*x)/c<sup>5</sup>

**mupad** [B] time = 0.12, size = 177, normalized size = 1.26

$$\frac{x\left(\frac{15Bb^4}{8} - \frac{11Ab^3c}{8}\right) - x^3\left(\frac{13Ab^2c^2}{8} - \frac{17Bb^3c}{8}\right) - x\left(\frac{3b\left(\frac{A}{c^3} - \frac{3Bb}{c^4}\right) + \frac{3Bb^2}{c^5}}{c} + x^3\left(\frac{A}{3c^3} - \frac{Bb}{c^4}\right) + \frac{Bx^5}{5c^3} - \frac{7b^{3/2}\operatorname{atan}\left(\frac{b^{3/2}\sqrt{c}x(5Ac-9Bb)}{9Bb^3-5Ab^2c}\right)}{8c^{11/2}}\right)(5Ac-9Bb)}{b^2c^5 + 2bc^6x^2 + c^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x<sup>14</sup>\*(A + B\*x<sup>2</sup>))/(b\*x<sup>2</sup> + c\*x<sup>4</sup>)<sup>3</sup>,x)

[Out]  $(x*((15*B*b^4)/8 - (11*A*b^3*c)/8) - x^3*((13*A*b^2*c^2)/8 - (17*B*b^3*c)/8)) / (b^2*c^5 + c^7*x^4 + 2*b*c^6*x^2) - x*((3*b*(A/c^3 - (3*B*b)/c^4))/c + (3*B*b^2)/c^5) + x^3*(A/(3*c^3) - (B*b)/c^4) + (B*x^5)/(5*c^3) - (7*b^{(3/2)}*atan((b^{(3/2)}*c^{(1/2)}*x*(5*A*c - 9*B*b))/(9*B*b^3 - 5*A*b^2*c))*(5*A*c - 9*B*b))/(8*c^{(11/2)})$

**sympy** [A] time = 1.40, size = 252, normalized size = 1.80

$$\frac{Bx^5}{5c^3} + x^3 \left( \frac{A}{3c^3} - \frac{Bb}{c^4} \right) + x \left( -\frac{3Ab}{c^4} + \frac{6Bb^2}{c^5} \right) + \frac{7\sqrt{-\frac{b^3}{c^3}}(-5Ac + 9Bb) \log\left( -\frac{7c^5\sqrt{-\frac{b^3}{c^3}}(-5Ac + 9Bb)}{-35Abc + 63Bb^2} + x \right)}{16} - \frac{7\sqrt{-\frac{b^3}{c^3}}(-5Ac + 9Bb) \log\left( \frac{7c^5\sqrt{-\frac{b^3}{c^3}}(-5Ac + 9Bb)}{-35Abc + 63Bb^2} + x \right)}{16} + \frac{x^3(-13Ab^2c^2 + 17Bb^3c) + x(-11Ab^3c + 15Bb^4)}{8b^2c^5 + 16bc^6x^2 + 8c^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*14\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out]  $B*x**5/(5*c**3) + x**3*(A/(3*c**3) - B*b/c**4) + x*(-3*A*b/c**4 + 6*B*b**2/c**5) + 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(-7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 - 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 + (x**3*(-13*A*b**2*c**2 + 17*B*b**3*c) + x*(-11*A*b**3*c + 15*B*b**4))/(8*b**2*c**5 + 16*b*c**6*x**2 + 8*c**7*x**4)$

$$3.74 \quad \int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=111

$$-\frac{b^3(bB - Ac)}{4c^5(b + cx^2)^2} + \frac{b^2(4bB - 3Ac)}{2c^5(b + cx^2)} + \frac{3b(2bB - Ac) \log(b + cx^2)}{2c^5} - \frac{x^2(3bB - Ac)}{2c^4} + \frac{Bx^4}{4c^3}$$

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$\frac{b^2(4bB - 3Ac)}{2c^5(b + cx^2)} - \frac{b^3(bB - Ac)}{4c^5(b + cx^2)^2} - \frac{x^2(3bB - Ac)}{2c^4} + \frac{3b(2bB - Ac) \log(b + cx^2)}{2c^5} + \frac{Bx^4}{4c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^13\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -((3\*b\*B - A\*c)\*x^2)/(2\*c^4) + (B\*x^4)/(4\*c^3) - (b^3\*(b\*B - A\*c))/(4\*c^5\*(b + c\*x^2)^2) + (b^2\*(4\*b\*B - 3\*A\*c))/(2\*c^5\*(b + c\*x^2)) + (3\*b\*(2\*b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^5)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^{13} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^7 (A + Bx^2)}{(b + cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x^3 (A + Bx)}{(b + cx)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{-3bB + Ac}{c^4} + \frac{Bx}{c^3} + \frac{b^3(bB - Ac)}{c^4(b + cx)^3} - \frac{b^2(4bB - 3Ac)}{c^4(b + cx)^2} + \frac{3b(2bB - Ac)}{c^4(b + cx)} \right) dx, x, x^2 \right) \\ &= -\frac{(3bB - Ac)x^2}{2c^4} + \frac{Bx^4}{4c^3} - \frac{b^3(bB - Ac)}{4c^5(b + cx^2)^2} + \frac{b^2(4bB - 3Ac)}{2c^5(b + cx^2)} + \frac{3b(2bB - Ac) \log(b + cx^2)}{2c^5} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 94, normalized size = 0.85

$$\frac{\frac{b^3(Ac-bB)}{(b+cx^2)^2} + \frac{2b^2(4bB-3Ac)}{b+cx^2} + 2cx^2(Ac-3bB) + 6b(2bB-Ac) \log(b+cx^2) + Bc^2x^4}{4c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^13\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*c\*(-3\*b\*B + A\*c)\*x^2 + B\*c^2\*x^4 + (b^3\*(-(b\*B) + A\*c))/(b + c\*x^2)^2 + (2\*b^2\*(4\*b\*B - 3\*A\*c))/(b + c\*x^2) + 6\*b\*(2\*b\*B - A\*c)\*Log[b + c\*x^2])/(4\*c^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13} (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^13\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^13\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.37, size = 179, normalized size = 1.61

$$\frac{Bc^4x^8 - 2(2Bbc^3 - Ac^4)x^6 + 7Bb^4 - 5Ab^3c - (11Bb^2c^2 - 4Abc^3)x^4 + 2(Bb^3c - 2Ab^2c^2)x^2 + 6(2Bb^4 - Ab^3c + (2Bb^2c^2 - Abc^3)x^4 + 2(2Bb^3c - Ab^2c^2)x^2) \log(cx^2 + b)}{4(c^7x^4 + 2bc^6x^2 + b^2c^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/4\*(B\*c^4\*x^8 - 2\*(2\*B\*b\*c^3 - A\*c^4)\*x^6 + 7\*B\*b^4 - 5\*A\*b^3\*c - (11\*B\*b^2\*c^2 - 4\*A\*b\*c^3)\*x^4 + 2\*(B\*b^3\*c - 2\*A\*b^2\*c^2)\*x^2 + 6\*(2\*B\*b^4 - A\*b^3\*c + (2\*B\*b^2\*c^2 - A\*b\*c^3)\*x^4 + 2\*(2\*B\*b^3\*c - A\*b^2\*c^2)\*x^2)\*log(c\*x^2 + b))/(c^7\*x^4 + 2\*b\*c^6\*x^2 + b^2\*c^5)

**giac [A]** time = 0.18, size = 132, normalized size = 1.19

$$\frac{3(2Bb^2 - Abc) \log(|cx^2 + b|)}{2c^5} + \frac{Bc^3x^4 - 6Bbc^2x^2 + 2Ac^3x^2}{4c^6} - \frac{18Bb^2c^2x^4 - 9Abc^3x^4 + 28Bb^3cx^2 - 12Ab^2c^2x^2 + 11Bb^4 - 4Ab^3c}{4(cx^2 + b)^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out]  $3/2*(2*B*b^2 - A*b*c)*\log(\text{abs}(c*x^2 + b))/c^5 + 1/4*(B*c^3*x^4 - 6*B*b*c^2*x^2 + 2*A*c^3*x^2)/c^6 - 1/4*(18*B*b^2*c^2*x^4 - 9*A*b*c^3*x^4 + 28*B*b^3*c*x^2 - 12*A*b^2*c^2*x^2 + 11*B*b^4 - 4*A*b^3*c)/((c*x^2 + b)^2*c^5)$

**maple [A]** time = 0.06, size = 134, normalized size = 1.21

$$\frac{Bx^4}{4c^3} + \frac{Ab^3}{4(cx^2+b)^2c^4} + \frac{Ax^2}{2c^3} - \frac{Bb^4}{4(cx^2+b)^2c^5} - \frac{3Bbx^2}{2c^4} - \frac{3Ab^2}{2(cx^2+b)c^4} - \frac{3Ab\ln(cx^2+b)}{2c^4} + \frac{2Bb^3}{(cx^2+b)c^5} + \frac{3Bb^2\ln(cx^2+b)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x)`

[Out]  $1/4*B*x^4/c^3 - 3/2/c^4*B*x^2*b + 1/2/c^3*A*x^2 - 3/2*b/c^4*\ln(c*x^2+b)*A + 3*b^2/c^5*\ln(c*x^2+b)*B - 3/2*b^2/c^4/(c*x^2+b)*A + 2*b^3/c^5/(c*x^2+b)*B + 1/4*b^3/c^4/(c*x^2+b)^2*A - 1/4*b^4/c^5/(c*x^2+b)^2*B$

**maxima [A]** time = 1.41, size = 116, normalized size = 1.05

$$\frac{7Bb^4 - 5Ab^3c + 2(4Bb^3c - 3Ab^2c^2)x^2}{4(c^7x^4 + 2bc^6x^2 + b^2c^5)} + \frac{Bcx^4 - 2(3Bb - Ac)x^2}{4c^4} + \frac{3(2Bb^2 - Abc)\log(cx^2 + b)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")`

[Out]  $1/4*(7*B*b^4 - 5*A*b^3*c + 2*(4*B*b^3*c - 3*A*b^2*c^2)*x^2)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5) + 1/4*(B*c*x^4 - 2*(3*B*b - A*c)*x^2)/c^4 + 3/2*(2*B*b^2 - A*b*c)*\log(c*x^2 + b)/c^5$

**mupad [B]** time = 0.12, size = 118, normalized size = 1.06

$$\frac{\frac{7Bb^4-5Ab^3c}{4c} + x^2\left(2Bb^3 - \frac{3Ab^2c}{2}\right)}{b^2c^4 + 2bc^5x^2 + c^6x^4} + x^2\left(\frac{A}{2c^3} - \frac{3Bb}{2c^4}\right) + \frac{\ln(cx^2 + b)(6Bb^2 - 3Abc)}{2c^5} + \frac{Bx^4}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)`

[Out]  $((7*B*b^4 - 5*A*b^3*c)/(4*c) + x^2*(2*B*b^3 - (3*A*b^2*c)/2))/(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x^2*(A/(2*c^3) - (3*B*b)/(2*c^4)) + (\log(b + c*x^2)*(6*B*b^2 - 3*A*b*c))/(2*c^5) + (B*x^4)/(4*c^3)$

**sympy [A]** time = 1.52, size = 119, normalized size = 1.07

$$\frac{Bx^4}{4c^3} + \frac{3b(-Ac + 2Bb)\log(b + cx^2)}{2c^5} + x^2\left(\frac{A}{2c^3} - \frac{3Bb}{2c^4}\right) + \frac{-5Ab^3c + 7Bb^4 + x^2(-6Ab^2c^2 + 8Bb^3c)}{4b^2c^5 + 8bc^6x^2 + 4c^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13*(B*x**2+A)/(c*x**4+b*x**2)**3,x)`

[Out]  $B*x**4/(4*c**3) + 3*b*(-A*c + 2*B*b)*\log(b + c*x**2)/(2*c**5) + x**2*(A/(2*c**3) - 3*B*b/(2*c**4)) + (-5*A*b**3*c + 7*B*b**4 + x**2*(-6*A*b**2*c**2 + 8*B*b**3*c))/(4*b**2*c**5 + 8*b*c**6*x**2 + 4*c**7*x**4)$

$$3.75 \quad \int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=118

$$\frac{b^2x(bB - Ac)}{4c^4(b + cx^2)^2} + \frac{5\sqrt{b}(7bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{8c^{9/2}} - \frac{bx(13bB - 9Ac)}{8c^4(b + cx^2)} - \frac{x(3bB - Ac)}{c^4} + \frac{Bx^3}{3c^3}$$

**Rubi [A]** time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1584, 455, 1814, 1153, 205}

$$\frac{b^2x(bB - Ac)}{4c^4(b + cx^2)^2} - \frac{bx(13bB - 9Ac)}{8c^4(b + cx^2)} - \frac{x(3bB - Ac)}{c^4} + \frac{5\sqrt{b}(7bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{8c^{9/2}} + \frac{Bx^3}{3c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -(((3\*b\*B - A\*c)\*x)/c^4) + (B\*x^3)/(3\*c^3) + (b^2\*(b\*B - A\*c)\*x)/(4\*c^4\*(b + c\*x^2)^2) - (b\*(13\*b\*B - 9\*A\*c)\*x)/(8\*c^4\*(b + c\*x^2)) + (5\*sqrt[b]\*(7\*b\*B - 3\*A\*c)\*ArcTan[(sqrt[c]\*x)/sqrt[b]])/(8\*c^(9/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1)/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

#### Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 1814

Int[(Pq)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int



$[(a + b*x^2)^{(p + 1)}*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /$   
 $; FreeQ[\{a, b\}, x] \&\& PolyQ[Pq, x] \&\& LtQ[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{x^{12} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^6 (A + Bx^2)}{(b + cx^2)^3} dx \\ &= \frac{b^2(bB - Ac)x}{4c^4 (b + cx^2)^2} - \frac{\int \frac{b^2(bB - Ac) - 4bc(bB - Ac)x^2 + 4c^2(bB - Ac)x^4 - 4Bc^3x^6}{(b + cx^2)^2} dx}{4c^4} \\ &= \frac{b^2(bB - Ac)x}{4c^4 (b + cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b + cx^2)} + \frac{\int \frac{b^2(11bB - 7Ac) - 8bc(2bB - Ac)x^2 + 8bBc^2x^4}{b + cx^2} dx}{8bc^4} \\ &= \frac{b^2(bB - Ac)x}{4c^4 (b + cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b + cx^2)} + \frac{\int \left( -8b(3bB - Ac) + 8bBcx^2 + \frac{5(7b^3B - 3Ab^2c)}{b + cx^2} \right) dx}{8bc^4} \\ &= -\frac{(3bB - Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB - Ac)x}{4c^4 (b + cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b + cx^2)} + \frac{(5b(7bB - 3Ac)) \int \frac{1}{b + cx^2} dx}{8c^4} \\ &= -\frac{(3bB - Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB - Ac)x}{4c^4 (b + cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b + cx^2)} + \frac{5\sqrt{b}(7bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 113, normalized size = 0.96

$$\frac{5b^2cx(9A - 35Bx^2) + bc^2x^3(75A - 56Bx^2) + 8c^3x^5(3A + Bx^2) - 105b^3Bx}{24c^4(b + cx^2)^2} + \frac{5\sqrt{b}(7bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (-105\*b^3\*B\*x + b\*c^2\*x^3\*(75\*A - 56\*B\*x^2) + 5\*b^2\*c\*x\*(9\*A - 35\*B\*x^2) + 8\*c^3\*x^5\*(3\*A + B\*x^2))/(24\*c^4\*(b + c\*x^2)^2) + (5\*sqrt[b]\*(7\*b\*B - 3\*A\*c)\*ArcTan[(sqrt[c]\*x)/sqrt[b]])/(8\*c^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12} (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.41, size = 358, normalized size = 3.03

$$\frac{16Bc^3x^2 - 16(7Bb^2 - 3Ac^2)x^3 - 50(7Bb^2 - 3Ac^2)x^4 - 15((7Bb^2 - 3Ac^2)^2 + 7Bb^3 - 3Ab^2c + 2(7Bb^2 - 3Ac^2)^2)\sqrt{\frac{c}{b}} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) - 30(7Bb^3 - 3Ab^2c)x - 8Bc^3x^2 - 8(7Bb^2 - 3Ac^2)x^3 - 25(7Bb^2 - 3Ac^2)x^4 + 15((7Bb^2 - 3Ac^2)^2 + 7Bb^3 - 3Ab^2c + 2(7Bb^2 - 3Ac^2)^2)\sqrt{\frac{c}{b}} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) - 15(7Bb^3 - 3Ab^2c)x}{48(c^2x^2 + 2bc^2x^2 + b^2c^2)} + \frac{24(c^2x^2 + 2bc^2x^2 + b^2c^2)}{24(c^2x^2 + 2bc^2x^2 + b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] [1/48\*(16\*B\*c^3\*x^7 - 16\*(7\*B\*b\*c^2 - 3\*A\*c^3)\*x^5 - 50\*(7\*B\*b^2\*c - 3\*A\*b\*c^2)\*x^3 - 15\*((7\*B\*b\*c^2 - 3\*A\*c^3)\*x^4 + 7\*B\*b^3 - 3\*A\*b^2\*c + 2\*(7\*B\*b^2\*c - 3\*A\*b\*c^2)\*x^2)\*sqrt(-b/c)\*log((c\*x^2 - 2\*c\*x\*sqrt(-b/c) - b)/(c\*x^2 + b)) - 30\*(7\*B\*b^3 - 3\*A\*b^2\*c)\*x)/(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4), 1/24\*(8\*B\*c^3\*x^7 - 8\*(7\*B\*b\*c^2 - 3\*A\*c^3)\*x^5 - 25\*(7\*B\*b^2\*c - 3\*A\*b\*c^2)\*x^3 + 15\*((7\*B\*b\*c^2 - 3\*A\*c^3)\*x^4 + 7\*B\*b^3 - 3\*A\*b^2\*c + 2\*(7\*B\*b^2\*c - 3\*A\*b\*c^2)\*x^2)\*sqrt(b/c)\*arctan(c\*x\*sqrt(b/c)/b) - 15\*(7\*B\*b^3 - 3\*A\*b^2\*c)\*x)/(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4)]

**giac** [A] time = 0.16, size = 111, normalized size = 0.94

$$\frac{5(7Bb^2 - 3Abc) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^4} - \frac{13Bb^2cx^3 - 9Abc^2x^3 + 11Bb^3x - 7Ab^2cx}{8(cx^2 + b)^2c^4} + \frac{Bc^6x^3 - 9Bbc^5x + 3Ac^6x}{3c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 5/8\*(7\*B\*b^2 - 3\*A\*b\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^4) - 1/8\*(13\*B\*b^2\*c\*x^3 - 9\*A\*b\*c^2\*x^3 + 11\*B\*b^3\*x - 7\*A\*b^2\*c\*x)/(c\*x^2 + b)^2\*c^4 + 1/3\*(B\*c^6\*x^3 - 9\*B\*b\*c^5\*x + 3\*A\*c^6\*x)/c^9

**maple** [A] time = 0.06, size = 147, normalized size = 1.25

$$\frac{9Abx^3}{8(cx^2 + b)^2c^2} - \frac{13Bb^2x^3}{8(cx^2 + b)^2c^3} + \frac{7Ab^2x}{8(cx^2 + b)^2c^3} - \frac{11Bb^3x}{8(cx^2 + b)^2c^4} + \frac{Bx^3}{3c^3} - \frac{15Ab \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^3} + \frac{35Bb^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^4} + \frac{Ax}{c^3} - \frac{3Bbx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 1/3\*B\*x^3/c^3+1/c^3\*A\*x-3/c^4\*b\*B\*x+9/8\*b/c^2/(c\*x^2+b)^2\*A\*x^3-13/8\*b^2/c^3/(c\*x^2+b)^2\*B\*x^3+7/8\*b^2/c^3/(c\*x^2+b)^2\*A\*x-11/8\*b^3/c^4/(c\*x^2+b)^2\*B\*x-15/8\*b/c^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A+35/8\*b^2/c^4/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B

**maxima** [A] time = 3.02, size = 120, normalized size = 1.02

$$\frac{(13Bb^2c - 9Abc^2)x^3 + (11Bb^3 - 7Ab^2c)x}{8(c^6x^4 + 2bc^5x^2 + b^2c^4)} + \frac{5(7Bb^2 - 3Abc) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^4} + \frac{Bcx^3 - 3(3Bb - Ac)x}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/8\*((13\*B\*b^2\*c - 9\*A\*b\*c^2)\*x^3 + (11\*B\*b^3 - 7\*A\*b^2\*c)\*x)/(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4) + 5/8\*(7\*B\*b^2 - 3\*A\*b\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*c^4) + 1/3\*(B\*c\*x^3 - 3\*(3\*B\*b - A\*c)\*x)/c^4

**mupad** [B] time = 0.08, size = 138, normalized size = 1.17

$$\frac{x^3 \left(\frac{9Abc^2}{8} - \frac{13Bb^2c}{8}\right) - x \left(\frac{11Bb^3}{8} - \frac{7Ab^2c}{8}\right)}{b^2c^4 + 2bc^5x^2 + c^6x^4} + x \left(\frac{A}{c^3} - \frac{3Bb}{c^4}\right) + \frac{Bx^3}{3c^3} + \frac{5\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c}x(3Ac-7Bb)}{7Bb^2-3Abc}\right)(3Ac-7Bb)}{8c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^12\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

```
[Out] (x^3*((9*A*b*c^2)/8 - (13*B*b^2*c)/8) - x*((11*B*b^3)/8 - (7*A*b^2*c)/8))/(
b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x*(A/c^3 - (3*B*b)/c^4) + (B*x^3)/(3*c^3
) + (5*b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 7*B*b))/(7*B*b^2 - 3*A*b*c
))* (3*A*c - 7*B*b))/(8*c^(9/2))
```

**sympy [A]** time = 1.30, size = 214, normalized size = 1.81

$$\frac{Bx^3}{3c^3} + x\left(\frac{A}{c^3} - \frac{3Bb}{c^4}\right) - \frac{5\sqrt{-\frac{b}{c^9}}(-3Ac + 7Bb)\log\left(-\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb} + x\right)}{16} + \frac{5\sqrt{-\frac{b}{c^9}}(-3Ac + 7Bb)\log\left(\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb} + x\right)}{16} + \frac{x^3(9Abc^2 - 13Bb^2c) + x(7Ab^2c - 11Bb^3)}{8b^2c^4 + 16bc^5x^2 + 8c^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**3/(3*c**3) + x*(A/c**3 - 3*B*b/c**4) - 5*sqrt(-b/c**9)*(-3*A*c + 7*B*b
)*log(-5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16 + 5
*sqrt(-b/c**9)*(-3*A*c + 7*B*b)*log(5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(
-15*A*c + 35*B*b) + x)/16 + (x**3*(9*A*b*c**2 - 13*B*b**2*c) + x*(7*A*b**2*
c - 11*B*b**3))/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4)
```

$$3.76 \quad \int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=89

$$\frac{b^2(bB - Ac)}{4c^4 (b + cx^2)^2} - \frac{b(3bB - 2Ac)}{2c^4 (b + cx^2)} - \frac{(3bB - Ac) \log(b + cx^2)}{2c^4} + \frac{Bx^2}{2c^3}$$

**Rubi [A]** time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$\frac{b^2(bB - Ac)}{4c^4 (b + cx^2)^2} - \frac{b(3bB - 2Ac)}{2c^4 (b + cx^2)} - \frac{(3bB - Ac) \log(b + cx^2)}{2c^4} + \frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (B\*x^2)/(2\*c^3) + (b^2\*(b\*B - A\*c))/(4\*c^4\*(b + c\*x^2)^2) - (b\*(3\*b\*B - 2\*A\*c))/(2\*c^4\*(b + c\*x^2)) - ((3\*b\*B - A\*c)\*Log[b + c\*x^2])/(2\*c^4)

#### Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{x^{11}(A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^5(A + Bx^2)}{(b + cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2(A + Bx)}{(b + cx)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{B}{c^3} - \frac{b^2(bB - Ac)}{c^3(b + cx)^3} + \frac{b(3bB - 2Ac)}{c^3(b + cx)^2} + \frac{-3bB + Ac}{c^3(b + cx)} \right) dx, x, x^2 \right) \\ &= \frac{Bx^2}{2c^3} + \frac{b^2(bB - Ac)}{4c^4 (b + cx^2)^2} - \frac{b(3bB - 2Ac)}{2c^4 (b + cx^2)} - \frac{(3bB - Ac) \log(b + cx^2)}{2c^4} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 92, normalized size = 1.03

$$\frac{2Abc - 3b^2B}{2c^4(b + cx^2)} + \frac{b^3B - Ab^2c}{4c^4(b + cx^2)^2} + \frac{(Ac - 3bB) \log(b + cx^2)}{2c^4} + \frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (B\*x^2)/(2\*c^3) + (b^3\*B - A\*b^2\*c)/(4\*c^4\*(b + c\*x^2)^2) + (-3\*b^2\*B + 2\*A\*b\*c)/(2\*c^4\*(b + c\*x^2)) + ((-3\*b\*B + A\*c)\*Log[b + c\*x^2])/(2\*c^4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.40, size = 142, normalized size = 1.60

$$\frac{2Bc^3x^6 + 4Bbc^2x^4 - 5Bb^3 + 3Ab^2c - 4(Bb^2c - Abc^2)x^2 - 2((3Bbc^2 - Ac^3)x^4 + 3Bb^3 - Ab^2c + 2(3Bb^2c - Abc^2)x^2) \log(cx^2 + b)}{4(c^6x^4 + 2bc^5x^2 + b^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/4\*(2\*B\*c^3\*x^6 + 4\*B\*b\*c^2\*x^4 - 5\*B\*b^3 + 3\*A\*b^2\*c - 4\*(B\*b^2\*c - A\*b\*c^2)\*x^2 - 2\*((3\*B\*b\*c^2 - A\*c^3)\*x^4 + 3\*B\*b^3 - A\*b^2\*c + 2\*(3\*B\*b^2\*c - A\*b\*c^2)\*x^2)\*log(c\*x^2 + b))/(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4)

**giac [A]** time = 0.16, size = 93, normalized size = 1.04

$$\frac{Bx^2}{2c^3} - \frac{(3Bb - Ac) \log(|cx^2 + b|)}{2c^4} + \frac{9Bbc^2x^4 - 3Ac^3x^4 + 12Bb^2cx^2 - 2Abc^2x^2 + 4Bb^3}{4(cx^2 + b)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 1/2\*B\*x^2/c^3 - 1/2\*(3\*B\*b - A\*c)\*log(abs(c\*x^2 + b))/c^4 + 1/4\*(9\*B\*b\*c^2\*x^4 - 3\*A\*c^3\*x^4 + 12\*B\*b^2\*c\*x^2 - 2\*A\*b\*c^2\*x^2 + 4\*B\*b^3)/((c\*x^2 + b)^2\*c^4)

**maple [A]** time = 0.06, size = 109, normalized size = 1.22

$$-\frac{Ab^2}{4(cx^2 + b)^2c^3} + \frac{Bb^3}{4(cx^2 + b)^2c^4} + \frac{Bx^2}{2c^3} + \frac{Ab}{(cx^2 + b)c^3} + \frac{A \ln(cx^2 + b)}{2c^3} - \frac{3Bb^2}{2(cx^2 + b)c^4} - \frac{3Bb \ln(cx^2 + b)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 1/2\*B\*x^2/c^3+1/2/c^3\*ln(c\*x^2+b)\*A-3/2/c^4\*ln(c\*x^2+b)\*b\*B+1/c^3\*b/(c\*x^2+b)\*A-3/2/c^4\*b^2/(c\*x^2+b)\*B-1/4/c^3\*b^2/(c\*x^2+b)^2\*A+1/4/c^4\*b^3/(c\*x^2+b)^2\*B

**maxima** [A] time = 1.36, size = 94, normalized size = 1.06

$$-\frac{5Bb^3 - 3Ab^2c + 2(3Bb^2c - 2Abc^2)x^2}{4(c^6x^4 + 2bc^5x^2 + b^2c^4)} + \frac{Bx^2}{2c^3} - \frac{(3Bb - Ac)\log(cx^2 + b)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/4\*(5\*B\*b^3 - 3\*A\*b^2\*c + 2\*(3\*B\*b^2\*c - 2\*A\*b\*c^2)\*x^2)/(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4) + 1/2\*B\*x^2/c^3 - 1/2\*(3\*B\*b - A\*c)\*log(c\*x^2 + b)/c^4

**mupad** [B] time = 0.09, size = 95, normalized size = 1.07

$$\frac{Bx^2}{2c^3} - \frac{x^2\left(\frac{3Bb^2}{2} - Abc\right) + \frac{5Bb^3 - 3Ab^2c}{4c}}{b^2c^3 + 2bc^4x^2 + c^5x^4} + \frac{\ln(cx^2 + b)(Ac - 3Bb)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] (B\*x^2)/(2\*c^3) - (x^2\*((3\*B\*b^2)/2 - A\*b\*c) + (5\*B\*b^3 - 3\*A\*b^2\*c)/(4\*c))/(b^2\*c^3 + c^5\*x^4 + 2\*b\*c^4\*x^2) + (log(b + c\*x^2)\*(A\*c - 3\*B\*b))/(2\*c^4)

**sympy** [A] time = 1.29, size = 94, normalized size = 1.06

$$\frac{Bx^2}{2c^3} + \frac{3Ab^2c - 5Bb^3 + x^2(4Abc^2 - 6Bb^2c)}{4b^2c^4 + 8bc^5x^2 + 4c^6x^4} - \frac{(-Ac + 3Bb)\log(b + cx^2)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*11\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] B\*x\*\*2/(2\*c\*\*3) + (3\*A\*b\*\*2\*c - 5\*B\*b\*\*3 + x\*\*2\*(4\*A\*b\*c\*\*2 - 6\*B\*b\*\*2\*c))/(4\*b\*\*2\*c\*\*4 + 8\*b\*c\*\*5\*x\*\*2 + 4\*c\*\*6\*x\*\*4) - (-A\*c + 3\*B\*b)\*log(b + c\*x\*\*2)/(2\*c\*\*4)

$$3.77 \quad \int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=95

$$-\frac{3(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}} + \frac{x(9bB - 5Ac)}{8c^3(b + cx^2)} - \frac{bx(bB - Ac)}{4c^3(b + cx^2)^2} + \frac{Bx}{c^3}$$

**Rubi [A]** time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1584, 455, 1157, 388, 205}

$$\frac{x(9bB - 5Ac)}{8c^3(b + cx^2)} - \frac{bx(bB - Ac)}{4c^3(b + cx^2)^2} - \frac{3(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}} + \frac{Bx}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (B\*x)/c^3 - (b\*(b\*B - A\*c)\*x)/(4\*c^3\*(b + c\*x^2)^2) + ((9\*b\*B - 5\*A\*c)\*x)/(8\*c^3\*(b + c\*x^2)) - (3\*(5\*b\*B - A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*Sqrt[b]\*c^(7/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

#### Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{10} (A + Bx^2)}{(bx^2 + cx^4)^3} dx = \int \frac{x^4 (A + Bx^2)}{(b + cx^2)^3} dx$$

$$= -\frac{b(bB - Ac)x}{4c^3 (b + cx^2)^2} - \frac{\int \frac{-b(bB - Ac) + 4c(bB - Ac)x^2 - 4Bc^2x^4}{(b + cx^2)^2} dx}{4c^3}$$

$$= -\frac{b(bB - Ac)x}{4c^3 (b + cx^2)^2} + \frac{(9bB - 5Ac)x}{8c^3 (b + cx^2)} + \frac{\int \frac{-b(7bB - 3Ac) + 8bBcx^2}{b + cx^2} dx}{8bc^3}$$

$$= \frac{Bx}{c^3} - \frac{b(bB - Ac)x}{4c^3 (b + cx^2)^2} + \frac{(9bB - 5Ac)x}{8c^3 (b + cx^2)} - \frac{(3(5bB - Ac)) \int \frac{1}{b + cx^2} dx}{8c^3}$$

$$= \frac{Bx}{c^3} - \frac{b(bB - Ac)x}{4c^3 (b + cx^2)^2} + \frac{(9bB - 5Ac)x}{8c^3 (b + cx^2)} - \frac{3(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

**Mathematica [A]** time = 0.11, size = 92, normalized size = 0.97

$$\frac{x (b (25Bcx^2 - 3Ac) + c^2x^2 (8Bx^2 - 5A) + 15b^2B)}{8c^3 (b + cx^2)^2} - \frac{3(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] (x*(15*b^2*B + c^2*x^2*(-5*A + 8*B*x^2) + b*(-3*A*c + 25*B*c*x^2)))/(8*c^3*(b + c*x^2)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c^(7/2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10} (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] IntegrateAlgebraic[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3, x]
```

**fricas [A]** time = 0.43, size = 328, normalized size = 3.45

$$\frac{16Bbc^3x^5 + 10(5Bb^2c^2 - Abc^3)x^3 + 3((5Bb^2c - Ac^3)x^4 + 5Bb^3 - Ab^2c + 2(5Bb^2c - Abc^2)x^2)\sqrt{-bc} \log\left(\frac{c^2x^2 + \sqrt{bc}x + b}{cx - b}\right) + 6(5Bb^2c - Ab^2c^2)x - 8Bbc^3x^5 + 5(5Bb^2c^2 - Abc^3)x^3 - 3((5Bb^2c - Ac^3)x^4 + 5Bb^3 - Ab^2c + 2(5Bb^2c - Abc^2)x^2)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right) + 3(5Bb^2c - Ab^2c^2)x}{16(bc^6x^4 + 2b^2c^5x^2 + b^3c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```



[Out]  $[1/16*(16*B*b*c^3*x^5 + 10*(5*B*b^2*c^2 - A*b*c^3)*x^3 + 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*\sqrt{-b*c}*\log((c*x^2 - 2*\sqrt{-b*c})*x - b)/(c*x^2 + b)) + 6*(5*B*b^3*c - A*b^2*c^2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4), 1/8*(8*B*b*c^3*x^5 + 5*(5*B*b^2*c^2 - A*b*c^3)*x^3 - 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*\sqrt{b*c}*\arctan(\sqrt{b*c}*x/b) + 3*(5*B*b^3*c - A*b^2*c^2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4)]$

**giac** [A] time = 0.18, size = 80, normalized size = 0.84

$$\frac{Bx}{c^3} - \frac{3(5Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^3} + \frac{9Bbcx^3 - 5Ac^2x^3 + 7Bb^2x - 3Abcx}{8(cx^2 + b)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out]  $B*x/c^3 - 3/8*(5*B*b - A*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*c^3) + 1/8*(9*B*b*c*x^3 - 5*A*c^2*x^3 + 7*B*b^2*x - 3*A*b*c*x)/((c*x^2 + b)^2*c^3)$

**maple** [A] time = 0.05, size = 122, normalized size = 1.28

$$-\frac{5Ax^3}{8(cx^2 + b)^2c} + \frac{9Bbx^3}{8(cx^2 + b)^2c^2} - \frac{3Abx}{8(cx^2 + b)^2c^2} + \frac{7Bb^2x}{8(cx^2 + b)^2c^3} + \frac{3A \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^2} - \frac{15Bb \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^3} + \frac{Bx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out]  $B*x/c^3 - 5/8/c/(c*x^2+b)^2*A*x^3 + 9/8/c^2/(c*x^2+b)^2*B*x^3*b - 3/8/c^2/(c*x^2+b)^2*A*b*x + 7/8/c^3/(c*x^2+b)^2*B*b^2*x + 3/8/c^2/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*A - 15/8/c^3/(b*c)^{(1/2)}*\arctan(1/(b*c)^{(1/2)}*c*x)*b*B$

**maxima** [A] time = 2.94, size = 94, normalized size = 0.99

$$\frac{(9Bbc - 5Ac^2)x^3 + (7Bb^2 - 3Abc)x}{8(c^5x^4 + 2bc^4x^2 + b^2c^3)} + \frac{Bx}{c^3} - \frac{3(5Bb - Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $1/8*((9*B*b*c - 5*A*c^2)*x^3 + (7*B*b^2 - 3*A*b*c)*x)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + B*x/c^3 - 3/8*(5*B*b - A*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c})*c^3)$

**mupad** [B] time = 0.15, size = 92, normalized size = 0.97

$$\frac{Bx}{c^3} - \frac{x^3 \left( \frac{5Ac^2}{8} - \frac{9Bbc}{8} \right) - x \left( \frac{7Bb^2}{8} - \frac{3Abc}{8} \right)}{b^2c^3 + 2bc^4x^2 + c^5x^4} + \frac{3 \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) (Ac - 5Bb)}{8\sqrt{b}c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^10\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out]  $(B*x)/c^3 - (x^3*((5*A*c^2)/8 - (9*B*b*c)/8) - x*((7*B*b^2)/8 - (3*A*b*c)/8))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (3*\operatorname{atan}((c^{(1/2)}*x)/b^{(1/2)})*(A*c - 5*B*b))/(8*b^{(1/2)}*c^{(7/2)})$

sympy [B] time = 1.11, size = 194, normalized size = 2.04

$$\frac{Bx}{c^3} + \frac{3\sqrt{-\frac{1}{bc^7}}(-Ac + 5Bb) \log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac + 5Bb)}{-3Ac + 15Bb} + x\right)}{16} - \frac{3\sqrt{-\frac{1}{bc^7}}(-Ac + 5Bb) \log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac + 5Bb)}{-3Ac + 15Bb} + x\right)}{16} + \frac{x^3(-5Ac^2 + 9Bbc) + x(-3Abc + 7Bb^2)}{8b^2c^3 + 16bc^4x^2 + 8c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*10\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] B\*x/c\*\*3 + 3\*sqrt(-1/(b\*c\*\*7))\*(-A\*c + 5\*B\*b)\*log(-3\*b\*c\*\*3\*sqrt(-1/(b\*c\*\*7)))\*(-A\*c + 5\*B\*b)/(-3\*A\*c + 15\*B\*b) + x)/16 - 3\*sqrt(-1/(b\*c\*\*7))\*(-A\*c + 5\*B\*b)\*log(3\*b\*c\*\*3\*sqrt(-1/(b\*c\*\*7)))\*(-A\*c + 5\*B\*b)/(-3\*A\*c + 15\*B\*b) + x)/16 + (x\*\*3\*(-5\*A\*c\*\*2 + 9\*B\*b\*c) + x\*(-3\*A\*b\*c + 7\*B\*b\*\*2))/(8\*b\*\*2\*c\*\*3 + 16\*b\*c\*\*4\*x\*\*2 + 8\*c\*\*5\*x\*\*4)

$$3.78 \quad \int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=67

$$-\frac{b(bB - Ac)}{4c^3(b + cx^2)^2} + \frac{2bB - Ac}{2c^3(b + cx^2)} + \frac{B \log(b + cx^2)}{2c^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$-\frac{b(bB - Ac)}{4c^3(b + cx^2)^2} + \frac{2bB - Ac}{2c^3(b + cx^2)} + \frac{B \log(b + cx^2)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -(b\*(b\*B - A\*c))/(4\*c^3\*(b + c\*x^2)^2) + (2\*b\*B - A\*c)/(2\*c^3\*(b + c\*x^2)) + (B\*Log[b + c\*x^2])/(2\*c^3)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3} dx &= \int \frac{x^3(A+Bx^2)}{(b+cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{x(A+Bx)}{(b+cx)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{b(bB - Ac)}{c^2(b+cx)^3} + \frac{-2bB + Ac}{c^2(b+cx)^2} + \frac{B}{c^2(b+cx)} \right) dx, x, x^2 \right) \\ &= -\frac{b(bB - Ac)}{4c^3(b+cx^2)^2} + \frac{2bB - Ac}{2c^3(b+cx^2)} + \frac{B \log(b+cx^2)}{2c^3} \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 64, normalized size = 0.96

$$\frac{-bc(A - 4Bx^2) - 2Ac^2x^2 + 3b^2B + 2B(b + cx^2)^2 \log(b + cx^2)}{4c^3(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (3\*b^2\*B - 2\*A\*c^2\*x^2 - b\*c\*(A - 4\*B\*x^2) + 2\*B\*(b + c\*x^2)^2\*Log[b + c\*x^2])/(4\*c^3\*(b + c\*x^2)^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9(A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas** [A] time = 0.39, size = 89, normalized size = 1.33

$$\frac{3Bb^2 - Abc + 2(2Bbc - Ac^2)x^2 + 2(Bc^2x^4 + 2Bbcx^2 + Bb^2) \log(cx^2 + b)}{4(c^5x^4 + 2bc^4x^2 + b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/4\*(3\*B\*b^2 - A\*b\*c + 2\*(2\*B\*b\*c - A\*c^2)\*x^2 + 2\*(B\*c^2\*x^4 + 2\*B\*b\*c\*x^2 + B\*b^2)\*log(c\*x^2 + b))/(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)

**giac** [A] time = 0.19, size = 55, normalized size = 0.82

$$\frac{B \log(|cx^2 + b|)}{2c^3} - \frac{3Bcx^4 + 2Bbx^2 + 2Acx^2 + Ab}{4(cx^2 + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 1/2\*B\*log(abs(c\*x^2 + b))/c^3 - 1/4\*(3\*B\*c\*x^4 + 2\*B\*b\*x^2 + 2\*A\*c\*x^2 + A\*b)/((c\*x^2 + b)^2\*c^2)

**maple** [A] time = 0.05, size = 80, normalized size = 1.19

$$\frac{Ab}{4(cx^2 + b)^2c^2} - \frac{Bb^2}{4(cx^2 + b)^2c^3} - \frac{A}{2(cx^2 + b)c^2} + \frac{Bb}{(cx^2 + b)c^3} + \frac{B \ln(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 1/2\*B\*ln(c\*x^2+b)/c^3-1/2/c^2/(c\*x^2+b)\*A+1/c^3/(c\*x^2+b)\*b\*B+1/4\*b/c^2/(c\*x^2+b)^2\*A-1/4\*b^2/c^3/(c\*x^2+b)^2\*B

**maxima** [A] time = 1.33, size = 72, normalized size = 1.07

$$\frac{3Bb^2 - Abc + 2(2Bbc - Ac^2)x^2}{4(c^5x^4 + 2bc^4x^2 + b^2c^3)} + \frac{B \log(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 1/4\*(3\*B\*b^2 - A\*b\*c + 2\*(2\*B\*b\*c - A\*c^2)\*x^2)/(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3) + 1/2\*B\*log(c\*x^2 + b)/c^3

**mupad** [B] time = 0.11, size = 70, normalized size = 1.04

$$\frac{\frac{3Bb^2 - Abc}{4c^3} - \frac{x^2(Ac - 2Bb)}{2c^2}}{b^2 + 2bcx^2 + c^2x^4} + \frac{B \ln(cx^2 + b)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] ((3\*B\*b^2 - A\*b\*c)/(4\*c^3) - (x^2\*(A\*c - 2\*B\*b))/(2\*c^2))/(b^2 + c^2\*x^4 + 2\*b\*c\*x^2) + (B\*log(b + c\*x^2))/(2\*c^3)

**sympy** [A] time = 0.93, size = 70, normalized size = 1.04

$$\frac{B \log(b + cx^2)}{2c^3} + \frac{-Abc + 3Bb^2 + x^2(-2Ac^2 + 4Bbc)}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*9\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] B\*log(b + c\*x\*\*2)/(2\*c\*\*3) + (-A\*b\*c + 3\*B\*b\*\*2 + x\*\*2\*(-2\*A\*c\*\*2 + 4\*B\*b\*c))/(4\*b\*\*2\*c\*\*3 + 8\*b\*c\*\*4\*x\*\*2 + 4\*c\*\*5\*x\*\*4)

$$3.79 \quad \int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=90

$$\frac{(Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}} - \frac{x(5bB - Ac)}{8bc^2(b + cx^2)} + \frac{x(bB - Ac)}{4c^2(b + cx^2)^2}$$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 455, 385, 205}

$$\frac{(Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}} - \frac{x(5bB - Ac)}{8bc^2(b + cx^2)} + \frac{x(bB - Ac)}{4c^2(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((b\*B - A\*c)\*x)/(4\*c^2\*(b + c\*x^2)^2) - ((5\*b\*B - A\*c)\*x)/(8\*b\*c^2\*(b + c\*x^2)) + ((3\*b\*B + A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(3/2)\*c^(5/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 455

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*x^2\*Together[(b^(m/2)\*x^(m - 2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)]/(a + b\*x^2)] - (-a)^(m/2 - 1)\*(b\*c - a\*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^8 (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^2 (A + Bx^2)}{(b + cx^2)^3} dx \\
&= \frac{(bB - Ac)x}{4c^2 (b + cx^2)^2} - \frac{\int \frac{bB - Ac - 4Bcx^2}{(b + cx^2)^2} dx}{4c^2} \\
&= \frac{(bB - Ac)x}{4c^2 (b + cx^2)^2} - \frac{(5bB - Ac)x}{8bc^2 (b + cx^2)} + \frac{(3bB + Ac) \int \frac{1}{b + cx^2} dx}{8bc^2} \\
&= \frac{(bB - Ac)x}{4c^2 (b + cx^2)^2} - \frac{(5bB - Ac)x}{8bc^2 (b + cx^2)} + \frac{(3bB + Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{8b^{3/2}c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 83, normalized size = 0.92

$$\frac{\frac{(Ac + 3bB) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right)}{b^{3/2}} + \frac{\sqrt{c}x(-bc(A + 5Bx^2) + Ac^2x^2 - 3b^2B)}{b(b + cx^2)^2}}{8c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] ((Sqrt[c]\*x\*(-3\*b^2\*B + A\*c^2\*x^2 - b\*c\*(A + 5\*B\*x^2)))/(b\*(b + c\*x^2)^2) + ((3\*b\*B + A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/b^(3/2))/(8\*c^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.41, size = 301, normalized size = 3.34

$$\left| \frac{2(5Bb^2c^2 - Abc^3)x^3 + ((3Bbc^2 + Ac^3)x^4 + 3Bb^3 + Ab^2c + 2(3Bb^2c + Abc^2)x^2)\sqrt{-bc} \log\left(\frac{x^2 - 2\sqrt{-bc}x + b}{cx^2 + b}\right) + 2(3Bb^2c + Ab^2c^2)x}{16(b^2c^5x^4 + 2b^3c^4x^2 + b^4c^3)}, \frac{(5Bb^2c^2 - Abc^3)x^3 - ((3Bbc^2 + Ac^3)x^4 + 3Bb^3 + Ab^2c + 2(3Bb^2c + Abc^2)x^2)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right) + (3Bb^2c + Ab^2c^2)x}{8(b^2c^5x^4 + 2b^3c^4x^2 + b^4c^3)} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3, x, algorithm="fricas")

[Out] [-1/16\*(2\*(5\*B\*b^2\*c^2 - A\*b\*c^3)\*x^3 + ((3\*B\*b\*c^2 + A\*c^3)\*x^4 + 3\*B\*b^3 + A\*b^2\*c + 2\*(3\*B\*b^2\*c + A\*b\*c^2)\*x^2)\*sqrt(-b\*c)\*log((c\*x^2 - 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)) + 2\*(3\*B\*b^3\*c + A\*b^2\*c^2)\*x)/(b^2\*c^5\*x^4 + 2\*b^3\*c^4\*x^2 + b^4\*c^3), -1/8\*((5\*B\*b^2\*c^2 - A\*b\*c^3)\*x^3 - ((3\*B\*b\*c^2 + A\*c^3)\*x^4 + 3\*B\*b^3 + A\*b^2\*c + 2\*(3\*B\*b^2\*c + A\*b\*c^2)\*x^2)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b) + (3\*B\*b^3\*c + A\*b^2\*c^2)\*x)/(b^2\*c^5\*x^4 + 2\*b^3\*c^4\*x^2 + b^4\*c^3)]

**giac [A]** time = 0.16, size = 78, normalized size = 0.87

$$\frac{(3Bb + Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}bc^2} - \frac{5Bbcx^3 - Ac^2x^3 + 3Bb^2x + Abcx}{8(cx^2 + b)^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out]  $\frac{1}{8}*(3*B*b + A*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b*c^2) - \frac{1}{8}*(5*B*b*c*x^3 - A*c^2*x^3 + 3*B*b^2*x + A*b*c*x)/((c*x^2 + b)^2*b*c^2)$

**maple** [A] time = 0.06, size = 89, normalized size = 0.99

$$\frac{A \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc} bc} + \frac{3B \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc} c^2} + \frac{\frac{(Ac-5bB)x^3}{8bc} - \frac{(Ac+3bB)x}{8c^2}}{(cx^2 + b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out]  $\frac{(1/8*(A*c-5*B*b))/b/c*x^3-1/8*(A*c+3*B*b)/c^2*x}{(c*x^2+b)^2} + \frac{1/8/c/b/(b*c)^{(1/2)*\arctan(1/(b*c)^{(1/2)*c*x)*A+3/8/c^2/(b*c)^{(1/2)*\arctan(1/(b*c)^{(1/2)*c*x)*B}}$

**maxima** [A] time = 2.98, size = 92, normalized size = 1.02

$$-\frac{(5Bbc - Ac^2)x^3 + (3Bb^2 + Abc)x}{8(bc^4x^4 + 2b^2c^3x^2 + b^3c^2)} + \frac{(3Bb + Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc} bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $-\frac{1}{8}*((5*B*b*c - A*c^2)*x^3 + (3*B*b^2 + A*b*c)*x)/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2) + \frac{1}{8}*(3*B*b + A*c)*\arctan(c*x/\sqrt{b*c})/(\sqrt{b*c}*b*c^2)$

**mupad** [B] time = 0.15, size = 82, normalized size = 0.91

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(Ac + 3Bb)}{8b^{3/2}c^{5/2}} - \frac{\frac{x(Ac+3Bb)}{8c^2} - \frac{x^3(Ac-5Bb)}{8bc}}{b^2 + 2bcx^2 + c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out]  $\frac{\operatorname{atan}((c^{1/2}*x)/b^{1/2})*(Ac + 3*B*b)}{(8*b^{3/2}*c^{5/2})} - \frac{(x*(Ac + 3*B*b))/(8*c^2) - (x^3*(Ac - 5*B*b))/(8*b*c)}{(b^2 + c^2*x^4 + 2*b*c*x^2)}$

**sympy** [A] time = 0.76, size = 155, normalized size = 1.72

$$-\frac{\sqrt{-\frac{1}{b^3c^5}}(Ac + 3Bb) \log\left(-b^2c^2\sqrt{-\frac{1}{b^3c^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{b^3c^5}}(Ac + 3Bb) \log\left(b^2c^2\sqrt{-\frac{1}{b^3c^5}} + x\right)}{16} + \frac{x^3(Ac^2 - 5Bbc) + x(-Abc - 3Bb^2)}{8b^3c^2 + 16b^2c^3x^2 + 8bc^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out]  $-\sqrt{-1/(b**3*c**5)}*(Ac + 3*B*b)*\log(-b**2*c**2*\sqrt{-1/(b**3*c**5)} + x)/16 + \sqrt{-1/(b**3*c**5)}*(Ac + 3*B*b)*\log(b**2*c**2*\sqrt{-1/(b**3*c**5)} + x)/16 + (x**3*(A*c**2 - 5*B*b*c) + x*(-A*b*c - 3*B*b**2))/(8*b**3*c**2 + 16*b**2*c**3*x**2 + 8*b*c**4*x**4)$



$$3.80 \quad \int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=32

$$\frac{(A+Bx^2)^2}{4(b+cx^2)^2(bB-Ac)}$$

**Rubi [A]** time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 444, 37}

$$\frac{(A+Bx^2)^2}{4(b+cx^2)^2(bB-Ac)}$$

Antiderivative was successfully verified.

[In] Int[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (A + B\*x^2)^2/(4\*(b\*B - A\*c)\*(b + c\*x^2)^2)

Rule 37

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Simp [((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3} dx &= \int \frac{x(A+Bx^2)}{(b+cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{(b+cx)^3} dx, x, x^2 \right) \\ &= \frac{(A+Bx^2)^2}{4(bB-Ac)(b+cx^2)^2} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 0.94

$$\frac{c(A+2Bx^2)+bB}{4c^2(b+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -1/4\*(b\*B + c\*(A + 2\*B\*x^2))/(c^2\*(b + c\*x^2)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

fricas [A] time = 0.39, size = 42, normalized size = 1.31

$$-\frac{2Bcx^2 + Bb + Ac}{4(c^4x^4 + 2bc^3x^2 + b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] -1/4\*(2\*B\*c\*x^2 + B\*b + A\*c)/(c^4\*x^4 + 2\*b\*c^3\*x^2 + b^2\*c^2)

giac [A] time = 0.16, size = 28, normalized size = 0.88

$$-\frac{2Bcx^2 + Bb + Ac}{4(cx^2 + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] -1/4\*(2\*B\*c\*x^2 + B\*b + A\*c)/((c\*x^2 + b)^2\*c^2)

maple [A] time = 0.05, size = 39, normalized size = 1.22

$$-\frac{B}{2(cx^2 + b)c^2} - \frac{Ac - bB}{4(cx^2 + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] -1/2\*B/c^2/(c\*x^2+b)-1/4\*(A\*c-B\*b)/c^2/(c\*x^2+b)^2

maxima [A] time = 1.35, size = 42, normalized size = 1.31

$$-\frac{2Bcx^2 + Bb + Ac}{4(c^4x^4 + 2bc^3x^2 + b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/4\*(2\*B\*c\*x^2 + B\*b + A\*c)/(c^4\*x^4 + 2\*b\*c^3\*x^2 + b^2\*c^2)

**mupad [B]** time = 0.08, size = 44, normalized size = 1.38

$$-\frac{\frac{Ac+Bb}{4c^2} + \frac{Bx^2}{2c}}{b^2 + 2bcx^2 + c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] -((A\*c + B\*b)/(4\*c^2) + (B\*x^2)/(2\*c))/(b^2 + c^2\*x^4 + 2\*b\*c\*x^2)

**sympy [A]** time = 0.54, size = 42, normalized size = 1.31

$$\frac{-Ac - Bb - 2Bcx^2}{4b^2c^2 + 8bc^3x^2 + 4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] (-A\*c - B\*b - 2\*B\*c\*x\*\*2)/(4\*b\*\*2\*c\*\*2 + 8\*b\*c\*\*3\*x\*\*2 + 4\*c\*\*4\*x\*\*4)

$$3.81 \quad \int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=92

$$\frac{(3Ac + bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}} + \frac{x(3Ac + bB)}{8b^2c(b + cx^2)} - \frac{x(bB - Ac)}{4bc(b + cx^2)^2}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 385, 199, 205}

$$\frac{(3Ac + bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}} + \frac{x(3Ac + bB)}{8b^2c(b + cx^2)} - \frac{x(bB - Ac)}{4bc(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -((b\*B - A\*c)\*x)/(4\*b\*c\*(b + c\*x^2)^2) + ((b\*B + 3\*A\*c)\*x)/(8\*b^2\*c\*(b + c\*x^2)) + ((b\*B + 3\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(5/2)\*c^(3/2))

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^6 (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{(b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)x}{4bc(b + cx^2)^2} + \frac{(bB + 3Ac) \int \frac{1}{(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)x}{4bc(b + cx^2)^2} + \frac{(bB + 3Ac)x}{8b^2c(b + cx^2)} + \frac{(bB + 3Ac) \int \frac{1}{b+cx^2} dx}{8b^2c} \\
&= -\frac{(bB - Ac)x}{4bc(b + cx^2)^2} + \frac{(bB + 3Ac)x}{8b^2c(b + cx^2)} + \frac{(bB + 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 84, normalized size = 0.91

$$\frac{(3Ac + bB) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}} + \frac{x(bc(5A + Bx^2) + 3Ac^2x^2 + b^2(-B))}{8b^2c(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (x\*(-(b^2\*B) + 3\*A\*c^2\*x^2 + b\*c\*(5\*A + B\*x^2)))/(8\*b^2\*c\*(b + c\*x^2)^2) + ((b\*B + 3\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(5/2)\*c^(3/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.42, size = 300, normalized size = 3.26

$$\frac{2(Bb^2c^2 + 3Abc^2)x^3 - ((Bbc^2 + 3Ac^2)x^4 + Bb^3 + 3Ab^2c + 2(Bb^2c + 3Abc^2)x^2)\sqrt{-bc} \log\left(\frac{x^2 - 2\sqrt{bc}x + b}{-c^2 + b^2}\right) - 2(Bb^3c - 5Ab^2c^2)x}{16(b^3c^4x^4 + 2b^4c^3x^2 + b^5c^2)} + \frac{(Bb^2c^2 + 3Abc^2)x^3 + ((Bbc^2 + 3Ac^2)x^4 + Bb^3 + 3Ab^2c + 2(Bb^2c + 3Abc^2)x^2)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{b}\right) - (Bb^3c - 5Ab^2c^2)x}{8(b^3c^4x^4 + 2b^4c^3x^2 + b^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] [1/16\*(2\*(B\*b^2\*c^2 + 3\*A\*b\*c^3)\*x^3 - ((B\*b\*c^2 + 3\*A\*c^3)\*x^4 + B\*b^3 + 3\*A\*b^2\*c + 2\*(B\*b^2\*c + 3\*A\*b\*c^2)\*x^2)\*sqrt(-b\*c)\*log((c\*x^2 - 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)) - 2\*(B\*b^3\*c - 5\*A\*b^2\*c^2)\*x)/(b^3\*c^4\*x^4 + 2\*b^4\*c^3\*x^2 + b^5\*c^2), 1/8\*((B\*b^2\*c^2 + 3\*A\*b\*c^3)\*x^3 + ((B\*b\*c^2 + 3\*A\*c^3)\*x^4 + B\*b^3 + 3\*A\*b^2\*c + 2\*(B\*b^2\*c + 3\*A\*b\*c^2)\*x^2)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b) - (B\*b^3\*c - 5\*A\*b^2\*c^2)\*x)/(b^3\*c^4\*x^4 + 2\*b^4\*c^3\*x^2 + b^5\*c^2)]

**giac [A]** time = 0.16, size = 78, normalized size = 0.85

$$\frac{(Bb + 3Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^2c} + \frac{Bbcx^3 + 3Ac^2x^3 - Bb^2x + 5Abcx}{8(cx^2 + b)^2b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 1/8\*(B\*b + 3\*A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^2\*c) + 1/8\*(B\*b\*c\*x^3 + 3\*A\*c^2\*x^3 - B\*b^2\*x + 5\*A\*b\*c\*x)/((c\*x^2 + b)^2\*b^2\*c)

maple [A] time = 0.06, size = 90, normalized size = 0.98

$$\frac{3A \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc} b^2} + \frac{B \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc} bc} + \frac{\frac{(3Ac+bB)x^3}{8b^2} + \frac{(5Ac-bB)x}{8bc}}{(cx^2 + b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] (1/8\*(3\*A\*c+B\*b)/b^2\*x^3+1/8\*(5\*A\*c-B\*b)/b/c\*x)/(c\*x^2+b)^2+3/8/b^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A+1/8/b/c/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B

maxima [A] time = 2.95, size = 92, normalized size = 1.00

$$\frac{(Bbc + 3Ac^2)x^3 - (Bb^2 - 5Abc)x}{8(b^2c^3x^4 + 2b^3c^2x^2 + b^4c)} + \frac{(Bb + 3Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc} b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 1/8\*((B\*b\*c + 3\*A\*c^2)\*x^3 - (B\*b^2 - 5\*A\*b\*c)\*x)/(b^2\*c^3\*x^4 + 2\*b^3\*c^2\*x^2 + b^4\*c) + 1/8\*(B\*b + 3\*A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^2\*c)

mupad [B] time = 0.14, size = 82, normalized size = 0.89

$$\frac{\frac{x^3(3Ac+Bb)}{8b^2} + \frac{x(5Ac-Bb)}{8bc}}{b^2 + 2bcx^2 + c^2x^4} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(3Ac + Bb)}{8b^{5/2}c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] ((x^3\*(3\*A\*c + B\*b))/(8\*b^2) + (x\*(5\*A\*c - B\*b))/(8\*b\*c))/(b^2 + c^2\*x^4 + 2\*b\*c\*x^2) + (atan((c^(1/2)\*x)/b^(1/2))\*(3\*A\*c + B\*b))/(8\*b^(5/2)\*c^(3/2))

sympy [A] time = 0.60, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c^3}}(3Ac + Bb)\log\left(-b^3c\sqrt{-\frac{1}{b^5c^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{b^5c^3}}(3Ac + Bb)\log\left(b^3c\sqrt{-\frac{1}{b^5c^3}} + x\right)}{16} + \frac{x^3(3Ac^2 + Bbc) + x(5Abc - Bb^2)}{8b^4c + 16b^3c^2x^2 + 8b^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] -sqrt(-1/(b\*\*5\*c\*\*3))\*(3\*A\*c + B\*b)\*log(-b\*\*3\*c\*sqrt(-1/(b\*\*5\*c\*\*3)) + x)/16 + sqrt(-1/(b\*\*5\*c\*\*3))\*(3\*A\*c + B\*b)\*log(b\*\*3\*c\*sqrt(-1/(b\*\*5\*c\*\*3)) + x)/16 + (x\*\*3\*(3\*A\*c\*\*2 + B\*b\*c) + x\*(5\*A\*b\*c - B\*b\*\*2))/(8\*b\*\*4\*c + 16\*b\*\*3\*c\*\*2\*x\*\*2 + 8\*b\*\*2\*c\*\*3\*x\*\*4)

$$3.82 \quad \int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=68

$$-\frac{A \log(b+cx^2)}{2b^3} + \frac{A \log(x)}{b^3} + \frac{A}{2b^2(b+cx^2)} - \frac{bB-Ac}{4bc(b+cx^2)^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$\frac{A}{2b^2(b+cx^2)} - \frac{A \log(b+cx^2)}{2b^3} + \frac{A \log(x)}{b^3} - \frac{bB-Ac}{4bc(b+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -(b\*B - A\*c)/(4\*b\*c\*(b + c\*x^2)^2) + A/(2\*b^2\*(b + c\*x^2)) + (A\*Log[x])/b^3 - (A\*Log[b + c\*x^2])/(2\*b^3)

Rule 77

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3} dx &= \int \frac{A+Bx^2}{x(b+cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x(b+cx)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^3x} + \frac{bB-Ac}{b(b+cx)^3} - \frac{Ac}{b^2(b+cx)^2} - \frac{Ac}{b^3(b+cx)} \right) dx, x, x^2 \right) \\ &= -\frac{bB-Ac}{4bc(b+cx^2)^2} + \frac{A}{2b^2(b+cx^2)} + \frac{A \log(x)}{b^3} - \frac{A \log(b+cx^2)}{2b^3} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 59, normalized size = 0.87

$$\frac{\frac{b(3Abc+2Ac^2x^2+b^2(-B))}{c(b+cx^2)^2} - 2A \log(b+cx^2) + 4A \log(x)}{4b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((b\*(-(b^2\*B) + 3\*A\*b\*c + 2\*A\*c^2\*x^2))/(c\*(b + c\*x^2)^2) + 4\*A\*Log[x] - 2\*A\*Log[b + c\*x^2])/(4\*b^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.40, size = 119, normalized size = 1.75

$$\frac{2Abc^2x^2 - Bb^3 + 3Ab^2c - 2(Ac^3x^4 + 2Abc^2x^2 + Ab^2c) \log(cx^2 + b) + 4(Ac^3x^4 + 2Abc^2x^2 + Ab^2c) \log(x)}{4(b^3c^3x^4 + 2b^4c^2x^2 + b^5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/4\*(2\*A\*b\*c^2\*x^2 - B\*b^3 + 3\*A\*b^2\*c - 2\*(A\*c^3\*x^4 + 2\*A\*b\*c^2\*x^2 + A\*b^2\*c)\*log(c\*x^2 + b) + 4\*(A\*c^3\*x^4 + 2\*A\*b\*c^2\*x^2 + A\*b^2\*c)\*log(x))/(b^3\*c^3\*x^4 + 2\*b^4\*c^2\*x^2 + b^5\*c)

**giac [A]** time = 0.16, size = 76, normalized size = 1.12

$$\frac{A \log(x^2)}{2b^3} - \frac{A \log(|cx^2 + b|)}{2b^3} + \frac{3Ac^3x^4 + 8Abc^2x^2 - Bb^3 + 6Ab^2c}{4(cx^2 + b)^2 b^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 1/2\*A\*log(x^2)/b^3 - 1/2\*A\*log(abs(c\*x^2 + b))/b^3 + 1/4\*(3\*A\*c^3\*x^4 + 8\*A\*b\*c^2\*x^2 - B\*b^3 + 6\*A\*b^2\*c)/((c\*x^2 + b)^2\*b^3\*c)

**maple [A]** time = 0.06, size = 68, normalized size = 1.00

$$\frac{A}{4(cx^2 + b)^2 b} - \frac{B}{4(cx^2 + b)^2 c} + \frac{A}{2(cx^2 + b)b^2} + \frac{A \ln(x)}{b^3} - \frac{A \ln(cx^2 + b)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] -1/2\*A\*ln(c\*x^2+b)/b^3+1/2\*A/b^2/(c\*x^2+b)+1/4/b/(c\*x^2+b)^2\*A-1/4/c/(c\*x^2+b)^2\*B+A\*ln(x)/b^3



**maxima [A]** time = 1.42, size = 77, normalized size = 1.13

$$\frac{2Ac^2x^2 - Bb^2 + 3Abc}{4(b^2c^3x^4 + 2b^3c^2x^2 + b^4c)} - \frac{A \log(cx^2 + b)}{2b^3} + \frac{A \log(x^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 1/4\*(2\*A\*c^2\*x^2 - B\*b^2 + 3\*A\*b\*c)/(b^2\*c^3\*x^4 + 2\*b^3\*c^2\*x^2 + b^4\*c) - 1/2\*A\*log(c\*x^2 + b)/b^3 + 1/2\*A\*log(x^2)/b^3

**mupad [B]** time = 0.18, size = 71, normalized size = 1.04

$$\frac{\frac{3Ac-Bb}{4bc} + \frac{Acx^2}{2b^2}}{b^2 + 2bcx^2 + c^2x^4} - \frac{A \ln(cx^2 + b)}{2b^3} + \frac{A \ln(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] ((3\*A\*c - B\*b)/(4\*b\*c) + (A\*c\*x^2)/(2\*b^2))/(b^2 + c^2\*x^4 + 2\*b\*c\*x^2) - (A\*log(b + c\*x^2))/(2\*b^3) + (A\*log(x))/b^3

**sympy [A]** time = 0.59, size = 75, normalized size = 1.10

$$\frac{A \log(x)}{b^3} - \frac{A \log\left(\frac{b}{c} + x^2\right)}{2b^3} + \frac{3Abc + 2Ac^2x^2 - Bb^2}{4b^4c + 8b^3c^2x^2 + 4b^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] A\*log(x)/b\*\*3 - A\*log(b/c + x\*\*2)/(2\*b\*\*3) + (3\*A\*b\*c + 2\*A\*c\*\*2\*x\*\*2 - B\*b\*\*2)/(4\*b\*\*4\*c + 8\*b\*\*3\*c\*\*2\*x\*\*2 + 4\*b\*\*2\*c\*\*3\*x\*\*4)

$$3.83 \quad \int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=96

$$\frac{3(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}} + \frac{x(3bB - 7Ac)}{8b^3(b + cx^2)} - \frac{A}{b^3x} + \frac{x(bB - Ac)}{4b^2(b + cx^2)^2}$$

**Rubi [A]** time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1584, 456, 453, 205}

$$\frac{x(3bB - 7Ac)}{8b^3(b + cx^2)} + \frac{x(bB - Ac)}{4b^2(b + cx^2)^2} + \frac{3(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}} - \frac{A}{b^3x}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -(A/(b^3\*x)) + ((b\*B - A\*c)\*x)/(4\*b^2\*(b + c\*x^2)^2) + ((3\*b\*B - 7\*A\*c)\*x)/(8\*b^3\*(b + c\*x^2)) + (3\*(b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(7/2)\*Sqrt[c])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e^(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p+1)/(2\*b^(m/2 + 1)\*(p+1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p+1)), Int[x^m\*(a + b\*x^2)^(p+1)\*ExpandToSum[2\*b\*(p+1)\*Together[(b^(m/2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)\*x^(-m+2)]/(a + b\*x^2)] - ((-a)^(m/2 - 1)\*(b\*c - a\*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m+2\*p+1, 0])

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m+n\*p)\*(a + b\*x^(q-p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^2 (b + cx^2)^3} dx \\
&= \frac{(bB - Ac)x}{4b^2 (b + cx^2)^2} - \frac{1}{4} \int \frac{-\frac{4A}{b} - \frac{3(bB - Ac)x^2}{b^2}}{x^2 (b + cx^2)^2} dx \\
&= \frac{(bB - Ac)x}{4b^2 (b + cx^2)^2} + \frac{(3bB - 7Ac)x}{8b^3 (b + cx^2)} + \frac{1}{8} \int \frac{\frac{8A}{b^2} + \frac{(3bB - 7Ac)x^2}{b^3}}{x^2 (b + cx^2)} dx \\
&= -\frac{A}{b^3 x} + \frac{(bB - Ac)x}{4b^2 (b + cx^2)^2} + \frac{(3bB - 7Ac)x}{8b^3 (b + cx^2)} + \frac{(3(bB - 5Ac)) \int \frac{1}{b + cx^2} dx}{8b^3} \\
&= -\frac{A}{b^3 x} + \frac{(bB - Ac)x}{4b^2 (b + cx^2)^2} + \frac{(3bB - 7Ac)x}{8b^3 (b + cx^2)} + \frac{3(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2} \sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 96, normalized size = 1.00

$$\frac{3(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2} \sqrt{c}} + \frac{x(3bB - 7Ac)}{8b^3 (b + cx^2)} - \frac{A}{b^3 x} + \frac{x(bB - Ac)}{4b^2 (b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -(A/(b^3\*x)) + ((b\*B - A\*c)\*x)/(4\*b^2\*(b + c\*x^2)^2) + (((3\*b\*B - 7\*A\*c)\*x)/(8\*b^3\*(b + c\*x^2)) + (3\*(b\*B - 5\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(7/2)\*Sqrt[c])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.42, size = 324, normalized size = 3.38

$$\frac{16Ab^3c - 6(Bb^2c^2 - 5Abc^2)x^4 - 10(Bb^2c - 5Ab^2c^2)x^2 - 3((Bb^2c - 5Ac^2)x^2 + 2(Bb^2c - 5Abc^2)x^2 + (Bb^2c - 5Ab^2c^2)x)\sqrt{-bc} \log\left(\frac{x^2 + 2\sqrt{-bc}x + b}{x^2 + b}\right) - 8Ab^3c - 3(Bb^2c^2 - 5Abc^2)x^4 - 5(Bb^2c - 5Ab^2c^2)x^2 - 3((Bb^2c - 5Ac^2)x^2 + 2(Bb^2c - 5Abc^2)x^2 + (Bb^2c - 5Ab^2c^2)x)\sqrt{bc} \arctan\left(\frac{\sqrt{bc}x}{\sqrt{b}}\right)}{16(b^3c^3 + 2b^2c^2x^3 + b^2cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] [-1/16\*(16\*A\*b^3\*c - 6\*(B\*b^2\*c^2 - 5\*A\*b\*c^3)\*x^4 - 10\*(B\*b^3\*c - 5\*A\*b^2\*c^2)\*x^2 - 3\*((B\*b\*c^2 - 5\*A\*c^3)\*x^5 + 2\*(B\*b^2\*c - 5\*A\*b\*c^2)\*x^3 + (B\*b^3 - 5\*A\*b^2\*c)\*x)\*sqrt(-b\*c)\*log((c\*x^2 + 2\*sqrt(-b\*c)\*x - b)/(c\*x^2 + b)))/(b^4\*c^3\*x^5 + 2\*b^5\*c^2\*x^3 + b^6\*c\*x), -1/8\*(8\*A\*b^3\*c - 3\*(B\*b^2\*c^2 - 5\*A\*b\*c^3)\*x^4 - 5\*(B\*b^3\*c - 5\*A\*b^2\*c^2)\*x^2 - 3\*((B\*b\*c^2 - 5\*A\*c^3)\*x^5 + 2\*(B\*b^2\*c - 5\*A\*b\*c^2)\*x^3 + (B\*b^3 - 5\*A\*b^2\*c)\*x)\*sqrt(b\*c)\*arctan(sqrt(b\*c)\*x/b)/(b^4\*c^3\*x^5 + 2\*b^5\*c^2\*x^3 + b^6\*c\*x)]

**giac** [A] time = 0.17, size = 82, normalized size = 0.85

$$\frac{3(Bb - 5Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^3} - \frac{A}{b^3x} + \frac{3Bbcx^3 - 7Ac^2x^3 + 5Bb^2x - 9Abcx}{8(cx^2 + b)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 3/8\*(B\*b - 5\*A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^3) - A/(b^3\*x) + 1/8\*(3\*B\*b\*c\*x^3 - 7\*A\*c^2\*x^3 + 5\*B\*b^2\*x - 9\*A\*b\*c\*x)/((c\*x^2 + b)^2\*b^3)

**maple** [A] time = 0.06, size = 125, normalized size = 1.30

$$-\frac{7Ac^2x^3}{8(cx^2+b)^2b^3} + \frac{3Bcx^3}{8(cx^2+b)^2b^2} - \frac{9Acx}{8(cx^2+b)^2b^2} + \frac{5Bx}{8(cx^2+b)^2b} - \frac{15Ac \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^3} + \frac{3B \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^2} - \frac{A}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] -7/8/b^3/(c\*x^2+b)^2\*A\*x^3\*c^2+3/8/b^2/(c\*x^2+b)^2\*B\*x^3\*c-9/8/b^2/(c\*x^2+b)^2\*A\*c\*x+5/8/b/(c\*x^2+b)^2\*B\*x-15/8/b^3/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A\*c+3/8/b^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B-A/b^3/x

**maxima** [A] time = 2.97, size = 96, normalized size = 1.00

$$\frac{3(Bbc - 5Ac^2)x^4 - 8Ab^2 + 5(Bb^2 - 5Abc)x^2}{8(b^3c^2x^5 + 2b^4cx^3 + b^5x)} + \frac{3(Bb - 5Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 1/8\*(3\*(B\*b\*c - 5\*A\*c^2)\*x^4 - 8\*A\*b^2 + 5\*(B\*b^2 - 5\*A\*b\*c)\*x^2)/(b^3\*c^2\*x^5 + 2\*b^4\*c\*x^3 + b^5\*x) + 3/8\*(B\*b - 5\*A\*c)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^3)

**mupad** [B] time = 0.18, size = 113, normalized size = 1.18

$$-\frac{\frac{A}{b} + \frac{5x^2(5Ac-Bb)}{8b^2} + \frac{3cx^4(5Ac-Bb)}{8b^3}}{b^2x + 2bcx^3 + c^2x^5} - \frac{3 \operatorname{atan}\left(\frac{3\sqrt{c}x(5Ac-Bb)}{\sqrt{b}(15Ac-3Bb)}\right)(5Ac-Bb)}{8b^{7/2}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] -(A/b + (5\*x^2\*(5\*A\*c - B\*b))/(8\*b^2) + (3\*c\*x^4\*(5\*A\*c - B\*b))/(8\*b^3))/(b^2\*x + c^2\*x^5 + 2\*b\*c\*x^3) - (3\*atan((3\*c^(1/2)\*x\*(5\*A\*c - B\*b))/(b^(1/2)\*(15\*A\*c - 3\*B\*b)))\*(5\*A\*c - B\*b))/(8\*b^(7/2)\*c^(1/2))

**sympy** [B] time = 0.73, size = 194, normalized size = 2.02

$$-\frac{3\sqrt{\frac{1}{b^3c}}(-5Ac+Bb)\log\left(-\frac{3b^4\sqrt{\frac{1}{b^3c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16} + \frac{3\sqrt{\frac{1}{b^3c}}(-5Ac+Bb)\log\left(\frac{3b^4\sqrt{\frac{1}{b^3c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16} + \frac{-8Ab^2+x^4(-15Ac^2+3Bbc)+x^2(-25Abc+5Bb^2)}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out]  $-3\sqrt{-1/(b**7*c)}*(-5*A*c + B*b)*\log(-3*b**4*\sqrt{-1/(b**7*c)}*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + 3*\sqrt{-1/(b**7*c)}*(-5*A*c + B*b)*\log(3*b**4*\sqrt{-1/(b**7*c)}*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + (-8*A*b**2 + x**4*(-15*A*c**2 + 3*B*b*c) + x**2*(-25*A*b*c + 5*B*b**2))/(8*b**5*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)$

$$3.84 \quad \int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=97

$$-\frac{(bB-3Ac)\log(b+cx^2)}{2b^4} + \frac{\log(x)(bB-3Ac)}{b^4} + \frac{bB-2Ac}{2b^3(b+cx^2)} - \frac{A}{2b^3x^2} + \frac{bB-Ac}{4b^2(b+cx^2)^2}$$

**Rubi [A]** time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$\frac{bB-2Ac}{2b^3(b+cx^2)} + \frac{bB-Ac}{4b^2(b+cx^2)^2} - \frac{(bB-3Ac)\log(b+cx^2)}{2b^4} + \frac{\log(x)(bB-3Ac)}{b^4} - \frac{A}{2b^3x^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -A/(2\*b^3\*x^2) + (b\*B - A\*c)/(4\*b^2\*(b + c\*x^2)^2) + (b\*B - 2\*A\*c)/(2\*b^3\*(b + c\*x^2)) + ((b\*B - 3\*A\*c)\*Log[x])/b^4 - ((b\*B - 3\*A\*c)\*Log[b + c\*x^2])/(2\*b^4)

#### Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx &= \int \frac{A+Bx^2}{x^3(b+cx^2)^3} dx \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x^2(b+cx)^3} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^3x^2} + \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)}{b^2(b+cx)^3} - \frac{c(bB-2Ac)}{b^3(b+cx)^2} - \frac{c(bB-3Ac)}{b^4(b+cx)} \right) dx, x, x^2 \right) \\
&= -\frac{A}{2b^3x^2} + \frac{bB-Ac}{4b^2(b+cx^2)^2} + \frac{bB-2Ac}{2b^3(b+cx^2)} + \frac{(bB-3Ac)\log(x)}{b^4} - \frac{(bB-3Ac)\log(b+cx^2)}{2b^4}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 86, normalized size = 0.89

$$\frac{\frac{b^2(bB-Ac)}{(b+cx^2)^2} + \frac{2b(bB-2Ac)}{b+cx^2} - 2(bB-3Ac)\log(b+cx^2) + 4\log(x)(bB-3Ac) - \frac{2Ab}{x^2}}{4b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] ((-2\*A\*b)/x^2 + (b^2\*(b\*B - A\*c))/(b + c\*x^2)^2 + (2\*b\*(b\*B - 2\*A\*c))/(b + c\*x^2) + 4\*(b\*B - 3\*A\*c)\*Log[x] - 2\*(b\*B - 3\*A\*c)\*Log[b + c\*x^2])/(4\*b^4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [B]** time = 0.40, size = 197, normalized size = 2.03

$$\frac{2(Bb^2c - 3Abc^2)x^4 - 2Ab^3 + 3(Bb^3 - 3Ab^2c)x^2 - 2((Bbc^2 - 3Ac^3)x^6 + 2(Bb^2c - 3Abc^2)x^4 + (Bb^3 - 3Ab^2c)x^2)\log(cx^2 + b) + 4((Bbc^2 - 3Ac^3)x^6 + 2(Bb^2c - 3Abc^2)x^4 + (Bb^3 - 3Ab^2c)x^2)\log(x)}{4(b^4cx^6 + 2b^5cx^4 + b^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3, x, algorithm="fricas")

[Out] 1/4\*(2\*(B\*b^2\*c - 3\*A\*b\*c^2)\*x^4 - 2\*A\*b^3 + 3\*(B\*b^3 - 3\*A\*b^2\*c)\*x^2 - 2\*((B\*b\*c^2 - 3\*A\*c^3)\*x^6 + 2\*(B\*b^2\*c - 3\*A\*b\*c^2)\*x^4 + (B\*b^3 - 3\*A\*b^2\*c)\*x^2)\*log(c\*x^2 + b) + 4\*((B\*b\*c^2 - 3\*A\*c^3)\*x^6 + 2\*(B\*b^2\*c - 3\*A\*b\*c^2)\*x^4 + (B\*b^3 - 3\*A\*b^2\*c)\*x^2)\*log(x)/(b^4\*c^2\*x^6 + 2\*b^5\*c\*x^4 + b^6\*x^2)

**giac [A]** time = 0.18, size = 105, normalized size = 1.08

$$\frac{(Bb - 3Ac)\log(|x|)}{b^4} - \frac{(Bbc - 3Ac^2)\log(|cx^2 + b|)}{2b^4c} + \frac{2(Bb^2c - 3Abc^2)x^4 - 2Ab^3 + 3(Bb^3 - 3Ab^2c)x^2}{4(cx^2 + b)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3, x, algorithm="giac")

[Out]  $(B*b - 3*A*c)*\log(\text{abs}(x))/b^4 - 1/2*(B*b*c - 3*A*c^2)*\log(\text{abs}(c*x^2 + b))/(b^4*c) + 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2)/((c*x^2 + b)^2*b^4*x^2)$

**maple** [A] time = 0.06, size = 118, normalized size = 1.22

$$-\frac{Ac}{4(cx^2+b)^2b^2} + \frac{B}{4(cx^2+b)^2b} - \frac{Ac}{(cx^2+b)b^3} - \frac{3Ac\ln(x)}{b^4} + \frac{3Ac\ln(cx^2+b)}{2b^4} + \frac{B}{2(cx^2+b)b^2} + \frac{B\ln(x)}{b^3} - \frac{B\ln(cx^2+b)}{2b^3} - \frac{A}{2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3, x)$

[Out]  $3/2/b^4*c*\ln(cx^2+b)*A - 1/2/b^3*\ln(cx^2+b)*B - 1/b^3*c/(cx^2+b)*A + 1/2/b^2/(cx^2+b)*B - 1/4/b^2*c/(cx^2+b)^2*A + 1/4/b/(cx^2+b)^2*B - 1/2*A/b^3/x^2 - 3/b^4*\ln(x)*A*c + 1/b^3*\ln(x)*B$

**maxima** [A] time = 1.37, size = 109, normalized size = 1.12

$$\frac{2(Bbc - 3Ac^2)x^4 - 2Ab^2 + 3(Bb^2 - 3Abc)x^2}{4(b^3c^2x^6 + 2b^4cx^4 + b^5x^2)} - \frac{(Bb - 3Ac)\log(cx^2 + b)}{2b^4} + \frac{(Bb - 3Ac)\log(x^2)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3, x, \text{algorithm}="maxima")$

[Out]  $1/4*(2*(B*b*c - 3*A*c^2)*x^4 - 2*A*b^2 + 3*(B*b^2 - 3*A*b*c)*x^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2) - 1/2*(B*b - 3*A*c)*\log(c*x^2 + b)/b^4 + 1/2*(B*b - 3*A*c)*\log(x^2)/b^4$

**mupad** [B] time = 0.15, size = 107, normalized size = 1.10

$$\frac{\ln(cx^2 + b)(3Ac - Bb)}{2b^4} - \frac{\frac{A}{2b} + \frac{3x^2(3Ac - Bb)}{4b^2} + \frac{cx^4(3Ac - Bb)}{2b^3}}{b^2x^2 + 2bcx^4 + c^2x^6} - \frac{\ln(x)(3Ac - Bb)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3, x)$

[Out]  $(\log(b + c*x^2)*(3*A*c - B*b))/(2*b^4) - (A/(2*b) + (3*x^2*(3*A*c - B*b)))/(4*b^2) + (c*x^4*(3*A*c - B*b))/(2*b^3)/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (\log(x)*(3*A*c - B*b))/b^4$

**sympy** [A] time = 1.11, size = 107, normalized size = 1.10

$$\frac{-2Ab^2 + x^4(-6Ac^2 + 2Bbc) + x^2(-9Abc + 3Bb^2)}{4b^5x^2 + 8b^4cx^4 + 4b^3c^2x^6} + \frac{(-3Ac + Bb)\log(x)}{b^4} - \frac{(-3Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x**3*(B*x**2+A)/(c*x**4+b*x**2)**3, x)$

[Out]  $(-2*A*b**2 + x**4*(-6*A*c**2 + 2*B*b*c) + x**2*(-9*A*b*c + 3*B*b**2))/(4*b**5*x**2 + 8*b**4*c*x**4 + 4*b**3*c**2*x**6) + (-3*A*c + B*b)*\log(x)/b**4 - (-3*A*c + B*b)*\log(b/c + x**2)/(2*b**4)$



$$3.85 \quad \int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=117

$$-\frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}} - \frac{cx(7bB-11Ac)}{8b^4(b+cx^2)} - \frac{bB-3Ac}{b^4x} - \frac{cx(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{A}{3b^3x^3}$$

**Rubi [A]** time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1584, 456, 1259, 1261, 205}

$$-\frac{cx(7bB-11Ac)}{8b^4(b+cx^2)} - \frac{cx(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{bB-3Ac}{b^4x} - \frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}} - \frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -A/(3\*b^3\*x^3) - (b\*B - 3\*A\*c)/(b^4\*x) - (c\*(b\*B - A\*c)\*x)/(4\*b^3\*(b + c\*x^2)^2) - (c\*(7\*b\*B - 11\*A\*c)\*x)/(8\*b^4\*(b + c\*x^2)) - (5\*Sqrt[c]\*(3\*b\*B - 7\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(9/2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 456**

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[x^m\*(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*Together[(b^(m/2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)\*x^(-m + 2)]/(a + b\*x^2)] - ((-a)^(m/2 - 1)\*(b\*c - a\*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

**Rule 1259**

Int[(x\_)^(m\_)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[((-d)^(m/2 - 1)\*(c\*d^2 - b\*d\*e + a\*e^2)^p\*x\*(d + e\*x^2)^(q + 1))/(2\*e^(2\*p + m/2)\*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2\*e^(2\*p)\*(q + 1)), Int[x^m\*(d + e\*x^2)^(q + 1)\*ExpandToSum[Together[(1\*(2\*(-d)^(m/2 - 1)\*e^(2\*p)\*(q + 1)\*(a + b\*x^2 + c\*x^4)^p - ((c\*d^2 - b\*d\*e + a\*e^2)^p/(e^(m/2)\*x^m))\*(d + e\*(2\*q + 3)\*x^2))]/(d + e\*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

**Rule 1261**

Int[((f\_.)\*(x\_)^(m\_)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(f\*x)^(m)\*(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rule 1584**

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned} \int \frac{x^2 (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^4 (b + cx^2)^3} dx \\ &= -\frac{c(bB - Ac)x}{4b^3 (b + cx^2)^2} - \frac{1}{4}c \int \frac{-\frac{4A}{bc} - \frac{4(bB - Ac)x^2}{b^2c} + \frac{3(bB - Ac)x^4}{b^3}}{x^4 (b + cx^2)^2} dx \\ &= -\frac{c(bB - Ac)x}{4b^3 (b + cx^2)^2} - \frac{c(7bB - 11Ac)x}{8b^4 (b + cx^2)} - \frac{\int \frac{-8Abc - 8c(bB - 2Ac)x^2 + \frac{c^2(7bB - 11Ac)x^4}{b}}{x^4 (b + cx^2)} dx}{8b^3c} \\ &= -\frac{c(bB - Ac)x}{4b^3 (b + cx^2)^2} - \frac{c(7bB - 11Ac)x}{8b^4 (b + cx^2)} - \frac{\int \left( -\frac{8Ac}{x^4} - \frac{8c(bB - 3Ac)}{bx^2} + \frac{5c^2(3bB - 7Ac)}{b(b + cx^2)} \right) dx}{8b^3c} \\ &= -\frac{A}{3b^3x^3} - \frac{bB - 3Ac}{b^4x} - \frac{c(bB - Ac)x}{4b^3 (b + cx^2)^2} - \frac{c(7bB - 11Ac)x}{8b^4 (b + cx^2)} - \frac{(5c(3bB - 7Ac)) \int \frac{1}{b + cx^2} dx}{8b^4} \\ &= -\frac{A}{3b^3x^3} - \frac{bB - 3Ac}{b^4x} - \frac{c(bB - Ac)x}{4b^3 (b + cx^2)^2} - \frac{c(7bB - 11Ac)x}{8b^4 (b + cx^2)} - \frac{5\sqrt{c}(3bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 119, normalized size = 1.02

$$-\frac{5\sqrt{c}(3bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}} - \frac{x(7bBc - 11Ac^2)}{8b^4 (b + cx^2)} + \frac{3Ac - bB}{b^4x} - \frac{cx(bB - Ac)}{4b^3 (b + cx^2)^2} - \frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] -1/3\*A/(b^3\*x^3) + (-b\*B) + 3\*A\*c)/(b^4\*x) - (c\*(b\*B - A\*c)\*x)/(4\*b^3\*(b + c\*x^2)^2) - ((7\*b\*B\*c - 11\*A\*c^2)\*x)/(8\*b^4\*(b + c\*x^2)) - (5\*sqrt[c]\*(3\*b\*B - 7\*A\*c)\*ArcTan[(sqrt[c]\*x)/sqrt[b]])/(8\*b^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [A]** time = 0.41, size = 368, normalized size = 3.15

$$\frac{30(3Bc^2 - 7Ac^2)x^6 + 50(3Bc^2 - 7Ac^2)x^4 + 16Ab^3 + 16(3Bb^3 - 7Ab^2c)x^2 + 15((3Bc^2 - 7Ac^2)x^2 + 2(3Bb^2c - 7Ab^2c^2))\sqrt{\frac{c}{b}} \log\left(\frac{c^2 - 2cx + b}{-cx}\right) - 15(3Bb^2 - 7Ac^2)x^4 + 25(3Bb^2c - 7Ab^2c^2)x^2 + 8Ab^3 + 8(3Bb^3 - 7Ab^2c)^2 + 15((3Bc^2 - 7Ac^2)x^2 + 2(3Bb^2c - 7Ab^2c^2))\sqrt{\frac{c}{b}} \arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{48(b^2c^2 + 2b^2cx + b^2c^2)} - \frac{5\sqrt{c}(3bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{24(b^2c^2 + 2b^2cx + b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] [-1/48\*(30\*(3\*B\*b\*c^2 - 7\*A\*c^3)\*x^6 + 50\*(3\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^4 + 16\*A\*b^3 + 16\*(3\*B\*b^3 - 7\*A\*b^2\*c)\*x^2 + 15\*((3\*B\*b\*c^2 - 7\*A\*c^3)\*x^7 + 2\*(3\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^5 + (3\*B\*b^3 - 7\*A\*b^2\*c)\*x^3)\*sqrt(-c/b)\*log((c\*x^2 + 2\*b\*x\*sqrt(-c/b) - b)/(c\*x^2 + b)))/(b^4\*c^2\*x^7 + 2\*b^5\*c\*x^5 + b^6\*x^3), -1/24\*(15\*(3\*B\*b\*c^2 - 7\*A\*c^3)\*x^6 + 25\*(3\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^4 + 8\*A\*b^3 + 8\*(3\*B\*b^3 - 7\*A\*b^2\*c)\*x^2 + 15\*((3\*B\*b\*c^2 - 7\*A\*c^3)\*x^7 + 2\*(3\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^5 + (3\*B\*b^3 - 7\*A\*b^2\*c)\*x^3)\*sqrt(c/b)\*arctan(x\*sqrt(c/b)))/(b^4\*c^2\*x^7 + 2\*b^5\*c\*x^5 + b^6\*x^3)]

**giac** [A] time = 0.16, size = 108, normalized size = 0.92

$$\frac{5(3Bbc - 7Ac^2) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^4} - \frac{7Bbc^2x^3 - 11Ac^3x^3 + 9Bb^2cx - 13Abc^2x}{8(cx^2 + b)^2b^4} - \frac{3Bbx^2 - 9Acx^2 + Ab}{3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] -5/8\*(3\*B\*b\*c - 7\*A\*c^2)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^4) - 1/8\*(7\*B\*b\*c^2\*x^3 - 11\*A\*c^3\*x^3 + 9\*B\*b^2\*c\*x - 13\*A\*b\*c^2\*x)/((c\*x^2 + b)^2\*b^4) - 1/3\*(3\*B\*b\*x^2 - 9\*A\*c\*x^2 + A\*b)/(b^4\*x^3)

**maple** [A] time = 0.06, size = 152, normalized size = 1.30

$$\frac{11Ac^3x^3}{8(cx^2 + b)^2b^4} - \frac{7Bc^2x^3}{8(cx^2 + b)^2b^3} + \frac{13Ac^2x}{8(cx^2 + b)^2b^3} - \frac{9Bcx}{8(cx^2 + b)^2b^2} + \frac{35Ac^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^4} - \frac{15Bc \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^3} + \frac{3Ac}{b^4x} - \frac{B}{b^3x} - \frac{A}{3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 11/8/b^4\*c^3/(c\*x^2+b)^2\*A\*x^3-7/8/b^3\*c^2/(c\*x^2+b)^2\*B\*x^3+13/8/b^3\*c^2/(c\*x^2+b)^2\*A\*x-9/8/b^2\*c/(c\*x^2+b)^2\*B\*x+35/8/b^4\*c^2/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*A-15/8/b^3\*c/(b\*c)^(1/2)\*arctan(1/(b\*c)^(1/2)\*c\*x)\*B-1/3\*A/b^3/x^3+3/b^4/x\*A\*c-1/b^3/x\*B

**maxima** [A] time = 3.11, size = 128, normalized size = 1.09

$$\frac{15(3Bbc^2 - 7Ac^3)x^6 + 25(3Bb^2c - 7Abc^2)x^4 + 8Ab^3 + 8(3Bb^3 - 7Ab^2c)x^2}{24(b^4c^2x^7 + 2b^5cx^5 + b^6x^3)} - \frac{5(3Bbc - 7Ac^2) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/24\*(15\*(3\*B\*b\*c^2 - 7\*A\*c^3)\*x^6 + 25\*(3\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^4 + 8\*A\*b^3 + 8\*(3\*B\*b^3 - 7\*A\*b^2\*c)\*x^2)/(b^4\*c^2\*x^7 + 2\*b^5\*c\*x^5 + b^6\*x^3) - 5/8\*(3\*B\*b\*c - 7\*A\*c^2)\*arctan(c\*x/sqrt(b\*c))/(sqrt(b\*c)\*b^4)

**mupad** [B] time = 0.18, size = 114, normalized size = 0.97

$$\frac{\frac{x^2(7Ac-3Bb)}{3b^2} - \frac{A}{3b} + \frac{5c^2x^6(7Ac-3Bb)}{8b^4} + \frac{25cx^4(7Ac-3Bb)}{24b^3}}{b^2x^3 + 2bcx^5 + c^2x^7} + \frac{5\sqrt{c} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(7Ac-3Bb)}{8b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out]  $((x^2(7Ac - 3Bb))/(3b^2) - A/(3b) + (5c^2x^6(7Ac - 3Bb))/(8b^4) + (25c^4x^4(7Ac - 3Bb))/(24b^3))/(b^2x^3 + c^2x^7 + 2b^2cx^5) + (5c^{1/2}) \operatorname{atan}((c^{1/2}x)/b^{1/2}) \cdot (7Ac - 3Bb)/(8b^{9/2})$

**sympy [B]** time = 0.83, size = 226, normalized size = 1.93

$$\frac{5\sqrt{\frac{c}{b^9}}(-7Ac + 3Bb) \log\left(\frac{5b^5\sqrt{\frac{c}{b^9}}(-7Ac + 3Bb)}{-35Ac^2 + 15Bbc} + x\right)}{16} - \frac{5\sqrt{\frac{c}{b^9}}(-7Ac + 3Bb) \log\left(\frac{5b^5\sqrt{\frac{c}{b^9}}(-7Ac + 3Bb)}{-35Ac^2 + 15Bbc} + x\right)}{16} + \frac{-8Ab^3 + x^6(105Ac^3 - 45Bbc^2) + x^4(175Abc^2 - 75Bb^2c) + x^2(56Ab^2c - 24Bb^3)}{24b^6x^3 + 48b^5cx^5 + 24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out]  $5\sqrt{c/b^9}(-7Ac + 3Bb) \log(-5b^5\sqrt{c/b^9}(-7Ac + 3Bb)/(-35Ac^2 + 15Bbc) + x)/16 - 5\sqrt{c/b^9}(-7Ac + 3Bb) \log(5b^5\sqrt{c/b^9}(-7Ac + 3Bb)/(-35Ac^2 + 15Bbc) + x)/16 + (-8Ab^3 + x^6(105Ac^3 - 45Bbc^2) + x^4(175Abc^2 - 75Bb^2c) + x^2(56Ab^2c - 24Bb^3))/(24b^6x^3 + 48b^5cx^5 + 24b^4c^2x^7)$

$$3.86 \quad \int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=121

$$\frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5} - \frac{3c\log(x)(bB-2Ac)}{b^5} - \frac{c(2bB-3Ac)}{2b^4(b+cx^2)} - \frac{bB-3Ac}{2b^4x^2} - \frac{c(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{A}{4b^3x^4}$$

**Rubi [A]** time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1584, 446, 77}

$$\frac{c(2bB-3Ac)}{2b^4(b+cx^2)} - \frac{bB-3Ac}{2b^4x^2} - \frac{c(bB-Ac)}{4b^3(b+cx^2)^2} + \frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5} - \frac{3c\log(x)(bB-2Ac)}{b^5} - \frac{A}{4b^3x^4}$$

Antiderivative was successfully verified.

[In] Int[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -A/(4\*b^3\*x^4) - (b\*B - 3\*A\*c)/(2\*b^4\*x^2) - (c\*(b\*B - A\*c))/(4\*b^3\*(b + c\*x^2)^2) - (c\*(2\*b\*B - 3\*A\*c))/(2\*b^4\*(b + c\*x^2)) - (3\*c\*(b\*B - 2\*A\*c)\*Log[x])/b^5 + (3\*c\*(b\*B - 2\*A\*c)\*Log[b + c\*x^2])/(2\*b^5)

**Rule 77**

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 446**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3} dx &= \int \frac{A+Bx^2}{x^5(b+cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x^3(b+cx)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^3x^3} + \frac{bB-3Ac}{b^4x^2} - \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)}{b^3(b+cx)^3} + \frac{c^2(2bB-3Ac)}{b^4(b+cx)^2} + \frac{3c^2}{b^5} \right) dx, x, x^2 \right) \\ &= -\frac{A}{4b^3x^4} - \frac{bB-3Ac}{2b^4x^2} - \frac{c(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{c(2bB-3Ac)}{2b^4(b+cx^2)} - \frac{3c(bB-2Ac)\log(x)}{b^5} + \frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 108, normalized size = 0.89

$$\frac{\frac{b^2c(Ac-bB)}{(b+cx^2)^2} - \frac{Ab^2}{x^4} + \frac{2bc(3Ac-2bB)}{b+cx^2} - \frac{2b(bB-3Ac)}{x^2} + 6c(bB-2Ac)\log(b+cx^2) + 12c\log(x)(2Ac-bB)}{4b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (-((A\*b^2)/x^4) - (2\*b\*(b\*B - 3\*A\*c))/x^2 + (b^2\*c\*(-(b\*B) + A\*c))/(b + c\*x^2)^2 + (2\*b\*c\*(-2\*b\*B + 3\*A\*c))/(b + c\*x^2) + 12\*c\*(-(b\*B) + 2\*A\*c)\*Log[x] + 6\*c\*(b\*B - 2\*A\*c)\*Log[b + c\*x^2])/(4\*b^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

**fricas [B]** time = 0.41, size = 229, normalized size = 1.89

$$\frac{6(Bb^2c^2 - 2Abc^3)x^6 + Ab^4 + 9(Bb^3c - 2Ab^2c^2)x^4 + 2(Bb^4 - 2Ab^3c)x^2 - 6((Bbc^3 - 2Ac^4)x^8 + 2(Bb^2c^2 - 2Abc^3)x^6 + (Bb^3c - 2Ab^2c^2)x^4)\log(cx^2 + b) + 12((Bbc^3 - 2Ac^4)x^8 + 2(Bb^2c^2 - 2Abc^3)x^6 + (Bb^3c - 2Ab^2c^2)x^4)\log(x)}{4(b^5cx^8 + 2b^6cx^6 + b^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] -1/4\*(6\*(B\*b^2\*c^2 - 2\*A\*b\*c^3)\*x^6 + A\*b^4 + 9\*(B\*b^3\*c - 2\*A\*b^2\*c^2)\*x^4 + 2\*(B\*b^4 - 2\*A\*b^3\*c)\*x^2 - 6\*((B\*b\*c^3 - 2\*A\*c^4)\*x^8 + 2\*(B\*b^2\*c^2 - 2\*A\*b\*c^3)\*x^6 + (B\*b^3\*c - 2\*A\*b^2\*c^2)\*x^4)\*log(cx^2 + b) + 12\*((B\*b\*c^3 - 2\*A\*c^4)\*x^8 + 2\*(B\*b^2\*c^2 - 2\*A\*b\*c^3)\*x^6 + (B\*b^3\*c - 2\*A\*b^2\*c^2)\*x^4)\*log(x))/(b^5\*c^2\*x^8 + 2\*b^6\*c\*x^6 + b^7\*x^4)

**giac [A]** time = 0.17, size = 132, normalized size = 1.09

$$-\frac{3(Bbc - 2Ac^2)\log(|x|)}{b^5} + \frac{3(Bbc^2 - 2Ac^3)\log(|cx^2 + b|)}{2b^5c} - \frac{6Bbcx^6 - 12Ac^3x^6 + 9Bb^2cx^4 - 18Abc^2x^4 + 2Bb^3x^2 - 4Ab^2cx^2 + Ab^3}{4(cx^4 + bx^2)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] -3\*(B\*b\*c - 2\*A\*c^2)\*log(abs(x))/b^5 + 3/2\*(B\*b\*c^2 - 2\*A\*c^3)\*log(abs(cx^2 + b))/(b^5\*c) - 1/4\*(6\*B\*b\*c^2\*x^6 - 12\*A\*c^3\*x^6 + 9\*B\*b^2\*c\*x^4 - 18\*A\*b\*c^2\*x^4 + 2\*B\*b^3\*x^2 - 4\*A\*b^2\*c\*x^2 + A\*b^3)/((cx^4 + bx^2)^2\*b^4)

**maple [A]** time = 0.06, size = 150, normalized size = 1.24

$$\frac{Ac^2}{4(cx^2 + b)^2b^3} - \frac{Bc}{4(cx^2 + b)^2b^2} + \frac{3Ac^2}{2(cx^2 + b)b^4} + \frac{6Ac^2\ln(x)}{b^5} - \frac{3Ac^2\ln(cx^2 + b)}{b^5} - \frac{Bc}{(cx^2 + b)b^3} - \frac{3Bc\ln(x)}{b^4} + \frac{3Bc\ln(cx^2 + b)}{2b^4} + \frac{3Ac}{2b^4x^2} - \frac{B}{2b^3x^2} - \frac{A}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] -3/b^5\*c^2\*ln(cx^2+b)\*A+3/2/b^4\*c\*ln(cx^2+b)\*B+3/2/b^4\*c^2/(cx^2+b)\*A-1/b^3\*c/(cx^2+b)\*B+1/4/b^3\*c^2/(cx^2+b)^2\*A-1/4/b^2\*c/(cx^2+b)^2\*B-1/4\*A/b^3/x^4+3/2/b^4/x^2\*A\*c-1/2/b^3/x^2\*B+6\*c^2/b^5\*ln(x)\*A-3\*c/b^4\*ln(x)\*B

**maxima [A]** time = 1.43, size = 137, normalized size = 1.13

$$\frac{6(Bbc^2 - 2Ac^3)x^6 + 9(Bb^2c - 2Abc^2)x^4 + Ab^3 + 2(Bb^3 - 2Ab^2c)x^2}{4(b^4c^2x^8 + 2b^5cx^6 + b^6x^4)} + \frac{3(Bbc - 2Ac^2)\log(cx^2 + b)}{2b^5} - \frac{3(Bbc - 2Ac^2)\log(x^2)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/4\*(6\*(B\*b\*c^2 - 2\*A\*c^3)\*x^6 + 9\*(B\*b^2\*c - 2\*A\*b\*c^2)\*x^4 + A\*b^3 + 2\*(B\*b^3 - 2\*A\*b^2\*c)\*x^2)/(b^4\*c^2\*x^8 + 2\*b^5\*c\*x^6 + b^6\*x^4) + 3/2\*(B\*b\*c - 2\*A\*c^2)\*log(c\*x^2 + b)/b^5 - 3/2\*(B\*b\*c - 2\*A\*c^2)\*log(x^2)/b^5

**mupad [B]** time = 0.17, size = 131, normalized size = 1.08

$$\frac{\frac{x^2(2Ac-Bb)}{2b^2} - \frac{A}{4b} + \frac{3c^2x^6(2Ac-Bb)}{2b^4} + \frac{9cx^4(2Ac-Bb)}{4b^3}}{b^2x^4 + 2bcx^6 + c^2x^8} - \frac{\ln(cx^2 + b)(6Ac^2 - 3Bbc)}{2b^5} + \frac{\ln(x)(6Ac^2 - 3Bbc)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] ((x^2\*(2\*A\*c - B\*b))/(2\*b^2) - A/(4\*b) + (3\*c^2\*x^6\*(2\*A\*c - B\*b))/(2\*b^4) + (9\*c\*x^4\*(2\*A\*c - B\*b))/(4\*b^3))/(b^2\*x^4 + c^2\*x^8 + 2\*b\*c\*x^6) - (log(b + c\*x^2)\*(6\*A\*c^2 - 3\*B\*b\*c))/(2\*b^5) + (log(x)\*(6\*A\*c^2 - 3\*B\*b\*c))/b^5

**sympy [A]** time = 1.25, size = 136, normalized size = 1.12

$$\frac{-Ab^3 + x^6(12Ac^3 - 6Bbc^2) + x^4(18Abc^2 - 9Bb^2c) + x^2(4Ab^2c - 2Bb^3)}{4b^6x^4 + 8b^5cx^6 + 4b^4c^2x^8} - \frac{3c(-2Ac + Bb)\log(x)}{b^5} + \frac{3c(-2Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] (-A\*b\*\*3 + x\*\*6\*(12\*A\*c\*\*3 - 6\*B\*b\*c\*\*2) + x\*\*4\*(18\*A\*b\*c\*\*2 - 9\*B\*b\*\*2\*c) + x\*\*2\*(4\*A\*b\*\*2\*c - 2\*B\*b\*\*3))/(4\*b\*\*6\*x\*\*4 + 8\*b\*\*5\*c\*x\*\*6 + 4\*b\*\*4\*c\*\*2\*x\*\*8) - 3\*c\*(-2\*A\*c + B\*b)\*log(x)/b\*\*5 + 3\*c\*(-2\*A\*c + B\*b)\*log(b/c + x\*\*2)/(2\*b\*\*5)

$$3.87 \quad \int \frac{A+Bx^2}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=140

$$\frac{7c^{3/2}(5bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}} + \frac{c^2x(11bB - 15Ac)}{8b^5(b + cx^2)} + \frac{3c(bB - 2Ac)}{b^5x} + \frac{c^2x(bB - Ac)}{4b^4(b + cx^2)^2} - \frac{bB - 3Ac}{3b^4x^3} - \frac{A}{5b^3x^5}$$

**Rubi [A]** time = 0.33, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1593, 456, 1805, 1802, 205}

$$\frac{c^2x(11bB - 15Ac)}{8b^5(b + cx^2)} + \frac{c^2x(bB - Ac)}{4b^4(b + cx^2)^2} + \frac{7c^{3/2}(5bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}} - \frac{bB - 3Ac}{3b^4x^3} + \frac{3c(bB - 2Ac)}{b^5x} - \frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(b\*x^2 + c\*x^4)^3,x]

[Out] -A/(5\*b^3\*x^5) - (b\*B - 3\*A\*c)/(3\*b^4\*x^3) + (3\*c\*(b\*B - 2\*A\*c))/(b^5\*x) + (c^2\*(b\*B - A\*c)\*x)/(4\*b^4\*(b + c\*x^2)^2) + (c^2\*(11\*b\*B - 15\*A\*c)\*x)/(8\*b^5\*(b + c\*x^2)) + (7\*c^(3/2)\*(5\*b\*B - 9\*A\*c)\*ArcTan[(Sqrt[c]\*x)/Sqrt[b]])/(8\*b^(11/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 456

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_)\*((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[((-a)^(m/2 - 1)\*(b\*c - a\*d)\*x\*(a + b\*x^2)^(p + 1))/(2\*b^(m/2 + 1)\*(p + 1)), x] + Dist[1/(2\*b^(m/2 + 1)\*(p + 1)), Int[x^m\*(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*b\*(p + 1)\*Together[(b^(m/2)\*(c + d\*x^2) - (-a)^(m/2 - 1)\*(b\*c - a\*d)\*x^(-m + 2)]/(a + b\*x^2)] - ((-a)^(m/2 - 1)\*(b\*c - a\*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2\*p + 1, 0])

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^n, x\_Symbol] := Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq\_)\*((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1805

Int[(Pq\_)\*((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[(c\*x)^m\*Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[(c\*x)^m\*Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c\*x)^m\*Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(c\*x)^m\*(a + b\*x^2)^(p + 1)\*Exp



andToSum[(2\*a\*(p + 1)\*Q)/(c\*x)^m + (f\*(2\*p + 3))/(c\*x)^m, x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^6 (b + cx^2)^3} dx \\ &= \frac{c^2(bB - Ac)x}{4b^4 (b + cx^2)^2} - \frac{1}{4}c^2 \int \frac{\frac{4A}{bc^2} - \frac{4(bB - Ac)x^2}{b^2c^2} + \frac{4(bB - Ac)x^4}{b^3c} - \frac{3(bB - Ac)x^6}{b^4}}{x^6 (b + cx^2)^2} dx \\ &= \frac{c^2(bB - Ac)x}{4b^4 (b + cx^2)^2} + \frac{c^2(11bB - 15Ac)x}{8b^5 (b + cx^2)} + \frac{c^2 \int \frac{\frac{8A}{bc^2} + \frac{8(bB - 2Ac)x^2}{b^2c^2} - \frac{8(2bB - 3Ac)x^4}{b^3c} + \frac{(11bB - 15Ac)x^6}{b^4}}{x^6 (b + cx^2)} dx}{8b} \\ &= \frac{c^2(bB - Ac)x}{4b^4 (b + cx^2)^2} + \frac{c^2(11bB - 15Ac)x}{8b^5 (b + cx^2)} + \frac{c^2 \int \left( \frac{8A}{b^2c^2x^6} + \frac{8(bB - 3Ac)}{b^3c^2x^4} - \frac{24(bB - 2Ac)}{b^4cx^2} + \frac{7(5bB - 9Ac)}{b^4(b + cx^2)} \right) dx}{8b} \\ &= -\frac{A}{5b^3x^5} - \frac{bB - 3Ac}{3b^4x^3} + \frac{3c(bB - 2Ac)}{b^5x} + \frac{c^2(bB - Ac)x}{4b^4 (b + cx^2)^2} + \frac{c^2(11bB - 15Ac)x}{8b^5 (b + cx^2)} + \frac{(7c^2(5bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right))}{8b^{11/2}} \\ &= -\frac{A}{5b^3x^5} - \frac{bB - 3Ac}{3b^4x^3} + \frac{3c(bB - 2Ac)}{b^5x} + \frac{c^2(bB - Ac)x}{4b^4 (b + cx^2)^2} + \frac{c^2(11bB - 15Ac)x}{8b^5 (b + cx^2)} + \frac{7c^{3/2}(5bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 140, normalized size = 1.00

$$\frac{7c^{3/2}(5bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}} + \frac{c^2x(11bB - 15Ac)}{8b^5 (b + cx^2)} + \frac{3c(bB - 2Ac)}{b^5x} + \frac{c^2x(bB - Ac)}{4b^4 (b + cx^2)^2} - \frac{bB - 3Ac}{3b^4x^3} - \frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^3, x]
[Out] -1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x) + (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(11/2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(b*x^2 + c*x^4)^3, x]
[Out] IntegrateAlgebraic[(A + B*x^2)/(b*x^2 + c*x^4)^3, x]
```

**fricas [A]** time = 0.43, size = 426, normalized size = 3.04

$\frac{210(5bB^2 - 9Ac^2)^3 + 350(5bB^2 - 9Ac^2)^2 - 48Ab^4 + 112(5bB^2 - 9Ac^2)^2 - 16(5bB^2 - 9Ac^2)^2 + 2(5bB^2 - 9Ac^2)^2 + (5bB^2 - 9Ac^2)^2}{240(b^2c^2 + 2b^2c^2 + b^2c^2)} \sqrt{\frac{c^2 - 11c}{-25c^2}} \log\left(\frac{c^2 - 11c}{-25c^2}\right) + \frac{105(5bB^2 - 9Ac^2)^3 + 175(5bB^2 - 9Ac^2)^2 - 24Ab^4 + 56(5bB^2 - 9Ac^2)^2 - 8(5bB^2 - 9Ac^2)^2 + 105((5bB^2 - 9Ac^2)^2 + 2(5bB^2 - 9Ac^2)^2 + (5bB^2 - 9Ac^2)^2) \sqrt{\frac{c^2 - 11c}{-25c^2}} \operatorname{arctan}\left(\frac{\sqrt{c}}{\sqrt{b}}\right)}{120(b^2c^2 + 2b^2c^2 + b^2c^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out]  $\frac{1}{240} \cdot (210 \cdot (5 \cdot B \cdot b \cdot c^3 - 9 \cdot A \cdot c^4) \cdot x^8 + 350 \cdot (5 \cdot B \cdot b^2 \cdot c^2 - 9 \cdot A \cdot b \cdot c^3) \cdot x^6 - 48 \cdot A \cdot b^4 + 112 \cdot (5 \cdot B \cdot b^3 \cdot c - 9 \cdot A \cdot b^2 \cdot c^2) \cdot x^4 - 16 \cdot (5 \cdot B \cdot b^4 - 9 \cdot A \cdot b^3 \cdot c) \cdot x^2 - 105 \cdot ((5 \cdot B \cdot b \cdot c^3 - 9 \cdot A \cdot c^4) \cdot x^9 + 2 \cdot (5 \cdot B \cdot b^2 \cdot c^2 - 9 \cdot A \cdot b \cdot c^3) \cdot x^7 + (5 \cdot B \cdot b^3 \cdot c - 9 \cdot A \cdot b^2 \cdot c^2) \cdot x^5) \cdot \sqrt{-c/b} \cdot \log((c \cdot x^2 - 2 \cdot b \cdot x \cdot \sqrt{-c/b}) - b) / (c \cdot x^2 + b)) / (b^5 \cdot c^2 \cdot x^9 + 2 \cdot b^6 \cdot c \cdot x^7 + b^7 \cdot x^5), \frac{1}{120} \cdot (105 \cdot (5 \cdot B \cdot b \cdot c^3 - 9 \cdot A \cdot c^4) \cdot x^8 + 175 \cdot (5 \cdot B \cdot b^2 \cdot c^2 - 9 \cdot A \cdot b \cdot c^3) \cdot x^6 - 24 \cdot A \cdot b^4 + 56 \cdot (5 \cdot B \cdot b^3 \cdot c - 9 \cdot A \cdot b^2 \cdot c^2) \cdot x^4 - 8 \cdot (5 \cdot B \cdot b^4 - 9 \cdot A \cdot b^3 \cdot c) \cdot x^2 + 105 \cdot ((5 \cdot B \cdot b \cdot c^3 - 9 \cdot A \cdot c^4) \cdot x^9 + 2 \cdot (5 \cdot B \cdot b^2 \cdot c^2 - 9 \cdot A \cdot b \cdot c^3) \cdot x^7 + (5 \cdot B \cdot b^3 \cdot c - 9 \cdot A \cdot b^2 \cdot c^2) \cdot x^5) \cdot \sqrt{c/b} \cdot \arctan(x \cdot \sqrt{c/b})) / (b^5 \cdot c^2 \cdot x^9 + 2 \cdot b^6 \cdot c \cdot x^7 + b^7 \cdot x^5]$

**giac [A]** time = 0.16, size = 135, normalized size = 0.96

$$\frac{7(5Bbc^2 - 9Ac^3) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^5} + \frac{11Bbc^3x^3 - 15Ac^4x^3 + 13Bb^2c^2x - 17Abc^3x}{8(cx^2 + b)^2b^5} + \frac{45Bbcx^4 - 90Ac^2x^4 - 5Bb^2x^2 + 15Abcx^2 - 3Ab^2}{15b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out]  $\frac{7}{8} \cdot (5 \cdot B \cdot b \cdot c^2 - 9 \cdot A \cdot c^3) \cdot \arctan(c \cdot x / \sqrt{b \cdot c}) / (\sqrt{b \cdot c} \cdot b^5) + \frac{1}{8} \cdot (11 \cdot B \cdot b \cdot c^3 \cdot x^3 - 15 \cdot A \cdot c^4 \cdot x^3 + 13 \cdot B \cdot b^2 \cdot c^2 \cdot x - 17 \cdot A \cdot b \cdot c^3 \cdot x) / ((c \cdot x^2 + b)^2 \cdot b^5) + \frac{1}{15} \cdot (45 \cdot B \cdot b \cdot c \cdot x^4 - 90 \cdot A \cdot c^2 \cdot x^4 - 5 \cdot B \cdot b^2 \cdot x^2 + 15 \cdot A \cdot b \cdot c \cdot x^2 - 3 \cdot A \cdot b^2) / (b^5 \cdot x^5)$

**maple [A]** time = 0.07, size = 177, normalized size = 1.26

$$-\frac{15A^4x^3}{8(cx^2+b)^2b^5} + \frac{11Bc^3x^3}{8(cx^2+b)^2b^4} - \frac{17Ac^3x}{8(cx^2+b)^2b^4} + \frac{13Bc^2x}{8(cx^2+b)^2b^3} - \frac{63A^3c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^5} + \frac{35Bc^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^4} - \frac{6Ac^2}{b^5x} + \frac{3Bc}{b^4x} + \frac{Ac}{b^4x^3} - \frac{B}{3b^3x^3} - \frac{A}{5b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out]  $-\frac{15}{8} \cdot \frac{1}{b^5 \cdot c^4} / (c \cdot x^2 + b)^2 \cdot A \cdot x^3 + \frac{11}{8} \cdot \frac{1}{b^4 \cdot c^3} / (c \cdot x^2 + b)^2 \cdot B \cdot x^3 - \frac{17}{8} \cdot \frac{1}{b^4 \cdot c^3} / (c \cdot x^2 + b)^2 \cdot A \cdot x + \frac{13}{8} \cdot \frac{1}{b^3 \cdot c^2} / (c \cdot x^2 + b)^2 \cdot B \cdot x - \frac{63}{8} \cdot \frac{1}{b^5 \cdot c^3} / (b \cdot c)^{(1/2)} \cdot \arctan(1 / (b \cdot c)^{(1/2)} \cdot c \cdot x) \cdot A + \frac{35}{8} \cdot \frac{1}{b^4 \cdot c^2} / (b \cdot c)^{(1/2)} \cdot \arctan(1 / (b \cdot c)^{(1/2)} \cdot c \cdot x) \cdot B - \frac{1}{5} \cdot \frac{A}{b^3 \cdot x^5} + \frac{1}{b^4 \cdot x^3} \cdot A \cdot c - \frac{1}{3} \cdot \frac{1}{b^3 \cdot x^3} \cdot B - \frac{6}{c^2} \cdot \frac{1}{b^5 \cdot x} \cdot A + \frac{3}{c} \cdot \frac{1}{b^4 \cdot x} \cdot B$

**maxima [A]** time = 3.02, size = 154, normalized size = 1.10

$$\frac{105(5Bbc^3 - 9Ac^4)x^8 + 175(5Bb^2c^2 - 9Abc^3)x^6 - 24Ab^4 + 56(5Bb^3c - 9Ab^2c^2)x^4 - 8(5Bb^4 - 9Ab^3c)x^2}{120(b^5c^2x^9 + 2b^6cx^7 + b^7x^5)} + \frac{7(5Bbc^2 - 9Ac^3) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $\frac{1}{120} \cdot (105 \cdot (5 \cdot B \cdot b \cdot c^3 - 9 \cdot A \cdot c^4) \cdot x^8 + 175 \cdot (5 \cdot B \cdot b^2 \cdot c^2 - 9 \cdot A \cdot b \cdot c^3) \cdot x^6 - 24 \cdot A \cdot b^4 + 56 \cdot (5 \cdot B \cdot b^3 \cdot c - 9 \cdot A \cdot b^2 \cdot c^2) \cdot x^4 - 8 \cdot (5 \cdot B \cdot b^4 - 9 \cdot A \cdot b^3 \cdot c) \cdot x^2) / (b^5 \cdot c^2 \cdot x^9 + 2 \cdot b^6 \cdot c \cdot x^7 + b^7 \cdot x^5) + \frac{7}{8} \cdot (5 \cdot B \cdot b \cdot c^2 - 9 \cdot A \cdot c^3) \cdot \arctan(c \cdot x / \sqrt{b \cdot c}) / (\sqrt{b \cdot c} \cdot b^5)$

**mupad [B]** time = 0.19, size = 135, normalized size = 0.96

$$-\frac{A}{5b} - \frac{x^2(9Ac-5Bb)}{15b^2} + \frac{35c^2x^6(9Ac-5Bb)}{24b^4} + \frac{7c^3x^8(9Ac-5Bb)}{8b^5} + \frac{7cx^4(9Ac-5Bb)}{15b^3} - \frac{7c^{3/2} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)(9Ac-5Bb)}{8b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(b\*x^2 + c\*x^4)^3, x)

[Out]  $-\frac{A}{5b} - \frac{x^2(9Ac - 5Bb)}{(15b^2)} + \frac{(35c^2x^6(9Ac - 5Bb))}{(24b^4)} + \frac{(7c^3x^8(9Ac - 5Bb))}{(8b^5)} + \frac{(7cx^4(9Ac - 5Bb))}{(15b^3)} / (b^2x^5 + c^2x^9 + 2bcx^7) - \frac{(7c^{3/2}) \operatorname{atan}((c^{1/2})x/b^{1/2}) (9Ac - 5Bb)}{(8b^{11/2})}$

**sympy [A]** time = 0.92, size = 260, normalized size = 1.86

$$\frac{7\sqrt{\frac{c}{b^3}}(-9Ac + 5Bb) \log\left(\frac{7b^6\sqrt{\frac{c}{b^3}}(-9Ac + 5Bb)}{-63Ac^3 + 3588b^2} + x\right)}{16} + \frac{7\sqrt{\frac{c}{b^3}}(-9Ac + 5Bb) \log\left(\frac{7b^6\sqrt{\frac{c}{b^3}}(-9Ac + 5Bb)}{-63Ac^3 + 3588b^2} + x\right)}{16} + \frac{-24Ab^4 + x^5(-945Ac^4 + 525Bbc^2) + x^6(-1575Abc^3 + 875Bb^2c^2) + x^4(-504Ab^2c^2 + 280Bb^3c) + x^2(72Ab^3c - 40Bb^4)}{120b^7x^5 + 240b^6cx^7 + 120b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3, x)

[Out]  $-7\sqrt{c/b^3}(-9Ac + 5Bb) \log(-7b^6\sqrt{c/b^3}(-9Ac + 5Bb)/(-63Ac^3 + 3588b^2) + x)/16 + 7\sqrt{c/b^3}(-9Ac + 5Bb) \log(7b^6\sqrt{c/b^3}(-9Ac + 5Bb)/(-63Ac^3 + 3588b^2) + x)/16 + (-24Ab^4 + x^5(-945Ac^4 + 525Bbc^2) + x^6(-1575Abc^3 + 875Bb^2c^2) + x^4(-504Ab^2c^2 + 280Bb^3c) + x^2(72Ab^3c - 40Bb^4))/(120b^7x^5 + 240b^6cx^7 + 120b^5c^2x^9)$

$$3.88 \quad \int \frac{A+Bx^2}{x(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=148

$$-\frac{c^2(3bB-5Ac)\log(b+cx^2)}{b^6} + \frac{2c^2\log(x)(3bB-5Ac)}{b^6} + \frac{c^2(3bB-4Ac)}{2b^5(b+cx^2)} + \frac{3c(bB-2Ac)}{2b^5x^2} + \frac{c^2(bB-Ac)}{4b^4(b+cx^2)^2} - \frac{bB-3Ac}{4b^4x^4}$$

**Rubi [A]** time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1584, 446, 77}

$$\frac{c^2(3bB-4Ac)}{2b^5(b+cx^2)} + \frac{c^2(bB-Ac)}{4b^4(b+cx^2)^2} - \frac{c^2(3bB-5Ac)\log(b+cx^2)}{b^6} + \frac{2c^2\log(x)(3bB-5Ac)}{b^6} + \frac{3c(bB-2Ac)}{2b^5x^2} - \frac{bB-3Ac}{4b^4x^4} - \frac{A}{6b^3x^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^3), x]

[Out] -A/(6\*b^3\*x^6) - (b\*B - 3\*A\*c)/(4\*b^4\*x^4) + (3\*c\*(b\*B - 2\*A\*c))/(2\*b^5\*x^2) + (c^2\*(b\*B - A\*c))/(4\*b^4\*(b + c\*x^2)^2) + (c^2\*(3\*b\*B - 4\*A\*c))/(2\*b^5\*(b + c\*x^2)) + (2\*c^2\*(3\*b\*B - 5\*A\*c)\*Log[x])/b^6 - (c^2\*(3\*b\*B - 5\*A\*c)\*Log[b + c\*x^2])/b^6

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{A+Bx^2}{x(bx^2+cx^4)^3} dx &= \int \frac{A+Bx^2}{x^7(b+cx^2)^3} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x^4(b+cx)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{A}{b^3x^4} + \frac{bB-3Ac}{b^4x^3} - \frac{3c(bB-2Ac)}{b^5x^2} + \frac{2c^2(3bB-5Ac)}{b^6x} - \frac{c^3(bB-Ac)}{b^4(b+cx)^3} - \frac{c^3(3bB-4Ac)}{b^5(b+cx)^2} \right) dx, x, x^2 \right) \\ &= -\frac{A}{6b^3x^6} - \frac{bB-3Ac}{4b^4x^4} + \frac{3c(bB-2Ac)}{2b^5x^2} + \frac{c^2(bB-Ac)}{4b^4(b+cx^2)^2} + \frac{c^2(3bB-4Ac)}{2b^5(b+cx^2)} + \frac{2c^2(3bB-5Ac)}{b^6} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 135, normalized size = 0.91

$$\frac{\frac{2Ab^3}{x^6} + \frac{3b^2c^2(bB-Ac)}{(b+cx^2)^2} - \frac{3b^2(bB-3Ac)}{x^4} + \frac{6bc^2(3bB-4Ac)}{b+cx^2} + 12c^2(5Ac-3bB)\log(b+cx^2) + 24c^2\log(x)(3bB-5Ac) + \frac{18bc(bB-2Ac)}{x^2}}{12b^6}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^3), x]

[Out] ((-2\*A\*b^3)/x^6 - (3\*b^2\*(b\*B - 3\*A\*c))/x^4 + (18\*b\*c\*(b\*B - 2\*A\*c))/x^2 + (3\*b^2\*c^2\*(b\*B - A\*c))/(b + c\*x^2)^2 + (6\*b\*c^2\*(3\*b\*B - 4\*A\*c))/(b + c\*x^2) + 24\*c^2\*(3\*b\*B - 5\*A\*c)\*Log[x] + 12\*c^2\*(-3\*b\*B + 5\*A\*c)\*Log[b + c\*x^2])/(12\*b^6)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^3), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^3), x]

**fricas [A]** time = 0.41, size = 267, normalized size = 1.80

$$\frac{12(3Bb^2c^3 - 5Abc^4)x^8 + 18(3Bb^2c^2 - 5Ab^2c^2)x^6 - 2Ab^5 + 4(3Bb^4c - 5Ab^3c^2)x^4 - (3Bb^5 - 5Ab^4c)x^2 - 12((3Bb^4c - 5Ac^5)x^{10} + 2(3Bb^2c^3 - 5Abc^4)x^8 + (3Bb^2c^2 - 5Ab^2c^2)x^6 + 24((3Bb^4c - 5Ac^5)x^{10} + 2(3Bb^2c^3 - 5Abc^4)x^8 + (3Bb^2c^2 - 5Ab^2c^2)x^6)\log(x))}{12(b^6c^2x^{10} + 2b^7cx^8 + b^8x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/12\*(12\*(3\*B\*b^2\*c^3 - 5\*A\*b\*c^4)\*x^8 + 18\*(3\*B\*b^3\*c^2 - 5\*A\*b^2\*c^3)\*x^6 - 2\*A\*b^5 + 4\*(3\*B\*b^4\*c - 5\*A\*b^3\*c^2)\*x^4 - (3\*B\*b^5 - 5\*A\*b^4\*c)\*x^2 - 12\*((3\*B\*b\*c^4 - 5\*A\*c^5)\*x^10 + 2\*(3\*B\*b^2\*c^3 - 5\*A\*b\*c^4)\*x^8 + (3\*B\*b^3\*c^2 - 5\*A\*b^2\*c^3)\*x^6)\*log(c\*x^2 + b) + 24\*((3\*B\*b\*c^4 - 5\*A\*c^5)\*x^10 + 2\*(3\*B\*b^2\*c^3 - 5\*A\*b\*c^4)\*x^8 + (3\*B\*b^3\*c^2 - 5\*A\*b^2\*c^3)\*x^6)\*log(x))/(b^6\*c^2\*x^10 + 2\*b^7\*c\*x^8 + b^8\*x^6)

**giac [A]** time = 0.17, size = 201, normalized size = 1.36

$$\frac{(3Bbc^2 - 5Ac^3)\log(x^2)}{b^6} - \frac{(3Bbc^3 - 5Ac^4)\log(|cx^2 + b|)}{b^6c} + \frac{18Bbc^4x^4 - 30Ac^5x^4 + 42Bb^2c^3x^2 - 68Abc^4x^2 + 25Bb^3c^2 - 39Ab^2c^3}{4(cx^2 + b)^2b^6} - \frac{66Bbc^2x^6 - 110Ac^3x^6 - 18Bb^2cx^4 + 36Abc^2x^4 + 3Bb^3x^2 - 9Ab^2cx^2 + 2Ab^3}{12b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] (3\*B\*b\*c^2 - 5\*A\*c^3)\*log(x^2)/b^6 - (3\*B\*b\*c^3 - 5\*A\*c^4)\*log(abs(c\*x^2 + b))/(b^6\*c) + 1/4\*(18\*B\*b\*c^4\*x^4 - 30\*A\*c^5\*x^4 + 42\*B\*b^2\*c^3\*x^2 - 68\*A\*b\*c^4\*x^2 + 25\*B\*b^3\*c^2 - 39\*A\*b^2\*c^3)/((c\*x^2 + b)^2\*b^6) - 1/12\*(66\*B\*b\*c^2\*x^6 - 110\*A\*c^3\*x^6 - 18\*B\*b^2\*c\*x^4 + 36\*A\*b\*c^2\*x^4 + 3\*B\*b^3\*x^2 - 9\*A\*b^2\*c\*x^2 + 2\*A\*b^3)/(b^6\*x^6)

**maple [A]** time = 0.06, size = 180, normalized size = 1.22

$$-\frac{Ac^3}{4(cx^2 + b)^2b^4} + \frac{Bc^2}{4(cx^2 + b)^2b^3} - \frac{2Ac^3}{(cx^2 + b)b^5} - \frac{10Ac^3\ln(x)}{b^6} + \frac{5Ac^3\ln(cx^2 + b)}{b^6} + \frac{3Bc^2}{2(cx^2 + b)b^4} + \frac{6Bc^2\ln(x)}{b^5} - \frac{3Bc^2\ln(cx^2 + b)}{b^5} - \frac{3Ac^2}{b^5x^2} + \frac{3Bc}{2b^4x^2} + \frac{3Ac}{4b^4x^4} - \frac{B}{4b^3x^4} - \frac{A}{6b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x/(c\*x^4+b\*x^2)^3,x)

[Out]  $5/b^6*c^3*\ln(c*x^2+b)*A-3/b^5*c^2*\ln(c*x^2+b)*B-2/b^5*c^3/(c*x^2+b)*A+3/2/b^4*c^2/(c*x^2+b)*B-1/4/b^4*c^3/(c*x^2+b)^2*A+1/4/b^3*c^2/(c*x^2+b)^2*B-1/6*A/b^3/x^6+3/4/b^4/x^4*A*c-1/4/b^3/x^4*B-3*c^2/b^5/x^2*A+3/2*c/b^4/x^2*B-10*c^3/b^6*\ln(x)*A+6*c^2/b^5*\ln(x)*B$

**maxima [A]** time = 1.43, size = 170, normalized size = 1.15

$$\frac{12(3Bbc^3 - 5Ac^4)x^8 + 18(3Bb^2c^2 - 5Abc^3)x^6 - 2Ab^4 + 4(3Bb^3c - 5Ab^2c^2)x^4 - (3Bb^4 - 5Ab^3c)x^2 - \frac{(3Bbc^2 - 5Ac^3)\log(cx^2 + b)}{b^6} + \frac{(3Bbc^2 - 5Ac^3)\log(x^2)}{b^6}}{12(b^5c^2x^{10} + 2b^6cx^8 + b^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $1/12*(12*(3*B*b*c^3 - 5*A*c^4)*x^8 + 18*(3*B*b^2*c^2 - 5*A*b*c^3)*x^6 - 2*A*b^4 + 4*(3*B*b^3*c - 5*A*b^2*c^2)*x^4 - (3*B*b^4 - 5*A*b^3*c)*x^2)/(b^5*c^2*x^{10} + 2*b^6*c*x^8 + b^7*x^6) - (3*B*b*c^2 - 5*A*c^3)*\log(c*x^2 + b)/b^6 + (3*B*b*c^2 - 5*A*c^3)*\log(x^2)/b^6$

**mupad [B]** time = 0.18, size = 155, normalized size = 1.05

$$\frac{\ln(cx^2 + b)(5Ac^3 - 3Bbc^2)}{b^6} - \frac{\frac{A}{6b} - \frac{x^2(5Ac - 3Bb)}{12b^2} + \frac{3c^2x^6(5Ac - 3Bb)}{2b^4} + \frac{c^3x^8(5Ac - 3Bb)}{b^5} + \frac{cx^4(5Ac - 3Bb)}{3b^3}}{b^2x^6 + 2bcx^8 + c^2x^{10}} - \frac{\ln(x)(10Ac^3 - 6Bbc^2)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^3), x)

[Out]  $(\log(b + c*x^2)*(5*A*c^3 - 3*B*b*c^2))/b^6 - (A/(6*b) - (x^2*(5*A*c - 3*B*b)))/(12*b^2) + (3*c^2*x^6*(5*A*c - 3*B*b))/(2*b^4) + (c^3*x^8*(5*A*c - 3*B*b))/b^5 + (c*x^4*(5*A*c - 3*B*b))/(3*b^3))/(b^2*x^6 + c^2*x^{10} + 2*b*c*x^8) - (\log(x)*(10*A*c^3 - 6*B*b*c^2))/b^6$

**sympy [A]** time = 1.33, size = 165, normalized size = 1.11

$$\frac{-2Ab^4 + x^8(-60Ac^4 + 36Bbc^3) + x^6(-90Abc^3 + 54Bb^2c^2) + x^4(-20Ab^2c^2 + 12Bb^3c) + x^2(5Ab^3c - 3Bb^4)}{12b^7x^6 + 24b^6cx^8 + 12b^5c^2x^{10}} + \frac{2c^2(-5Ac + 3Bb)\log(x)}{b^6} - \frac{c^2(-5Ac + 3Bb)\log\left(\frac{b}{c} + x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out]  $(-2*A*b**4 + x**8*(-60*A*c**4 + 36*B*b*c**3) + x**6*(-90*A*b*c**3 + 54*B*b*c**2) + x**4*(-20*A*b**2*c**2 + 12*B*b**3*c) + x**2*(5*A*b**3*c - 3*B*b**4))/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) + 2*c**2*(-5*A*c + 3*B*b)*\log(x)/b**6 - c**2*(-5*A*c + 3*B*b)*\log(b/c + x**2)/b**6$

$$3.89 \quad \int x^7 (A + Bx^2) \sqrt{bx^2 + cx^4} dx$$

**Optimal.** Leaf size=218

$$\frac{7b^5(3bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}} + \frac{7b^3(b + 2cx^2) \sqrt{bx^2 + cx^4} (3bB - 4Ac)}{1024c^5} - \frac{7b^2(bx^2 + cx^4)^{3/2} (3bB - 4Ac)}{384c^4}$$

**Rubi [A]** time = 0.38, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$ , Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$\frac{7b^2(bx^2 + cx^4)^{3/2}(3bB - 4Ac)}{384c^4} + \frac{7b^3(b + 2cx^2)\sqrt{bx^2 + cx^4}(3bB - 4Ac)}{1024c^5} - \frac{7b^5(3bB - 4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}} - \frac{x^4(bx^2 + cx^4)^{3/2}(3bB - 4Ac)}{40c^2} + \frac{7bx^2(bx^2 + cx^4)^{3/2}(3bB - 4Ac)}{320c^3} + \frac{Bx^6(bx^2 + cx^4)^{3/2}}{12c}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] (7\*b^3\*(3\*b\*B - 4\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(1024\*c^5) - (7\*b^2\*(3\*b\*B - 4\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(384\*c^4) + (7\*b\*(3\*b\*B - 4\*A\*c)\*x^2\*(b\*x^2 + c\*x^4)^(3/2))/(320\*c^3) - ((3\*b\*B - 4\*A\*c)\*x^4\*(b\*x^2 + c\*x^4)^(3/2))/(40\*c^2) + (B\*x^6\*(b\*x^2 + c\*x^4)^(3/2))/(12\*c) - (7\*b^5\*(3\*b\*B - 4\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(1024\*c^(11/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_.))^(p_)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^7 (A + Bx^2) \sqrt{bx^2 + cx^4} dx = \frac{1}{2} \text{Subst} \left( \int x^3 (A + Bx) \sqrt{bx + cx^2} dx, x, x^2 \right)$$

$$= \frac{Bx^6 (bx^2 + cx^4)^{3/2}}{12c} + \frac{\left( 3(-bB + Ac) + \frac{3}{2}(-bB + 2Ac) \right) \text{Subst} \left( \int x^3 \sqrt{bx + cx^2} dx, x, x^2 \right)}{12c}$$

$$= -\frac{(3bB - 4Ac)x^4 (bx^2 + cx^4)^{3/2}}{40c^2} + \frac{Bx^6 (bx^2 + cx^4)^{3/2}}{12c} + \frac{(7b(3bB - 4Ac)) \text{Subst} \left( \int x^3 \sqrt{bx + cx^2} dx, x, x^2 \right)}{8c}$$

$$= \frac{7b(3bB - 4Ac)x^2 (bx^2 + cx^4)^{3/2}}{320c^3} - \frac{(3bB - 4Ac)x^4 (bx^2 + cx^4)^{3/2}}{40c^2} + \frac{Bx^6 (bx^2 + cx^4)^{3/2}}{12c}$$

$$= -\frac{7b^2(3bB - 4Ac) (bx^2 + cx^4)^{3/2}}{384c^4} + \frac{7b(3bB - 4Ac)x^2 (bx^2 + cx^4)^{3/2}}{320c^3} - \frac{(3bB - 4Ac)x^4 (bx^2 + cx^4)^{3/2}}{40c^2} + \frac{Bx^6 (bx^2 + cx^4)^{3/2}}{12c}$$

$$= \frac{7b^3(3bB - 4Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{1024c^5} - \frac{7b^2(3bB - 4Ac) (bx^2 + cx^4)^{3/2}}{384c^4} + \frac{7b(3bB - 4Ac)x^2 (bx^2 + cx^4)^{3/2}}{320c^3} - \frac{(3bB - 4Ac)x^4 (bx^2 + cx^4)^{3/2}}{40c^2} + \frac{Bx^6 (bx^2 + cx^4)^{3/2}}{12c}$$

$$= \frac{7b^3(3bB - 4Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{1024c^5} - \frac{7b^2(3bB - 4Ac) (bx^2 + cx^4)^{3/2}}{384c^4} + \frac{7b(3bB - 4Ac)x^2 (bx^2 + cx^4)^{3/2}}{320c^3} - \frac{(3bB - 4Ac)x^4 (bx^2 + cx^4)^{3/2}}{40c^2} + \frac{Bx^6 (bx^2 + cx^4)^{3/2}}{12c}$$

**Mathematica [A]** time = 0.32, size = 193, normalized size = 0.89

$$\frac{\sqrt{x^2(b + cx^2)} \left( \sqrt{cx^2/b + 1} (-210b^4c(2A + Bx^2) + 56b^3c^2x^2(5A + 3Bx^2) - 16b^2c^3x^4(14A + 9Bx^2) + 64bc^4x^6(3A + 2Bx^2) + 256c^5x^8(6A + 5Bx^2) + 315b^5B) - 105b^{9/2}(3bB - 4Ac) \sinh^{-1} \left( \frac{\sqrt{cx^2}}{\sqrt{b}} \right) \right)}{15360c^{11/2}x\sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4], x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(315*b^5*B - 210*b^4*c*(2*A + B*x^2) + 64*b*c^4*x^6*(3*A + 2*B*x^2) + 56*b^3*c^2*x^2*(5*A + 3*B*x^2) + 256*c^5*x^8*(6*A + 5*B*x^2) - 16*b^2*c^3*x^4*(14*A + 9*B*x^2)) - 105*b^(9/2)*(3*b*B - 4*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(15360*c^(11/2)*x*Sqrt[1 + (c*x^2)/b])
```

**IntegrateAlgebraic [A]** time = 0.74, size = 187, normalized size = 0.86

$$\frac{7(3b^6B - 4Ab^5c) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right) + \sqrt{bx^2 + cx^4} (-420Ab^4c + 280Ab^3c^2x^2 - 224Ab^2c^3x^4 + 192Abc^4x^6 + 1536Ac^5x^8 + 315b^5B - 210b^4Bcx^2 + 168b^3Bc^2x^4 - 144b^2Bc^3x^6 + 128bBc^4x^8 + 1280Bc^5x^{10})}{2048c^{11/2}} + \frac{15360c^5}{15360c^5}$$



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(315\*b^5\*B - 420\*A\*b^4\*c - 210\*b^4\*B\*c\*x^2 + 280\*A\*b^3\*c^2\*x^2 + 168\*b^3\*B\*c^2\*x^4 - 224\*A\*b^2\*c^3\*x^4 - 144\*b^2\*B\*c^3\*x^6 + 192\*A\*b\*c^4\*x^6 + 128\*b\*B\*c^4\*x^8 + 1536\*A\*c^5\*x^8 + 1280\*B\*c^5\*x^10))/(15360\*c^5) + (7\*(3\*b^6\*B - 4\*A\*b^5\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(2048\*c^(11/2))

**fricas** [A] time = 0.57, size = 368, normalized size = 1.69

$$\frac{105(3Bb^5 - 4Ac^2)\sqrt{c}\log(-2c^2 - 2\sqrt{c^2 + b^2}\sqrt{c}) - 2(1280Bc^5 + 128(Bb^5 + 12Ac^2)c^3 - 355Bb^4 - 420Ab^4c - 48(3Bb^3 - 4Ab^2c)^2 - 20(3Bb^2 - 4Ab^2c)^2)\sqrt{c^2 + b^2} + 105(3Bb^5 - 4Ac^2)\sqrt{c}\arctan\left(\frac{\sqrt{c^2 + b^2}}{c}\right) + (1280Bc^5 + 128(Bb^5 + 12Ac^2)c^3 - 355Bb^4 - 420Ab^4c - 48(3Bb^3 - 4Ab^2c)^2 + 56(3Bb^2 - 4Ab^2c)^2 - 70(3Bb^2 - 4Ab^2c)^2)\sqrt{c^2 + b^2}}{15360c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/30720\*(105\*(3\*B\*b^6 - 4\*A\*b^5\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(1280\*B\*c^6\*x^10 + 128\*(B\*b\*c^5 + 12\*A\*c^6)\*x^8 + 315\*B\*b^5\*c - 420\*A\*b^4\*c^2 - 48\*(3\*B\*b^2\*c^4 - 4\*A\*b\*c^5)\*x^6 + 56\*(3\*B\*b^3\*c^3 - 4\*A\*b^2\*c^4)\*x^4 - 70\*(3\*B\*b^4\*c^2 - 4\*A\*b^3\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^6, 1/15360\*(105\*(3\*B\*b^6 - 4\*A\*b^5\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + (1280\*B\*c^6\*x^10 + 128\*(B\*b\*c^5 + 12\*A\*c^6)\*x^8 + 315\*B\*b^5\*c - 420\*A\*b^4\*c^2 - 48\*(3\*B\*b^2\*c^4 - 4\*A\*b\*c^5)\*x^6 + 56\*(3\*B\*b^3\*c^3 - 4\*A\*b^2\*c^4)\*x^4 - 70\*(3\*B\*b^4\*c^2 - 4\*A\*b^3\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^6]

**giac** [A] time = 0.23, size = 245, normalized size = 1.12

$$\frac{1}{15360} \left( \frac{1}{2} \left( \frac{105 B b^5 \operatorname{sgn}(c) + 12 A c^2 \operatorname{sgn}(c)}{c^5} \log\left(\frac{-2 c x^2 - b - 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{2 (1280 B c^5 + 128 (B b^5 + 12 A c^2) c^3 - 355 B b^4 - 420 A b^4 c - 48 (3 B b^3 - 4 A b^2 c)^2 - 20 (3 B b^2 - 4 A b^2 c)^2) \sqrt{c^2 + b^2}}{c^5} + \frac{105 (3 B b^5 - 4 A c^2) \sqrt{c} \arctan\left(\frac{\sqrt{c^2 + b^2}}{c}\right) + (1280 B c^5 + 128 (B b^5 + 12 A c^2) c^3 - 355 B b^4 - 420 A b^4 c - 48 (3 B b^3 - 4 A b^2 c)^2 + 56 (3 B b^2 - 4 A b^2 c)^2 - 70 (3 B b^2 - 4 A b^2 c)^2) \sqrt{c^2 + b^2}}{15360 c^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] 1/15360\*(2\*(4\*(2\*(8\*(10\*B\*x^2\*sgn(x) + (B\*b\*c^9\*sgn(x) + 12\*A\*c^10\*sgn(x)))/c^10)\*x^2 - 3\*(3\*B\*b^2\*c^8\*sgn(x) - 4\*A\*b\*c^9\*sgn(x))/c^10)\*x^2 + 7\*(3\*B\*b^3\*c^7\*sgn(x) - 4\*A\*b^2\*c^8\*sgn(x))/c^10)\*x^2 - 35\*(3\*B\*b^4\*c^6\*sgn(x) - 4\*A\*b^3\*c^7\*sgn(x))/c^10)\*x^2 + 105\*(3\*B\*b^5\*c^5\*sgn(x) - 4\*A\*b^4\*c^6\*sgn(x))/c^10)\*sqrt(c\*x^2 + b)\*x + 7/1024\*(3\*B\*b^6\*sgn(x) - 4\*A\*b^5\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(11/2) - 7/2048\*(3\*B\*b^6\*log(abs(b)) - 4\*A\*b^5\*c\*log(abs(b)))\*sgn(x)/c^(11/2)

**maple** [A] time = 0.08, size = 290, normalized size = 1.33

$$\frac{\sqrt{c^2 + b^2} \left( 1280 (c^2 + b)^2 B c^2 x^2 + 1536 (c^2 + b)^2 A c^2 x - 1152 (c^2 + b)^2 B b c^2 x - 1344 (c^2 + b)^2 A b c^2 x + 1008 (c^2 + b)^2 B b^2 c^2 + 1120 (c^2 + b)^2 A b^2 c^2 - 840 (c^2 + b)^2 B b^3 c^2 + 420 A b^2 \ln(\sqrt{c^2 + b^2}) - 315 B b^4 \ln(\sqrt{c^2 + b^2}) + 420 \sqrt{c^2 + b^2} A b^5 c^2 - 315 \sqrt{c^2 + b^2} B b^4 c^2 - 840 (c^2 + b)^2 A b^5 c^2 + 420 (c^2 + b)^2 B b^4 c^2 \right)}{15360 \sqrt{c^2 + b^2} c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x)

[Out] 1/15360\*(c\*x^4+b\*x^2)^(1/2)\*(1280\*B\*(c\*x^2+b)^(3/2)\*c^(9/2)\*x^9+1536\*A\*(c\*x^2+b)^(3/2)\*c^(9/2)\*x^7-1152\*B\*(c\*x^2+b)^(3/2)\*c^(7/2)\*x^7\*b-1344\*A\*(c\*x^2+b)^(3/2)\*c^(7/2)\*x^5\*b+1008\*B\*(c\*x^2+b)^(3/2)\*c^(5/2)\*x^5\*b^2+1120\*A\*(c\*x^2+b)^(3/2)\*c^(5/2)\*x^3\*b^2-840\*B\*(c\*x^2+b)^(3/2)\*c^(3/2)\*x^3\*b^3-840\*A\*(c\*x^2+b)^(3/2)\*c^(3/2)\*x\*b^3+630\*B\*(c\*x^2+b)^(3/2)\*c^(1/2)\*x\*b^4+420\*A\*(c\*x^2+b)^(1/2)\*c^(3/2)\*x\*b^4-315\*B\*(c\*x^2+b)^(1/2)\*c^(1/2)\*x\*b^5+420\*A\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^5\*c-315\*B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^6)/x/(c\*x^2+b)^(1/2)/c^(11/2)

**maxima** [A] time = 1.62, size = 321, normalized size = 1.47

$$\frac{1}{7680} \left( \frac{768 (c^2 + b)^2 A^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{420 \sqrt{c^2 + b^2} B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{672 (c^2 + b)^2 B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{105 B^2 \log(2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c})}{c^5} - \frac{210 \sqrt{c^2 + b^2} B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{560 (c^2 + b)^2 B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) + \frac{1}{30720} \left( \frac{2560 (c^2 + b)^2 B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{2304 (c^2 + b)^2 B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{1260 \sqrt{c^2 + b^2} B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{2016 (c^2 + b)^2 B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{315 B^2 \log(2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c})}{c^5} - \frac{630 \sqrt{c^2 + b^2} B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) - \frac{1680 (c^2 + b)^2 B^2}{c^5} \log\left(\frac{2 c x^2 + b + 2 \sqrt{c^2 + b^2} \sqrt{c}}{c}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7680*(768*(c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^3 - 672*(c*x^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c*x^4 + b*x^2)^(3/2)*b^2/c^3)*A + 1/30720*(2560*(c*x^4 + b*x^2)^(3/2)*x^6/c - 2304*(c*x^4 + b*x^2)^(3/2)*b*x^4/c^2 + 1260*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^4 + 2016*(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^3 - 315*b^6*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(11/2) + 630*sqrt(c*x^4 + b*x^2)*b^5/c^5 - 1680*(c*x^4 + b*x^2)^(3/2)*b^3/c^4)*B
```

**mupad [B]** time = 1.47, size = 289, normalized size = 1.33

$$\frac{Ax^4(cx^4 + bx^2)^{3/2}}{10c} + \frac{Bx^6(cx^4 + bx^2)^{3/2}}{12c} - \frac{3Bb \left( \frac{5b \left( \frac{b^3 \ln(b+2cx^2+2\sqrt{c}\sqrt{cx^4+bx^2}) + \sqrt{cx^4+bx^2}(-3b^2+2b+c^2+8c^2x^4)}{16c^2} \right) + \sqrt{cx^4+bx^2}(-3b^2+2b+c^2+8c^2x^4)}{24c^2} \right) \cdot \frac{x^4(cx^4+bx^2)^{3/2}}{4c}}{8c} + \frac{x^4(cx^4+bx^2)^{3/2}}{5c} \right) + \frac{7Ab \left( \frac{5b \left( \frac{b^3 \ln(b+2cx^2+2\sqrt{c}\sqrt{cx^4+bx^2}) + \sqrt{cx^4+bx^2}(-3b^2+2b+c^2+8c^2x^4)}{16c^2} \right) + \sqrt{cx^4+bx^2}(-3b^2+2b+c^2+8c^2x^4)}{24c^2} \right) \cdot \frac{x^2(cx^4+bx^2)^{3/2}}{4c}}{8c} \right)}{20c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c) - (3*B*b*((7*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2)^(1/2))))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(10*c) + (x^4*(b*x^2 + c*x^4)^(3/2))/(5*c))/(8*c) + (7*A*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2)^(1/2))))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(20*c)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sqrt{x^2 (b + cx^2)} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

### 3.90 $\int x^5 (A + Bx^2) \sqrt{bx^2 + cx^4} dx$

**Optimal.** Leaf size=181

$$\frac{b^4(7bB - 10Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}} - \frac{b^2(b + 2cx^2) \sqrt{bx^2 + cx^4} (7bB - 10Ac)}{256c^4} + \frac{b(bx^2 + cx^4)^{3/2} (7bB - 10Ac)}{96c^3} - \frac{Bx^4(bx^2 + cx^4)^{3/2}}{10c}$$

**Rubi [A]** time = 0.33, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$ , Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$\frac{b^2(b + 2cx^2) \sqrt{bx^2 + cx^4} (7bB - 10Ac)}{256c^4} + \frac{b^4(7bB - 10Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}} + \frac{b(bx^2 + cx^4)^{3/2} (7bB - 10Ac)}{96c^3} - \frac{x^2(bx^2 + cx^4)^{3/2} (7bB - 10Ac)}{80c^2} + \frac{Bx^4(bx^2 + cx^4)^{3/2}}{10c}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] -(b^2\*(7\*b\*B - 10\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(256\*c^4) + (b\*(7\*b\*B - 10\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(96\*c^3) - ((7\*b\*B - 10\*A\*c)\*x^2\*(b\*x^2 + c\*x^4)^(3/2))/(80\*c^2) + (B\*x^4\*(b\*x^2 + c\*x^4)^(3/2))/(10\*c) + (b^4\*(7\*b\*B - 10\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(256\*c^(9/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 794

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)

```

)))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])

```

Rule 2034

```

Int[(x_)^(m_)*((b_)*(x_)^(k_) + (a_)*(x_)^(j_))^(p_)*((c_) + (d_)*(x_)
^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]

```

Rubi steps

$$\begin{aligned}
 \int x^5 (A + Bx^2) \sqrt{bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left( \int x^2 (A + Bx) \sqrt{bx + cx^2} dx, x, x^2 \right) \\
 &= \frac{Bx^4 (bx^2 + cx^4)^{3/2}}{10c} + \frac{\left( 2(-bB + Ac) + \frac{3}{2}(-bB + 2Ac) \right) \text{Subst} \left( \int x^2 \sqrt{bx + cx^2} dx \right)}{10c} \\
 &= -\frac{(7bB - 10Ac)x^2 (bx^2 + cx^4)^{3/2}}{80c^2} + \frac{Bx^4 (bx^2 + cx^4)^{3/2}}{10c} + \frac{(b(7bB - 10Ac)) \text{Subst} \left( \int x^2 \sqrt{bx + cx^2} dx \right)}{10c} \\
 &= \frac{b(7bB - 10Ac) (bx^2 + cx^4)^{3/2}}{96c^3} - \frac{(7bB - 10Ac)x^2 (bx^2 + cx^4)^{3/2}}{80c^2} + \frac{Bx^4 (bx^2 + cx^4)^{3/2}}{10c} \\
 &= -\frac{b^2(7bB - 10Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{256c^4} + \frac{b(7bB - 10Ac) (bx^2 + cx^4)^{3/2}}{96c^3} - \frac{(7bB - 10Ac)x^2 (bx^2 + cx^4)^{3/2}}{80c^2} \\
 &= -\frac{b^2(7bB - 10Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{256c^4} + \frac{b(7bB - 10Ac) (bx^2 + cx^4)^{3/2}}{96c^3} - \frac{(7bB - 10Ac)x^2 (bx^2 + cx^4)^{3/2}}{80c^2} \\
 &= -\frac{b^2(7bB - 10Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{256c^4} + \frac{b(7bB - 10Ac) (bx^2 + cx^4)^{3/2}}{96c^3} - \frac{(7bB - 10Ac)x^2 (bx^2 + cx^4)^{3/2}}{80c^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.30, size = 173, normalized size = 0.96

$$\frac{\sqrt{x^2(b + cx^2)} \left( 15b^{7/2}(7bB - 10Ac) \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b}} \right) + \sqrt{cx} \sqrt{\frac{cx^2}{b} + 1} (10b^3c(15A + 7Bx^2) - 4b^2c^2x^2(25A + 14Bx^2) + 16bc^3x^4(5A + 3Bx^2) + 96c^4x^6(5A + 4Bx^2) - 105b^4B) \right)}{3840c^{9/2}x\sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

```

[In] Integrate[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4], x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-105*b^4*B + 16*b*c^
3*x^4*(5*A + 3*B*x^2) + 96*c^4*x^6*(5*A + 4*B*x^2) + 10*b^3*c*(15*A + 7*B*x
^2) - 4*b^2*c^2*x^2*(25*A + 14*B*x^2)) + 15*b^(7/2)*(7*b*B - 10*A*c)*ArcSin
h[(Sqrt[c]*x)/Sqrt[b]])/(3840*c^(9/2)*x*Sqrt[1 + (c*x^2)/b])

```

**IntegrateAlgebraic [A]** time = 0.65, size = 163, normalized size = 0.90

$$\frac{(10Ab^4c - 7b^5B) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{512c^{9/2}} + \frac{\sqrt{bx^2 + cx^4} (150Ab^3c - 100Ab^2c^2x^2 + 80Abc^3x^4 + 480Ac^4x^6 - 105b^4B + 70b^3Bcx^2 - 56b^2Bc^2x^4 + 48bBc^3x^6 + 384Bc^4x^8)}{3840c^4}$$

Antiderivative was successfully verified.

```

[In] IntegrateAlgebraic[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4], x]

```

[Out]  $(\sqrt{bx^2 + cx^4}) * (-105b^4B + 150Ab^3c + 70b^3B^2cx^2 - 100Ab^2c^2x^2 - 56b^2B^2c^2x^4 + 80Ab^2c^3x^4 + 48b^2B^2c^3x^6 + 480Ab^2c^4x^6 + 384B^2c^4x^8) / (3840c^4) + ((-7b^5B + 10Ab^4c) * \text{Log}[b + 2cx^2 - 2\sqrt{c} * \sqrt{bx^2 + cx^4}]) / (512c^{(9/2)})$

**fricas** [A] time = 0.49, size = 321, normalized size = 1.77

$$\frac{15(7Bb^5 - 10Ab^4c)\sqrt{c} \log(-2cx^2 - b + 2\sqrt{cx^4 + bx^2}\sqrt{c}) - 2(384B^2c^4 + 48(Bb^4 + 10Ac^2)c^2 - 105Bb^4c + 150Ab^3c^2 - 8(7Bb^3c^2 - 10Ab^2c^3)c)\sqrt{cx^4 + bx^2} - 15(7Bb^5 - 10Ab^4c)\sqrt{c} \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (384B^2c^4 + 48(Bb^4 + 10Ac^2)c^2 - 105Bb^4c + 150Ab^3c^2 - 8(7Bb^3c^2 - 10Ab^2c^3)c)\sqrt{cx^4 + bx^2}}{7680c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out]  $[-1/7680 * (15 * (7 * B * b^5 - 10 * A * b^4 * c) * \text{sqrt}(c) * \log(-2 * c * x^2 - b + 2 * \text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(c)) - 2 * (384 * B * c^4 * x^8 + 48 * (B * b * c^4 + 10 * A * c^5) * x^6 - 105 * B * b^4 * c + 150 * A * b^3 * c^2 - 8 * (7 * B * b^3 * c^2 * c^3 - 10 * A * b * c^4) * x^4 + 10 * (7 * B * b^3 * c^2 - 10 * A * b^2 * c^3) * x^2) * \text{sqrt}(c * x^4 + b * x^2)) / c^5, -1/3840 * (15 * (7 * B * b^5 - 10 * A * b^4 * c) * \text{sqrt}(-c) * \arctan(\text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(-c) / (c * x^2 + b)) - (384 * B * c^4 * x^8 + 48 * (B * b * c^4 + 10 * A * c^5) * x^6 - 105 * B * b^4 * c + 150 * A * b^3 * c^2 - 8 * (7 * B * b^3 * c^2 * c^3 - 10 * A * b * c^4) * x^4 + 10 * (7 * B * b^3 * c^2 - 10 * A * b^2 * c^3) * x^2) * \text{sqrt}(c * x^4 + b * x^2)) / c^5]$

**giac** [A] time = 0.20, size = 211, normalized size = 1.17

$$\frac{1}{3840} \left( 2 \left( 6 \left( 8 B b^3 \text{sgn}(x) + \frac{B b c^7 \text{sgn}(x) + 10 A c^8 \text{sgn}(x)}{c^8} \right) x^2 - \frac{7 B b^2 c^6 \text{sgn}(x) - 10 A b c^7 \text{sgn}(x)}{c^8} \right) x^2 + \frac{5(7 B b^3 c^5 \text{sgn}(x) - 10 A b^2 c^6 \text{sgn}(x))}{c^8} \right) x^2 - \frac{15(7 B b^4 c^5 \text{sgn}(x) - 10 A b^3 c^6 \text{sgn}(x))}{c^8} \sqrt{cx^2 + b} - \frac{(7 B b^5 \text{sgn}(x) - 10 A b^4 c \text{sgn}(x)) \log\left(\frac{-\sqrt{cx^2 + b}}{256c^{\frac{1}{2}}}\right) + (7 B b^5 \log(|b|) - 10 A b^4 c \log(|b|)) \text{sgn}(x)}{512c^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $1/3840 * (2 * (4 * (6 * (8 * B * x^2 * \text{sgn}(x) + (B * b * c^7 * \text{sgn}(x) + 10 * A * c^8 * \text{sgn}(x)) / c^8) * x^2 - (7 * B * b^2 * c^6 * \text{sgn}(x) - 10 * A * b * c^7 * \text{sgn}(x)) / c^8) * x^2 + 5 * (7 * B * b^3 * c^5 * \text{sgn}(x) - 10 * A * b^2 * c^6 * \text{sgn}(x)) / c^8) * x^2 - 15 * (7 * B * b^4 * c^5 * \text{sgn}(x) - 10 * A * b^3 * c^6 * \text{sgn}(x)) / c^8) * \text{sqrt}(c * x^2 + b) * x - 1/256 * (7 * B * b^5 * \text{sgn}(x) - 10 * A * b^4 * c * \text{sgn}(x)) * \log(\text{abs}(-\text{sqrt}(c) * x + \text{sqrt}(c * x^2 + b))) / c^{(9/2)} + 1/512 * (7 * B * b^5 * \log(\text{abs}(b)) - 10 * A * b^4 * c * \log(\text{abs}(b))) * \text{sgn}(x) / c^{(9/2)})$

**maple** [A] time = 0.06, size = 248, normalized size = 1.37

$$\frac{\sqrt{cx^2 + b} \left( 384 (cx^2 + b)^{\frac{3}{2}} B c^{\frac{7}{2}} x^7 + 480 (cx^2 + b)^{\frac{3}{2}} A c^{\frac{7}{2}} x^5 - 336 (cx^2 + b)^{\frac{3}{2}} B b c^{\frac{7}{2}} x^5 - 400 (cx^2 + b)^{\frac{3}{2}} A b c^{\frac{7}{2}} x^3 + 280 (cx^2 + b)^{\frac{3}{2}} B b^2 c^{\frac{7}{2}} x^3 - 150 A b^3 c^{\frac{7}{2}} \ln(\sqrt{cx^2 + b}) + 105 B b^3 \ln(\sqrt{cx^2 + b}) - 150 \sqrt{cx^2 + b} A b^3 c^{\frac{7}{2}} x + 105 \sqrt{cx^2 + b} B b^4 c^{\frac{7}{2}} x + 300 (cx^2 + b)^{\frac{3}{2}} A b^2 c^{\frac{7}{2}} x - 210 (cx^2 + b)^{\frac{3}{2}} B b^2 c^{\frac{7}{2}} x \right)}{3840 \sqrt{cx^2 + b} c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $1/3840 * (c * x^4 + b * x^2)^{(1/2)} * (384 * B * (c * x^2 + b)^{(3/2)} * c^{(7/2)} * x^7 + 480 * A * (c * x^2 + b)^{(3/2)} * c^{(7/2)} * x^5 - 336 * B * (c * x^2 + b)^{(3/2)} * c^{(5/2)} * x^5 * b - 400 * A * (c * x^2 + b)^{(3/2)} * c^{(5/2)} * x^3 * b + 280 * B * (c * x^2 + b)^{(3/2)} * c^{(3/2)} * x^3 * b^2 + 300 * A * (c * x^2 + b)^{(3/2)} * c^{(3/2)} * x * b^2 - 210 * B * (c * x^2 + b)^{(3/2)} * c^{(1/2)} * x * b^3 - 150 * A * (c * x^2 + b)^{(1/2)} * c^{(3/2)} * x * b^3 + 105 * B * (c * x^2 + b)^{(1/2)} * c^{(1/2)} * x * b^4 - 150 * A * \ln(c^{(1/2)} * x + (c * x^2 + b)^{(1/2)}) * b^4 * c + 105 * B * \ln(c^{(1/2)} * x + (c * x^2 + b)^{(1/2)}) * b^5) / x / (c * x^2 + b)^{(1/2)} / c^{(9/2)}$

**maxima** [A] time = 1.51, size = 273, normalized size = 1.51

$$\frac{1}{768} \left( \frac{60 \sqrt{cx^2 + bx^2} b^2 x^2}{c^2} + \frac{96 (cx^4 + bx^2)^{\frac{3}{2}} x^2}{c} - \frac{15 b^4 \log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{1}{2}}} + \frac{30 \sqrt{cx^4 + bx^2} b^3}{c^3} - \frac{80 (cx^4 + bx^2)^{\frac{3}{2}} b}{c^2} \right) A + \frac{1}{7680} \left( \frac{768 (cx^4 + bx^2)^{\frac{3}{2}} x^4}{c} - \frac{420 \sqrt{cx^4 + bx^2} b^3 x^2}{c^3} - \frac{672 (cx^4 + bx^2)^{\frac{3}{2}} b x^2}{c^2} + \frac{105 b^5 \log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{1}{2}}} - \frac{210 \sqrt{cx^4 + bx^2} b^4}{c^4} + \frac{560 (cx^4 + bx^2)^{\frac{3}{2}} b^2}{c^3} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $1/768 * (60 * \text{sqrt}(c * x^4 + b * x^2) * b^2 * x^2 / c^2 + 96 * (c * x^4 + b * x^2)^{(3/2)} * x^2 / c - 15 * b^4 * \log(2 * c * x^2 + b + 2 * \text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(c)) / c^{(7/2)} + 30 * \text{sqrt}$

$(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^{(3/2)}*b/c^2)*A + 1/7680*(768*(c*x^4 + b*x^2)^{(3/2)}*x^4/c - 420*\sqrt{c*x^4 + b*x^2}*b^3*x^2/c^3 - 672*(c*x^4 + b*x^2)^{(3/2)}*b*x^2/c^2 + 105*b^5*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2})*\sqrt{c})/c^{(9/2)} - 210*\sqrt{c*x^4 + b*x^2}*b^4/c^4 + 560*(c*x^4 + b*x^2)^{(3/2)}*b^2/c^3)*B$

**mupad [B]** time = 0.89, size = 233, normalized size = 1.29

$$\frac{Ax^2(cx^4+bx^2)^{3/2}}{8c} - \frac{5Ab\left(\frac{b^3\ln(b+2cx^2+2\sqrt{c}\sqrt{cx^2+b})}{16c^{5/2}} + \frac{\sqrt{cx^4+bx^2}(-3b^2+2bcx^2+8c^2x^4)}{24c^2}\right)}{16c} + \frac{Bx^4(cx^4+bx^2)^{3/2}}{10c} + \frac{7Bb\left(\frac{5b\left(\frac{b^3\ln(b+2cx^2+2\sqrt{c}\sqrt{cx^2+b})}{16c^{5/2}} + \frac{\sqrt{cx^4+bx^2}(-3b^2+2bcx^2+8c^2x^4)}{24c^2}\right)}{8c} - \frac{x^2(cx^4+bx^2)^{3/2}}{4c}\right)}{20c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2), x)

[Out]  $(A*x^2*(b*x^2 + c*x^4)^{(3/2)})/(8*c) - (5*A*b*((b^3*\log(b + 2*c*x^2 + 2*c^{(1/2)}*abs(x)*(b + c*x^2)^{(1/2)})))/(16*c^{(5/2)}) + ((b*x^2 + c*x^4)^{(1/2)}*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (B*x^4*(b*x^2 + c*x^4)^{(3/2)})/(10*c) + (7*B*b*((5*b*((b^3*\log(b + 2*c*x^2 + 2*c^{(1/2)}*abs(x)*(b + c*x^2)^{(1/2)})))/(16*c^{(5/2)}) + ((b*x^2 + c*x^4)^{(1/2)}*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^{(3/2)})/(4*c)))/(20*c)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sqrt{x^2 (b + cx^2)} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2), x)

[Out] Integral(x\*\*5\*sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2), x)

### 3.91 $\int x^3 (A + Bx^2) \sqrt{bx^2 + cx^4} dx$

**Optimal.** Leaf size=125

$$\frac{b^3(5bB - 8Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}} + \frac{b(b + 2cx^2) \sqrt{bx^2 + cx^4} (5bB - 8Ac)}{128c^3} - \frac{(bx^2 + cx^4)^{3/2} (-8Ac + 5bB - 6Bcx^2)}{48c^2}$$

**Rubi [A]** time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 779, 612, 620, 206}

$$\frac{b^3(5bB - 8Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}} + \frac{b(b + 2cx^2) \sqrt{bx^2 + cx^4} (5bB - 8Ac)}{128c^3} - \frac{(bx^2 + cx^4)^{3/2} (-8Ac + 5bB - 6Bcx^2)}{48c^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] (b\*(5\*b\*B - 8\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(128\*c^3) - ((5\*b\*B - 8\*A\*c - 6\*B\*c\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/(48\*c^2) - (b^3\*(5\*b\*B - 8\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(128\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int x^3 (A + Bx^2) \sqrt{bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left( \int x(A + Bx) \sqrt{bx + cx^2} dx, x, x^2 \right) \\
&= -\frac{(5bB - 8Ac - 6Bcx^2)(bx^2 + cx^4)^{3/2}}{48c^2} + \frac{(b(5bB - 8Ac)) \text{Subst} \left( \int \sqrt{bx + cx^2} dx, x, x^2 \right)}{32c^2} \\
&= \frac{b(5bB - 8Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{128c^3} - \frac{(5bB - 8Ac - 6Bcx^2)(bx^2 + cx^4)^{3/2}}{48c^2} \\
&= \frac{b(5bB - 8Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{128c^3} - \frac{(5bB - 8Ac - 6Bcx^2)(bx^2 + cx^4)^{3/2}}{48c^2} \\
&= \frac{b(5bB - 8Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{128c^3} - \frac{(5bB - 8Ac - 6Bcx^2)(bx^2 + cx^4)^{3/2}}{48c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 151, normalized size = 1.21

$$\frac{\sqrt{x^2(b + cx^2)} \left( \sqrt{cx} \sqrt{\frac{cx^2}{b} + 1} (-2b^2c(12A + 5Bx^2) + 8bc^2x^2(2A + Bx^2) + 16c^3x^4(4A + 3Bx^2) + 15b^3B) - 3b^{5/2}(5bB - 8Ac) \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b}} \right) \right)}{384c^{7/2}x\sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(15\*b^3\*B + 8\*b\*c^2\*x^2\*(2\*A + B\*x^2) + 16\*c^3\*x^4\*(4\*A + 3\*B\*x^2) - 2\*b^2\*c\*(12\*A + 5\*B\*x^2)) - 3\*b^(5/2)\*(5\*b\*B - 8\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(384\*c^(7/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.52, size = 139, normalized size = 1.11

$$\frac{(5b^4B - 8Ab^3c) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{256c^{7/2}} + \frac{\sqrt{bx^2 + cx^4} (-24Ab^2c + 16Abc^2x^2 + 64Ac^3x^4 + 15b^3B - 10b^2Bcx^2 + 8bBc^2x^4 + 48Bc^3x^6)}{384c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(15\*b^3\*B - 24\*A\*b^2\*c - 10\*b^2\*B\*c\*x^2 + 16\*A\*b\*c^2\*x^2 + 8\*b\*B\*c^2\*x^4 + 64\*A\*c^3\*x^4 + 48\*B\*c^3\*x^6))/(384\*c^3) + ((5\*b^4\*B - 8\*A\*b^3\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(256\*c^(7/2))

**fricas [A]** time = 0.46, size = 272, normalized size = 2.18

$$\frac{3(5Bb^4 - 8Ab^3c)\sqrt{c} \log \left( -2cx^2 - b - 2\sqrt{cx^2 + bx^2} \sqrt{c} \right) - 2(48Bc^4x^6 + 15Bb^3c - 24Ab^2c^2 + 8(Bb^3c^3 - 8Ab^2c^2)x^2) \sqrt{cx^2 + bx^2} - 3(5Bb^4 - 8Ab^3c)\sqrt{-c} \arctan \left( \frac{\sqrt{cx^2 + bx^2} \sqrt{c}}{cx^2 + b} \right) + (48Bc^4x^6 + 15Bb^3c - 24Ab^2c^2 + 8(Bb^3c^3 - 8Ab^2c^2)x^2) \sqrt{cx^2 + bx^2}}{768c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/768\*(3\*(5\*B\*b^4 - 8\*A\*b^3\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(48\*B\*c^4\*x^6 + 15\*B\*b^3\*c - 24\*A\*b^2\*c^2 + 8\*(B\*b\*c^3 + 8\*A\*c^4)\*x^4 - 2\*(5\*B\*b^2\*c^2 - 8\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^4, 1/384\*(3\*(5\*B\*b^4 - 8\*A\*b^3\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + (48\*B\*c^4\*x^6 + 15\*B\*b^3\*c - 24\*A\*b^2\*c^2 + 8\*(B\*b\*c^3 + 8\*A\*c^4)\*x^4 - 2\*(5\*B\*b^2\*c^2 - 8\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^4]

**giac [A]** time = 0.22, size = 177, normalized size = 1.42

$$\frac{1}{384} \left( 2 \left( 6Bx^2 \operatorname{sgn}(x) + \frac{Bbc^5 \operatorname{sgn}(x) + 8Ac^6 \operatorname{sgn}(x)}{c^6} \right) x^2 - \frac{5Bb^2c^4 \operatorname{sgn}(x) - 8Abc^5 \operatorname{sgn}(x)}{c^6} x + \frac{3(5Bb^3c^3 \operatorname{sgn}(x) - 8Ab^2c^4 \operatorname{sgn}(x))}{c^6} \right) \sqrt{cx^2 + bx} + \frac{(5Bb^4 \operatorname{sgn}(x) - 8Ab^3c \operatorname{sgn}(x)) \log \left( \left| -\sqrt{cx^2 + bx} \right| \right)}{128c^2} - \frac{(5Bb^4 \log(|b|) - 8Ab^3c \log(|b|)) \operatorname{sgn}(x)}{256c^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{384} * (2 * (4 * (6 * B * x^2 * \text{sgn}(x) + (B * b * c^5 * \text{sgn}(x) + 8 * A * c^6 * \text{sgn}(x))) / c^6) * x^2 - (5 * B * b^2 * c^4 * \text{sgn}(x) - 8 * A * b * c^5 * \text{sgn}(x)) / c^6) * x^2 + 3 * (5 * B * b^3 * c^3 * \text{sgn}(x) - 8 * A * b^2 * c^4 * \text{sgn}(x)) / c^6) * \text{sqrt}(c * x^2 + b) * x + 1 / 128 * (5 * B * b^4 * \text{sgn}(x) - 8 * A * b^3 * c * \text{sgn}(x)) * \log(\text{abs}(-\text{sqrt}(c) * x + \text{sqrt}(c * x^2 + b))) / c^{7/2} - 1 / 256 * (5 * B * b^4 * \log(\text{abs}(b)) - 8 * A * b^3 * c * \log(\text{abs}(b))) * \text{sgn}(x) / c^{7/2}$

**maple [A]** time = 0.05, size = 206, normalized size = 1.65

$$\frac{\sqrt{cx^4+bx^2} \left( 48(cx^2+b)^{\frac{5}{2}} Bc^{\frac{5}{2}}x^5 + 64(cx^2+b)^{\frac{5}{2}} Ac^{\frac{5}{2}}x^3 - 40(cx^2+b)^{\frac{5}{2}} Bb^{\frac{3}{2}}x^3 + 24Ab^3c \ln(\sqrt{c}x + \sqrt{cx^2+b}) - 15Bb^4 \ln(\sqrt{c}x + \sqrt{cx^2+b}) + 24\sqrt{cx^2+b} Ab^2c^{\frac{3}{2}}x - 15\sqrt{cx^2+b} Bb^3\sqrt{c}x - 48(cx^2+b)^{\frac{3}{2}} Ab^{\frac{3}{2}}x + 30(cx^2+b)^{\frac{3}{2}} Bb^2\sqrt{c}x \right)}{384\sqrt{cx^2+b}c^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $\frac{1}{384} * (c * x^4 + b * x^2)^{1/2} * (48 * B * (c * x^2 + b)^{3/2} * c^{5/2} * x^5 + 64 * A * (c * x^2 + b)^{3/2} * c^{5/2} * x^3 - 40 * B * (c * x^2 + b)^{3/2} * c^{3/2} * x^3 * b - 48 * A * (c * x^2 + b)^{3/2} * c^{3/2} * x * b + 30 * B * (c * x^2 + b)^{3/2} * c^{1/2} * x * b^2 + 24 * A * (c * x^2 + b)^{1/2} * c^{3/2} * x * b^2 - 15 * B * (c * x^2 + b)^{1/2} * c^{1/2} * x * b^3 + 24 * A * \ln(c^{1/2} * x + (c * x^2 + b)^{1/2}) * b^3 * c - 15 * B * \ln(c^{1/2} * x + (c * x^2 + b)^{1/2}) * b^4) / x / (c * x^2 + b)^{1/2} / c^{7/2}$

**maxima [B]** time = 1.49, size = 225, normalized size = 1.80

$$\frac{1}{96} \left( \frac{12\sqrt{cx^4+bx^2}bx^2}{c} - \frac{3b^3 \log(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c})}{c^{\frac{5}{2}}} + \frac{6\sqrt{cx^4+bx^2}b^2}{c^2} - \frac{16(cx^4+bx^2)^{\frac{3}{2}}}{c} \right) A + \frac{1}{768} \left( \frac{60\sqrt{cx^4+bx^2}b^2x^2}{c^2} + \frac{96(cx^4+bx^2)^{\frac{3}{2}}x^2}{c} - \frac{15b^4 \log(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c})}{c^{\frac{7}{2}}} + \frac{30\sqrt{cx^4+bx^2}b^3}{c^3} - \frac{80(cx^4+bx^2)^{\frac{3}{2}}b}{c^2} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $-1/96 * (12 * \text{sqrt}(c * x^4 + b * x^2) * b * x^2 / c - 3 * b^3 * \log(2 * c * x^2 + b + 2 * \text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(c))) / c^{5/2} + 6 * \text{sqrt}(c * x^4 + b * x^2) * b^2 / c^2 - 16 * (c * x^4 + b * x^2)^{3/2} / c) * A + 1/768 * (60 * \text{sqrt}(c * x^4 + b * x^2) * b^2 * x^2 / c^2 + 96 * (c * x^4 + b * x^2)^{3/2} * x^2 / c - 15 * b^4 * \log(2 * c * x^2 + b + 2 * \text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(c))) / c^{7/2} + 30 * \text{sqrt}(c * x^4 + b * x^2) * b^3 / c^3 - 80 * (c * x^4 + b * x^2)^{3/2} * b / c^2) * B$

**mupad [B]** time = 0.74, size = 177, normalized size = 1.42

$$\frac{Bx^2(cx^4+bx^2)^{3/2}}{8c} - \frac{5Bb \left( \frac{b^3 \ln(b+2cx^2+2\sqrt{c}|x|\sqrt{cx^2+b})}{16c^{5/2}} + \frac{\sqrt{cx^4+bx^2}(-3b^2+2bcx^2+8c^2x^4)}{24c^2} \right)}{16c} + \frac{Ab^3 \ln(b+2cx^2+2\sqrt{c}|x|\sqrt{cx^2+b})}{32c^{5/2}} + \frac{A\sqrt{cx^4+bx^2}(-3b^2+2bcx^2+8c^2x^4)}{48c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2),x)

[Out]  $(B * x^2 * (b * x^2 + c * x^4)^{3/2}) / (8 * c) - (5 * B * b * ((b^3 * \log(b + 2 * c * x^2 + 2 * c^{1/2} * \text{abs}(x) * (b + c * x^2)^{1/2})) / (16 * c^{5/2}) + ((b * x^2 + c * x^4)^{1/2}) * (8 * c^2 * x^4 - 3 * b^2 + 2 * b * c * x^2)) / (24 * c^2))) / (16 * c) + (A * b^3 * \log(b + 2 * c * x^2 + 2 * c^{1/2} * \text{abs}(x) * (b + c * x^2)^{1/2})) / (32 * c^{5/2}) + (A * (b * x^2 + c * x^4)^{1/2} * (8 * c^2 * x^4 - 3 * b^2 + 2 * b * c * x^2)) / (48 * c^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{x^2 (b + cx^2)} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*3\*sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2), x)

### 3.92 $\int x (A + Bx^2) \sqrt{bx^2 + cx^4} dx$

**Optimal.** Leaf size=107

$$\frac{b^2(bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}} - \frac{(b + 2cx^2) \sqrt{bx^2 + cx^4} (bB - 2Ac)}{16c^2} + \frac{B (bx^2 + cx^4)^{3/2}}{6c}$$

**Rubi [A]** time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {2034, 640, 612, 620, 206}

$$\frac{b^2(bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}} - \frac{(b + 2cx^2) \sqrt{bx^2 + cx^4} (bB - 2Ac)}{16c^2} + \frac{B (bx^2 + cx^4)^{3/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[x\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] -((b\*B - 2\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(16\*c^2) + (B\*(b\*x^2 + c\*x^4)^(3/2))/(6\*c) + (b^2\*(b\*B - 2\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(16\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int x(A + Bx^2)\sqrt{bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst}\left(\int (A + Bx)\sqrt{bx + cx^2} dx, x, x^2\right) \\
&= \frac{B(bx^2 + cx^4)^{3/2}}{6c} + \frac{(-bB + 2Ac) \text{Subst}\left(\int \sqrt{bx + cx^2} dx, x, x^2\right)}{4c} \\
&= -\frac{(bB - 2Ac)(b + 2cx^2)\sqrt{bx^2 + cx^4}}{16c^2} + \frac{B(bx^2 + cx^4)^{3/2}}{6c} + \frac{(b^2(bB - 2Ac)) \text{Su}}{16c^2} \\
&= -\frac{(bB - 2Ac)(b + 2cx^2)\sqrt{bx^2 + cx^4}}{16c^2} + \frac{B(bx^2 + cx^4)^{3/2}}{6c} + \frac{(b^2(bB - 2Ac)) \text{Su}}{16c^2} \\
&= -\frac{(bB - 2Ac)(b + 2cx^2)\sqrt{bx^2 + cx^4}}{16c^2} + \frac{B(bx^2 + cx^4)^{3/2}}{6c} + \frac{b^2(bB - 2Ac) \tanh}{16c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 129, normalized size = 1.21

$$\frac{\sqrt{x^2(b + cx^2)} \left( 3b^{3/2}(bB - 2Ac) \sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right) + \sqrt{c}x\sqrt{\frac{cx^2}{b} + 1} (2bc(3A + Bx^2) + 4c^2x^2(3A + 2Bx^2) - 3b^2B) \right)}{48c^{5/2}x\sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(-3\*b^2\*B + 2\*b\*c\*(3\*A + B\*x^2) + 4\*c^2\*x^2\*(3\*A + 2\*B\*x^2)) + 3\*b^(3/2)\*(b\*B - 2\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(48\*c^(5/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.59, size = 115, normalized size = 1.07

$$\frac{\sqrt{bx^2 + cx^4} (6Abc + 12Ac^2x^2 - 3b^2B + 2bBcx^2 + 8Bc^2x^4)}{48c^2} + \frac{(2Ab^2c - b^3B) \log\left(-2\sqrt{c}\sqrt{bx^2 + cx^4} + b + 2cx^2\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-3\*b^2\*B + 6\*A\*b\*c + 2\*b\*B\*c\*x^2 + 12\*A\*c^2\*x^2 + 8\*B\*c^2\*x^4))/(48\*c^2) + ((-(b^3\*B) + 2\*A\*b^2\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(32\*c^(5/2))

**fricas [A]** time = 0.43, size = 223, normalized size = 2.08

$$\left[ \frac{3(Bb^3 - 2Ab^2c)\sqrt{c} \log\left(-2cx^2 - b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right) - 2(8Bc^3x^4 - 3Bb^2c + 6Abc^2 + 2(Bbc^2 + 6Ac^3)x^2)\sqrt{cx^4 + bx^2}}{96c^3}, \frac{3(Bb^3 - 2Ab^2c)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-c}}{cx^2 + b}\right) - (8Bc^3x^4 - 3Bb^2c + 6Abc^2 + 2(Bbc^2 + 6Ac^3)x^2)\sqrt{cx^4 + bx^2}}{48c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/96\*(3\*(B\*b^3 - 2\*A\*b^2\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(8\*B\*c^3\*x^4 - 3\*B\*b^2\*c + 6\*A\*b\*c^2 + 2\*(B\*b\*c^2 + 6\*A\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^3, -1/48\*(3\*(B\*b^3 - 2\*A\*b^2\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) - (8\*B\*c^3\*x^4 - 3\*B\*b^2\*c + 6\*A\*b\*c^2 + 2\*(B\*b\*c^2 + 6\*A\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^3]

**giac [A]** time = 0.19, size = 140, normalized size = 1.31

$$\frac{1}{48} \left( 2 \left( 4Bx^2 \operatorname{sgn}(x) + \frac{Bbc^3 \operatorname{sgn}(x) + 6Ac^4 \operatorname{sgn}(x)}{c^4} \right) x^2 - \frac{3(Bb^2c^2 \operatorname{sgn}(x) - 2Abc^3 \operatorname{sgn}(x))}{c^4} \right) \sqrt{cx^2 + bx} - \frac{(Bb^3 \operatorname{sgn}(x) - 2Ab^2c \operatorname{sgn}(x)) \log\left(-\sqrt{c}x + \sqrt{cx^2 + b}\right)}{16c^{5/2}} + \frac{(Bb^3 \log(|b|) - 2Ab^2c \log(|b|)) \operatorname{sgn}(x)}{32c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{48} * (2 * (4 * B * x^2 * \text{sgn}(x) + (B * b * c^3 * \text{sgn}(x) + 6 * A * c^4 * \text{sgn}(x))) / c^4) * x^2 - 3 * (B * b^2 * c^2 * \text{sgn}(x) - 2 * A * b * c^3 * \text{sgn}(x)) / c^4 * \text{sqrt}(c * x^2 + b) * x - 1 / 16 * (B * b^3 * \text{sgn}(x) - 2 * A * b^2 * c * \text{sgn}(x)) * \log(\text{abs}(-\text{sqrt}(c) * x + \text{sqrt}(c * x^2 + b))) / c^{5/2} + 1 / 32 * (B * b^3 * \log(\text{abs}(b)) - 2 * A * b^2 * c * \log(\text{abs}(b))) * \text{sgn}(x) / c^{5/2}$

**maple [A]** time = 0.05, size = 164, normalized size = 1.53

$$\frac{\sqrt{cx^4+bx^2} \left( 8(c^2x^2+b)^{\frac{3}{2}} Bc^2x^3 - 6A^2b^2c \ln(\sqrt{cx^2+b}) + 3Bb^3 \ln(\sqrt{cx^2+b}) - 6\sqrt{cx^2+b} Abc^{\frac{3}{2}}x + 3\sqrt{cx^2+b} Bb^2\sqrt{cx^2+b} + 12(c^2x^2+b)^{\frac{3}{2}} Ac^{\frac{3}{2}}x - 6(c^2x^2+b)^{\frac{3}{2}} Bb\sqrt{cx^2+b} \right)}{48\sqrt{cx^2+b} c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $\frac{1}{48} * (c * x^4 + b * x^2)^{1/2} * (8 * B * c^{3/2} * (c * x^2 + b)^{3/2} * x^3 + 12 * A * c^{3/2} * (c * x^2 + b)^{3/2} * x - 6 * B * c^{1/2} * (c * x^2 + b)^{3/2} * x * b - 6 * A * c^{3/2} * (c * x^2 + b)^{1/2} * x * b + 3 * B * c^{1/2} * (c * x^2 + b)^{1/2} * x * b^2 - 6 * A * \ln(c^{1/2} * x + (c * x^2 + b)^{1/2}) * b^2 * c + 3 * B * \ln(c^{1/2} * x + (c * x^2 + b)^{1/2}) * b^3) / x / (c * x^2 + b)^{1/2} / c^{5/2}$

**maxima [A]** time = 1.42, size = 177, normalized size = 1.65

$$\frac{1}{16} \left( 4\sqrt{cx^4+bx^2}x^2 - \frac{b^2 \log(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c})}{c^{\frac{3}{2}}} + \frac{2\sqrt{cx^4+bx^2}b}{c} \right) A - \frac{1}{96} \left( \frac{12\sqrt{cx^4+bx^2}bx^2}{c} - \frac{3b^3 \log(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c})}{c^{\frac{5}{2}}} + \frac{6\sqrt{cx^4+bx^2}b^2}{c^2} - \frac{16(cx^4+bx^2)^{\frac{3}{2}}}{c} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{16} * (4 * \text{sqrt}(c * x^4 + b * x^2) * x^2 - b^2 * \log(2 * c * x^2 + b + 2 * \text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(c))) / c^{3/2} + 2 * \text{sqrt}(c * x^4 + b * x^2) * b / c * A - 1 / 96 * (12 * \text{sqrt}(c * x^4 + b * x^2) * b * x^2 / c - 3 * b^3 * \log(2 * c * x^2 + b + 2 * \text{sqrt}(c * x^4 + b * x^2) * \text{sqrt}(c))) / c^{5/2} + 6 * \text{sqrt}(c * x^4 + b * x^2) * b^2 / c^2 - 16 * (c * x^4 + b * x^2)^{3/2} / c * B$

**mupad [B]** time = 0.75, size = 140, normalized size = 1.31

$$\frac{A \left( \frac{b}{4c} + \frac{x^2}{2} \right) \sqrt{cx^4+bx^2}}{2} + \frac{Bb^3 \ln(b+2cx^2+2\sqrt{c}|x|\sqrt{cx^2+b})}{32c^{5/2}} - \frac{Ab^2 \ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4+bx^2}\right)}{16c^{3/2}} + \frac{B\sqrt{cx^4+bx^2}(-3b^2+2bcx^2+8c^2x^4)}{48c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2),x)

[Out]  $(A * (b / (4 * c) + x^2 / 2) * (b * x^2 + c * x^4)^{1/2}) / 2 + (B * b^3 * \log(b + 2 * c * x^2 + 2 * c^{1/2} * \text{abs}(x) * (b + c * x^2)^{1/2})) / (32 * c^{5/2}) - (A * b^2 * \log((b / 2 + c * x^2) / c^{1/2} + (b * x^2 + c * x^4)^{1/2})) / (16 * c^{3/2}) + (B * (b * x^2 + c * x^4)^{1/2} * (8 * c^2 * x^4 - 3 * b^2 + 2 * b * c * x^2)) / (48 * c^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{x^2(b+cx^2)} (A+Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2), x)

$$3.93 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x} dx$$

**Optimal.** Leaf size=100

$$\frac{b(bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}} - \frac{\sqrt{bx^2 + cx^4}(bB - 4Ac)}{8c} + \frac{B(bx^2 + cx^4)^{3/2}}{4cx^2}$$

**Rubi [A]** time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 794, 664, 620, 206}

$$\frac{b(bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}} - \frac{\sqrt{bx^2 + cx^4}(bB - 4Ac)}{8c} + \frac{B(bx^2 + cx^4)^{3/2}}{4cx^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x,x]

[Out] -((b\*B - 4\*A\*c)\*Sqrt[b\*x^2 + c\*x^4]/(8\*c) + (B\*(b\*x^2 + c\*x^4)^(3/2))/(4\*c\*x^2) - (b\*(b\*B - 4\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(8\*c^(3/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] :> Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 664

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(p\*(2\*c\*d - b\*e))/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 794

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(c\*e\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[m + 2\*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^p)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +

1)/n]] && NeQ[n^2, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx) \sqrt{bx + cx^2}}{x} dx, x, x^2 \right) \\
 &= \frac{B (bx^2 + cx^4)^{3/2}}{4cx^2} + \frac{(bB - Ac + \frac{3}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x} dx, x, x^2 \right)}{4c} \\
 &= -\frac{(bB - 4Ac) \sqrt{bx^2 + cx^4}}{8c} + \frac{B (bx^2 + cx^4)^{3/2}}{4cx^2} - \frac{(b(bB - 4Ac)) \text{Subst} \left( \int \frac{1}{\sqrt{bx+cx^2}} dx, x, x^2 \right)}{16c} \\
 &= -\frac{(bB - 4Ac) \sqrt{bx^2 + cx^4}}{8c} + \frac{B (bx^2 + cx^4)^{3/2}}{4cx^2} - \frac{(b(bB - 4Ac)) \text{Subst} \left( \int \frac{1}{1-cx^2} dx, x, x^2 \right)}{8c} \\
 &= -\frac{(bB - 4Ac) \sqrt{bx^2 + cx^4}}{8c} + \frac{B (bx^2 + cx^4)^{3/2}}{4cx^2} - \frac{b(bB - 4Ac) \tanh^{-1} \left( \frac{\sqrt{c} x^2}{\sqrt{bx^2 + cx^4}} \right)}{8c^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 91, normalized size = 0.91

$$\frac{\sqrt{x^2 (b + cx^2)} \left( \sqrt{c} (4Ac + bB + 2Bcx^2) - \frac{\sqrt{b} (bB - 4Ac) \sinh^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right)}{x \sqrt{\frac{cx^2}{b} + 1}} \right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*(b\*B + 4\*A\*c + 2\*B\*c\*x^2) - (Sqrt[b]\*(b\*B - 4\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(x\*Sqrt[1 + (c\*x^2)/b]))/(8\*c^(3/2))

**IntegrateAlgebraic [A]** time = 0.50, size = 93, normalized size = 0.93

$$\frac{(b^2B - 4Abc) \log \left( -2c^{3/2} \sqrt{bx^2 + cx^4} + bc + 2c^2x^2 \right)}{16c^{3/2}} + \frac{\sqrt{bx^2 + cx^4} (4Ac + bB + 2Bcx^2)}{8c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x, x]

[Out] ((b\*B + 4\*A\*c + 2\*B\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(8\*c) + ((b^2\*B - 4\*A\*b\*c)\*Log[b\*c + 2\*c^2\*x^2 - 2\*c^(3/2)\*Sqrt[b\*x^2 + c\*x^4]])/(16\*c^(3/2))

**fricas [A]** time = 0.44, size = 172, normalized size = 1.72

$$\left[ \frac{(Bb^2 - 4Abc) \sqrt{c} \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2} \sqrt{c} \right) - 2(2Bc^2x^2 + Bbc + 4Ac^2) \sqrt{cx^4 + bx^2}}{16c^2}, \frac{(Bb^2 - 4Abc) \sqrt{-c} \arctan \left( \frac{\sqrt{cx^4 + bx^2} \sqrt{-c}}{cx^2 + b} \right) + (2Bc^2x^2 + Bbc + 4Ac^2) \sqrt{cx^4 + bx^2}}{8c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x,x, algorithm="fricas")

[Out] [-1/16\*((B\*b^2 - 4\*A\*b\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(2\*B\*c^2\*x^2 + B\*b\*c + 4\*A\*c^2)\*sqrt(c\*x^4 + b\*x^2))/c^2, 1/8\*

$$\left( (B*b^2 - 4*A*b*c)*\sqrt{-c}*\arctan(\sqrt{c*x^4 + b*x^2}*\sqrt{-c}/(c*x^2 + b)) + (2*B*c^2*x^2 + B*b*c + 4*A*c^2)*\sqrt{c*x^4 + b*x^2} \right) / c^2$$

**giac** [A] time = 0.18, size = 103, normalized size = 1.03

$$\frac{1}{8} \left( 2Bx^2 \operatorname{sgn}(x) + \frac{Bbc \operatorname{sgn}(x) + 4Ac^2 \operatorname{sgn}(x)}{c^2} \right) \sqrt{cx^2 + bx} + \frac{(Bb^2 \operatorname{sgn}(x) - 4Abc \operatorname{sgn}(x)) \log\left(\left| -\sqrt{c}x + \sqrt{cx^2 + b} \right|\right)}{8c^{\frac{3}{2}}} - \frac{(Bb^2 \log(|b|) - 4Abc \log(|b|)) \operatorname{sgn}(x)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/8\*(2\*B\*x^2\*sgn(x) + (B\*b\*c\*sgn(x) + 4\*A\*c^2\*sgn(x))/c^2)\*sqrt(c\*x^2 + b)\*x + 1/8\*(B\*b^2\*sgn(x) - 4\*A\*b\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(3/2) - 1/16\*(B\*b^2\*log(abs(b)) - 4\*A\*b\*c\*log(abs(b)))\*sgn(x)/c^(3/2)

**maple** [A] time = 0.05, size = 124, normalized size = 1.24

$$\frac{\sqrt{cx^4 + bx^2} \left( 4Abc \ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right) - Bb^2 \ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right) + 4\sqrt{cx^2 + b} Ac^{\frac{3}{2}}x - \sqrt{cx^2 + b} Bb\sqrt{c}x + 2(cx^2 + b)^{\frac{3}{2}} B\sqrt{c}x \right)}{8\sqrt{cx^2 + b} c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x,x)

[Out] 1/8\*(c\*x^4+b\*x^2)^(1/2)\*(2\*B\*c^(1/2)\*(c\*x^2+b)^(3/2)\*x+4\*A\*c^(3/2)\*(c\*x^2+b)^(1/2)\*x-B\*c^(1/2)\*(c\*x^2+b)^(1/2)\*x\*b+4\*A\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b\*c-B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^2)/c^(3/2)/(c\*x^2+b)^(1/2)/x

**maxima** [A] time = 1.45, size = 128, normalized size = 1.28

$$\frac{1}{4} \left( \frac{b \log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{\sqrt{c}} + 2\sqrt{cx^4 + bx^2} \right) A + \frac{1}{16} \left( 4\sqrt{cx^4 + bx^2}x^2 - \frac{b^2 \log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{cx^4 + bx^2}b}{c} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x,x, algorithm="maxima")

[Out] 1/4\*(b\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/sqrt(c) + 2\*sqrt(c\*x^4 + b\*x^2))\*A + 1/16\*(4\*sqrt(c\*x^4 + b\*x^2)\*x^2 - b^2\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(3/2) + 2\*sqrt(c\*x^4 + b\*x^2)\*b/c)\*B

**mapad** [B] time = 0.61, size = 117, normalized size = 1.17

$$\frac{A\sqrt{cx^4 + bx^2}}{2} + \frac{B\left(\frac{b}{4c} + \frac{x^2}{2}\right)\sqrt{cx^4 + bx^2}}{2} + \frac{Ab \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2}\right)}{4\sqrt{c}} - \frac{Bb^2 \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2}\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x,x)

[Out] (A\*(b\*x^2 + c\*x^4)^(1/2))/2 + (B\*(b/(4\*c) + x^2/2)\*(b\*x^2 + c\*x^4)^(1/2))/2 + (A\*b\*log((b/2 + c\*x^2)/c^(1/2) + (b\*x^2 + c\*x^4)^(1/2)))/(4\*c^(1/2)) - (B\*b^2\*log((b/2 + c\*x^2)/c^(1/2) + (b\*x^2 + c\*x^4)^(1/2)))/(16\*c^(3/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x, x)

$$3.94 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3} dx$$

**Optimal.** Leaf size=97

$$\frac{\sqrt{bx^2+cx^4}(2Ac+bB)}{2b} + \frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4}$$

**Rubi [A]** time = 0.21, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 792, 664, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}(2Ac+bB)}{2b} + \frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^3,x]

[Out] ((b\*B + 2\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(2\*b) - (A\*(b\*x^2 + c\*x^4)^(3/2))/(b\*x^4) + ((b\*B + 2\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(2\*Sqrt[c])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 664

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^p, x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(p\*(2\*c\*d - b\*e))/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_) + (a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^p, x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^m\*((b\_.)\*(x\_)^k + (a\_.)\*(x\_)^j)^p\*((c\_) + (d\_.)\*(x\_)^n)^q, x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In



tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^3} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx) \sqrt{bx + cx^2}}{x^2} dx, x, x^2 \right) \\
 &= -\frac{A(bx^2 + cx^4)^{3/2}}{bx^4} + \frac{(-2(-bB + Ac) + \frac{3}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x} dx, x \right)}{b} \\
 &= \frac{(bB + 2Ac) \sqrt{bx^2 + cx^4}}{2b} - \frac{A(bx^2 + cx^4)^{3/2}}{bx^4} + \frac{1}{4}(bB + 2Ac) \text{Subst} \left( \int \frac{1}{\sqrt{bx + c}} dx, x \right) \\
 &= \frac{(bB + 2Ac) \sqrt{bx^2 + cx^4}}{2b} - \frac{A(bx^2 + cx^4)^{3/2}}{bx^4} + \frac{1}{2}(bB + 2Ac) \text{Subst} \left( \int \frac{1}{1 - cx^2} dx, x \right) \\
 &= \frac{(bB + 2Ac) \sqrt{bx^2 + cx^4}}{2b} - \frac{A(bx^2 + cx^4)^{3/2}}{bx^4} + \frac{(bB + 2Ac) \tanh^{-1} \left( \frac{\sqrt{c} x^2}{\sqrt{bx^2 + cx^4}} \right)}{2\sqrt{c}}
 \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 78, normalized size = 0.80

$$\frac{\sqrt{x^2(b + cx^2)} \left( \frac{x(2Ac + bB) \sinh^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right) - 2A + Bx^2}{\sqrt{b} \sqrt{c} \sqrt{\frac{cx^2}{b} + 1}} \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^3, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-2\*A + B\*x^2 + ((b\*B + 2\*A\*c)\*x\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(Sqrt[b]\*Sqrt[c]\*Sqrt[1 + (c\*x^2)/b]))/(2\*x^2)

**IntegrateAlgebraic [A]** time = 0.39, size = 81, normalized size = 0.84

$$\frac{(Bx^2 - 2A) \sqrt{bx^2 + cx^4}}{2x^2} + \frac{(-2Ac - bB) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^3, x]

[Out] ((-2\*A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(2\*x^2) + (((-b\*B) - 2\*A\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(4\*Sqrt[c])

**fricas [A]** time = 0.42, size = 161, normalized size = 1.66

$$\left[ \frac{(Bb + 2Ac)\sqrt{c}x^2 \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c} \right) + 2\sqrt{cx^4 + bx^2}(Bcx^2 - 2Ac)}{4cx^2}, -\frac{(Bb + 2Ac)\sqrt{-c}x^2 \arctan \left( \frac{\sqrt{cx^4 + bx^2}\sqrt{-c}}{cx^2 + b} \right) - \sqrt{cx^4 + bx^2}(Bcx^2 - 2Ac)}{2cx^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^3,x, algorithm="fricas")

[Out]  $\left[ \frac{1}{4} * ((B * b + 2 * A * c) * \sqrt{c}) * x^2 * \log(-2 * c * x^2 - b - 2 * \sqrt{c * x^4 + b * x^2}) * \sqrt{c} + 2 * \sqrt{c * x^4 + b * x^2} * (B * c * x^2 - 2 * A * c) / (c * x^2), -1/2 * ((B * b + 2 * A * c) * \sqrt{-c}) * x^2 * \arctan(\sqrt{c * x^4 + b * x^2} * \sqrt{-c} / (c * x^2 + b)) - \sqrt{c * x^4 + b * x^2} * (B * c * x^2 - 2 * A * c) / (c * x^2) \right]$

**giac** [A] time = 0.27, size = 92, normalized size = 0.95

$$\frac{1}{2} \sqrt{cx^2 + b} B x \operatorname{sgn}(x) + \frac{2 A b \sqrt{c} \operatorname{sgn}(x)}{(\sqrt{c} x - \sqrt{cx^2 + b})^2 - b} - \frac{(B b \sqrt{c} \operatorname{sgn}(x) + 2 A c^{\frac{3}{2}} \operatorname{sgn}(x)) \log\left(\left(\sqrt{c} x - \sqrt{cx^2 + b}\right)^2\right)}{4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")`

[Out]  $\frac{1}{2} * \sqrt{c * x^2 + b} * B * x * \operatorname{sgn}(x) + 2 * A * b * \sqrt{c} * \operatorname{sgn}(x) / ((\sqrt{c}) * x - \sqrt{c * x^2 + b})^2 - b - \frac{1}{4} * (B * b * \sqrt{c} * \operatorname{sgn}(x) + 2 * A * c^{\frac{3}{2}} * \operatorname{sgn}(x)) * \log((\sqrt{c}) * x - \sqrt{c * x^2 + b})^2 / c$

**maple** [A] time = 0.06, size = 130, normalized size = 1.34

$$\frac{\sqrt{cx^4 + bx^2} \left( 2Abcx \ln(\sqrt{c}x + \sqrt{cx^2 + b}) + Bb^2x \ln(\sqrt{c}x + \sqrt{cx^2 + b}) + 2\sqrt{cx^2 + b} Ac^{\frac{3}{2}}x^2 + \sqrt{cx^2 + b} Bb\sqrt{c}x^2 - 2(cx^2 + b)^{\frac{3}{2}}A\sqrt{c} \right)}{2\sqrt{cx^2 + b} b\sqrt{c}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x)`

[Out]  $\frac{1}{2} * (c * x^4 + b * x^2)^{\frac{1}{2}} * (2 * A * c^{\frac{3}{2}} * (c * x^2 + b)^{\frac{1}{2}} * x^2 + B * c^{\frac{1}{2}} * (c * x^2 + b)^{\frac{1}{2}} * x^2 * b - 2 * A * c^{\frac{1}{2}} * (c * x^2 + b)^{\frac{3}{2}} + 2 * A * \ln(c^{\frac{1}{2}} * x + (c * x^2 + b)^{\frac{1}{2}})) * x * b * c + B * \ln(c^{\frac{1}{2}} * x + (c * x^2 + b)^{\frac{1}{2}})) * x * b^2 / x^2 / (c * x^2 + b)^{\frac{1}{2}} / b / c^{\frac{1}{2}}$

**maxima** [A] time = 1.48, size = 105, normalized size = 1.08

$$\frac{1}{2} \left( \sqrt{c} \log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right) - \frac{2\sqrt{cx^4 + bx^2}}{x^2} \right) A + \frac{1}{4} \left( \frac{b \log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{\sqrt{c}} + 2\sqrt{cx^4 + bx^2} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (\sqrt{c}) * \log(2 * c * x^2 + b + 2 * \sqrt{c * x^4 + b * x^2}) * \sqrt{c} - 2 * \sqrt{c * x^4 + b * x^2} / x^2 * A + \frac{1}{4} * (b * \log(2 * c * x^2 + b + 2 * \sqrt{c * x^4 + b * x^2}) * \sqrt{c} / \sqrt{c} + 2 * \sqrt{c * x^4 + b * x^2}) * B$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A) \sqrt{cx^4 + bx^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3,x)`

[Out] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**3, x)
```

$$3.95 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5} dx$$

**Optimal.** Leaf size=80

$$-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6} - \frac{B\sqrt{bx^2+cx^4}}{x^2} + B\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

**Rubi [A]** time = 0.20, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 792, 662, 620, 206}

$$-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6} - \frac{B\sqrt{bx^2+cx^4}}{x^2} + B\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^5,x]

[Out] -((B\*Sqrt[b\*x^2 + c\*x^4])/x^2) - (A\*(b\*x^2 + c\*x^4)^(3/2))/(3\*b\*x^6) + B\*Sqrt[c]\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

Rule 662

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + p + 1)), x] - Dist[(c\*p)/(e^2\*(m + p + 1)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2\*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2\*p]

Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In

tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^5} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)\sqrt{bx + cx^2}}{x^3} dx, x, x^2 \right) \\
 &= -\frac{A(bx^2 + cx^4)^{3/2}}{3bx^6} + \frac{1}{2}B \text{Subst} \left( \int \frac{\sqrt{bx + cx^2}}{x^2} dx, x, x^2 \right) \\
 &= -\frac{B\sqrt{bx^2 + cx^4}}{x^2} - \frac{A(bx^2 + cx^4)^{3/2}}{3bx^6} + \frac{1}{2}(Bc) \text{Subst} \left( \int \frac{1}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\
 &= -\frac{B\sqrt{bx^2 + cx^4}}{x^2} - \frac{A(bx^2 + cx^4)^{3/2}}{3bx^6} + (Bc) \text{Subst} \left( \int \frac{1}{1 - cx^2} dx, x, \frac{x^2}{\sqrt{bx^2 + cx^4}} \right) \\
 &= -\frac{B\sqrt{bx^2 + cx^4}}{x^2} - \frac{A(bx^2 + cx^4)^{3/2}}{3bx^6} + B\sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c}x^2}{\sqrt{bx^2 + cx^4}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 86, normalized size = 1.08

$$\frac{\sqrt{x^2(b + cx^2)} \left( -A(b + cx^2) + \frac{3\sqrt{b}B\sqrt{c}x^3 \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) - 3bBx^2}{\sqrt{\frac{cx^2}{b} + 1}} \right)}{3bx^4}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^5, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-3\*b\*B\*x^2 - A\*(b + c\*x^2) + (3\*Sqrt[b]\*B\*Sqrt[c]\*x^3\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/Sqrt[1 + (c\*x^2)/b]))/(3\*b\*x^4)

**IntegrateAlgebraic [A]** time = 0.37, size = 86, normalized size = 1.08

$$\frac{\sqrt{bx^2 + cx^4}(-Ab - Acx^2 - 3bBx^2)}{3bx^4} - \frac{1}{2}B\sqrt{c} \log\left(-2\sqrt{c}\sqrt{bx^2 + cx^4} + b + 2cx^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^5, x]

[Out] ((-(A\*b) - 3\*b\*B\*x^2 - A\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(3\*b\*x^4) - (B\*Sqrt[c]\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/2

**fricas [A]** time = 0.44, size = 160, normalized size = 2.00

$$\left[ \frac{3Bb\sqrt{c}x^4 \log\left(-2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c}\right) - 2\sqrt{cx^4 + bx^2}\left((3Bb + Ac)x^2 + Ab\right)}{6bx^4}, -\frac{3Bb\sqrt{-c}x^4 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-c}}{cx^2 + b}\right) + \sqrt{cx^4 + bx^2}\left((3Bb + Ac)x^2 + Ab\right)}{3bx^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^5,x, algorithm="fricas")

[Out] [1/6\*(3\*B\*b\*sqrt(c)\*x^4\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*sqrt(c\*x^4 + b\*x^2)\*((3\*B\*b + A\*c)\*x^2 + A\*b))/(b\*x^4), -1/3\*(3\*B\*b\*sqrt

$(-c)*x^4*\arctan(\sqrt{c*x^4 + b*x^2}*\sqrt{-c}/(c*x^2 + b)) + \sqrt{c*x^4 + b*x^2}*((3*B*b + A*c)*x^2 + A*b)/(b*x^4)]$

**giac** [B] time = 0.50, size = 163, normalized size = 2.04

$$\frac{\frac{1}{2}B\sqrt{c}\log\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 Bb\sqrt{c}\operatorname{sgn}(x) + 3\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 Ac^{\frac{3}{2}}\operatorname{sgn}(x) - 6\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 Bb^2\sqrt{c}\operatorname{sgn}(x) + 3Bb^3\sqrt{c}\operatorname{sgn}(x) + Ab^2c^{\frac{3}{2}}\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^5,x, algorithm="giac")

[Out]  $-1/2*B*\sqrt{c}*\log((\sqrt{c}*x - \sqrt{c*x^2 + b})^2)*\operatorname{sgn}(x) + 2/3*(3*(\sqrt{c}(c*x - \sqrt{c*x^2 + b}))^4*B*b*\sqrt{c}*\operatorname{sgn}(x) + 3*(\sqrt{c}*x - \sqrt{c*x^2 + b}))^4*A*c^{(3/2)}*\operatorname{sgn}(x) - 6*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*B*b^2*\sqrt{c}*\operatorname{sgn}(x) + 3*B*b^3*\sqrt{c}*\operatorname{sgn}(x) + A*b^2*c^{(3/2)}*\operatorname{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + b})^2 - b)^3$

**maple** [A] time = 0.06, size = 109, normalized size = 1.36

$$\frac{\sqrt{cx^4 + bx^2} \left( -3Bbcx^3 \ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right) - 3\sqrt{cx^2 + b} Bc^{\frac{3}{2}}x^4 + 3(cx^2 + b)^{\frac{3}{2}} B\sqrt{c}x^2 + (cx^2 + b)^{\frac{3}{2}} A\sqrt{c} \right)}{3\sqrt{cx^2 + b} b\sqrt{c}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^5,x)

[Out]  $-1/3*(c*x^4+b*x^2)^{(1/2)}*(-3*B*c^{(3/2)}*(c*x^2+b)^{(1/2)}*x^4+3*B*c^{(1/2)}*(c*x^2+b)^{(3/2)}*x^2-3*B*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*x^3*b*c+(c*x^2+b)^{(3/2)}*A*c^{(1/2)})/x^4/(c*x^2+b)^{(1/2)}/b/c^{(1/2)}$

**maxima** [A] time = 1.42, size = 96, normalized size = 1.20

$$\frac{1}{2} \left( \sqrt{c} \log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2} \sqrt{c}\right) - \frac{2\sqrt{cx^4 + bx^2}}{x^2} \right) B - \frac{1}{3} A \left( \frac{\sqrt{cx^4 + bx^2} c}{bx^2} + \frac{\sqrt{cx^4 + bx^2}}{x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^5,x, algorithm="maxima")

[Out]  $1/2*(\sqrt{c}*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2}*\sqrt{c})) - 2*\sqrt{c*x^4 + b*x^2}/x^2)*B - 1/3*A*(\sqrt{c*x^4 + b*x^2}*c/(b*x^2) + \sqrt{c*x^4 + b*x^2}/x^4)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A) \sqrt{cx^4 + bx^2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^5,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*5,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*5, x)

$$3.96 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7} dx$$

**Optimal.** Leaf size=61

$$-\frac{(bx^2 + cx^4)^{3/2} (5bB - 2Ac)}{15b^2x^6} - \frac{A(bx^2 + cx^4)^{3/2}}{5bx^8}$$

**Rubi [A]** time = 0.16, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2034, 792, 650}

$$-\frac{(bx^2 + cx^4)^{3/2} (5bB - 2Ac)}{15b^2x^6} - \frac{A(bx^2 + cx^4)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^7,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(3/2))/(5\*b\*x^8) - ((5\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(15\*b^2\*x^6)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^7} dx = \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)\sqrt{bx + cx^2}}{x^4} dx, x, x^2 \right)$$

$$= -\frac{A(bx^2 + cx^4)^{3/2}}{5bx^8} + \frac{\left(-4(-bB + Ac) + \frac{3}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x^3} dx, x, x^2 \right)}{5b}$$

$$= -\frac{A(bx^2 + cx^4)^{3/2}}{5bx^8} - \frac{(5bB - 2Ac)(bx^2 + cx^4)^{3/2}}{15b^2x^6}$$

**Mathematica** [A] time = 0.02, size = 44, normalized size = 0.72

$$-\frac{(x^2(b + cx^2))^{3/2}(3Ab - 2Acx^2 + 5bBx^2)}{15b^2x^8}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^7, x]

[Out] -1/15\*((x^2\*(b + c\*x^2))^(3/2)\*(3\*A\*b + 5\*b\*B\*x^2 - 2\*A\*c\*x^2))/(b^2\*x^8)

**IntegrateAlgebraic** [A] time = 0.34, size = 66, normalized size = 1.08

$$\frac{\sqrt{bx^2 + cx^4}(-3Ab^2 - Abcx^2 + 2Ac^2x^4 - 5b^2Bx^2 - 5bBcx^4)}{15b^2x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^7, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-3\*A\*b^2 - 5\*b^2\*B\*x^2 - A\*b\*c\*x^2 - 5\*b\*B\*c\*x^4 + 2\*A\*c^2\*x^4))/(15\*b^2\*x^6)

**fricas** [A] time = 0.41, size = 59, normalized size = 0.97

$$\frac{\left((5Bbc - 2Ac^2)x^4 + 3Ab^2 + (5Bb^2 + Abc)x^2\right)\sqrt{cx^4 + bx^2}}{15b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^7,x, algorithm="fricas")

[Out] -1/15\*((5\*B\*b\*c - 2\*A\*c^2)\*x^4 + 3\*A\*b^2 + (5\*B\*b^2 + A\*b\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^2\*x^6)

**giac** [B] time = 0.92, size = 250, normalized size = 4.10

$$\frac{2(15(\sqrt{cx - \sqrt{cx^2 + b}})^6 Bc^{\frac{1}{2}} \text{sgn}(x) - 30(\sqrt{cx - \sqrt{cx^2 + b}})^6 Bbc^{\frac{1}{2}} \text{sgn}(x) + 30(\sqrt{cx - \sqrt{cx^2 + b}})^6 Ac^{\frac{1}{2}} \text{sgn}(x) + 20(\sqrt{cx - \sqrt{cx^2 + b}})^6 Bb^{\frac{1}{2}} c^{\frac{1}{2}} \text{sgn}(x) + 10(\sqrt{cx - \sqrt{cx^2 + b}})^6 Abc^{\frac{1}{2}} \text{sgn}(x) - 10(\sqrt{cx - \sqrt{cx^2 + b}})^6 Bb^{\frac{1}{2}} c^{\frac{1}{2}} \text{sgn}(x) + 10(\sqrt{cx - \sqrt{cx^2 + b}})^6 Ab^{\frac{1}{2}} c^{\frac{1}{2}} \text{sgn}(x) + 5Bb^{\frac{1}{2}} c^{\frac{1}{2}} \text{sgn}(x) - 2Ab^{\frac{1}{2}} c^{\frac{1}{2}} \text{sgn}(x))}{15((\sqrt{cx - \sqrt{cx^2 + b}})^2 - b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^7,x, algorithm="giac")

[Out] 2/15\*(15\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^8\*B\*c^(3/2)\*sgn(x) - 30\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^6\*B\*b\*c^(3/2)\*sgn(x) + 30\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^6\*A\*c^(5/2)\*sgn(x) + 20\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*B\*b^2\*c^(3/2)\*sgn(x) + 10\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*A\*b\*c^(5/2)\*sgn(x) - 10\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2\*B\*b^3\*c^(3/2)\*sgn(x) + 10\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2



$2Ab^2c^{5/2}\operatorname{sgn}(x) + 5Bb^4c^{3/2}\operatorname{sgn}(x) - 2A^3b^3c^{5/2}\operatorname{sgn}(x) / ((\sqrt{c}x - \sqrt{cx^2 + b})^2 - b)^5$

**maple [A]** time = 0.05, size = 48, normalized size = 0.79

$$\frac{(cx^2 + b)(-2Acx^2 + 5Bbx^2 + 3Ab)\sqrt{cx^4 + bx^2}}{15b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^7, x)

[Out] -1/15\*(c\*x^2+b)\*(-2\*A\*c\*x^2+5\*B\*b\*x^2+3\*A\*b)\*(c\*x^4+b\*x^2)^(1/2)/b^2/x^6

**maxima [B]** time = 1.50, size = 111, normalized size = 1.82

$$-\frac{1}{3}B\left(\frac{\sqrt{cx^4 + bx^2}c}{bx^2} + \frac{\sqrt{cx^4 + bx^2}}{x^4}\right) + \frac{1}{15}A\left(\frac{2\sqrt{cx^4 + bx^2}c^2}{b^2x^2} - \frac{\sqrt{cx^4 + bx^2}c}{bx^4} - \frac{3\sqrt{cx^4 + bx^2}}{x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^7, x, algorithm="maxima")

[Out] -1/3\*B\*(sqrt(c\*x^4 + b\*x^2)\*c/(b\*x^2) + sqrt(c\*x^4 + b\*x^2)/x^4) + 1/15\*A\*(2\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b^2\*x^2) - sqrt(c\*x^4 + b\*x^2)\*c/(b\*x^4) - 3\*sqrt(c\*x^4 + b\*x^2)/x^6)

**mupad [B]** time = 0.47, size = 113, normalized size = 1.85

$$\frac{(Ac^2 + Bbc)\sqrt{cx^4 + bx^2}}{5b^2x^2} - \frac{(5Bb^2 + Acb)\sqrt{cx^4 + bx^2}}{15b^2x^4} - \frac{A\sqrt{cx^4 + bx^2}}{5x^6} - \frac{c(Ac + 8Bb)\sqrt{cx^4 + bx^2}}{15b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^7, x)

[Out] ((A\*c^2 + B\*b\*c)\*(b\*x^2 + c\*x^4)^(1/2))/(5\*b^2\*x^2) - ((5\*B\*b^2 + A\*b\*c)\*(b\*x^2 + c\*x^4)^(1/2))/(15\*b^2\*x^4) - (A\*(b\*x^2 + c\*x^4)^(1/2))/(5\*x^6) - (c\*(A\*c + 8\*B\*b)\*(b\*x^2 + c\*x^4)^(1/2))/(15\*b^2\*x^2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*7, x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*7, x)

$$3.97 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9} dx$$

**Optimal.** Leaf size=96

$$\frac{2c(bx^2+cx^4)^{3/2}(7bB-4Ac)}{105b^3x^6} - \frac{(bx^2+cx^4)^{3/2}(7bB-4Ac)}{35b^2x^8} - \frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}}$$

**Rubi [A]** time = 0.21, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{2c(bx^2+cx^4)^{3/2}(7bB-4Ac)}{105b^3x^6} - \frac{(bx^2+cx^4)^{3/2}(7bB-4Ac)}{35b^2x^8} - \frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^9,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(3/2))/(7\*b\*x^10) - ((7\*b\*B - 4\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(35\*b^2\*x^8) + (2\*c\*(7\*b\*B - 4\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(105\*b^3\*x^6)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^9} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)\sqrt{bx + cx^2}}{x^5} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{7bx^{10}} + \frac{\left(-5(-bB + Ac) + \frac{3}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x^4} dx, x \right)}{7b} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB - 4Ac)(bx^2 + cx^4)^{3/2}}{35b^2x^8} - \frac{(c(7bB - 4Ac)) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x^4} dx, x \right)}{35b^2} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB - 4Ac)(bx^2 + cx^4)^{3/2}}{35b^2x^8} + \frac{2c(7bB - 4Ac)(bx^2 + cx^4)^{3/2}}{105b^3x^6}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.69

$$\frac{(x^2(b + cx^2))^{3/2} (A(-15b^2 + 12bcx^2 - 8c^2x^4) + 7bBx^2(2cx^2 - 3b))}{105b^3x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^9, x]

[Out] ((x^2\*(b + c\*x^2))^(3/2)\*(7\*b\*B\*x^2\*(-3\*b + 2\*c\*x^2) + A\*(-15\*b^2 + 12\*b\*c\*x^2 - 8\*c^2\*x^4)))/(105\*b^3\*x^10)

**IntegrateAlgebraic [A]** time = 0.38, size = 90, normalized size = 0.94

$$\frac{\sqrt{bx^2 + cx^4} (-15Ab^3 - 3Ab^2cx^2 + 4Abc^2x^4 - 8Ac^3x^6 - 21b^3Bx^2 - 7b^2Bcx^4 + 14bBc^2x^6)}{105b^3x^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^9, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-15\*A\*b^3 - 21\*b^3\*B\*x^2 - 3\*A\*b^2\*c\*x^2 - 7\*b^2\*B\*c\*x^4 + 4\*A\*b\*c^2\*x^4 + 14\*b\*B\*c^2\*x^6 - 8\*A\*c^3\*x^6))/(105\*b^3\*x^8)

**fricas [A]** time = 0.43, size = 85, normalized size = 0.89

$$\frac{(2(7Bbc^2 - 4Ac^3)x^6 - (7Bb^2c - 4Abc^2)x^4 - 15Ab^3 - 3(7Bb^3 + Ab^2c)x^2)\sqrt{cx^4 + bx^2}}{105b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^9, x, algorithm="fricas")

[Out] 1/105\*(2\*(7\*B\*b\*c^2 - 4\*A\*c^3)\*x^6 - (7\*B\*b^2\*c - 4\*A\*b\*c^2)\*x^4 - 15\*A\*b^3 - 3\*(7\*B\*b^3 + A\*b^2\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^3\*x^8)

**giac [B]** time = 1.36, size = 310, normalized size = 3.23

$$\frac{4(105(\sqrt{c} - \sqrt{c^2+b})^{10} Bc^{\frac{5}{2}} \text{sgn}(c) - 175(\sqrt{c} - \sqrt{c^2+b})^8 Bc^{\frac{3}{2}} \text{sgn}(c) + 280(\sqrt{c} - \sqrt{c^2+b})^6 Bc^{\frac{1}{2}} \text{sgn}(c) + 70(\sqrt{c} - \sqrt{c^2+b})^4 Bc^{\frac{1}{2}} \text{sgn}(c) + 140(\sqrt{c} - \sqrt{c^2+b})^2 Bc^{\frac{1}{2}} \text{sgn}(c) - 42(\sqrt{c} - \sqrt{c^2+b}) Bc^{\frac{1}{2}} \text{sgn}(c) + 84(\sqrt{c} - \sqrt{c^2+b}) A c^{\frac{1}{2}} \text{sgn}(c) + 49(\sqrt{c} - \sqrt{c^2+b})^2 B b^{\frac{1}{2}} \text{sgn}(c) - 28(\sqrt{c} - \sqrt{c^2+b}) A b^{\frac{1}{2}} \text{sgn}(c) - 7 B b^{\frac{1}{2}} \text{sgn}(c) + 4 A b^{\frac{1}{2}} \text{sgn}(c))}{105((\sqrt{c} - \sqrt{c^2+b})^5 - c^{\frac{5}{2}})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^9, x, algorithm="giac")

[Out] 4/105\*(105\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^10\*B\*c^(5/2)\*sgn(x) - 175\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^8\*B\*b\*c^(5/2)\*sgn(x) + 280\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^6\*B\*b\*c^(5/2)\*sgn(x) + 70\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*B\*b\*c^(5/2)\*sgn(x) + 140\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2\*B\*b\*c^(5/2)\*sgn(x) - 42\*(sqrt(c)\*x - sqrt(c\*x^2 + b))\*B\*b\*c^(5/2)\*sgn(x) + 84\*(sqrt(c)\*x - sqrt(c\*x^2 + b))\*A\*c^(5/2)\*sgn(x) + 49\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2\*B\*b\*c^(5/2)\*sgn(x) - 28\*(sqrt(c)\*x - sqrt(c\*x^2 + b))\*A\*b\*c^(5/2)\*sgn(x) - 7\*B\*b\*c^(5/2)\*sgn(x) + 4\*A\*b\*c^(5/2)\*sgn(x))

$b))^8 A c^{7/2} \operatorname{sgn}(x) + 70(\sqrt{c}x - \sqrt{c x^2 + b})^6 B b^2 c^{5/2} \operatorname{sgn}(x) + 140(\sqrt{c}x - \sqrt{c x^2 + b})^6 A b c^{7/2} \operatorname{sgn}(x) - 42(\sqrt{c}x - \sqrt{c x^2 + b})^4 B b^3 c^{5/2} \operatorname{sgn}(x) + 84(\sqrt{c}x - \sqrt{c x^2 + b})^4 A b^2 c^{7/2} \operatorname{sgn}(x) + 49(\sqrt{c}x - \sqrt{c x^2 + b})^2 B b^4 c^{5/2} \operatorname{sgn}(x) - 28(\sqrt{c}x - \sqrt{c x^2 + b})^2 A b^3 c^{7/2} \operatorname{sgn}(x) - 7 B b^5 c^{5/2} \operatorname{sgn}(x) + 4 A b^4 c^{7/2} \operatorname{sgn}(x) / ((\sqrt{c}x - \sqrt{c x^2 + b})^2 - b)^7$

**maple [A]** time = 0.04, size = 70, normalized size = 0.73

$$\frac{(c x^2 + b)(8 A c^2 x^4 - 14 B b c x^4 - 12 A b c x^2 + 21 B b^2 x^2 + 15 b^2 A) \sqrt{c x^4 + b x^2}}{105 b^3 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^9,x)

[Out] -1/105\*(c\*x^2+b)\*(8\*A\*c^2\*x^4-14\*B\*b\*c\*x^4-12\*A\*b\*c\*x^2+21\*B\*b^2\*x^2+15\*A\*b^2)\*(c\*x^4+b\*x^2)^(1/2)/b^3/x^8

**maxima [A]** time = 1.48, size = 161, normalized size = 1.68

$$\frac{1}{15} B \left( \frac{2 \sqrt{c x^4 + b x^2} c^2}{b^2 x^2} - \frac{\sqrt{c x^4 + b x^2} c}{b x^4} - \frac{3 \sqrt{c x^4 + b x^2}}{x^6} \right) - \frac{1}{105} A \left( \frac{8 \sqrt{c x^4 + b x^2} c^3}{b^3 x^2} - \frac{4 \sqrt{c x^4 + b x^2} c^2}{b^2 x^4} + \frac{3 \sqrt{c x^4 + b x^2} c}{b x^6} + \frac{15 \sqrt{c x^4 + b x^2}}{x^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^9,x, algorithm="maxima")

[Out] 1/15\*B\*(2\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b^2\*x^2) - sqrt(c\*x^4 + b\*x^2)\*c/(b\*x^4) - 3\*sqrt(c\*x^4 + b\*x^2)/x^6) - 1/105\*A\*(8\*sqrt(c\*x^4 + b\*x^2)\*c^3/(b^3\*x^2) - 4\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b^2\*x^4) + 3\*sqrt(c\*x^4 + b\*x^2)\*c/(b\*x^6) + 15\*sqrt(c\*x^4 + b\*x^2)/x^8)

**mupad [B]** time = 0.68, size = 160, normalized size = 1.67

$$\frac{4 A c^2 \sqrt{c x^4 + b x^2}}{105 b^2 x^4} - \frac{B \sqrt{c x^4 + b x^2}}{5 x^6} - \frac{A c \sqrt{c x^4 + b x^2}}{35 b x^6} - \frac{B c \sqrt{c x^4 + b x^2}}{15 b x^4} - \frac{A \sqrt{c x^4 + b x^2}}{7 x^8} - \frac{8 A c^3 \sqrt{c x^4 + b x^2}}{105 b^3 x^2} + \frac{2 B c^2 \sqrt{c x^4 + b x^2}}{15 b^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^9,x)

[Out] (4\*A\*c^2\*(b\*x^2 + c\*x^4)^(1/2))/(105\*b^2\*x^4) - (B\*(b\*x^2 + c\*x^4)^(1/2))/(5\*x^6) - (A\*c\*(b\*x^2 + c\*x^4)^(1/2))/(35\*b\*x^6) - (B\*c\*(b\*x^2 + c\*x^4)^(1/2))/(15\*b\*x^4) - (A\*(b\*x^2 + c\*x^4)^(1/2))/(7\*x^8) - (8\*A\*c^3\*(b\*x^2 + c\*x^4)^(1/2))/(105\*b^3\*x^2) + (2\*B\*c^2\*(b\*x^2 + c\*x^4)^(1/2))/(15\*b^2\*x^2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*9,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*9, x)

$$3.98 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}} dx$$

**Optimal.** Leaf size=133

$$\frac{8c^2 (bx^2 + cx^4)^{3/2} (3bB - 2Ac)}{315b^4x^6} + \frac{4c (bx^2 + cx^4)^{3/2} (3bB - 2Ac)}{105b^3x^8} - \frac{(bx^2 + cx^4)^{3/2} (3bB - 2Ac)}{21b^2x^{10}} - \frac{A (bx^2 + cx^4)^{3/2}}{9bx^{12}}$$

**Rubi [A]** time = 0.24, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{8c^2 (bx^2 + cx^4)^{3/2} (3bB - 2Ac)}{315b^4x^6} + \frac{4c (bx^2 + cx^4)^{3/2} (3bB - 2Ac)}{105b^3x^8} - \frac{(bx^2 + cx^4)^{3/2} (3bB - 2Ac)}{21b^2x^{10}} - \frac{A (bx^2 + cx^4)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^11,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(3/2))/(9\*b\*x^12) - ((3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(21\*b^2\*x^10) + (4\*c\*(3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(105\*b^3\*x^8) - (8\*c^2\*(3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(315\*b^4\*x^6)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((f\_.) + (g\_.)\*(x\_.))\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{11}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)\sqrt{bx + cx^2}}{x^6} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{9bx^{12}} + \frac{\left(-6(-bB + Ac) + \frac{3}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x^5} dx, x, x^2 \right)}{9b} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{3/2}}{21b^2x^{10}} - \frac{(2c(3bB - 2Ac)) \text{Subst} \left( \int \frac{\sqrt{bx}}{x} \right)}{21b^2} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB - 2Ac)(bx^2 + cx^4)^{3/2}}{105b^3x^8} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB - 2Ac)(bx^2 + cx^4)^{3/2}}{105b^3x^8}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 88, normalized size = 0.66

$$\frac{(x^2(b + cx^2))^{3/2} (A(35b^3 - 30b^2cx^2 + 24bc^2x^4 - 16c^3x^6) + 3bBx^2(15b^2 - 12bcx^2 + 8c^2x^4))}{315b^4x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^11, x]

[Out] -1/315\*((x^2\*(b + c\*x^2))^(3/2)\*(3\*b\*B\*x^2\*(15\*b^2 - 12\*b\*c\*x^2 + 8\*c^2\*x^4) + A\*(35\*b^3 - 30\*b^2\*c\*x^2 + 24\*b\*c^2\*x^4 - 16\*c^3\*x^6)))/(b^4\*x^12)

**IntegrateAlgebraic [A]** time = 0.40, size = 114, normalized size = 0.86

$$\frac{\sqrt{bx^2 + cx^4} (-35Ab^4 - 5Ab^3cx^2 + 6Ab^2c^2x^4 - 8Abc^3x^6 + 16Ac^4x^8 - 45b^4Bx^2 - 9b^3Bcx^4 + 12b^2Bc^2x^6 - 24bBc^3x^8)}{315b^4x^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^11, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-35\*A\*b^4 - 45\*b^4\*B\*x^2 - 5\*A\*b^3\*c\*x^2 - 9\*b^3\*B\*c\*x^4 + 6\*A\*b^2\*c^2\*x^4 + 12\*b^2\*B\*c^2\*x^6 - 8\*A\*b\*c^3\*x^6 - 24\*b\*B\*c^3\*x^8 + 16\*A\*c^4\*x^8))/(315\*b^4\*x^10)

**fricas [A]** time = 0.47, size = 109, normalized size = 0.82

$$\frac{(8(3Bbc^3 - 2Ac^4)x^8 - 4(3Bb^2c^2 - 2Abc^3)x^6 + 35Ab^4 + 3(3Bb^3c - 2Ab^2c^2)x^4 + 5(9Bb^4 + Ab^3c)x^2)\sqrt{cx^4 + bx^2}}{315b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^11, x, algorithm="fricas")

[Out] -1/315\*(8\*(3\*B\*b\*c^3 - 2\*A\*c^4)\*x^8 - 4\*(3\*B\*b^2\*c^2 - 2\*A\*b\*c^3)\*x^6 + 35\*A\*b^4 + 3\*(3\*B\*b^3\*c - 2\*A\*b^2\*c^2)\*x^4 + 5\*(9\*B\*b^4 + A\*b^3\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^4\*x^10)

**giac [B]** time = 1.97, size = 370, normalized size = 2.78

$$\frac{16\sqrt{c}\sqrt{bx^2+cx^4}^{11} - 315\sqrt{c}\sqrt{bx^2+cx^4}^{10} + 630\sqrt{c}\sqrt{bx^2+cx^4}^9 - 42\sqrt{c}\sqrt{bx^2+cx^4}^8 + 168\sqrt{c}\sqrt{bx^2+cx^4}^7 - 22\sqrt{c}\sqrt{bx^2+cx^4}^6 + 18\sqrt{c}\sqrt{bx^2+cx^4}^5 - 7\sqrt{c}\sqrt{bx^2+cx^4}^4 - 2\sqrt{c}\sqrt{bx^2+cx^4}^3 + 3Bb^4\sqrt{c}\sqrt{bx^2+cx^4} - 2Ab^3c\sqrt{c}\sqrt{bx^2+cx^4}}{315\sqrt{c}\sqrt{bx^2+cx^4}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^11,x, algorithm="giac")

[Out]  $16/315*(210*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*B*c^{(7/2)}*\text{sgn}(x) - 315*(\sqrt{c}) * x - \sqrt{c*x^2 + b})^{10}*B*b*c^{(7/2)}*\text{sgn}(x) + 630*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*A*c^{(9/2)}*\text{sgn}(x) + 63*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*B*b^2*c^{(7/2)}*\text{sgn}(x) + 378*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*A*b*c^{(9/2)}*\text{sgn}(x) - 42*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*B*b^3*c^{(7/2)}*\text{sgn}(x) + 168*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*A*b^2*c^{(9/2)}*\text{sgn}(x) + 108*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*B*b^4*c^{(7/2)}*\text{sgn}(x) - 72*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*A*b^3*c^{(9/2)}*\text{sgn}(x) - 27*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*B*b^5*c^{(7/2)}*\text{sgn}(x) + 18*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*A*b^4*c^{(9/2)}*\text{sgn}(x) + 3*B*b^6*c^{(7/2)}*\text{sgn}(x) - 2*A*b^5*c^{(9/2)}*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + b})^2 - b)^9$

**maple** [A] time = 0.05, size = 94, normalized size = 0.71

$$\frac{(cx^2 + b)(-16Ac^3x^6 + 24Bbc^2x^6 + 24Abc^2x^4 - 36Bb^2cx^4 - 30Ab^2cx^2 + 45Bb^3x^2 + 35Ab^3)\sqrt{cx^4 + bx^2}}{315b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^11,x)

[Out]  $-1/315*(c*x^2+b)*(-16*A*c^3*x^6+24*B*b*c^2*x^6+24*A*b*c^2*x^4-36*B*b^2*c*x^4-30*A*b^2*c*x^2+45*B*b^3*x^2+35*A*b^3)*(c*x^4+b*x^2)^(1/2)/x^{10}/b^4$

**maxima** [A] time = 1.49, size = 209, normalized size = 1.57

$$\frac{1}{105}B\left(\frac{8\sqrt{cx^4+bx^2}c^3}{b^3x^2} - \frac{4\sqrt{cx^4+bx^2}c^2}{b^2x^4} + \frac{3\sqrt{cx^4+bx^2}c}{bx^6} + \frac{15\sqrt{cx^4+bx^2}}{x^8}\right) + \frac{1}{315}A\left(\frac{16\sqrt{cx^4+bx^2}c^4}{b^4x^2} - \frac{8\sqrt{cx^4+bx^2}c^3}{b^3x^4} + \frac{6\sqrt{cx^4+bx^2}c^2}{b^2x^6} - \frac{5\sqrt{cx^4+bx^2}c}{bx^8} - \frac{35\sqrt{cx^4+bx^2}}{x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^11,x, algorithm="maxima")

[Out]  $-1/105*B*(8*\sqrt{c*x^4 + b*x^2})*c^3/(b^3*x^2) - 4*\sqrt{c*x^4 + b*x^2})*c^2/(b^2*x^4) + 3*\sqrt{c*x^4 + b*x^2})*c/(b*x^6) + 15*\sqrt{c*x^4 + b*x^2}/x^8) + 1/315*A*(16*\sqrt{c*x^4 + b*x^2})*c^4/(b^4*x^2) - 8*\sqrt{c*x^4 + b*x^2})*c^3/(b^3*x^4) + 6*\sqrt{c*x^4 + b*x^2})*c^2/(b^2*x^6) - 5*\sqrt{c*x^4 + b*x^2})*c/(b*x^8) - 35*\sqrt{c*x^4 + b*x^2}/x^{10})$

**mupad** [B] time = 1.04, size = 210, normalized size = 1.58

$$\frac{2Ac^2\sqrt{cx^4+bx^2}}{105b^2x^6} - \frac{B\sqrt{cx^4+bx^2}}{7x^8} - \frac{Ac\sqrt{cx^4+bx^2}}{63bx^8} - \frac{Bc\sqrt{cx^4+bx^2}}{35bx^6} - \frac{A\sqrt{cx^4+bx^2}}{9x^{10}} - \frac{8Ac^3\sqrt{cx^4+bx^2}}{315b^3x^4} + \frac{16Ac^4\sqrt{cx^4+bx^2}}{315b^4x^2} + \frac{4Bc^2\sqrt{cx^4+bx^2}}{105b^2x^4} - \frac{8Bc^3\sqrt{cx^4+bx^2}}{105b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^11,x)

[Out]  $(2*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (B*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (A*c*(b*x^2 + c*x^4)^(1/2))/(63*b*x^8) - (B*c*(b*x^2 + c*x^4)^(1/2))/(35*b*x^6) - (A*(b*x^2 + c*x^4)^(1/2))/(9*x^{10}) - (8*A*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^4) + (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2) + (4*B*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (8*B*c^3*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x^2)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*11,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*11, x)

$$3.99 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}} dx$$

**Optimal.** Leaf size=170

$$\frac{16c^3 (bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{3465b^5x^6} - \frac{8c^2 (bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{1155b^4x^8} + \frac{2c (bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{231b^3x^{10}} - \frac{(bx^2 + cx^4)^3}{99}$$

**Rubi [A]** time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{16c^3 (bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{3465b^5x^6} - \frac{8c^2 (bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{1155b^4x^8} + \frac{2c (bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{231b^3x^{10}} - \frac{(bx^2 + cx^4)^{3/2} (11bB - 8Ac)}{99b^2x^{12}} - \frac{A (bx^2 + cx^4)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^13,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(3/2))/(11\*b\*x^14) - ((11\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(99\*b^2\*x^12) + (2\*c\*(11\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(231\*b^3\*x^10) - (8\*c^2\*(11\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(1155\*b^4\*x^8) + (16\*c^3\*(11\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(3465\*b^5\*x^6)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]



Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{13}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)\sqrt{bx + cx^2}}{x^7} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{11bx^{14}} + \frac{\left(-7(-bB + Ac) + \frac{3}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x^6} dx, x \right)}{11b} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{99b^2x^{12}} - \frac{(c(11bB - 8Ac)) \text{Subst} \left( \int \frac{\sqrt{bx+cx^2}}{x^6} dx, x \right)}{33b^2} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{231b^3x^{10}} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{231b^3x^{10}} \\
&= -\frac{A(bx^2 + cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB - 8Ac)(bx^2 + cx^4)^{3/2}}{231b^3x^{10}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 94, normalized size = 0.55

$$\frac{\sqrt{x^2(b + cx^2)} \left( x^2 \left( \frac{cx^2}{b} + 1 \right) (35b^3 - 30b^2cx^2 + 24bc^2x^4 - 16c^3x^6) (8Ac - 11bB) - 315Ab^3(b + cx^2) \right)}{3465b^4x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^13,x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-315\*A\*b^3\*(b + c\*x^2) + (-11\*b\*B + 8\*A\*c)\*x^2\*(1 + (c\*x^2)/b)\*(35\*b^3 - 30\*b^2\*c\*x^2 + 24\*b\*c^2\*x^4 - 16\*c^3\*x^6)))/(3465\*b^4\*x^12)

**IntegrateAlgebraic [A]** time = 0.45, size = 138, normalized size = 0.81

$$\frac{\sqrt{bx^2 + cx^4} (-315Ab^5 - 35Ab^4cx^2 + 40Ab^3c^2x^4 - 48Ab^2c^3x^6 + 64Abc^4x^8 - 128Ac^5x^{10} - 385b^5Bx^2 - 55b^4Bcx^4 + 66b^3Bc^2x^6 - 88b^2Bc^3x^8 + 176bBc^4x^{10})}{3465b^5x^{12}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^13,x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-315\*A\*b^5 - 385\*b^5\*B\*x^2 - 35\*A\*b^4\*c\*x^2 - 55\*b^4\*B\*c\*x^4 + 40\*A\*b^3\*c^2\*x^4 + 66\*b^3\*B\*c^2\*x^6 - 48\*A\*b^2\*c^3\*x^6 - 88\*b^2\*B\*c^3\*x^8 + 64\*A\*b\*c^4\*x^8 + 176\*b\*B\*c^4\*x^{10} - 128\*A\*c^5\*x^{10}))/ (3465\*b^5\*x^{12})

**fricas [A]** time = 0.50, size = 133, normalized size = 0.78

$$\frac{(16(11Bbc^4 - 8Ac^5)x^{10} - 8(11Bb^2c^3 - 8Abc^4)x^8 + 6(11Bb^3c^2 - 8Ab^2c^3)x^6 - 315Ab^5 - 5(11Bb^4c - 8Ab^3c^2)x^4 - 35(11Bb^5 + Ab^4c)x^2)\sqrt{cx^4 + bx^2}}{3465b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^13,x, algorithm="fricas")

[Out] 1/3465\*(16\*(11\*B\*b\*c^4 - 8\*A\*c^5)\*x^10 - 8\*(11\*B\*b^2\*c^3 - 8\*A\*b\*c^4)\*x^8 + 6\*(11\*B\*b^3\*c^2 - 8\*A\*b^2\*c^3)\*x^6 - 315\*A\*b^5 - 5\*(11\*B\*b^4\*c - 8\*A\*b^3\*c^2)\*x^4 - 35\*(11\*B\*b^5 + A\*b^4\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^5\*x^12)

**giac [B]** time = 2.97, size = 430, normalized size = 2.53

$$\frac{32(3465\sqrt{c}x - \sqrt{cx^2 + b})^{14}Bc^{9/2}\operatorname{sgn}(x) - 4851(\sqrt{c}x - \sqrt{cx^2 + b})^{12}B^2b^2c^{9/2}\operatorname{sgn}(x) + 11088(\sqrt{c}x - \sqrt{cx^2 + b})^{10}B^2b^2c^{9/2}\operatorname{sgn}(x) + 7392(\sqrt{c}x - \sqrt{cx^2 + b})^{10}A^2b^2c^{11/2}\operatorname{sgn}(x) - 165(\sqrt{c}x - \sqrt{cx^2 + b})^8B^2b^3c^{9/2}\operatorname{sgn}(x) + 2640(\sqrt{c}x - \sqrt{cx^2 + b})^8A^2b^2c^{11/2}\operatorname{sgn}(x) + 1815(\sqrt{c}x - \sqrt{cx^2 + b})^6B^2b^4c^{9/2}\operatorname{sgn}(x) - 1320(\sqrt{c}x - \sqrt{cx^2 + b})^6A^2b^3c^{11/2}\operatorname{sgn}(x) - 605(\sqrt{c}x - \sqrt{cx^2 + b})^4B^2b^5c^{9/2}\operatorname{sgn}(x) + 440(\sqrt{c}x - \sqrt{cx^2 + b})^4A^2b^4c^{11/2}\operatorname{sgn}(x) + 121(\sqrt{c}x - \sqrt{cx^2 + b})^2B^2b^6c^{9/2}\operatorname{sgn}(x) - 88(\sqrt{c}x - \sqrt{cx^2 + b})^2A^2b^5c^{11/2}\operatorname{sgn}(x) - 11B^2b^7c^{9/2}\operatorname{sgn}(x) + 8A^2b^6c^{11/2}\operatorname{sgn}(x)}{(\sqrt{c}x - \sqrt{cx^2 + b})^2 - b}^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^13,x, algorithm="giac")

[Out] 32/3465\*(3465\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^14\*B\*c^(9/2)\*sgn(x) - 4851\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^12\*B\*b\*c^(9/2)\*sgn(x) + 11088\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^10\*B\*b^2\*c^(9/2)\*sgn(x) + 7392\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^10\*A\*b\*c^(11/2)\*sgn(x) - 165\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^8\*B\*b^3\*c^(9/2)\*sgn(x) + 2640\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^8\*A\*b^2\*c^(11/2)\*sgn(x) + 1815\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^6\*B\*b^4\*c^(9/2)\*sgn(x) - 1320\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^6\*A\*b^3\*c^(11/2)\*sgn(x) - 605\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*B\*b^5\*c^(9/2)\*sgn(x) + 440\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*A\*b^4\*c^(11/2)\*sgn(x) + 121\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2\*B\*b^6\*c^(9/2)\*sgn(x) - 88\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2\*A\*b^5\*c^(11/2)\*sgn(x) - 11\*B\*b^7\*c^(9/2)\*sgn(x) + 8\*A\*b^6\*c^(11/2)\*sgn(x))/((sqrt(c)\*x - sqrt(c\*x^2 + b))^2 - b)^11

**maple [A]** time = 0.05, size = 118, normalized size = 0.69

$$\frac{(cx^2 + b)(128A^4c^4x^8 - 176Bb^2c^3x^8 - 192Ab^2c^3x^6 + 264B^2b^2c^2x^6 + 240A^2b^2c^2x^4 - 330B^2b^3cx^4 - 280A^2b^3cx^2 + 385B^2b^4x^2 + 315A^2b^4)\sqrt{cx^2 + b}}{3465b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^13,x)

[Out] -1/3465\*(c\*x^2+b)\*(128\*A\*c^4\*x^8-176\*B\*b\*c^3\*x^8-192\*A\*b\*c^3\*x^6+264\*B\*b^2\*c^2\*x^6+240\*A\*b^2\*c^2\*x^4-330\*B\*b^3\*c\*x^4-280\*A\*b^3\*c\*x^2+385\*B\*b^4\*x^2+315\*A\*b^4)\*(c\*x^4+b\*x^2)^(1/2)/b^5/x^12

**maxima [A]** time = 1.57, size = 257, normalized size = 1.51

$$\frac{1}{315}B\left(\frac{16\sqrt{cx^4 + bx^2}c^4}{b^3x^2} - \frac{8\sqrt{cx^4 + bx^2}c^3}{b^3x^4} + \frac{6\sqrt{cx^4 + bx^2}c^2}{b^2x^6} - \frac{5\sqrt{cx^4 + bx^2}c}{bx^8} - \frac{35\sqrt{cx^4 + bx^2}}{x^{10}}\right) - \frac{1}{3465}A\left(\frac{128\sqrt{cx^4 + bx^2}c^5}{b^5x^2} - \frac{64\sqrt{cx^4 + bx^2}c^4}{b^4x^4} + \frac{48\sqrt{cx^4 + bx^2}c^3}{b^3x^6} - \frac{40\sqrt{cx^4 + bx^2}c^2}{b^2x^8} + \frac{35\sqrt{cx^4 + bx^2}c}{bx^{10}} + \frac{315\sqrt{cx^4 + bx^2}}{x^{12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^13,x, algorithm="maxima")

[Out] 1/315\*B\*(16\*sqrt(c\*x^4 + b\*x^2)\*c^4/(b^4\*x^2) - 8\*sqrt(c\*x^4 + b\*x^2)\*c^3/(b^3\*x^4) + 6\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b^2\*x^6) - 5\*sqrt(c\*x^4 + b\*x^2)\*c/(b\*x^8) - 35\*sqrt(c\*x^4 + b\*x^2)/x^10) - 1/3465\*A\*(128\*sqrt(c\*x^4 + b\*x^2)\*c^5/(b^5\*x^2) - 64\*sqrt(c\*x^4 + b\*x^2)\*c^4/(b^4\*x^4) + 48\*sqrt(c\*x^4 + b\*x^2)\*c^3/(b^3\*x^6) - 40\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b^2\*x^8) + 35\*sqrt(c\*x^4 + b\*x^2)\*c/(b\*x^10) + 315\*sqrt(c\*x^4 + b\*x^2)/x^12)

**mupad [B]** time = 1.41, size = 260, normalized size = 1.53

$$\frac{8A^2\sqrt{cx^4 + bx^2}}{693b^2x^8} - \frac{B\sqrt{cx^4 + bx^2}}{9x^{10}} - \frac{Ac\sqrt{cx^4 + bx^2}}{99b^2x^{10}} - \frac{Bc\sqrt{cx^4 + bx^2}}{63bx^8} - \frac{A\sqrt{cx^4 + bx^2}}{11x^{12}} - \frac{16A^2\sqrt{cx^4 + bx^2}}{1155b^3x^6} + \frac{64A^2\sqrt{cx^4 + bx^2}}{3465b^4x^4} - \frac{128A^2\sqrt{cx^4 + bx^2}}{3465b^5x^2} + \frac{2Bc^2\sqrt{cx^4 + bx^2}}{105b^2x^6} - \frac{8Bc^3\sqrt{cx^4 + bx^2}}{315b^3x^4} + \frac{16Bc^4\sqrt{cx^4 + bx^2}}{315b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^13,x)

[Out] (8\*A\*c^2\*(b\*x^2 + c\*x^4)^(1/2))/(693\*b^2\*x^8) - (B\*(b\*x^2 + c\*x^4)^(1/2))/(9\*x^10) - (A\*c\*(b\*x^2 + c\*x^4)^(1/2))/(99\*b\*x^10) - (B\*c\*(b\*x^2 + c\*x^4)^(1/2))/(63\*b\*x^8) - (A\*(b\*x^2 + c\*x^4)^(1/2))/(11\*x^12) - (16\*A\*c^3\*(b\*x^2 + c\*x^4)^(1/2))/(1155\*b^3\*x^6) + (64\*A\*c^4\*(b\*x^2 + c\*x^4)^(1/2))/(3465\*b^4\*x^4) - (128\*A\*c^5\*(b\*x^2 + c\*x^4)^(1/2))/(3465\*b^5\*x^2) + (2\*B\*c^2\*(b\*x^2 + c\*x^4)^(1/2))/(105\*b^2\*x^6) - (8\*B\*c^3\*(b\*x^2 + c\*x^4)^(1/2))/(315\*b^3\*x^4) + (16\*B\*c^4\*(b\*x^2 + c\*x^4)^(1/2))/(315\*b^4\*x^2)

$c*x^4)^{(1/2))/(105*b^2*x^6) - (8*B*c^3*(b*x^2 + c*x^4)^{(1/2))/(315*b^3*x^4)$   
 $+ (16*B*c^4*(b*x^2 + c*x^4)^{(1/2))/(315*b^4*x^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)}(A + Bx^2)}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*13,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*13, x)

### 3.100 $\int x^4 (A + Bx^2) \sqrt{bx^2 + cx^4} dx$

**Optimal.** Leaf size=131

$$-\frac{8b^2 (bx^2 + cx^4)^{3/2} (2bB - 3Ac)}{315c^4x^3} + \frac{4b (bx^2 + cx^4)^{3/2} (2bB - 3Ac)}{105c^3x} - \frac{x (bx^2 + cx^4)^{3/2} (2bB - 3Ac)}{21c^2} + \frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c}$$

**Rubi [A]** time = 0.22, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2039, 2016, 2000}

$$-\frac{8b^2 (bx^2 + cx^4)^{3/2} (2bB - 3Ac)}{315c^4x^3} - \frac{x (bx^2 + cx^4)^{3/2} (2bB - 3Ac)}{21c^2} + \frac{4b (bx^2 + cx^4)^{3/2} (2bB - 3Ac)}{105c^3x} + \frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] (-8\*b^2\*(2\*b\*B - 3\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(315\*c^4\*x^3) + (4\*b\*(2\*b\*B - 3\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(105\*c^3\*x) - ((2\*b\*B - 3\*A\*c)\*x\*(b\*x^2 + c\*x^4)^(3/2))/(21\*c^2) + (B\*x^3\*(b\*x^2 + c\*x^4)^(3/2))/(9\*c)

#### Rule 2000

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a\*x^j + b\*x^n)^(p + 1)/(b\*(n - j)\*(p + 1)\*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j\*p - n + j + 1, 0]

#### Rule 2016

Int[((c\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

#### Rule 2039

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned} \int x^4 (A + Bx^2) \sqrt{bx^2 + cx^4} dx &= \frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} - \frac{(6bB - 9Ac) \int x^4 \sqrt{bx^2 + cx^4} dx}{9c} \\ &= -\frac{(2bB - 3Ac)x (bx^2 + cx^4)^{3/2}}{21c^2} + \frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} + \frac{(4b(2bB - 3Ac)) \int x^2 \sqrt{bx^2 + cx^4} dx}{21c^2} \\ &= \frac{4b(2bB - 3Ac) (bx^2 + cx^4)^{3/2}}{105c^3x} - \frac{(2bB - 3Ac)x (bx^2 + cx^4)^{3/2}}{21c^2} + \frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} \\ &= -\frac{8b^2(2bB - 3Ac) (bx^2 + cx^4)^{3/2}}{315c^4x^3} + \frac{4b(2bB - 3Ac) (bx^2 + cx^4)^{3/2}}{105c^3x} - \frac{(2bB - 3Ac) \int x^2 \sqrt{bx^2 + cx^4} dx}{21c^2} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 82, normalized size = 0.63

$$\frac{(x^2(b + cx^2))^{3/2} (24b^2c(A + Bx^2) - 6bc^2x^2(6A + 5Bx^2) + 5c^3x^4(9A + 7Bx^2) - 16b^3B)}{315c^4x^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] ((x^2\*(b + c\*x^2))^(3/2)\*(-16\*b^3\*B + 24\*b^2\*c\*(A + B\*x^2) - 6\*b\*c^2\*x^2\*(6\*A + 5\*B\*x^2) + 5\*c^3\*x^4\*(9\*A + 7\*B\*x^2)))/(315\*c^4\*x^3)

**IntegrateAlgebraic [A]** time = 0.11, size = 87, normalized size = 0.66

$$\frac{(bx^2 + cx^4)^{3/2} (24Ab^2c - 36Abc^2x^2 + 45Ac^3x^4 - 16b^3B + 24b^2Bcx^2 - 30bBc^2x^4 + 35Bc^3x^6)}{315c^4x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] ((b\*x^2 + c\*x^4)^(3/2)\*(-16\*b^3\*B + 24\*A\*b^2\*c + 24\*b^2\*B\*c\*x^2 - 36\*A\*b\*c^2\*x^2 - 30\*b\*B\*c^2\*x^4 + 45\*A\*c^3\*x^4 + 35\*B\*c^3\*x^6))/(315\*c^4\*x^3)

**fricas [A]** time = 0.41, size = 106, normalized size = 0.81

$$\frac{(35Bc^4x^8 + 5(Bbc^3 + 9Ac^4)x^6 - 16Bb^4 + 24Ab^3c - 3(2Bb^2c^2 - 3Abc^3)x^4 + 4(2Bb^3c - 3Ab^2c^2)x^2)\sqrt{cx^4 + bx^2}}{315c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/315\*(35\*B\*c^4\*x^8 + 5\*(B\*b\*c^3 + 9\*A\*c^4)\*x^6 - 16\*B\*b^4 + 24\*A\*b^3\*c - 3\*(2\*B\*b^2\*c^2 - 3\*A\*b\*c^3)\*x^4 + 4\*(2\*B\*b^3\*c - 3\*A\*b^2\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^4\*x)

**giac [A]** time = 0.17, size = 140, normalized size = 1.07

$$\frac{8(2Bb^2 - 3Ab^2c)\operatorname{sgn}(x)}{315c^4} + \frac{35(c^2 + b)^2 B \operatorname{sgn}(x) - 135(c^2 + b)^2 B b \operatorname{sgn}(x) + 189(c^2 + b)^2 B b^2 \operatorname{sgn}(x) - 105(c^2 + b)^2 B b^3 \operatorname{sgn}(x) + 45(c^2 + b)^2 A c \operatorname{sgn}(x) - 126(c^2 + b)^2 A b c \operatorname{sgn}(x) + 105(c^2 + b)^2 A b^2 c \operatorname{sgn}(x)}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out] 8/315\*(2\*B\*b^(9/2) - 3\*A\*b^(7/2)\*c)\*sgn(x)/c^4 + 1/315\*(35\*(c\*x^2 + b)^(9/2)\*B\*sgn(x) - 135\*(c\*x^2 + b)^(7/2)\*B\*b\*sgn(x) + 189\*(c\*x^2 + b)^(5/2)\*B\*b^2\*sgn(x) - 105\*(c\*x^2 + b)^(3/2)\*B\*b^3\*sgn(x) + 45\*(c\*x^2 + b)^(7/2)\*A\*c\*sgn(x) - 126\*(c\*x^2 + b)^(5/2)\*A\*b\*c\*sgn(x) + 105\*(c\*x^2 + b)^(3/2)\*A\*b^2\*c\*sgn(x))/c^4

**maple [A]** time = 0.05, size = 91, normalized size = 0.69

$$\frac{(cx^2 + b)(35Bc^3x^6 + 45Ac^3x^4 - 30Bbc^2x^4 - 36Abc^2x^2 + 24Bb^2cx^2 + 24Ab^2c - 16Bb^3)\sqrt{cx^4 + bx^2}}{315c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x)

[Out] 1/315\*(c\*x^2+b)\*(35\*B\*c^3\*x^6+45\*A\*c^3\*x^4-30\*B\*b\*c^2\*x^4-36\*A\*b\*c^2\*x^2+24\*B\*b^2\*c\*x^2+24\*A\*b^2\*c-16\*B\*b^3)\*(c\*x^4+b\*x^2)^(1/2)/c^4/x

**maxima** [A] time = 1.42, size = 106, normalized size = 0.81

$$\frac{(15c^3x^6 + 3bc^2x^4 - 4b^2cx^2 + 8b^3)\sqrt{cx^2 + b}A}{105c^3} + \frac{(35c^4x^8 + 5bc^3x^6 - 6b^2c^2x^4 + 8b^3cx^2 - 16b^4)\sqrt{cx^2 + b}B}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/105\*(15\*c^3\*x^6 + 3\*b\*c^2\*x^4 - 4\*b^2\*c\*x^2 + 8\*b^3)\*sqrt(c\*x^2 + b)\*A/c^3 + 1/315\*(35\*c^4\*x^8 + 5\*b\*c^3\*x^6 - 6\*b^2\*c^2\*x^4 + 8\*b^3\*c\*x^2 - 16\*b^4)\*sqrt(c\*x^2 + b)\*B/c^4

**mupad** [B] time = 0.25, size = 103, normalized size = 0.79

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{Bx^8}{9} - \frac{16Bb^4 - 24Ab^3c}{315c^4} + \frac{x^6(45Ac^4 + 5Bbc^3)}{315c^4} - \frac{4b^2x^2(3Ac - 2Bb)}{315c^3} + \frac{bx^4(3Ac - 2Bb)}{105c^2} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2),x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*((B\*x^8)/9 - (16\*B\*b^4 - 24\*A\*b^3\*c)/(315\*c^4) + (x^6\*(45\*A\*c^4 + 5\*B\*b\*c^3))/(315\*c^4) - (4\*b^2\*x^2\*(3\*A\*c - 2\*B\*b))/(315\*c^3) + (b\*x^4\*(3\*A\*c - 2\*B\*b))/(105\*c^2))/x

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{x^2(b + cx^2)} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*4\*sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2), x)

### 3.101 $\int x^2 (A + Bx^2) \sqrt{bx^2 + cx^4} dx$

**Optimal.** Leaf size=94

$$\frac{2b(bx^2 + cx^4)^{3/2}(4bB - 7Ac)}{105c^3x^3} - \frac{(bx^2 + cx^4)^{3/2}(4bB - 7Ac)}{35c^2x} + \frac{Bx(bx^2 + cx^4)^{3/2}}{7c}$$

**Rubi [A]** time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2039, 2016, 2000}

$$-\frac{(bx^2 + cx^4)^{3/2}(4bB - 7Ac)}{35c^2x} + \frac{2b(bx^2 + cx^4)^{3/2}(4bB - 7Ac)}{105c^3x^3} + \frac{Bx(bx^2 + cx^4)^{3/2}}{7c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] (2\*b\*(4\*b\*B - 7\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(105\*c^3\*x^3) - ((4\*b\*B - 7\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(35\*c^2\*x) + (B\*x\*(b\*x^2 + c\*x^4)^(3/2))/(7\*c)

#### Rule 2000

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a\*x^j + b\*x^n)^(p + 1)/(b\*(n - j)\*(p + 1)\*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j\*p - n + j + 1, 0]

#### Rule 2016

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

#### Rule 2039

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned} \int x^2 (A + Bx^2) \sqrt{bx^2 + cx^4} dx &= \frac{Bx(bx^2 + cx^4)^{3/2}}{7c} - \frac{(4bB - 7Ac) \int x^2 \sqrt{bx^2 + cx^4} dx}{7c} \\ &= -\frac{(4bB - 7Ac)(bx^2 + cx^4)^{3/2}}{35c^2x} + \frac{Bx(bx^2 + cx^4)^{3/2}}{7c} + \frac{(2b(4bB - 7Ac)) \int \sqrt{bx^2 + cx^4} dx}{35c^2} \\ &= \frac{2b(4bB - 7Ac)(bx^2 + cx^4)^{3/2}}{105c^3x^3} - \frac{(4bB - 7Ac)(bx^2 + cx^4)^{3/2}}{35c^2x} + \frac{Bx(bx^2 + cx^4)^{3/2}}{7c} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 64, normalized size = 0.68

$$\frac{(x^2(b + cx^2))^{3/2}(-2bc(7A + 6Bx^2) + 3c^2x^2(7A + 5Bx^2) + 8b^2B)}{105c^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] ((x^2\*(b + c\*x^2))^(3/2)\*(8\*b^2\*B + 3\*c^2\*x^2\*(7\*A + 5\*B\*x^2) - 2\*b\*c\*(7\*A + 6\*B\*x^2)))/(105\*c^3\*x^3)

**IntegrateAlgebraic [A]** time = 0.09, size = 63, normalized size = 0.67

$$\frac{(bx^2 + cx^4)^{3/2}(-14Abc + 21Ac^2x^2 + 8b^2B - 12bBcx^2 + 15Bc^2x^4)}{105c^3x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2\*(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out] ((b\*x^2 + c\*x^4)^(3/2)\*(8\*b^2\*B - 14\*A\*b\*c - 12\*b\*B\*c\*x^2 + 21\*A\*c^2\*x^2 + 15\*B\*c^2\*x^4))/(105\*c^3\*x^3)

**fricas [A]** time = 0.41, size = 82, normalized size = 0.87

$$\frac{(15Bc^3x^6 + 3(Bbc^2 + 7Ac^3)x^4 + 8Bb^3 - 14Ab^2c - (4Bb^2c - 7Abc^2)x^2)\sqrt{cx^4 + bx^2}}{105c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/105\*(15\*B\*c^3\*x^6 + 3\*(B\*b\*c^2 + 7\*A\*c^3)\*x^4 + 8\*B\*b^3 - 14\*A\*b^2\*c - (4\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^3\*x)

**giac [A]** time = 0.21, size = 105, normalized size = 1.12

$$-\frac{2(4Bb^{\frac{7}{2}} - 7Ab^{\frac{5}{2}}c)\operatorname{sgn}(x)}{105c^3} + \frac{15(cx^2 + b)^{\frac{7}{2}}B\operatorname{sgn}(x) - 42(cx^2 + b)^{\frac{5}{2}}Bb\operatorname{sgn}(x) + 35(cx^2 + b)^{\frac{3}{2}}Bb^2\operatorname{sgn}(x) + 21(cx^2 + b)^{\frac{5}{2}}Ac\operatorname{sgn}(x) - 35(cx^2 + b)^{\frac{3}{2}}Abc\operatorname{sgn}(x)}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out] -2/105\*(4\*B\*b^(7/2) - 7\*A\*b^(5/2)\*c)\*sgn(x)/c^3 + 1/105\*(15\*(c\*x^2 + b)^(7/2)\*B\*sgn(x) - 42\*(c\*x^2 + b)^(5/2)\*B\*b\*sgn(x) + 35\*(c\*x^2 + b)^(3/2)\*B\*b^2\*sgn(x) + 21\*(c\*x^2 + b)^(5/2)\*A\*c\*sgn(x) - 35\*(c\*x^2 + b)^(3/2)\*A\*b\*c\*sgn(x))/c^3

**maple [A]** time = 0.05, size = 67, normalized size = 0.71

$$-\frac{(cx^2 + b)(-15Bc^2x^4 - 21Ac^2x^2 + 12Bbcx^2 + 14Abc - 8Bb^2)\sqrt{cx^4 + bx^2}}{105c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x)

[Out] -1/105\*(c\*x^2+b)\*(-15\*B\*c^2\*x^4-21\*A\*c^2\*x^2+12\*B\*b\*c\*x^2+14\*A\*b\*c-8\*B\*b^2)\*(c\*x^4+b\*x^2)^(1/2)/c^3/x



**maxima [A]** time = 1.54, size = 83, normalized size = 0.88

$$\frac{(3c^2x^4 + bcx^2 - 2b^2)\sqrt{cx^2 + b}A}{15c^2} + \frac{(15c^3x^6 + 3bc^2x^4 - 4b^2cx^2 + 8b^3)\sqrt{cx^2 + b}B}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/15\*(3\*c^2\*x^4 + b\*c\*x^2 - 2\*b^2)\*sqrt(c\*x^2 + b)\*A/c^2 + 1/105\*(15\*c^3\*x^6 + 3\*b\*c^2\*x^4 - 4\*b^2\*c\*x^2 + 8\*b^3)\*sqrt(c\*x^2 + b)\*B/c^3

**mupad [B]** time = 0.19, size = 83, normalized size = 0.88

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{Bx^6}{7} + \frac{8Bb^3 - 14Ab^2c}{105c^3} + \frac{x^4(21Ac^3 + 3Bbc^2)}{105c^3} + \frac{bx^2(7Ac - 4Bb)}{105c^2} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2),x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*((B\*x^6)/7 + (8\*B\*b^3 - 14\*A\*b^2\*c)/(105\*c^3) + (x^4\*(21\*A\*c^3 + 3\*B\*b\*c^2))/(105\*c^3) + (b\*x^2\*(7\*A\*c - 4\*B\*b))/(105\*c^2)))/x

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{x^2 (b + cx^2)} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*2\*sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2), x)

### 3.102 $\int (A + Bx^2) \sqrt{bx^2 + cx^4} dx$

Optimal. Leaf size=61

$$\frac{B(bx^2 + cx^4)^{3/2}}{5cx} - \frac{(bx^2 + cx^4)^{3/2} (2bB - 5Ac)}{15c^2x^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {1145, 2000}

$$\frac{B(bx^2 + cx^4)^{3/2}}{5cx} - \frac{(bx^2 + cx^4)^{3/2} (2bB - 5Ac)}{15c^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] -((2\*b\*B - 5\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(15\*c^2\*x^3) + (B\*(b\*x^2 + c\*x^4)^(3/2))/(5\*c\*x)

#### Rule 1145

Int[((d\_) + (e\_.)\*(x\_)^2)\*((b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> Simp[(e\*(b\*x^2 + c\*x^4)^(p + 1))/(c\*(4\*p + 3)\*x), x] - Dist[(b\*e\*(2\*p + 1) - c\*d\*(4\*p + 3))/(c\*(4\*p + 3)), Int[(b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{b, c, d, e, p}, x] && !IntegerQ[p] && NeQ[4\*p + 3, 0] && NeQ[b\*e\*(2\*p + 1) - c\*d\*(4\*p + 3), 0]

#### Rule 2000

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a\*x^j + b\*x^n)^(p + 1)/(b\*(n - j)\*(p + 1)\*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j\*p - n + j + 1, 0]

#### Rubi steps

$$\begin{aligned} \int (A + Bx^2) \sqrt{bx^2 + cx^4} dx &= \frac{B(bx^2 + cx^4)^{3/2}}{5cx} - \frac{(2bB - 5Ac) \int \sqrt{bx^2 + cx^4} dx}{5c} \\ &= -\frac{(2bB - 5Ac)(bx^2 + cx^4)^{3/2}}{15c^2x^3} + \frac{B(bx^2 + cx^4)^{3/2}}{5cx} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 0.67

$$\frac{(x^2(b + cx^2))^{3/2} (5Ac - 2bB + 3Bcx^2)}{15c^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4],x]

[Out] ((x^2\*(b + c\*x^2))^(3/2)\*(-2\*b\*B + 5\*A\*c + 3\*B\*c\*x^2))/(15\*c^2\*x^3)

**IntegrateAlgebraic [A]** time = 0.07, size = 41, normalized size = 0.67

$$\frac{(bx^2 + cx^4)^{3/2} (5Ac - 2bB + 3Bcx^2)}{15c^2x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4], x]

[Out]  $((-2*b*B + 5*A*c + 3*B*c*x^2)*(b*x^2 + c*x^4)^{(3/2)})/(15*c^2*x^3)$

**fricas** [A] time = 0.41, size = 57, normalized size = 0.93

$$\frac{(3 B c^2 x^4 - 2 B b^2 + 5 A b c + (B b c + 5 A c^2) x^2) \sqrt{c x^4 + b x^2}}{15 c^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out]  $1/15*(3*B*c^2*x^4 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x^2)*\text{sqrt}(c*x^4 + b*x^2)/(c^2*x)$

**giac** [A] time = 0.16, size = 72, normalized size = 1.18

$$\frac{(2 B b^{\frac{5}{2}} - 5 A b^{\frac{3}{2}} c) \text{sgn}(x)}{15 c^2} + \frac{3 (c x^2 + b)^{\frac{5}{2}} B \text{sgn}(x) - 5 (c x^2 + b)^{\frac{3}{2}} B b \text{sgn}(x) + 5 (c x^2 + b)^{\frac{3}{2}} A c \text{sgn}(x)}{15 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out]  $1/15*(2*B*b^{(5/2)} - 5*A*b^{(3/2)}*c)*\text{sgn}(x)/c^2 + 1/15*(3*(c*x^2 + b)^{(5/2)}*B*\text{sgn}(x) - 5*(c*x^2 + b)^{(3/2)}*B*b*\text{sgn}(x) + 5*(c*x^2 + b)^{(3/2)}*A*c*\text{sgn}(x))/c^2$

**maple** [A] time = 0.05, size = 45, normalized size = 0.74

$$\frac{(c x^2 + b) (3 B c x^2 + 5 A c - 2 b B) \sqrt{c x^4 + b x^2}}{15 c^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x)

[Out]  $1/15*(c*x^2+b)*(3*B*c*x^2+5*A*c-2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x$

**maxima** [A] time = 1.46, size = 51, normalized size = 0.84

$$\frac{(c x^2 + b)^{\frac{3}{2}} A}{3 c} + \frac{(3 c^2 x^4 + b c x^2 - 2 b^2) \sqrt{c x^2 + b} B}{15 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2), x, algorithm="maxima")

[Out]  $1/3*(c*x^2 + b)^{(3/2)}*A/c + 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*\text{sqrt}(c*x^2 + b)*B/c^2$

**mupad** [B] time = 0.15, size = 60, normalized size = 0.98

$$\frac{\sqrt{c x^4 + b x^2} \left( \frac{B x^4}{5} - \frac{2 B b^2 - 5 A b c}{15 c^2} + \frac{x^2 (5 A c^2 + B b c)}{15 c^2} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^4)/5 - (2*B*b^2 - 5*A*b*c)/(15*c^2) + (x^2*(5*A*c^2 + B*b*c))/(15*c^2)))/x
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{x^2(b + cx^2)} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

$$3.103 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} dx$$

**Optimal.** Leaf size=78

$$\frac{A\sqrt{bx^2+cx^4}}{x} - A\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

**Rubi [A]** time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2039, 2021, 2008, 206}

$$\frac{A\sqrt{bx^2+cx^4}}{x} - A\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^2,x]

[Out] (A\*Sqrt[b\*x^2 + c\*x^4])/x + (B\*(b\*x^2 + c\*x^4)^(3/2))/(3\*c\*x^3) - A\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2021

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+n\*p+1)), x] + Dist[(a\*(n-j)\*p)/(c^j\*(m+n\*p+1)), Int[(c\*x)^(m+j)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m+n\*p+1, 0]

#### Rule 2039

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(b\*(m+n+p\*(j+n)+1)), x] - Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(b\*(m+n+p\*(j+n)+1)), Int[(e\*x)^m\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m+n+p\*(j+n)+1, 0] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^2} dx &= \frac{B(bx^2 + cx^4)^{3/2}}{3cx^3} + A \int \frac{\sqrt{bx^2 + cx^4}}{x^2} dx \\
&= \frac{A\sqrt{bx^2 + cx^4}}{x} + \frac{B(bx^2 + cx^4)^{3/2}}{3cx^3} + (Ab) \int \frac{1}{\sqrt{bx^2 + cx^4}} dx \\
&= \frac{A\sqrt{bx^2 + cx^4}}{x} + \frac{B(bx^2 + cx^4)^{3/2}}{3cx^3} - (Ab) \operatorname{Subst} \left( \int \frac{1}{1 - bx^2} dx, x, \frac{x}{\sqrt{bx^2 + cx^4}} \right) \\
&= \frac{A\sqrt{bx^2 + cx^4}}{x} + \frac{B(bx^2 + cx^4)^{3/2}}{3cx^3} - A\sqrt{b} \tanh^{-1} \left( \frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 84, normalized size = 1.08

$$\frac{x \left( (b + cx^2)(3Ac + bB + Bcx^2) - 3A\sqrt{b}c\sqrt{b + cx^2} \tanh^{-1} \left( \frac{\sqrt{b+cx^2}}{\sqrt{b}} \right) \right)}{3c\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^2,x]

[Out] (x\*((b + c\*x^2)\*(b\*B + 3\*A\*c + B\*c\*x^2) - 3\*A\*Sqrt[b]\*c\*Sqrt[b + c\*x^2]\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]]))/(3\*c\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.14, size = 71, normalized size = 0.91

$$\frac{\sqrt{bx^2 + cx^4} (3Ac + bB + Bcx^2)}{3cx} - A\sqrt{b} \tanh^{-1} \left( \frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^2,x]

[Out] ((b\*B + 3\*A\*c + B\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/((3\*c\*x) - A\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])

**fricas [A]** time = 0.41, size = 159, normalized size = 2.04

$$\left[ \frac{3A\sqrt{b}cx \log\left(\frac{-cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4+bx^2}(Bcx^2+Bb+3Ac)}{6cx}, \frac{3A\sqrt{-b}cx \arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) + \sqrt{cx^4+bx^2}(Bcx^2+Bb+3Ac)}{3cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/6\*(3\*A\*sqrt(b)\*c\*x\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*(B\*c\*x^2 + B\*b + 3\*A\*c))/(c\*x), 1/3\*(3\*A\*sqrt(-b)\*c\*x\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + sqrt(c\*x^4 + b\*x^2)\*(B\*c\*x^2 + B\*b + 3\*A\*c))/(c\*x)]

**giac [A]** time = 0.20, size = 116, normalized size = 1.49

$$\frac{Ab \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right) \operatorname{sgn}(x)}{\sqrt{-b}} - \frac{(3Abc \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + B\sqrt{-b}b^{\frac{3}{2}} + 3A\sqrt{-b}\sqrt{b}c) \operatorname{sgn}(x)}{3\sqrt{-b}c} + \frac{(cx^2 + b)^{\frac{3}{2}} Bc^2 \operatorname{sgn}(x) + 3\sqrt{cx^2 + b} Ac^3 \operatorname{sgn}(x)}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^2,x, algorithm="giac")

[Out] A\*b\*arctan(sqrt(c\*x^2 + b)/sqrt(-b))\*sgn(x)/sqrt(-b) - 1/3\*(3\*A\*b\*c\*arctan(sqrt(b)/sqrt(-b)) + B\*sqrt(-b)\*b^(3/2) + 3\*A\*sqrt(-b)\*sqrt(b)\*c)\*sgn(x)/(sqrt(-b)\*c) + 1/3\*((c\*x^2 + b)^(3/2)\*B\*c^2\*sgn(x) + 3\*sqrt(c\*x^2 + b)\*A\*c^3\*sgn(x))/c^3

**maple** [A] time = 0.05, size = 85, normalized size = 1.09

$$\frac{\sqrt{cx^4 + bx^2} \left( 3A\sqrt{b} c \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 3\sqrt{cx^2 + b} Ac - (cx^2 + b)^{\frac{3}{2}} B \right)}{3\sqrt{cx^2 + b} cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^2,x)

[Out] -1/3\*(c\*x^4+b\*x^2)^(1/2)\*(3\*A\*ln(2\*(b^(1/2)\*(c\*x^2+b)^(1/2)+b)/x)\*b^(1/2)\*c - B\*(c\*x^2+b)^(3/2)-3\*A\*(c\*x^2+b)^(1/2)\*c)/x/(c\*x^2+b)^(1/2)/c

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$A \int \frac{\sqrt{cx^2 + b}}{x} dx + \frac{(cx^2 + b)^{\frac{3}{2}} B}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^2,x, algorithm="maxima")

[Out] A\*integrate(sqrt(c\*x^2 + b)/x, x) + 1/3\*(c\*x^2 + b)^(3/2)\*B/c

**mupad** [B] time = 0.50, size = 99, normalized size = 1.27

$$\frac{A \sqrt{cx^4 + bx^2}}{x} + \frac{B (cx^2 + b) \sqrt{cx^4 + bx^2}}{3cx} + \frac{A \sqrt{b} \operatorname{asin}\left(\frac{\sqrt{b} 1i}{\sqrt{c} x}\right) \sqrt{cx^4 + bx^2} 1i}{\sqrt{c} x^2 \sqrt{\frac{b}{cx^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^2,x)

[Out] (A\*(b\*x^2 + c\*x^4)^(1/2))/x + (B\*(b + c\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/(3\*c\*x) + (A\*b^(1/2)\*asin((b^(1/2)\*1i)/(c^(1/2)\*x))\*(b\*x^2 + c\*x^4)^(1/2)\*1i)/(c^(1/2)\*x^2\*(b/(c\*x^2) + 1)^(1/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)} (A + Bx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*2,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*2, x)

$$3.104 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4} dx$$

**Optimal.** Leaf size=100

$$\frac{\sqrt{bx^2+cx^4}(Ac+2bB)}{2bx} - \frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

**Rubi [A]** time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2038, 2021, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}(Ac+2bB)}{2bx} - \frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^4,x]

[Out] ((2\*b\*B + A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(2\*b\*x) - (A\*(b\*x^2 + c\*x^4)^(3/2))/(2\*b\*x^5) - ((2\*b\*B + A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(2\*Sqrt[b])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2021

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+n\*p+1)), x] + Dist[(a\*(n-j)\*p)/(c^j\*(m+n\*p+1)), Int[(c\*x)^(m+j)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m+n\*p+1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m+j\*p, -1] || (IntegersQ[m-1/2, p-1/2] && LtQ[p, 0] && LtQ[m, -(n\*p)-1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m+j\*p+1, 0] && NeQ[m-n+j\*p+1, 0]

#### Rubi steps



$$\begin{aligned}
\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^4} dx &= -\frac{A(bx^2 + cx^4)^{3/2}}{2bx^5} - \frac{(-2bB - Ac) \int \frac{\sqrt{bx^2 + cx^4}}{x^2} dx}{2b} \\
&= \frac{(2bB + Ac)\sqrt{bx^2 + cx^4}}{2bx} - \frac{A(bx^2 + cx^4)^{3/2}}{2bx^5} - \frac{1}{2}(-2bB - Ac) \int \frac{1}{\sqrt{bx^2 + cx^4}} dx \\
&= \frac{(2bB + Ac)\sqrt{bx^2 + cx^4}}{2bx} - \frac{A(bx^2 + cx^4)^{3/2}}{2bx^5} - \frac{1}{2}(2bB + Ac) \operatorname{Subst}\left(\int \frac{1}{1 - bx^2} dx, \frac{\sqrt{bx^2 + cx^4}}{x}\right) \\
&= \frac{(2bB + Ac)\sqrt{bx^2 + cx^4}}{2bx} - \frac{A(bx^2 + cx^4)^{3/2}}{2bx^5} - \frac{(2bB + Ac) \tanh^{-1}\left(\frac{\sqrt{bx^2 + cx^4}}{x}\right)}{2\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 94, normalized size = 0.94

$$\frac{\sqrt{x^2(b + cx^2)} \left( \sqrt{b} (A - 2Bx^2) \sqrt{b + cx^2} + x^2(Ac + 2bB) \tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right) \right)}{2\sqrt{b} x^3 \sqrt{b + cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^4,x]

[Out] -1/2\*(Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[b]\*(A - 2\*B\*x^2)\*Sqrt[b + c\*x^2] + (2\*b\*B + A\*c)\*x^2\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]]))/(Sqrt[b]\*x^3\*Sqrt[b + c\*x^2])

**IntegrateAlgebraic [A]** time = 0.18, size = 74, normalized size = 0.74

$$\frac{(-Ac - 2bB) \tanh^{-1}\left(\frac{\sqrt{bx^2 + cx^4}}{x}\right)}{2\sqrt{b}} + \frac{(2Bx^2 - A) \sqrt{bx^2 + cx^4}}{2x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^4,x]

[Out] (((-A + 2\*B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(2\*x^3) + ((-2\*b\*B - A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(2\*Sqrt[b]))

**fricas [A]** time = 0.43, size = 169, normalized size = 1.69

$$\left[ \frac{(2Bb + Ac)\sqrt{b}x^3 \log\left(-\frac{cx^3 + 2bx - 2\sqrt{cx^4 + bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4 + bx^2}(2Bbx^2 - Ab)}{4bx^3}, \frac{(2Bb + Ac)\sqrt{-b}x^3 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx}\right) + \sqrt{cx^4 + bx^2}(2Bbx^2 - Ab)}{2bx^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] [1/4\*((2\*B\*b + A\*c)\*sqrt(b)\*x^3\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*(2\*B\*b\*x^2 - A\*b)/(b\*x^3), 1/2\*((2\*B\*b + A\*c)\*sqrt(-b)\*x^3\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + sqrt(c\*x^4 + b\*x^2)\*(2\*B\*b\*x^2 - A\*b)/(b\*x^3)]

**giac [A]** time = 0.22, size = 76, normalized size = 0.76

$$\frac{2\sqrt{cx^2 + b} Bc \operatorname{sgn}(x) + \frac{(2Bb \operatorname{sgn}(x) + Ac^2 \operatorname{sgn}(x)) \arctan\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx^2 + b} Ac \operatorname{sgn}(x)}{x^2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^4,x, algorithm="giac")

[Out]  $\frac{1}{2}*(2*\sqrt{c*x^2 + b}*B*c*\text{sgn}(x) + (2*B*b*c*\text{sgn}(x) + A*c^2*\text{sgn}(x))*\arctan(\sqrt{c*x^2 + b}/\sqrt{-b}))/\sqrt{-b} - \sqrt{c*x^2 + b}*A*c*\text{sgn}(x)/x^2)/c$

**maple** [A] time = 0.05, size = 135, normalized size = 1.35

$$\frac{\sqrt{cx^4 + bx^2} \left( A\sqrt{b} c x^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) + 2B b^{\frac{3}{2}} x^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - \sqrt{cx^2 + b} A c x^2 - 2\sqrt{cx^2 + b} B b x^2 + (cx^2 + b)^{\frac{3}{2}} A \right)}{2\sqrt{cx^2 + b} b x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^4,x)

[Out]  $-1/2*(c*x^4+b*x^2)^{(1/2)}*(A*b^{(1/2)}*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)}))/x)*x^2*c+2*B*b^{(3/2)}*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)}))/x)*x^2-A*(c*x^2+b)^{(1/2)}*x^2*c-2*B*(c*x^2+b)^{(1/2)}*x^2*b+A*(c*x^2+b)^{(3/2)}/x^3/(c*x^2+b)^{(1/2)}/b$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^4 + b\*x^2)\*(B\*x^2 + A)/x^4, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A) \sqrt{cx^4 + bx^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^4,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(b + cx^2)} (A + Bx^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*4,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*4, x)

$$3.105 \quad \int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6} dx$$

**Optimal.** Leaf size=103

$$\frac{c(4bB - Ac) \tanh^{-1}\left(\frac{\sqrt{bx^2+cx^4}}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}} - \frac{\sqrt{bx^2+cx^4}(4bB - Ac)}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

**Rubi [A]** time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2038, 2020, 2008, 206}

$$\frac{c(4bB - Ac) \tanh^{-1}\left(\frac{\sqrt{bx^2+cx^4}}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}} - \frac{\sqrt{bx^2+cx^4}(4bB - Ac)}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^6,x]

[Out] -((4\*b\*B - A\*c)\*Sqrt[b\*x^2 + c\*x^4]/(8\*b\*x^3) - (A\*(b\*x^2 + c\*x^4)^(3/2))/(4\*b\*x^7) - (c\*(4\*b\*B - A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]]/(8\*b^(3/2)))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2020

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+j\*p+1)), x] - Dist[(b\*p\*(n-j))/(c^n\*(m+j\*p+1)), Int[(c\*x)^(m+n)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j\*p+1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m+j\*p, -1] || (IntegersQ[m-1/2, p-1/2] && LtQ[p, 0] && LtQ[m, -(n\*p)-1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m+j\*p+1, 0] && NeQ[m-n+j\*p+1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^6} dx &= -\frac{A(bx^2 + cx^4)^{3/2}}{4bx^7} - \frac{(-4bB + Ac) \int \frac{\sqrt{bx^2 + cx^4}}{x^4} dx}{4b} \\
&= -\frac{(4bB - Ac)\sqrt{bx^2 + cx^4}}{8bx^3} - \frac{A(bx^2 + cx^4)^{3/2}}{4bx^7} + \frac{(c(4bB - Ac)) \int \frac{1}{\sqrt{bx^2 + cx^4}} dx}{8b} \\
&= -\frac{(4bB - Ac)\sqrt{bx^2 + cx^4}}{8bx^3} - \frac{A(bx^2 + cx^4)^{3/2}}{4bx^7} - \frac{(c(4bB - Ac)) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x\right)}{8b} \\
&= -\frac{(4bB - Ac)\sqrt{bx^2 + cx^4}}{8bx^3} - \frac{A(bx^2 + cx^4)^{3/2}}{4bx^7} - \frac{c(4bB - Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}}\right)}{8b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 95, normalized size = 0.92

$$-\frac{(b + cx^2)(2Ab + Acx^2 + 4bBx^2) + cx^4 \sqrt{\frac{cx^2}{b} + 1} (4bB - Ac) \tanh^{-1}\left(\sqrt{\frac{cx^2}{b} + 1}\right)}{8bx^3 \sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^6,x]

[Out] -1/8\*((b + c\*x^2)\*(2\*A\*b + 4\*b\*B\*x^2 + A\*c\*x^2) + c\*(4\*b\*B - A\*c)\*x^4\*Sqrt[1 + (c\*x^2)/b]\*ArcTanh[Sqrt[1 + (c\*x^2)/b]])/(b\*x^3\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.21, size = 88, normalized size = 0.85

$$\frac{(Ac^2 - 4bBc) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}}\right)}{8b^{3/2}} + \frac{\sqrt{bx^2 + cx^4} (-2Ab - Acx^2 - 4bBx^2)}{8bx^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/x^6,x]

[Out] ((-2\*A\*b - 4\*b\*B\*x^2 - A\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(8\*b\*x^5) + ((-4\*b\*B\*c + A\*c^2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(8\*b^(3/2))

**fricas [A]** time = 0.43, size = 198, normalized size = 1.92

$$\left[ \frac{(4Bbc - Ac^2)\sqrt{b}x^5 \log\left(\frac{-cx^3 + 2bx^2 + \sqrt{cx^4 + bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4 + bx^2}(2Ab^2 + (4Bb^2 + Abc)x^2)}{16b^2x^5}, \frac{(4Bbc - Ac^2)\sqrt{-b}x^5 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx}\right) - \sqrt{cx^4 + bx^2}(2Ab^2 + (4Bb^2 + Abc)x^2)}{8b^2x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^6,x, algorithm="fricas")

[Out] [-1/16\*((4\*B\*b\*c - A\*c^2)\*sqrt(b)\*x^5\*log(-(c\*x^3 + 2\*b\*x + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*(2\*A\*b^2 + (4\*B\*b^2 + A\*b\*c)\*x^2))/(b^2\*x^5), 1/8\*((4\*B\*b\*c - A\*c^2)\*sqrt(-b)\*x^5\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) - sqrt(c\*x^4 + b\*x^2)\*(2\*A\*b^2 + (4\*B\*b^2 + A\*b\*c)\*x^2))/(b^2\*x^5)]

**giac [A]** time = 0.30, size = 132, normalized size = 1.28

$$\frac{(4Bbc^2\text{sgn}(x) - Ac^3\text{sgn}(x)) \arctan\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4(cx^2 + b)^{\frac{3}{2}} Bbc^2\text{sgn}(x) - 4\sqrt{cx^2 + b} Bb^2c^2\text{sgn}(x) + (cx^2 + b)^{\frac{3}{2}} Ac^3\text{sgn}(x) + \sqrt{cx^2 + b} Abc^3\text{sgn}(x)}{bc^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^6,x, algorithm="giac")

[Out]  $\frac{1}{8} * ((4 * B * b * c^2 * \text{sgn}(x) - A * c^3 * \text{sgn}(x)) * \arctan(\sqrt{c * x^2 + b} / \sqrt{-b}) / (\sqrt{-b} * b) - (4 * (c * x^2 + b)^{(3/2)} * B * b * c^2 * \text{sgn}(x) - 4 * \sqrt{c * x^2 + b} * B * b^2 * c^2 * \text{sgn}(x) + (c * x^2 + b)^{(3/2)} * A * c^3 * \text{sgn}(x) + \sqrt{c * x^2 + b} * A * b * c^3 * \text{sgn}(x)) / (b * c^2 * x^4)) / c$

**maple [A]** time = 0.06, size = 174, normalized size = 1.69

$$\frac{\sqrt{cx^4 + bx^2} \left( A\sqrt{b} c^2 x^4 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 4Bb^3 c x^4 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - \sqrt{cx^2+b} A c^2 x^4 + 4\sqrt{cx^2+b} B b c x^4 + (cx^2+b)^{\frac{3}{2}} A c x^2 - 4(cx^2+b)^{\frac{3}{2}} B b x^2 - 2(cx^2+b)^{\frac{3}{2}} A b \right)}{8\sqrt{cx^2+b} b^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^6,x)

[Out]  $\frac{1}{8} * (c * x^4 + b * x^2)^{(1/2)} * (A * b^{(1/2)} * \ln(2 * (b + (c * x^2 + b)^{(1/2)} * b^{(1/2)}) / x) * x^4 * c^2 - 4 * B * b^{(3/2)} * \ln(2 * (b + (c * x^2 + b)^{(1/2)} * b^{(1/2)}) / x) * x^4 * c - A * (c * x^2 + b)^{(1/2)} * x^4 * c^2 + 4 * B * (c * x^2 + b)^{(1/2)} * x^4 * b * c + A * (c * x^2 + b)^{(3/2)} * x^2 * c - 4 * B * (c * x^2 + b)^{(3/2)} * x^2 * b - 2 * A * (c * x^2 + b)^{(3/2)} * b) / x^5 / (c * x^2 + b)^{(1/2)} / b^2$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2} (Bx^2 + A)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(1/2)/x^6,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^4 + b\*x^2)\*(B\*x^2 + A)/x^6, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A) \sqrt{cx^4 + bx^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^6,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(1/2))/x^6, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)} (A + Bx^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(1/2)/x\*\*6,x)

[Out] Integral(sqrt(x\*\*2\*(b + c\*x\*\*2))\*(A + B\*x\*\*2)/x\*\*6, x)

$$3.106 \quad \int x^5 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$$

**Optimal.** Leaf size=223

$$\frac{b^6(9bB - 14Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}} + \frac{b^4(b + 2cx^2)\sqrt{bx^2 + cx^4}(9bB - 14Ac)}{2048c^5} - \frac{b^2(b + 2cx^2)(bx^2 + cx^4)^{3/2}(9bB - 14Ac)}{768c^4}$$

**Rubi [A]** time = 0.40, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$ , Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$\frac{b^4(b + 2cx^2)\sqrt{bx^2 + cx^4}(9bB - 14Ac)}{2048c^5} - \frac{b^2(b + 2cx^2)(bx^2 + cx^4)^{3/2}(9bB - 14Ac)}{768c^4} - \frac{b^6(9bB - 14Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}} + \frac{b(bx^2 + cx^4)^{5/2}(9bB - 14Ac)}{240c^3} - \frac{x^2(bx^2 + cx^4)^{5/2}(9bB - 14Ac)}{168c^2} + \frac{Bx^4(bx^2 + cx^4)^{5/2}}{14c}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x]

[Out] (b^4\*(9\*b\*B - 14\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(2048\*c^5) - (b^2\*(9\*b\*B - 14\*A\*c)\*(b + 2\*c\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/(768\*c^4) + (b\*(9\*b\*B - 14\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(240\*c^3) - ((9\*b\*B - 14\*A\*c)\*x^2\*(b\*x^2 + c\*x^4)^(5/2))/(168\*c^2) + (B\*x^4\*(b\*x^2 + c\*x^4)^(5/2))/(14\*c) - (b^6\*(9\*b\*B - 14\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(2048\*c^(11/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

### Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

### Rubi steps

$$\begin{aligned} \int x^5 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left( \int x^2 (A + Bx) (bx + cx^2)^{3/2} dx, x, x^2 \right) \\ &= \frac{Bx^4 (bx^2 + cx^4)^{5/2}}{14c} + \frac{\left( 2(-bB + Ac) + \frac{5}{2}(-bB + 2Ac) \right) \text{Subst} \left( \int x^2 (bx + cx^2)^{3/2} dx, x, x^2 \right)}{14c} \\ &= -\frac{(9bB - 14Ac)x^2 (bx^2 + cx^4)^{5/2}}{168c^2} + \frac{Bx^4 (bx^2 + cx^4)^{5/2}}{14c} + \frac{b(9bB - 14Ac)}{14c} \text{Subst} \left( \int x^2 (bx + cx^2)^{3/2} dx, x, x^2 \right) \\ &= \frac{b(9bB - 14Ac) (bx^2 + cx^4)^{5/2}}{240c^3} - \frac{(9bB - 14Ac)x^2 (bx^2 + cx^4)^{5/2}}{168c^2} + \frac{Bx^4 (bx^2 + cx^4)^{5/2}}{14c} \\ &= -\frac{b^2(9bB - 14Ac) (b + 2cx^2) (bx^2 + cx^4)^{3/2}}{768c^4} + \frac{b(9bB - 14Ac) (bx^2 + cx^4)^{5/2}}{240c^3} \\ &= \frac{b^4(9bB - 14Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{2048c^5} - \frac{b^2(9bB - 14Ac) (b + 2cx^2) (bx^2 + cx^4)^{5/2}}{768c^4} \\ &= \frac{b^4(9bB - 14Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{2048c^5} - \frac{b^2(9bB - 14Ac) (b + 2cx^2) (bx^2 + cx^4)^{5/2}}{768c^4} \\ &= \frac{b^4(9bB - 14Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{2048c^5} - \frac{b^2(9bB - 14Ac) (b + 2cx^2) (bx^2 + cx^4)^{5/2}}{768c^4} \end{aligned}$$

**Mathematica [A]** time = 0.36, size = 215, normalized size = 0.96

$$\frac{\sqrt{x^2(b+cx^2)} \left( \sqrt{cx^2+1} (-210b^5c(7A+3Bx^2) + 28b^4c^2x^2(35A+18Bx^2) - 16b^3c^3x^4(49A+27Bx^2) + 96b^2c^4x^6(7A+4Bx^2) + 256bc^5x^8(91A+75Bx^2) + 2560c^6x^{10}(7A+6Bx^2) + 945b^6B) - 105b^{11/2}(9bB-14Ac) \sinh^{-1}\left(\frac{\sqrt{cx^2+1}}{\sqrt{b}}\right) \right)}{215040c^{11/2}x\sqrt{cx^2+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(945\*b^6\*B - 210\*b^5\*c\*(7\*A + 3\*B\*x^2) + 96\*b^2\*c^4\*x^6\*(7\*A + 4\*B\*x^2) + 2560\*c^6\*x^10\*(7\*A + 6\*B\*x^2) + 28\*b^4\*c^2\*x^2\*(35\*A + 18\*B\*x^2) - 16\*b^3\*c^3\*x^4\*(49\*A + 27\*B\*x^2) + 256\*b\*c^5\*x^8\*(91\*A + 75\*B\*x^2)) - 105\*b^(11/2)\*(9\*b\*B - 14\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(215040\*c^(11/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.82, size = 211, normalized size = 0.95

$$\frac{(9b^7B - 14Ab^6c) \log\left(-2\sqrt{c}\sqrt{bx^2 + cx^4 + b + 2cx^2}\right) + \sqrt{bx^2 + cx^4} (-1470Ab^5c + 980Ab^4c^2x^2 - 784Ab^3c^3x^4 + 672Ab^2c^4x^6 + 23296Abc^5x^8 + 17920Ac^6x^{10} + 945b^6B - 630b^5Bcx^2 + 504b^4Bc^2x^4 - 432b^3Bc^3x^6 + 384b^2Bc^4x^8 + 19200bBc^5x^{10} + 15360Bc^6x^{12})}{4096c^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(945\*b^6\*B - 1470\*A\*b^5\*c - 630\*b^5\*B\*c\*x^2 + 980\*A\*b^4\*c^2\*x^2 + 504\*b^4\*B\*c^2\*x^4 - 784\*A\*b^3\*c^3\*x^4 - 432\*b^3\*B\*c^3\*x^6 + 672\*A\*b^2\*c^4\*x^6 + 384\*b^2\*B\*c^4\*x^8 + 23296\*A\*b\*c^5\*x^8 + 19200\*b\*B\*c^5\*x^10 + 17920\*A\*c^6\*x^10 + 15360\*B\*c^6\*x^12))/(215040\*c^5) + ((9\*b^7\*B - 14\*A\*b^6\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(4096\*c^(11/2))

**fricas** [A] time = 0.69, size = 418, normalized size = 1.87

$$\frac{105(9b^7 - 14Ab^6c)\sqrt{c}\log\left(\frac{b + 2cx^2 - 2\sqrt{c}\sqrt{bx^2 + cx^4}}{b + 2cx^2}\right) - 2(15360Bc^6 + 14A^2c^7)x^{10} + 128(3Bb^2c^5 + 182A^2b^2c^6)x^8 + 945Bb^6c - 1470A^2b^5c^2 - 48(9Bb^3c^4 - 14A^2b^2c^5)x^6 + 56(9Bb^4c^3 - 14A^2b^3c^4)x^4 - 70(9Bb^5c^2 - 14A^2b^4c^3)x^2}{215040c^5} + \frac{(9b^7B - 14Ab^6c)\sqrt{c}\sqrt{bx^2 + cx^4}}{4096c^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/430080\*(105\*(9\*B\*b^7 - 14\*A\*b^6\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(15360\*B\*c^7\*x^12 + 1280\*(15\*B\*b\*c^6 + 14\*A\*c^7)\*x^10 + 128\*(3\*B\*b^2\*c^5 + 182\*A\*b\*c^6)\*x^8 + 945\*B\*b^6\*c - 1470\*A\*b^5\*c^2 - 48\*(9\*B\*b^3\*c^4 - 14\*A\*b^2\*c^5)\*x^6 + 56\*(9\*B\*b^4\*c^3 - 14\*A\*b^3\*c^4)\*x^4 - 70\*(9\*B\*b^5\*c^2 - 14\*A\*b^4\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^6, 1/215040\*(105\*(9\*B\*b^7 - 14\*A\*b^6\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + (15360\*B\*c^7\*x^12 + 1280\*(15\*B\*b\*c^6 + 14\*A\*c^7)\*x^10 + 128\*(3\*B\*b^2\*c^5 + 182\*A\*b\*c^6)\*x^8 + 945\*B\*b^6\*c - 1470\*A\*b^5\*c^2 - 48\*(9\*B\*b^3\*c^4 - 14\*A\*b^2\*c^5)\*x^6 + 56\*(9\*B\*b^4\*c^3 - 14\*A\*b^3\*c^4)\*x^4 - 70\*(9\*B\*b^5\*c^2 - 14\*A\*b^4\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^6]

**giac** [A] time = 0.34, size = 280, normalized size = 1.26

$$\frac{1}{215040} \left[ \frac{105(9b^7 - 14Ab^6c)\sqrt{c}\log\left(\frac{-2cx^2 - b - 2\sqrt{c}\sqrt{bx^2 + cx^4}}{b + 2cx^2}\right)}{215040c^5} + \frac{(9b^7B - 14Ab^6c)\sqrt{c}\sqrt{bx^2 + cx^4}}{4096c^{11/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] 1/215040\*(2\*(4\*(2\*(8\*(10\*(12\*B\*c\*x^2\*sgn(x) + (15\*B\*b\*c^12\*sgn(x) + 14\*A\*c^13\*sgn(x)))/c^12)\*x^2 + (3\*B\*b^2\*c^11\*sgn(x) + 182\*A\*b\*c^12\*sgn(x)))/c^12)\*x^2 - 3\*(9\*B\*b^3\*c^10\*sgn(x) - 14\*A\*b^2\*c^11\*sgn(x))/c^12)\*x^2 + 7\*(9\*B\*b^4\*c^9\*sgn(x) - 14\*A\*b^3\*c^10\*sgn(x))/c^12)\*x^2 - 35\*(9\*B\*b^5\*c^8\*sgn(x) - 14\*A\*b^4\*c^9\*sgn(x))/c^12)\*sqrt(c\*x^2 + b)\*x + 1/2048\*(9\*B\*b^7\*sgn(x) - 14\*A\*b^6\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(11/2) - 1/4096\*(9\*B\*b^7\*log(abs(b)) - 14\*A\*b^6\*c\*log(abs(b)))\*sgn(x)/c^(11/2)

**maple** [A] time = 0.08, size = 328, normalized size = 1.47

$$\frac{(c^2 + b)^2(15360Bc^6 + 14A^2c^7)x^{12} + 128(3Bb^2c^5 + 182A^2b^2c^6)x^8 + 945Bb^6c - 1470A^2b^5c^2 - 48(9Bb^3c^4 - 14A^2b^2c^5)x^6 + 56(9Bb^4c^3 - 14A^2b^3c^4)x^4 - 70(9Bb^5c^2 - 14A^2b^4c^3)x^2}{215040c^5} + \frac{(9b^7B - 14Ab^6c)\sqrt{c}\sqrt{bx^2 + cx^4}}{4096c^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x)

[Out] 1/215040\*(c\*x^4+b\*x^2)^(3/2)\*(15360\*B\*(c\*x^2+b)^(5/2)\*c^(9/2)\*x^9+17920\*A\*(c\*x^2+b)^(5/2)\*c^(9/2)\*x^7-11520\*B\*(c\*x^2+b)^(5/2)\*c^(7/2)\*x^7\*b-12544\*A\*(c\*x^2+b)^(5/2)\*c^(7/2)\*x^5\*b+8064\*B\*(c\*x^2+b)^(5/2)\*c^(5/2)\*x^5\*b^2+7840\*A\*(c\*x^2+b)^(5/2)\*c^(5/2)\*x^3\*b^2-5040\*B\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x^3\*b^3-3920\*A\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x\*b^3+2520\*B\*(c\*x^2+b)^(5/2)\*c^(1/2)\*x\*b^4+980\*A\*(c\*x^2+b)^(3/2)\*c^(3/2)\*x\*b^4-630\*B\*(c\*x^2+b)^(3/2)\*c^(1/2)\*x\*b^5+1470\*A\*(c\*x^2+b)^(1/2)\*c^(3/2)\*x\*b^5-945\*B\*(c\*x^2+b)^(1/2)\*c^(1/2)\*x\*b^6+1470\*A\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^6\*c-945\*B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^7)/x^3/(c\*x^2+b)^(3/2)/c^(11/2)



**maxima [A]** time = 1.58, size = 363, normalized size = 1.63

$$\frac{1}{3078} \left( \frac{420\sqrt{c^2+32b^2}b^2}{c^2} - \frac{1120(c^4+b^2)^{3/2}}{c^2} - \frac{2560(c^4+b^2)^{3/2}}{c^2} - \frac{105b^6 \log(2c^2+b+2\sqrt{c^2+32b^2})}{c^2} - \frac{210\sqrt{c^2+32b^2}b^6}{c^2} - \frac{560(c^4+b^2)^{3/2}}{c^2} - \frac{1792(c^4+b^2)^{3/2}}{c^2} \right) A + \frac{1}{143360} \left( \frac{10240(c^4+b^2)^{5/2}}{c^2} - \frac{1260\sqrt{c^2+32b^2}b^2}{c^2} - \frac{3360(c^4+b^2)^{3/2}}{c^2} - \frac{7680(c^4+b^2)^{3/2}}{c^2} - \frac{335b^7 \log(2c^2+b+2\sqrt{c^2+32b^2})}{c^2} - \frac{630\sqrt{c^2+32b^2}b^6}{c^2} - \frac{1680(c^4+b^2)^{3/2}}{c^2} - \frac{5376(c^4+b^2)^{3/2}}{c^2} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out]  $-1/30720*(420*\sqrt{c*x^4 + b*x^2}*b^4*x^2/c^3 - 1120*(c*x^4 + b*x^2)^{(3/2)}*b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^{(5/2)}*x^2/c - 105*b^6*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2}*\sqrt{c}))/c^{(9/2)} + 210*\sqrt{c*x^4 + b*x^2}*b^5/c^4 - 560*(c*x^4 + b*x^2)^{(3/2)}*b^3/c^3 + 1792*(c*x^4 + b*x^2)^{(5/2)}*b/c^2)*A + 1/143360*(10240*(c*x^4 + b*x^2)^{(5/2)}*x^4/c + 1260*\sqrt{c*x^4 + b*x^2}*b^5*x^2/c^4 - 3360*(c*x^4 + b*x^2)^{(3/2)}*b^3*x^2/c^3 - 7680*(c*x^4 + b*x^2)^{(5/2)}*b*x^2/c^2 - 315*b^7*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2}*\sqrt{c}))/c^{(11/2)} + 630*\sqrt{c*x^4 + b*x^2}*b^6/c^5 - 1680*(c*x^4 + b*x^2)^{(3/2)}*b^4/c^4 + 5376*(c*x^4 + b*x^2)^{(5/2)}*b^2/c^3)*B$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 (Bx^2 + A) (cx^4 + bx^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x)

[Out] int(x^5\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (x^2 (b + cx^2))^{\frac{3}{2}} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*\*5\*(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2), x)

$$3.107 \quad \int x^3 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$$

**Optimal.** Leaf size=167

$$\frac{b^5(7bB - 12Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}} - \frac{b^3(b + 2cx^2) \sqrt{bx^2 + cx^4} (7bB - 12Ac)}{1024c^4} + \frac{b(b + 2cx^2) (bx^2 + cx^4)^{3/2} (7bB - 12Ac)}{384c^3}$$

**Rubi [A]** time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 779, 612, 620, 206}

$$-\frac{b^3(b + 2cx^2) \sqrt{bx^2 + cx^4} (7bB - 12Ac)}{1024c^4} + \frac{b^5(7bB - 12Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}} + \frac{b(b + 2cx^2) (bx^2 + cx^4)^{3/2} (7bB - 12Ac)}{384c^3} - \frac{(bx^2 + cx^4)^{5/2} (-12Ac + 7bB - 10Bcx^2)}{120c^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x]

[Out]  $-(b^3*(7*b*B - 12*A*c)*(b + 2*c*x^2)*\text{Sqrt}[b*x^2 + c*x^4])/(1024*c^4) + (b*(7*b*B - 12*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(384*c^3) - ((7*b*B - 12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/(120*c^2) + (b^5*(7*b*B - 12*A*c)*\text{ArcTanh}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[b*x^2 + c*x^4]])/(1024*c^(9/2))$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int x^3 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left( \int x(A + Bx) (bx + cx^2)^{3/2} dx, x, x^2 \right) \\
&= -\frac{(7bB - 12Ac - 10Bcx^2) (bx^2 + cx^4)^{5/2}}{120c^2} + \frac{(b(7bB - 12Ac)) \text{Subst} \left( \int (bx + cx^2)^{3/2} dx, x, x^2 \right)}{48c^2} \\
&= \frac{b(7bB - 12Ac) (b + 2cx^2) (bx^2 + cx^4)^{3/2}}{384c^3} - \frac{(7bB - 12Ac - 10Bcx^2) (bx^2 + cx^4)^{5/2}}{120c^2} \\
&= -\frac{b^3(7bB - 12Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{1024c^4} + \frac{b(7bB - 12Ac) (b + 2cx^2) (bx^2 + cx^4)^{3/2}}{384c^3} \\
&= -\frac{b^3(7bB - 12Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{1024c^4} + \frac{b(7bB - 12Ac) (b + 2cx^2) (bx^2 + cx^4)^{3/2}}{384c^3} \\
&= -\frac{b^3(7bB - 12Ac) (b + 2cx^2) \sqrt{bx^2 + cx^4}}{1024c^4} + \frac{b(7bB - 12Ac) (b + 2cx^2) (bx^2 + cx^4)^{3/2}}{384c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.32, size = 193, normalized size = 1.16

$$\frac{\sqrt{x^2(b+cx^2)} \left( 15b^{9/2}(7bB-12Ac) \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b}} \right) + \sqrt{cx} \sqrt{\frac{cx^2}{b}+1} (10b^4c(18A+7Bx^2) - 8b^3c^2x^2(15A+7Bx^2) + 48b^2c^3x^4(2A+Bx^2) + 64bc^4x^6(33A+26Bx^2) + 256c^5x^8(6A+5Bx^2) - 105b^5B) \right)}{15360c^{9/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(-105\*b^5\*B + 48\*b^2\*c^3\*x^4\*(2\*A + B\*x^2) + 256\*c^5\*x^8\*(6\*A + 5\*B\*x^2) - 8\*b^3\*c^2\*x^2\*(15\*A + 7\*B\*x^2) + 10\*b^4\*c\*(18\*A + 7\*B\*x^2) + 64\*b\*c^4\*x^6\*(33\*A + 26\*B\*x^2)) + 15\*b^(9/2)\*(7\*b\*B - 12\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(15360\*c^(9/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.70, size = 187, normalized size = 1.12

$$\frac{(12Ab^5c - 7b^6B) \log(-2\sqrt{c}\sqrt{bx^2 + cx^4} + b + 2cx^2)}{2048c^{9/2}} + \frac{\sqrt{bx^2 + cx^4} (180Ab^4c - 120Ab^3c^2x^2 + 96Ab^2c^3x^4 + 2112Abc^4x^6 + 1536Ac^5x^8 - 105b^5B + 70b^4Bcx^2 - 56b^3Bc^2x^4 + 48b^2Bc^3x^6 + 1664bBc^4x^8 + 1280Bc^5x^{10})}{15360c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-105\*b^5\*B + 180\*A\*b^4\*c + 70\*b^4\*B\*c\*x^2 - 120\*A\*b^3\*c^2\*x^2 - 56\*b^3\*B\*c^2\*x^4 + 96\*A\*b^2\*c^3\*x^4 + 48\*b^2\*B\*c^3\*x^6 + 2112\*A\*b\*c^4\*x^6 + 1664\*b\*B\*c^4\*x^8 + 1536\*A\*c^5\*x^8 + 1280\*B\*c^5\*x^10))/(15360\*c^4) + ((-7\*b^6\*B + 12\*A\*b^5\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(2048\*c^(9/2))

**fricas [A]** time = 0.56, size = 369, normalized size = 2.21

$$\frac{15(7b^6 - 12Ab^5c)\sqrt{c}\log(-2x^2 - b + 2\sqrt{c}\sqrt{bx^2 + cx^4}) - 2(1280Bc^5x^{10} + 128(13Bb^3c^5 + 12Ac^6)x^8 - 105Bb^5c + 180Ab^4c^2 + 48(7b^4c^3 - 12Ab^3c^2)x^4 + 10(7b^4c^3 - 12Ab^3c^2)x^4)\sqrt{c}\sqrt{bx^2 + cx^4} - 15(7b^6 - 12Ab^5c)\sqrt{c}\operatorname{arcsinh}\left(\frac{\sqrt{cx}}{\sqrt{b}}\right) - (1280Bc^5x^{10} + 128(13Bb^3c^5 + 12Ac^6)x^8 - 105Bb^5c + 180Ab^4c^2 + 48(7b^4c^3 - 12Ab^3c^2)x^4 + 10(7b^4c^3 - 12Ab^3c^2)x^4)\sqrt{c}\sqrt{bx^2 + cx^4}}{15360c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/30720\*(15\*(7\*B\*b^6 - 12\*A\*b^5\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(1280\*B\*c^6\*x^10 + 128\*(13\*B\*b\*c^5 + 12\*A\*c^6)\*x^8 - 105\*B\*b^5\*c + 180\*A\*b^4\*c^2 + 48\*(B\*b^2\*c^4 + 44\*A\*b\*c^5)\*x^6 - 8\*(7\*B\*b^3\*c^3 - 12\*A\*b^2\*c^4)\*x^4 + 10\*(7\*B\*b^4\*c^2 - 12\*A\*b^3\*c^3)\*x^2)\*sqrt(c\*x^4

+ b\*x^2))/c^5, -1/15360\*(15\*(7\*B\*b^6 - 12\*A\*b^5\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2))\*sqrt(-c)/(c\*x^2 + b) - (1280\*B\*c^6\*x^10 + 128\*(13\*B\*b\*c^5 + 12\*A\*c^6)\*x^8 - 105\*B\*b^5\*c + 180\*A\*b^4\*c^2 + 48\*(B\*b^2\*c^4 + 44\*A\*b\*c^5)\*x^6 - 8\*(7\*B\*b^3\*c^3 - 12\*A\*b^2\*c^4)\*x^4 + 10\*(7\*B\*b^4\*c^2 - 12\*A\*b^3\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^5]

**giac** [A] time = 0.23, size = 246, normalized size = 1.47

$$\frac{1}{15360} \left( \frac{1}{2} \left( \frac{1}{c^2} \left( 8 \left( 10 B c^2 \operatorname{sgn}(x) + \frac{13 B b c^3 \operatorname{sgn}(x) + 12 A c^4 \operatorname{sgn}(x)}{c^{10}} \right) \right) + \frac{3 \left( B b^2 c^9 \operatorname{sgn}(x) + 44 A b c^{10} \operatorname{sgn}(x) \right)}{c^{10}} \right) - \frac{7 B b^3 c^8 \operatorname{sgn}(x) - 12 A b^2 c^9 \operatorname{sgn}(x)}{c^{10}} \right) - \frac{5 \left( 7 B b^4 c^7 \operatorname{sgn}(x) - 12 A b^3 c^8 \operatorname{sgn}(x) \right)}{c^{10}} - \frac{15 \left( 7 B b^5 c^6 \operatorname{sgn}(x) - 12 A b^4 c^7 \operatorname{sgn}(x) \right)}{c^{10}} \right) \sqrt{c x^2 + b} - \frac{\left( 7 B b^6 \operatorname{sgn}(x) - 12 A b^5 c \operatorname{sgn}(x) \right) \log \left( \frac{-\sqrt{c} x + \sqrt{c x^2 + b}}{1024 c^2} \right) + \frac{\left( 7 B b^6 \log(0) - 12 A b^5 c \log(0) \right) \operatorname{sgn}(x)}{2048 c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] 1/15360\*(2\*(4\*(2\*(8\*(10\*B\*c\*x^2\*sgn(x) + (13\*B\*b\*c^10\*sgn(x) + 12\*A\*c^11\*sgn(x))/c^10)\*x^2 + 3\*(B\*b^2\*c^9\*sgn(x) + 44\*A\*b\*c^10\*sgn(x))/c^10)\*x^2 - (7\*B\*b^3\*c^8\*sgn(x) - 12\*A\*b^2\*c^9\*sgn(x))/c^10)\*x^2 + 5\*(7\*B\*b^4\*c^7\*sgn(x) - 12\*A\*b^3\*c^8\*sgn(x))/c^10)\*x^2 - 15\*(7\*B\*b^5\*c^6\*sgn(x) - 12\*A\*b^4\*c^7\*sgn(x))/c^10)\*sqrt(c\*x^2 + b)\*x - 1/1024\*(7\*B\*b^6\*sgn(x) - 12\*A\*b^5\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(9/2) + 1/2048\*(7\*B\*b^6\*log(abs(b)) - 12\*A\*b^5\*c\*log(abs(b)))\*sgn(x)/c^(9/2)

**maple** [A] time = 0.07, size = 286, normalized size = 1.71

$$\frac{(c x^4 + b x^2)^2 \left( 1280 (c x^2 + b)^2 B c^2 x^2 + 1536 (c x^2 + b)^2 A c^2 x^2 - 896 (c x^2 + b)^2 B b c^2 x^2 - 180 A b^3 c \ln(\sqrt{c} x + \sqrt{c x^2 + b}) + 105 B b^4 \ln(\sqrt{c} x + \sqrt{c x^2 + b}) - 180 \sqrt{c x^2 + b} A b^2 c^2 x - 960 (c x^2 + b)^2 A b c^2 x^2 + 105 \sqrt{c x^2 + b} B b^3 \sqrt{c} x + 560 (c x^2 + b)^2 B b^2 c^2 x^2 - 120 (c x^2 + b)^2 A b^3 c^2 x + 70 (c x^2 + b)^2 B b^4 \sqrt{c} x + 480 (c x^2 + b)^2 A b^2 c^2 x - 280 (c x^2 + b)^2 B b^3 \sqrt{c} x \right)}{15360 (c x^2 + b)^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x)

[Out] 1/15360\*(c\*x^4+b\*x^2)^(3/2)\*(1280\*B\*(c\*x^2+b)^(5/2)\*c^(7/2)\*x^7+1536\*A\*(c\*x^2+b)^(5/2)\*c^(7/2)\*x^5-896\*B\*(c\*x^2+b)^(5/2)\*c^(5/2)\*x^5\*b-960\*A\*(c\*x^2+b)^(5/2)\*c^(5/2)\*x^3\*b+560\*B\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x^3\*b^2+480\*A\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x\*b^2-280\*B\*(c\*x^2+b)^(5/2)\*c^(1/2)\*x\*b^3-120\*(c\*x^2+b)^(3/2)\*A\*b^3\*c^(3/2)\*x+70\*(c\*x^2+b)^(3/2)\*B\*b^4\*c^(1/2)\*x-180\*(c\*x^2+b)^(1/2)\*A\*b^4\*c^(3/2)\*x+105\*(c\*x^2+b)^(1/2)\*B\*b^5\*c^(1/2)\*x-180\*A\*b^5\*c\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))+105\*B\*b^6\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2)))/x^3/(c\*x^2+b)^(3/2)/c^(9/2)

**maxima** [B] time = 1.56, size = 315, normalized size = 1.89

$$\frac{1}{2560} \left( \frac{60 \sqrt{c^2 + b x^2} b^2}{c^2} - \frac{160 (c^2 + b x^2)^2 b^2}{c^2} - \frac{15 b^3 \log(2 c x^2 + b + 2 \sqrt{c^2 + b x^2} \sqrt{c})}{c^2} + \frac{30 \sqrt{c^2 + b x^2} b^4}{c^2} - \frac{80 (c^2 + b x^2)^2 b^2}{c^2} + \frac{256 (c^2 + b x^2)^2}{c^2} \right) A - \frac{1}{30720} \left( \frac{420 \sqrt{c^2 + b x^2} b^2}{c^2} - \frac{1120 (c^2 + b x^2)^2 b^2}{c^2} - \frac{2560 (c^2 + b x^2)^2}{c^2} - \frac{105 b^3 \log(2 c x^2 + b + 2 \sqrt{c^2 + b x^2} \sqrt{c})}{c^2} + \frac{210 \sqrt{c^2 + b x^2} b^5}{c^2} - \frac{560 (c^2 + b x^2)^2 b^3}{c^2} + \frac{1792 (c^2 + b x^2)^2 b}{c^2} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/2560\*(60\*sqrt(c\*x^4 + b\*x^2)\*b^3\*x^2/c^2 - 160\*(c\*x^4 + b\*x^2)^(3/2)\*b\*x^2/c - 15\*b^5\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(c))/c^(7/2) + 30\*sqrt(c\*x^4 + b\*x^2)\*b^4/c^3 - 80\*(c\*x^4 + b\*x^2)^(3/2)\*b^2/c^2 + 256\*(c\*x^4 + b\*x^2)^(5/2)/c)\*A - 1/30720\*(420\*sqrt(c\*x^4 + b\*x^2)\*b^4\*x^2/c^3 - 1120\*(c\*x^4 + b\*x^2)^(3/2)\*b^2\*x^2/c^2 - 2560\*(c\*x^4 + b\*x^2)^(5/2)\*x^2/c - 105\*b^6\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(c))/c^(9/2) + 210\*sqrt(c\*x^4 + b\*x^2)\*b^5/c^4 - 560\*(c\*x^4 + b\*x^2)^(3/2)\*b^3/c^3 + 1792\*(c\*x^4 + b\*x^2)^(5/2)\*b/c^2)\*B

**mapad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (B x^2 + A) (c x^4 + b x^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)`

[Out] `int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left( x^2 (b + cx^2) \right)^{\frac{3}{2}} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(3/2), x)`

[Out] `Integral(x**3*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)`

$$3.108 \quad \int x (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$$

**Optimal.** Leaf size=148

$$\frac{3b^4(bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}} + \frac{3b^2(b + 2cx^2) \sqrt{bx^2 + cx^4} (bB - 2Ac)}{256c^3} - \frac{(b + 2cx^2) (bx^2 + cx^4)^{3/2} (bB - 2Ac)}{32c^2}$$

**Rubi [A]** time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {2034, 640, 612, 620, 206}

$$\frac{3b^2(b + 2cx^2) \sqrt{bx^2 + cx^4} (bB - 2Ac)}{256c^3} - \frac{3b^4(bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}} - \frac{(b + 2cx^2) (bx^2 + cx^4)^{3/2} (bB - 2Ac)}{32c^2} + \frac{B(bx^2 + cx^4)^{5/2}}{10c}$$

Antiderivative was successfully verified.

[In] Int[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x]

[Out] (3\*b^2\*(b\*B - 2\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4]/(256\*c^3) - ((b\*B - 2\*A\*c)\*(b + 2\*c\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/(32\*c^2) + (B\*(b\*x^2 + c\*x^4)^(5/2))/(10\*c) - (3\*b^4\*(b\*B - 2\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]]/(256\*c^(7/2)))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int x(A+Bx^2)(bx^2+cx^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left( \int (A+Bx)(bx+cx^2)^{3/2} dx, x, x^2 \right) \\
&= \frac{B(bx^2+cx^4)^{5/2}}{10c} + \frac{(-bB+2Ac) \text{Subst} \left( \int (bx+cx^2)^{3/2} dx, x, x^2 \right)}{4c} \\
&= -\frac{(bB-2Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{32c^2} + \frac{B(bx^2+cx^4)^{5/2}}{10c} + \frac{(3b^2(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4})}{32c^2} \\
&= \frac{3b^2(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3} - \frac{(bB-2Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{32c^2} \\
&= \frac{3b^2(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3} - \frac{(bB-2Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{32c^2} \\
&= \frac{3b^2(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3} - \frac{(bB-2Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{32c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.27, size = 171, normalized size = 1.16

$$\frac{\sqrt{x^2(b+cx^2)} \left( \sqrt{cx^2/b+1} (-10b^3c(3A+Bx^2) + 4b^2c^2x^2(5A+2Bx^2) + 16bc^3x^4(15A+11Bx^2) + 32c^4x^6(5A+4Bx^2) + 15b^4B) - 15b^{7/2}(bB-2Ac) \sinh^{-1} \left( \frac{\sqrt{cx^2/b+1}}{\sqrt{b}} \right) \right)}{1280c^{7/2}x\sqrt{cx^2/b+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(15\*b^4\*B - 10\*b^3\*c\*(3\*A + B\*x^2) + 4\*b^2\*c^2\*x^2\*(5\*A + 2\*B\*x^2) + 32\*c^4\*x^6\*(5\*A + 4\*B\*x^2) + 16\*b\*c^3\*x^4\*(15\*A + 11\*B\*x^2)) - 15\*b^(7/2)\*(b\*B - 2\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(1280\*c^(7/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.70, size = 162, normalized size = 1.09

$$\frac{3(b^5B-2Ab^4c) \log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{512c^{7/2}} + \frac{\sqrt{bx^2+cx^4}(-30Ab^3c+20Ab^2c^2x^2+240Abc^3x^4+160Ac^4x^6+15b^4B-10b^3Bcx^2+8b^2Bc^2x^4+176bBc^3x^6+128Bc^4x^8)}{1280c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(15\*b^4\*B - 30\*A\*b^3\*c - 10\*b^3\*B\*c\*x^2 + 20\*A\*b^2\*c^2\*x^2 + 8\*b^2\*B\*c^2\*x^4 + 240\*A\*b\*c^3\*x^4 + 176\*b\*B\*c^3\*x^6 + 160\*A\*c^4\*x^6 + 128\*B\*c^4\*x^8))/(1280\*c^3) + (3\*(b^5\*B - 2\*A\*b^4\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(512\*c^(7/2))

**fricas [A]** time = 0.48, size = 316, normalized size = 2.14

$$\frac{15(b^5B-2Ab^4c)\sqrt{c}\log\left(-2cx^2-b-2\sqrt{cx^2+bx^4}\sqrt{c}\right)-2(128Bb^5x^8+16(11Bb^4c+10Ac^5)x^6+15Bb^4c-30A*b^3*c^2+8(Bb^2c^3+30A*b*c^4)x^4-10(Bb^3c^2-2Ab^2c^2)\sqrt{cx^2+bx^4})\sqrt{c}+15(b^5B-2Ab^4c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx^4}}{\sqrt{-c}}\right)+\frac{(128Bb^5x^8+16(11Bb^4c+10Ac^5)x^6+15Bb^4c-30A*b^3*c^2+8(Bb^2c^3+30A*b*c^4)x^4-10(Bb^3c^2-2Ab^2c^2)\sqrt{cx^2+bx^4})\sqrt{-c}}{1280c^4}}{2560c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/2560\*(15\*(B\*b^5 - 2\*A\*b^4\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(128\*B\*c^5\*x^8 + 16\*(11\*B\*b\*c^4 + 10\*A\*c^5)\*x^6 + 15\*B\*b^4\*c - 30\*A\*b^3\*c^2 + 8\*(B\*b^2\*c^3 + 30\*A\*b\*c^4)\*x^4 - 10\*(B\*b^3\*c^2 - 2\*A\*b^2\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^4, 1/1280\*(15\*(B\*b^5 - 2\*A\*b^4\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + (128\*B\*c^5\*x^8 + 1

$$6*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]$$

**giac** [A] time = 0.22, size = 207, normalized size = 1.40

$$\frac{1}{1280} \left( 4 \left( 2 \left( 8 B c^2 \operatorname{sgn}(x) + \frac{11 B b^6 \operatorname{sgn}(x) + 10 A c^5 \operatorname{sgn}(x)}{c^6} \right) x^2 + \frac{B b^2 c^7 \operatorname{sgn}(x) + 30 A b c^8 \operatorname{sgn}(x)}{c^8} \right) x^2 - \frac{5 (B b^3 c^2 \operatorname{sgn}(x) - 2 A b^2 c^3 \operatorname{sgn}(x))}{c^8} \right) x^2 + \frac{15 (B b^4 c^5 \operatorname{sgn}(x) - 2 A b^3 c^6 \operatorname{sgn}(x))}{c^8} \sqrt{c x^2 + b} x + \frac{3 (B b^5 \operatorname{sgn}(x) - 2 A b^4 c \operatorname{sgn}(x)) \log \left( \frac{-\sqrt{c x^2 + b}}{256 c^2} \right) - \frac{3 (B b^5 \log(|b|) - 2 A b^4 \log(|b|)) \operatorname{sgn}(x)}{512 c^2}}{512 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] 1/1280\*(2\*(4\*(2\*(8\*B\*c\*x^2\*sgn(x) + (11\*B\*b\*c^8\*sgn(x) + 10\*A\*c^9\*sgn(x)))/c^8)\*x^2 + (B\*b^2\*c^7\*sgn(x) + 30\*A\*b\*c^8\*sgn(x))/c^8)\*x^2 - 5\*(B\*b^3\*c^6\*sgn(x) - 2\*A\*b^2\*c^7\*sgn(x))/c^8)\*x^2 + 15\*(B\*b^4\*c^5\*sgn(x) - 2\*A\*b^3\*c^6\*sgn(x))/c^8)\*sqrt(c\*x^2 + b)\*x + 3/256\*(B\*b^5\*sgn(x) - 2\*A\*b^4\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(7/2) - 3/512\*(B\*b^5\*log(abs(b)) - 2\*A\*b^4\*c\*log(abs(b)))\*sgn(x)/c^(7/2)

**maple** [A] time = 0.06, size = 244, normalized size = 1.65

$$\frac{(c x^4 + b x^2)^{\frac{5}{2}} \left( 128 (c x^2 + b)^{\frac{5}{2}} B c^2 x^5 + 30 A b^4 c \ln(\sqrt{c} x + \sqrt{c x^2 + b}) - 15 B b^5 \ln(\sqrt{c} x + \sqrt{c x^2 + b}) + 30 \sqrt{c x^2 + b} A b^3 c^2 x + 160 (c x^2 + b)^{\frac{5}{2}} A c^2 x^3 - 15 \sqrt{c x^2 + b} B b^4 \sqrt{c} x - 80 (c x^2 + b)^{\frac{5}{2}} B b c^2 x^3 + 20 (c x^2 + b)^{\frac{5}{2}} A b^2 c^2 x - 10 (c x^2 + b)^{\frac{5}{2}} B b^3 \sqrt{c} x - 80 (c x^2 + b)^{\frac{5}{2}} A b c^2 x + 40 (c x^2 + b)^{\frac{5}{2}} B b^2 \sqrt{c} x \right)}{1280 (c x^2 + b)^{\frac{5}{2}} c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x)

[Out] 1/1280\*(c\*x^4+b\*x^2)^(3/2)\*(128\*B\*(c\*x^2+b)^(5/2)\*c^(5/2)\*x^5+160\*A\*(c\*x^2+b)^(5/2)\*c^(5/2)\*x^3-80\*B\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x^3\*b-80\*A\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x\*b+40\*B\*(c\*x^2+b)^(5/2)\*c^(1/2)\*x\*b^2+20\*(c\*x^2+b)^(3/2)\*A\*b^2\*c^(3/2)\*x-10\*(c\*x^2+b)^(3/2)\*B\*b^3\*c^(1/2)\*x+30\*(c\*x^2+b)^(1/2)\*A\*b^3\*c^(3/2)\*x-15\*(c\*x^2+b)^(1/2)\*B\*b^4\*c^(1/2)\*x+30\*A\*b^4\*c\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))-15\*B\*b^5\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2)))/x^3/(c\*x^2+b)^(3/2)/c^(7/2)

**maxima** [B] time = 1.51, size = 267, normalized size = 1.80

$$\frac{1}{256} \left( 32 (c x^4 + b x^2)^{\frac{5}{2}} x^2 - \frac{12 \sqrt{c x^4 + b x^2} b^2 x^2}{c} + \frac{3 b^4 \log(2 c x^2 + b + 2 \sqrt{c x^4 + b x^2} \sqrt{c})}{c^2} - \frac{6 \sqrt{c x^4 + b x^2} b^3}{c^2} + \frac{16 (c x^4 + b x^2)^{\frac{5}{2}} b}{c} \right) A + \frac{1}{2560} \left( \frac{60 \sqrt{c x^4 + b x^2} b^2 x^2}{c^2} - \frac{160 (c x^4 + b x^2)^{\frac{5}{2}} b x^2}{c} - \frac{15 b^5 \log(2 c x^2 + b + 2 \sqrt{c x^4 + b x^2} \sqrt{c})}{c^2} + \frac{30 \sqrt{c x^4 + b x^2} b^4}{c^3} - \frac{80 (c x^4 + b x^2)^{\frac{5}{2}} b^2}{c^2} + \frac{256 (c x^4 + b x^2)^{\frac{5}{2}}}{c} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/256\*(32\*(c\*x^4 + b\*x^2)^(3/2)\*x^2 - 12\*sqrt(c\*x^4 + b\*x^2)\*b^2\*x^2/c + 3\*b^4\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(5/2) - 6\*sqrt(c\*x^4 + b\*x^2)\*b^3/c^2 + 16\*(c\*x^4 + b\*x^2)^(3/2)\*b/c)\*A + 1/2560\*(60\*sqrt(c\*x^4 + b\*x^2)\*b^3\*x^2/c^2 - 160\*(c\*x^4 + b\*x^2)^(3/2)\*b\*x^2/c - 15\*b^5\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(7/2) + 30\*sqrt(c\*x^4 + b\*x^2)\*b^4/c^3 - 80\*(c\*x^4 + b\*x^2)^(3/2)\*b^2/c^2 + 256\*(c\*x^4 + b\*x^2)^(5/2)/c)\*B

**mupad** [B] time = 1.01, size = 236, normalized size = 1.59

$$\frac{B(c x^4 + b x^2)^{\frac{5}{2}}}{10 c} + \frac{A(c x^4 + b x^2)^{\frac{3}{2}} \left( c x^2 + \frac{b}{2} \right)}{8 c} - \frac{3 A b^2 \left( \left( \frac{b}{4 c} + \frac{x^2}{2} \right) \sqrt{c x^4 + b x^2} - \frac{b^2 \ln \left( \frac{c x^2 + \frac{b}{2} + \sqrt{c x^4 + b x^2}}{\sqrt{c}} + \sqrt{c x^4 + b x^2} \right)}{8 c^{\frac{3}{2}}} \right)}{32 c} - \frac{B b \left( \frac{x^2 (c x^4 + b x^2)^{\frac{3}{2}}}{4} - \frac{3 b^2 \left( \frac{(2 c x^2 + b) \sqrt{c x^4 + b x^2}}{4 c} - \frac{b^2 \ln \left( \frac{c x^2 + \frac{b}{2} + \sqrt{c x^4 + b x^2}}{\sqrt{c}} + \sqrt{c x^4 + b x^2} \right)}{8 c^{\frac{3}{2}}} \right)}{16 c} + \frac{b (c x^4 + b x^2)^{\frac{3}{2}}}{8 c} \right)}{4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x)

[Out] (B\*(b\*x^2 + c\*x^4)^(5/2))/(10\*c) + (A\*(b\*x^2 + c\*x^4)^(3/2)\*(b/2 + c\*x^2))/(8\*c) - (3\*A\*b^2\*((b/(4\*c) + x^2/2)\*(b\*x^2 + c\*x^4)^(1/2) - (b^2\*log((b/2 +



$$\frac{c*x^2/c^{(1/2)} + (b*x^2 + c*x^4)^{(1/2)}}{(8*c^{(3/2)})} / (32*c) - (B*b*((x^2 * (b*x^2 + c*x^4)^{(3/2)})/4 - (3*b^2*((b + 2*c*x^2)*(b*x^2 + c*x^4)^{(1/2)})/(4*c) - (b^2*\log((b/2 + c*x^2)/c^{(1/2)} + (b*x^2 + c*x^4)^{(1/2)})))/(8*c^{(3/2)})))/(16*c) + (b*(b*x^2 + c*x^4)^{(3/2)})/(8*c)))/(4*c)$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \left( x^2 (b + cx^2) \right)^{\frac{3}{2}} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*(x\*\*2\*(b + c\*x\*\*2))\*\* (3/2)\*(A + B\*x\*\*2), x)

$$3.109 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x} dx$$

**Optimal.** Leaf size=144

$$\frac{b^3(3bB - 8Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}} - \frac{b(b + 2cx^2)\sqrt{bx^2 + cx^4}(3bB - 8Ac)}{128c^2} - \frac{(bx^2 + cx^4)^{3/2}(3bB - 8Ac)}{48c} + \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2}$$

**Rubi [A]** time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 794, 664, 612, 620, 206}

$$\frac{b^3(3bB - 8Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}} - \frac{b(b + 2cx^2)\sqrt{bx^2 + cx^4}(3bB - 8Ac)}{128c^2} - \frac{(bx^2 + cx^4)^{3/2}(3bB - 8Ac)}{48c} + \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x,x]

[Out] -(b\*(3\*b\*B - 8\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(128\*c^2) - ((3\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(48\*c) + (B\*(b\*x^2 + c\*x^4)^(5/2))/(8\*c\*x^2) + (b^3\*(3\*b\*B - 8\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(128\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 664

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(p\*(2\*c\*d - b\*e))/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 794

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(c\*e\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[m + 2\*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])

Rule 2034

```
Int[(x_)^(m_)*((b_)*(x_)^(k_) + (a_)*(x_)^(j_))^(p_)*((c_) + (d_)*(x_)
^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x} dx = \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x} dx, x, x^2 \right)$$

$$= \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2} + \frac{(bB - Ac + \frac{5}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x} dx, x, x^2 \right)}{8c}$$

$$= -\frac{(3bB - 8Ac)(bx^2 + cx^4)^{3/2}}{48c} + \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2} - \frac{(b(3bB - 8Ac)) \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x} dx, x, x^2 \right)}{32c}$$

$$= -\frac{b(3bB - 8Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{128c^2} - \frac{(3bB - 8Ac)(bx^2 + cx^4)^{3/2}}{48c} + \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2}$$

$$= -\frac{b(3bB - 8Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{128c^2} - \frac{(3bB - 8Ac)(bx^2 + cx^4)^{3/2}}{48c} + \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2}$$

$$= -\frac{b(3bB - 8Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{128c^2} - \frac{(3bB - 8Ac)(bx^2 + cx^4)^{3/2}}{48c} + \frac{B(bx^2 + cx^4)^{5/2}}{8cx^2}$$

**Mathematica [A]** time = 0.24, size = 151, normalized size = 1.05

$$\frac{\sqrt{x^2(b + cx^2)} \left( 3b^{5/2}(3bB - 8Ac) \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b}} \right) + \sqrt{c} x \sqrt{\frac{cx^2}{b} + 1} (6b^2c(4A + Bx^2) + 8bc^2x^2(14A + 9Bx^2) + 16c^3x^4(4A + 3Bx^2) - 9b^3B) \right)}{384c^{5/2} x \sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x,x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(-9\*b^3\*B + 6\*b^2\*c\*(4\*A + B\*x^2) + 16\*c^3\*x^4\*(4\*A + 3\*B\*x^2) + 8\*b\*c^2\*x^2\*(14\*A + 9\*B\*x^2)) + 3\*b^(5/2)\*(3\*b\*B - 8\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(384\*c^(5/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.71, size = 139, normalized size = 0.97

$$\frac{(8Ab^3c - 3b^4B) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{256c^{5/2}} + \frac{\sqrt{bx^2 + cx^4} (24Ab^2c + 112Abc^2x^2 + 64Ac^3x^4 - 9b^3B + 6b^2Bcx^2 + 72bBc^2x^4 + 48Bc^3x^6)}{384c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x,x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-9\*b^3\*B + 24\*A\*b^2\*c + 6\*b^2\*B\*c\*x^2 + 112\*A\*b\*c^2\*x^2 + 72\*b\*B\*c^2\*x^4 + 64\*A\*c^3\*x^4 + 48\*B\*c^3\*x^6))/(384\*c^2) + ((-3\*b^4\*B + 8\*A\*b^3\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(256\*c^(5/2))

**fricas** [A] time = 0.47, size = 275, normalized size = 1.91

$$\frac{3(3Bb^4 - 8Ab^3c)\sqrt{c}\log(-2cx^2 - b + 2\sqrt{cx^4 + bx^2}\sqrt{c}) - 2(48Bc^4x^6 - 9B^2b^3c + 24A^2b^2c^2 + 8(9Bb^3 + 8A^4)x^4 + 2(3Bb^2c^2 + 56Abc^3)c^2)\sqrt{cx^4 + bx^2} - 3(3Bb^4 - 8Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (48Bc^4x^6 - 9B^2b^3c + 24A^2b^2c^2 + 8(9Bb^3 + 8A^4)x^4 + 2(3Bb^2c^2 + 56Abc^3)c^2)\sqrt{cx^4 + bx^2}}{768c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x,x, algorithm="fricas")

[Out] [-1/768\*(3\*(3\*B\*b^4 - 8\*A\*b^3\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(48\*B\*c^4\*x^6 - 9\*B\*b^3\*c + 24\*A\*b^2\*c^2 + 8\*(9\*B\*b\*c^3 + 8\*A\*c^4)\*x^4 + 2\*(3\*B\*b^2\*c^2 + 56\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^3, -1/384\*(3\*(3\*B\*b^4 - 8\*A\*b^3\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) - (48\*B\*c^4\*x^6 - 9\*B\*b^3\*c + 24\*A\*b^2\*c^2 + 8\*(9\*B\*b\*c^3 + 8\*A\*c^4)\*x^4 + 2\*(3\*B\*b^2\*c^2 + 56\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^3]

**giac** [A] time = 0.20, size = 178, normalized size = 1.24

$$\frac{1}{384}\left(2\left(4\left(6Bcx^2\operatorname{sgn}(x) + \frac{9Bbc^2\operatorname{sgn}(x) + 8Ac^2\operatorname{sgn}(x)}{c^6}\right)x^2 + \frac{3Bb^2c^2\operatorname{sgn}(x) + 56Abc^3\operatorname{sgn}(x)}{c^6}\right)x^2 - \frac{3(3Bb^4\operatorname{sgn}(x) - 8Ab^3c\operatorname{sgn}(x))}{c^6}\right)\sqrt{cx^2 + b} - \frac{(3Bb^4\operatorname{sgn}(x) - 8Ab^3c\operatorname{sgn}(x))\log\left(\frac{-\sqrt{cx^2 + b}}{\sqrt{cx^2 + b}}\right)}{128c^2} + \frac{(3Bb^4\log(|b|) - 8Ab^3c\log(|b|))\operatorname{sgn}(x)}{256c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x,x, algorithm="giac")

[Out] 1/384\*(2\*(4\*(6\*B\*c\*x^2\*sgn(x) + (9\*B\*b\*c^6\*sgn(x) + 8\*A\*c^7\*sgn(x))/c^6)\*x^2 + (3\*B\*b^2\*c^5\*sgn(x) + 56\*A\*b\*c^6\*sgn(x))/c^6)\*x^2 - 3\*(3\*B\*b^3\*c^4\*sgn(x) - 8\*A\*b^2\*c^5\*sgn(x))/c^6)\*sqrt(c\*x^2 + b)\*x - 1/128\*(3\*B\*b^4\*sgn(x) - 8\*A\*b^3\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(5/2) + 1/256\*(3\*B\*b^4\*log(abs(b)) - 8\*A\*b^3\*c\*log(abs(b)))\*sgn(x)/c^(5/2)

**maple** [A] time = 0.06, size = 202, normalized size = 1.40

$$\frac{(cx^4 + bx^2)^{\frac{3}{2}}\left(-24Ab^3c\ln(\sqrt{cx^2 + b}) + 9Bb^4\ln(\sqrt{cx^2 + b}) - 24\sqrt{cx^2 + b}Ab^2c^2x + 9\sqrt{cx^2 + b}Bb^3\sqrt{cx^2 + b} + 48(cx^2 + b)^{\frac{5}{2}}Bc^2x^3 - 16(cx^2 + b)^{\frac{3}{2}}Abc^2x + 6(cx^2 + b)^{\frac{3}{2}}Bb^2\sqrt{cx^2 + b} + 64(cx^2 + b)^{\frac{5}{2}}Ac^3x - 24(cx^2 + b)^{\frac{5}{2}}Bb\sqrt{cx^2 + b}\right)}{384(cx^2 + b)^{\frac{3}{2}}c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x,x)

[Out] 1/384\*(c\*x^4+b\*x^2)^(3/2)\*(48\*B\*c^(3/2)\*(c\*x^2+b)^(5/2)\*x^3+64\*A\*c^(3/2)\*(c\*x^2+b)^(5/2)\*x-24\*B\*c^(1/2)\*(c\*x^2+b)^(5/2)\*x\*b-16\*(c\*x^2+b)^(3/2)\*A\*b\*c^(3/2)\*x+6\*(c\*x^2+b)^(3/2)\*B\*b^2\*c^(1/2)\*x-24\*(c\*x^2+b)^(1/2)\*A\*b^2\*c^(3/2)\*x+9\*(c\*x^2+b)^(1/2)\*B\*b^3\*c^(1/2)\*x-24\*A\*b^3\*c\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))+9\*B\*b^4\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2)))/x^3/(c\*x^2+b)^(3/2)/c^(5/2)

**maxima** [A] time = 1.49, size = 216, normalized size = 1.50

$$\frac{1}{96}\left(12\sqrt{cx^4 + bx^2}bx^2 - \frac{3b^3\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{3}{2}}} + 16(cx^4 + bx^2)^{\frac{3}{2}} + \frac{6\sqrt{cx^4 + bx^2}b^2}{c}\right)A + \frac{1}{256}\left(32(cx^4 + bx^2)^{\frac{3}{2}}x^2 - \frac{12\sqrt{cx^4 + bx^2}b^2x^2}{c} + \frac{3b^4\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{3}{2}}} - \frac{6\sqrt{cx^4 + bx^2}b^3}{c^2} + \frac{16(cx^4 + bx^2)^{\frac{3}{2}}b}{c}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x,x, algorithm="maxima")

[Out] 1/96\*(12\*sqrt(c\*x^4 + b\*x^2)\*b\*x^2 - 3\*b^3\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(3/2) + 16\*(c\*x^4 + b\*x^2)^(3/2) + 6\*sqrt(c\*x^4 + b\*x^2)\*b^2/c)\*A + 1/256\*(32\*(c\*x^4 + b\*x^2)^(3/2)\*x^2 - 12\*sqrt(c\*x^4 + b\*x^2)\*b^2\*x^2/c + 3\*b^4\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(5/2) - 6\*sqrt(c\*x^4 + b\*x^2)\*b^3/c^2 + 16\*(c\*x^4 + b\*x^2)^(3/2)\*b/c)\*B

**mapad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x, x)`

[Out] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x, x)`

[Out] `Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x, x)`

$$3.110 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} dx$$

**Optimal.** Leaf size=137

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}} + \frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b} + \frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c} + \frac{A(bx^2+cx^4)}{bx^4}$$

**Rubi [A]** time = 0.30, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 792, 664, 612, 620, 206}

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}} + \frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b} + \frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^3,x]

[Out] ((b\*B - 6\*A\*c)\*(b + 2\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(16\*c) + ((b\*B - 6\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(6\*b) + (A\*(b\*x^2 + c\*x^4)^(5/2))/(b\*x^4) - (b^2\*(b\*B - 6\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(16\*c^(3/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 664

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(p\*(2\*c\*d - b\*e))/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0])

) && NeQ[m + p + 1, 0]

### Rule 2034

Int[(x\_)^(m\_)\*((b\_)\*(x\_)^(k\_) + (a\_)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^3} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^2} dx, x, x^2 \right) \\ &= \frac{A(bx^2 + cx^4)^{5/2}}{bx^4} - \frac{\left(-2(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x} dx, x, x^2 \right)}{b} \\ &= \frac{(bB - 6Ac)(bx^2 + cx^4)^{3/2}}{6b} + \frac{A(bx^2 + cx^4)^{5/2}}{bx^4} - \frac{1}{4}(-bB + 6Ac) \text{Subst} \left( \int \sqrt{bx + cx^2} dx, x, x^2 \right) \\ &= \frac{(bB - 6Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{16c} + \frac{(bB - 6Ac)(bx^2 + cx^4)^{3/2}}{6b} + \frac{A(bx^2 + cx^4)^{5/2}}{bx^4} \\ &= \frac{(bB - 6Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{16c} + \frac{(bB - 6Ac)(bx^2 + cx^4)^{3/2}}{6b} + \frac{A(bx^2 + cx^4)^{5/2}}{bx^4} \\ &= \frac{(bB - 6Ac)(b + 2cx^2) \sqrt{bx^2 + cx^4}}{16c} + \frac{(bB - 6Ac)(bx^2 + cx^4)^{3/2}}{6b} + \frac{A(bx^2 + cx^4)^{5/2}}{bx^4} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 130, normalized size = 0.95

$$\frac{\sqrt{x^2(b + cx^2)} \left( \sqrt{cx} \sqrt{\frac{cx^2}{b} + 1} (2bc(15A + 7Bx^2) + 4c^2x^2(3A + 2Bx^2) + 3b^2B) - 3b^{3/2}(bB - 6Ac) \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b}} \right) \right)}{48c^{3/2}x \sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^3, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/b]\*(3\*b^2\*B + 4\*c^2\*x^2\*(3\*A + 2\*B\*x^2) + 2\*b\*c\*(15\*A + 7\*B\*x^2)) - 3\*b^(3/2)\*(b\*B - 6\*A\*c)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(48\*c^(3/2)\*x\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.55, size = 118, normalized size = 0.86

$$\frac{\sqrt{bx^2 + cx^4} (30Abc + 12Ac^2x^2 + 3b^2B + 14bBcx^2 + 8Bc^2x^4)}{48c} + \frac{(b^3B - 6Ab^2c) \log \left( -2c^{3/2} \sqrt{bx^2 + cx^4} + bc + 2c^2x^2 \right)}{32c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^3, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(3\*b^2\*B + 30\*A\*b\*c + 14\*b\*B\*c\*x^2 + 12\*A\*c^2\*x^2 + 8\*B\*c^2\*x^4))/(48\*c) + ((b^3\*B - 6\*A\*b^2\*c)\*Log[b\*c + 2\*c^2\*x^2 - 2\*c^(3/2)\*Sqrt[b\*x^2 + c\*x^4]])/(32\*c^(3/2))

**fricas** [A] time = 0.44, size = 224, normalized size = 1.64

$$\frac{3(Bb^3 - 6Ab^2c)\sqrt{c} \log(-2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c}) - 2(8Bc^3x^4 + 3Bb^2c + 30Abc^2 + 2(7Bbc^2 + 6Ac^3)x^2)\sqrt{cx^4 + bx^2} - 3(Bb^3 - 6Ab^2c)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) + (8Bc^3x^4 + 3Bb^2c + 30Abc^2 + 2(7Bbc^2 + 6Ac^3)x^2)\sqrt{cx^4 + bx^2}}{96c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] [-1/96\*(3\*(B\*b^3 - 6\*A\*b^2\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(8\*B\*c^3\*x^4 + 3\*B\*b^2\*c + 30\*A\*b\*c^2 + 2\*(7\*B\*b\*c^2 + 6\*A\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^2, 1/48\*(3\*(B\*b^3 - 6\*A\*b^2\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + (8\*B\*c^3\*x^4 + 3\*B\*b^2\*c + 30\*A\*b\*c^2 + 2\*(7\*B\*b\*c^2 + 6\*A\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^2]

**giac** [A] time = 0.21, size = 142, normalized size = 1.04

$$\frac{1}{48} \left( 2 \left( 4 B c x^2 \operatorname{sgn}(x) + \frac{7 B b c^4 \operatorname{sgn}(x) + 6 A c^5 \operatorname{sgn}(x)}{c^4} \right) x^2 + \frac{3 (B b^2 c^3 \operatorname{sgn}(x) + 10 A b c^4 \operatorname{sgn}(x))}{c^4} \right) \sqrt{c x^2 + b x} + \frac{(B b^3 \operatorname{sgn}(x) - 6 A b^2 c \operatorname{sgn}(x)) \log(|-\sqrt{c} x + \sqrt{c x^2 + b}|)}{16 c^{\frac{3}{2}}} - \frac{(B b^3 \log(|b|) - 6 A b^2 c \log(|b|)) \operatorname{sgn}(x)}{32 c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^3,x, algorithm="giac")

[Out] 1/48\*(2\*(4\*B\*c\*x^2\*sgn(x) + (7\*B\*b\*c^4\*sgn(x) + 6\*A\*c^5\*sgn(x))/c^4)\*x^2 + 3\*(B\*b^2\*c^3\*sgn(x) + 10\*A\*b\*c^4\*sgn(x))/c^4)\*sqrt(c\*x^2 + b)\*x + 1/16\*(B\*b^3\*sgn(x) - 6\*A\*b^2\*c\*sgn(x))\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + b)))/c^(3/2) - 1/32\*(B\*b^3\*log(abs(b)) - 6\*A\*b^2\*c\*log(abs(b)))\*sgn(x)/c^(3/2)

**maple** [A] time = 0.05, size = 162, normalized size = 1.18

$$\frac{(c x^4 + b x^2)^{\frac{3}{2}} \left( 18 A b^2 c \ln(\sqrt{c} x + \sqrt{c x^2 + b}) - 3 B b^3 \ln(\sqrt{c} x + \sqrt{c x^2 + b}) + 18 \sqrt{c x^2 + b} A b c^{\frac{3}{2}} x - 3 \sqrt{c x^2 + b} B b^2 \sqrt{c} x + 12 (c x^2 + b)^{\frac{3}{2}} A c^{\frac{3}{2}} x - 2 (c x^2 + b)^{\frac{3}{2}} B b \sqrt{c} x + 8 (c x^2 + b)^{\frac{5}{2}} B \sqrt{c} x \right)}{48 (c x^2 + b)^{\frac{3}{2}} c^{\frac{3}{2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^3,x)

[Out] 1/48\*(c\*x^4+b\*x^2)^(3/2)\*(8\*B\*c^(1/2)\*(c\*x^2+b)^(5/2)\*x+12\*(c\*x^2+b)^(3/2)\*A\*c^(3/2)\*x-2\*(c\*x^2+b)^(3/2)\*B\*b\*c^(1/2)\*x+18\*(c\*x^2+b)^(1/2)\*A\*b\*c^(3/2)\*x-3\*(c\*x^2+b)^(1/2)\*B\*b^2\*c^(1/2)\*x+18\*A\*b^2\*c\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))-3\*B\*b^3\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2)))/x^3/(c\*x^2+b)^(3/2)/c^(3/2)

**maxima** [A] time = 1.43, size = 168, normalized size = 1.23

$$\frac{1}{16} \left( \frac{3 b^2 \log(2 c x^2 + b + 2 \sqrt{c x^4 + b x^2} \sqrt{c})}{\sqrt{c}} + 6 \sqrt{c x^4 + b x^2} b + \frac{4 (c x^4 + b x^2)^{\frac{3}{2}}}{x^2} \right) A + \frac{1}{96} \left( 12 \sqrt{c x^4 + b x^2} b x^2 - \frac{3 b^3 \log(2 c x^2 + b + 2 \sqrt{c x^4 + b x^2} \sqrt{c})}{c^{\frac{3}{2}}} + 16 (c x^4 + b x^2)^{\frac{3}{2}} + \frac{6 \sqrt{c x^4 + b x^2} b^2}{c} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^3,x, algorithm="maxima")

[Out] 1/16\*(3\*b^2\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/sqrt(c) + 6\*sqrt(c\*x^4 + b\*x^2)\*b + 4\*(c\*x^4 + b\*x^2)^(3/2)/x^2)\*A + 1/96\*(12\*sqrt(c\*x^4 + b\*x^2)\*b\*x^2 - 3\*b^3\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(3/2) + 16\*(c\*x^4 + b\*x^2)^(3/2) + 6\*sqrt(c\*x^4 + b\*x^2)\*b^2/c)\*B

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B x^2 + A) (c x^4 + b x^2)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3, x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 (b + cx^2)\right)^{\frac{3}{2}} (A + Bx^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**3, x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**3, x)
```

$$3.111 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx$$

**Optimal.** Leaf size=128

$$\frac{(bx^2 + cx^4)^{3/2} (4Ac + bB)}{4bx^2} + \frac{3}{8} \sqrt{bx^2 + cx^4} (4Ac + bB) + \frac{3b(4Ac + bB) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2 + cx^4}}\right)}{8\sqrt{c}} - \frac{A(bx^2 + cx^4)^{5/2}}{bx^6}$$

**Rubi [A]** time = 0.28, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 792, 664, 620, 206}

$$\frac{(bx^2 + cx^4)^{3/2} (4Ac + bB)}{4bx^2} + \frac{3}{8} \sqrt{bx^2 + cx^4} (4Ac + bB) + \frac{3b(4Ac + bB) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2 + cx^4}}\right)}{8\sqrt{c}} - \frac{A(bx^2 + cx^4)^{5/2}}{bx^6}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^5,x]

[Out] (3\*(b\*B + 4\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/8 + ((b\*B + 4\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(4\*b\*x^2) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(b\*x^6) + (3\*b\*(b\*B + 4\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(8\*Sqrt[c])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 664

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(p\*(2\*c\*d - b\*e))/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; F

reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^5} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^3} dx, x, x^2 \right) \\ &= -\frac{A(bx^2 + cx^4)^{5/2}}{bx^6} + \frac{(-3(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^2} dx \right)}{b} \\ &= \frac{(bB + 4Ac)(bx^2 + cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2 + cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3(bB + 4Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^2} dx \right) \\ &= \frac{3}{8}(bB + 4Ac)\sqrt{bx^2 + cx^4} + \frac{(bB + 4Ac)(bx^2 + cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2 + cx^4)^{5/2}}{bx^6} + \\ &= \frac{3}{8}(bB + 4Ac)\sqrt{bx^2 + cx^4} + \frac{(bB + 4Ac)(bx^2 + cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2 + cx^4)^{5/2}}{bx^6} + \\ &= \frac{3}{8}(bB + 4Ac)\sqrt{bx^2 + cx^4} + \frac{(bB + 4Ac)(bx^2 + cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2 + cx^4)^{5/2}}{bx^6} + \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 96, normalized size = 0.75

$$\frac{\sqrt{x^2(b + cx^2)} \left( \frac{3\sqrt{b}x(4Ac + bB) \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right) - 8Ab + 4Acx^2 + 5bBx^2 + 2Bcx^4}{\sqrt{c}\sqrt{\frac{cx^2}{b} + 1}} \right)}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^5, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-8\*A\*b + 5\*b\*B\*x^2 + 4\*A\*c\*x^2 + 2\*B\*c\*x^4 + (3\*Sqrt[b]\*(b\*B + 4\*A\*c))\*x\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(Sqrt[c]\*Sqrt[1 + (c\*x^2)/b]))/(8\*x^2)

**IntegrateAlgebraic [A]** time = 0.53, size = 100, normalized size = 0.78

$$\frac{\sqrt{bx^2 + cx^4}(-8Ab + 4Acx^2 + 5bBx^2 + 2Bcx^4)}{8x^2} - \frac{3(4Abc + b^2B) \log\left(-2\sqrt{c}\sqrt{bx^2 + cx^4} + b + 2cx^2\right)}{16\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^5, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-8\*A\*b + 5\*b\*B\*x^2 + 4\*A\*c\*x^2 + 2\*B\*c\*x^4))/(8\*x^2) - (3\*(b^2\*B + 4\*A\*b\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(16\*Sqrt[c])

**fricas [A]** time = 0.44, size = 209, normalized size = 1.63

$$\left[ \frac{3(Bb^2 + 4Abc)\sqrt{c}x^2 \log\left(-2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c}\right) + 2(2Bc^2x^4 - 8Abc + (5Bbc + 4Ac^2)x^2)\sqrt{cx^4 + bx^2}}{16cx^2}, -\frac{3(Bb^2 + 4Abc)\sqrt{-c}x^2 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (2Bc^2x^4 - 8Abc + (5Bbc + 4Ac^2)x^2)\sqrt{cx^4 + bx^2}}{8cx^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^5,x, algorithm="fricas")

[Out] [1/16\*(3\*(B\*b^2 + 4\*A\*b\*c)\*sqrt(c)\*x^2\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) + 2\*(2\*B\*c^2\*x^4 - 8\*A\*b\*c + (5\*B\*b\*c + 4\*A\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(c\*x^2), -1/8\*(3\*(B\*b^2 + 4\*A\*b\*c)\*sqrt(-c)\*x^2\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) - (2\*B\*c^2\*x^4 - 8\*A\*b\*c + (5\*B\*b\*c + 4\*A\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(c\*x^2)]

**giac** [A] time = 0.35, size = 126, normalized size = 0.98

$$\frac{2Ab^2\sqrt{c}\operatorname{sgn}(x)}{(\sqrt{cx}-\sqrt{cx^2+b})^2-b} + \frac{1}{8}\left(2Bcx^2\operatorname{sgn}(x) + \frac{5Bbc^2\operatorname{sgn}(x) + 4Ac^3\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+bx} - \frac{3\left(Bb^2\sqrt{c}\operatorname{sgn}(x) + 4Abc^2\operatorname{sgn}(x)\right)\log\left(\left(\sqrt{cx}-\sqrt{cx^2+b}\right)^2\right)}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^5,x, algorithm="giac")

[Out] 2\*A\*b^2\*sqrt(c)\*sgn(x)/((sqrt(c)\*x - sqrt(c\*x^2 + b))^2 - b) + 1/8\*(2\*B\*c\*x^2\*sgn(x) + (5\*B\*b\*c^2\*sgn(x) + 4\*A\*c^3\*sgn(x))/c^2)\*sqrt(c\*x^2 + b)\*x - 3/16\*(B\*b^2\*sqrt(c)\*sgn(x) + 4\*A\*b\*c^(3/2)\*sgn(x))\*log((sqrt(c)\*x - sqrt(c\*x^2 + b))^2)/c

**maple** [A] time = 0.06, size = 174, normalized size = 1.36

$$\frac{(cx^4+bx^2)^{\frac{3}{2}}\left(12Ab^2cx\ln(\sqrt{cx}+\sqrt{cx^2+b})+3Bb^3x\ln(\sqrt{cx}+\sqrt{cx^2+b})+12\sqrt{cx^2+b}Abc^{\frac{3}{2}}x^2+3\sqrt{cx^2+b}Bb^2\sqrt{c}x^2+8(cx^2+b)^{\frac{3}{2}}Ac^{\frac{3}{2}}x^2+2(cx^2+b)^{\frac{3}{2}}Bb\sqrt{c}x^2-8(cx^2+b)^{\frac{5}{2}}A\sqrt{c}\right)}{8(cx^2+b)^{\frac{3}{2}}b\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^5,x)

[Out] 1/8\*(c\*x^4+b\*x^2)^(3/2)\*(8\*A\*c^(3/2)\*(c\*x^2+b)^(3/2)\*x^2+12\*A\*c^(3/2)\*(c\*x^2+b)^(1/2)\*x^2\*b+2\*B\*c^(1/2)\*(c\*x^2+b)^(3/2)\*x^2\*b-8\*A\*c^(1/2)\*(c\*x^2+b)^(5/2)+3\*B\*c^(1/2)\*(c\*x^2+b)^(1/2)\*x^2\*b^2+12\*A\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*x\*b^2\*c+3\*B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*x\*b^3)/x^4/(c\*x^2+b)^(3/2)/b/c^(1/2)

**maxima** [A] time = 1.49, size = 148, normalized size = 1.16

$$\frac{1}{4}\left(3b\sqrt{c}\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{6\sqrt{cx^4+bx^2}b}{x^2}+\frac{2(cx^4+bx^2)^{\frac{3}{2}}}{x^4}\right)A+\frac{1}{16}\left(\frac{3b^2\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+6\sqrt{cx^4+bx^2}b+\frac{4(cx^4+bx^2)^{\frac{3}{2}}}{x^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] 1/4\*(3\*b\*sqrt(c)\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 6\*sqrt(c\*x^4 + b\*x^2)\*b/x^2 + 2\*(c\*x^4 + b\*x^2)^(3/2)/x^4)\*A + 1/16\*(3\*b^2\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/sqrt(c) + 6\*sqrt(c\*x^4 + b\*x^2)\*b + 4\*(c\*x^4 + b\*x^2)^(3/2)/x^2)\*B

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^5,x)

[Out] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**5, x)`

[Out] `Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**5, x)`

$$3.112 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7} dx$$

**Optimal.** Leaf size=136

$$-\frac{(bx^2+cx^4)^{3/2}(2Ac+3bB)}{3bx^4} + \frac{c\sqrt{bx^2+cx^4}(2Ac+3bB)}{2b} + \frac{1}{2}\sqrt{c}(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8}$$

**Rubi [A]** time = 0.27, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 792, 662, 664, 620, 206}

$$-\frac{(bx^2+cx^4)^{3/2}(2Ac+3bB)}{3bx^4} + \frac{c\sqrt{bx^2+cx^4}(2Ac+3bB)}{2b} + \frac{1}{2}\sqrt{c}(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^7,x]

[Out] (c\*(3\*b\*B + 2\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(2\*b) - ((3\*b\*B + 2\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(3\*b\*x^4) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(3\*b\*x^8) + (Sqrt[c]\*(3\*b\*B + 2\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/2

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 662

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + p + 1)), x] - Dist[(c\*p)/(e^2\*(m + p + 1)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2\*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2\*p]

#### Rule 664

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(p\*(2\*c\*d - b\*e))/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1]

&& !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

### Rule 2034

Int[(x\_)^(m\_)\*((b\_)\*(x\_)^(k\_) + (a\_)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x^q, x), x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^7} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^4} dx, x, x^2 \right) \\ &= -\frac{A(bx^2 + cx^4)^{5/2}}{3bx^8} + \frac{\left(-4(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x^3} dx, x, x^2 \right)}{3b} \\ &= -\frac{(3bB + 2Ac)(bx^2 + cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2 + cx^4)^{5/2}}{3bx^8} + \frac{\left(c\left(-4(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)\right)\right) \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x^3} dx, x, x^2 \right)}{3b} \\ &= \frac{c(3bB + 2Ac)\sqrt{bx^2 + cx^4}}{2b} - \frac{(3bB + 2Ac)(bx^2 + cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2 + cx^4)^{5/2}}{3bx^8} \\ &= \frac{c(3bB + 2Ac)\sqrt{bx^2 + cx^4}}{2b} - \frac{(3bB + 2Ac)(bx^2 + cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2 + cx^4)^{5/2}}{3bx^8} \\ &= \frac{c(3bB + 2Ac)\sqrt{bx^2 + cx^4}}{2b} - \frac{(3bB + 2Ac)(bx^2 + cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2 + cx^4)^{5/2}}{3bx^8} \end{aligned}$$

**Mathematica [C]** time = 0.05, size = 98, normalized size = 0.72

$$\frac{\sqrt{x^2(b + cx^2)} \left( bx^2(2Ac + 3bB) {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{cx^2}{b}\right) + A(b + cx^2)^2 \sqrt{\frac{cx^2}{b} + 1} \right)}{3bx^4 \sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^7, x]

[Out] -1/3\*(Sqrt[x^2\*(b + c\*x^2)]\*(A\*(b + c\*x^2)^2\*Sqrt[1 + (c\*x^2)/b] + b\*(3\*b\*B + 2\*A\*c)\*x^2\*Hypergeometric2F1[-3/2, -1/2, 1/2, -(c\*x^2)/b]))/(b\*x^4\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.53, size = 102, normalized size = 0.75

$$\frac{1}{4}(-2Ac^{3/2} - 3bB\sqrt{c}) \log\left(-2\sqrt{c}\sqrt{bx^2 + cx^4} + b + 2cx^2\right) + \frac{\sqrt{bx^2 + cx^4}(-2Ab - 8Acx^2 - 6bBx^2 + 3Bcx^4)}{6x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^7, x]

[Out]  $(\text{Sqrt}[b*x^2 + c*x^4]*(-2*A*b - 6*b*B*x^2 - 8*A*c*x^2 + 3*B*c*x^4))/(6*x^4) + ((-3*b*B*\text{Sqrt}[c] - 2*A*c^{(3/2)})*\text{Log}[b + 2*c*x^2 - 2*\text{Sqrt}[c]*\text{Sqrt}[b*x^2 + c*x^4]])/4$

**fricas** [A] time = 0.44, size = 189, normalized size = 1.39

$$\frac{3(3Bb + 2Ac)\sqrt{c}x^4 \log(-2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c}) + 2(3Bcx^4 - 2(3Bb + 4Ac)x^2 - 2Ab)\sqrt{cx^4 + bx^2} - 3(3Bb + 2Ac)\sqrt{-c}x^4 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (3Bcx^4 - 2(3Bb + 4Ac)x^2 - 2Ab)\sqrt{cx^4 + bx^2}}{12x^4} - \frac{3(3Bb + 2Ac)\sqrt{-c}x^4 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (3Bcx^4 - 2(3Bb + 4Ac)x^2 - 2Ab)\sqrt{cx^4 + bx^2}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")`

[Out]  $[1/12*(3*(3*B*b + 2*A*c)*\text{sqrt}(c)*x^4*\log(-2*c*x^2 - b - 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c)) + 2*(3*B*c*x^4 - 2*(3*B*b + 4*A*c))*x^2 - 2*A*b)*\text{sqrt}(c*x^4 + b*x^2))/x^4, -1/6*(3*(3*B*b + 2*A*c)*\text{sqrt}(-c)*x^4*\arctan(\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(-c)/(c*x^2 + b)) - (3*B*c*x^4 - 2*(3*B*b + 4*A*c))*x^2 - 2*A*b)*\text{sqrt}(c*x^4 + b*x^2))/x^4]$

**giac** [A] time = 0.53, size = 225, normalized size = 1.65

$$\frac{1}{2}\sqrt{cx^2 + b}Bc\text{sgn}(x) - \frac{1}{4}(3Bb\sqrt{c}\text{sgn}(x) + 2Ac^3\text{sgn}(x))\log\left(\left(\sqrt{cx - \sqrt{cx^2 + b}}\right)^2\right) + \frac{2\left(3\left(\sqrt{cx - \sqrt{cx^2 + b}}\right)^4 Bb^2\sqrt{c}\text{sgn}(x) + 6\left(\sqrt{cx - \sqrt{cx^2 + b}}\right)^4 Abc^3\text{sgn}(x) - 6\left(\sqrt{cx - \sqrt{cx^2 + b}}\right)^2 Bb^3\sqrt{c}\text{sgn}(x) - 6\left(\sqrt{cx - \sqrt{cx^2 + b}}\right)^2 Ab^2c^3\text{sgn}(x) + 3Bb^4\sqrt{c}\text{sgn}(x) + 4Ab^3c^3\text{sgn}(x)\right)}{3\left(\left(\sqrt{cx - \sqrt{cx^2 + b}}\right)^2 - b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")`

[Out]  $1/2*\text{sqrt}(c*x^2 + b)*B*c*x*\text{sgn}(x) - 1/4*(3*B*b*\text{sqrt}(c)*\text{sgn}(x) + 2*A*c^{(3/2)}*\text{sgn}(x))*\log((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^2) + 2/3*(3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^4*B*b^2*\text{sqrt}(c)*\text{sgn}(x) + 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^4*A*b*c^{(3/2)}*\text{sgn}(x) - 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^2*B*b^3*\text{sqrt}(c)*\text{sgn}(x) - 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^2*A*b^2*c^{(3/2)}*\text{sgn}(x) + 3*B*b^4*\text{sqrt}(c)*\text{sgn}(x) + 4*A*b^3*c^{(3/2)}*\text{sgn}(x))/((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^2 - b)^3$

**maple** [A] time = 0.06, size = 219, normalized size = 1.61

$$\frac{(cx^4 + bx^2)^{\frac{3}{2}}\left(-6Ab^2c^2x^3\ln(\sqrt{cx + \sqrt{cx^2 + b}}) - 9Bb^3cx^3\ln(\sqrt{cx + \sqrt{cx^2 + b}}) - 6\sqrt{cx^2 + b}Abc^2x^4 - 9\sqrt{cx^2 + b}Bb^2c^2x^4 - 4(cx^2 + b)^{\frac{3}{2}}Ac^2x^4 - 6(cx^2 + b)^{\frac{3}{2}}Bb^2c^2x^4 + 4(cx^2 + b)^{\frac{5}{2}}Ac^2x^2 + 6(cx^2 + b)^{\frac{5}{2}}Bb\sqrt{c}x^2 + 2(cx^2 + b)^{\frac{5}{2}}Ab\sqrt{c}\right)}{6(cx^2 + b)^{\frac{3}{2}}b^2\sqrt{c}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x)`

[Out]  $-1/6*(c*x^4+b*x^2)^(3/2)*(-4*A*(c*x^2+b)^(3/2)*c^(5/2)*x^4-6*B*(c*x^2+b)^(3/2)*c^(3/2)*x^4*b+4*A*(c*x^2+b)^(5/2)*c^(3/2)*x^2-6*A*(c*x^2+b)^(1/2)*c^(5/2)*x^4*b+6*B*(c*x^2+b)^(5/2)*c^(1/2)*x^2*b-9*B*(c*x^2+b)^(1/2)*c^(3/2)*x^4*b^2-6*A*\ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x^3*b^2*c^2-9*B*\ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x^3*b^3*c+2*A*(c*x^2+b)^(5/2)*c^(1/2)*b)/x^6/(c*x^2+b)^(3/2)/b^2/c^(1/2)$

**maxima** [A] time = 1.50, size = 167, normalized size = 1.23

$$\frac{1}{6}\left(3c^{\frac{3}{2}}\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}) - \frac{7\sqrt{cx^4 + bx^2}c}{x^2} - \frac{\sqrt{cx^4 + bx^2}b}{x^4} - \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^6}\right)A + \frac{1}{4}\left(3b\sqrt{c}\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}) - \frac{6\sqrt{cx^4 + bx^2}b}{x^2} + \frac{2(cx^4 + bx^2)^{\frac{3}{2}}}{x^4}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")`

[Out]  $1/6*(3*c^{(3/2)}*\log(2*c*x^2 + b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c)) - 7*\text{sqrt}(c*x^4 + b*x^2)*c/x^2 - \text{sqrt}(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)*A + 1/4*(3*b*\text{sqrt}(c)*\log(2*c*x^2 + b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c)) - 6*\text{sqrt}(c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*B$



**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^7, x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^7, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{3/2}(A + Bx^2)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*7, x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\* (3/2)\*(A + B\*x\*\*2)/x\*\*7, x)

$$3.113 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9} dx$$

**Optimal.** Leaf size=104

$$-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6}$$

**Rubi [A]** time = 0.25, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 792, 662, 620, 206}

$$-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{Bc\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^9,x]

[Out] -((B\*c\*Sqrt[b\*x^2 + c\*x^4])/x^2) - (B\*(b\*x^2 + c\*x^4)^(3/2))/(3\*x^6) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(5\*b\*x^10) + B\*c^(3/2)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 662

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p)/(e\*(m + p + 1)), x] - Dist[(c\*p)/(e^2\*(m + p + 1)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2\*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2\*p]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; F

```

reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^9} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^5} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2} B \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x^4} dx, x, x^2 \right) \\
&= -\frac{B(bx^2 + cx^4)^{3/2}}{3x^6} - \frac{A(bx^2 + cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2} (Bc) \text{Subst} \left( \int \frac{\sqrt{bx + cx^2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{Bc\sqrt{bx^2 + cx^4}}{x^2} - \frac{B(bx^2 + cx^4)^{3/2}}{3x^6} - \frac{A(bx^2 + cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2} (Bc^2) \text{Subst} \left( \int \frac{1}{1 + \sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{Bc\sqrt{bx^2 + cx^4}}{x^2} - \frac{B(bx^2 + cx^4)^{3/2}}{3x^6} - \frac{A(bx^2 + cx^4)^{5/2}}{5bx^{10}} + (Bc^2) \text{Subst} \left( \int \frac{1}{1 + \sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{Bc\sqrt{bx^2 + cx^4}}{x^2} - \frac{B(bx^2 + cx^4)^{3/2}}{3x^6} - \frac{A(bx^2 + cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2} \tanh^{-1} \left( \frac{\sqrt{bx + cx^2}}{\sqrt{bx}} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 94, normalized size = 0.90

$$\frac{\sqrt{x^2(b + cx^2)} \left( 3A(b + cx^2)^2 \sqrt{\frac{cx^2}{b} + 1} + 5b^2 Bx^2 {}_2F_1 \left( -\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{cx^2}{b} \right) \right)}{15bx^6 \sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x]
```

```
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b^2
*B*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(c*x^2)/b]))/(b*x^6*Sqrt[1 +
(c*x^2)/b])
```

**IntegrateAlgebraic [A]** time = 0.45, size = 108, normalized size = 1.04

$$\frac{\sqrt{bx^2 + cx^4} (-3Ab^2 - 6Abcx^2 - 3Ac^2x^4 - 5b^2Bx^2 - 20bBcx^4)}{15bx^6} - \frac{1}{2} Bc^{3/2} \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-3*A*b^2 - 5*b^2*B*x^2 - 6*A*b*c*x^2 - 20*b*B*c*x^4 -
3*A*c^2*x^4))/(15*b*x^6) - (B*c^(3/2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x
^2 + c*x^4]])/2
```

**fricas [A]** time = 0.45, size = 207, normalized size = 1.99

$$\left| \frac{15Bbc^{\frac{3}{2}}x^6 \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2} \sqrt{c} \right) - 2 \left( (20Bbc + 3Ac^2)x^4 + 3Ab^2 + (5Bb^2 + 6Abc)x^2 \right) \sqrt{cx^4 + bx^2}}{30bx^6} - \frac{15Bb\sqrt{-c}cx^6 \arctan \left( \frac{\sqrt{cx^4 + bx^2} \sqrt{-c}}{cx^2 + b} \right) + \left( (20Bbc + 3Ac^2)x^4 + 3Ab^2 + (5Bb^2 + 6Abc)x^2 \right) \sqrt{cx^4 + bx^2}}{15bx^6} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] [1/30\*(15\*B\*b\*c^(3/2)\*x^6\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*((20\*B\*b\*c + 3\*A\*c^2)\*x^4 + 3\*A\*b^2 + (5\*B\*b^2 + 6\*A\*b\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(b\*x^6), -1/15\*(15\*B\*b\*sqrt(-c)\*c\*x^6\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + ((20\*B\*b\*c + 3\*A\*c^2)\*x^4 + 3\*A\*b^2 + (5\*B\*b^2 + 6\*A\*b\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(b\*x^6)]

**giac** [B] time = 1.02, size = 254, normalized size = 2.44

$$\frac{\frac{1}{2} B c^{\frac{3}{2}} \log\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^2 \operatorname{sgn}(x)+\frac{2\left(30\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^5 B b c^{\frac{3}{2}} \operatorname{sgn}(x)+15\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^5 A c^{\frac{3}{2}} \operatorname{sgn}(x)-90\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^5 B b^2 c^{\frac{3}{2}} \operatorname{sgn}(x)+110\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^4 B b^2 c^{\frac{3}{2}} \operatorname{sgn}(x)+30\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^4 A b^2 c^{\frac{3}{2}} \operatorname{sgn}(x)-70\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^3 B b^4 c^{\frac{3}{2}} \operatorname{sgn}(x)+20 B b^5 c^{\frac{3}{2}} \operatorname{sgn}(x)+3 A b^4 c^{\frac{3}{2}} \operatorname{sgn}(x)\right)}{15\left(\sqrt{c x-\sqrt{c x^2+b}}\right)^2-b}}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^9,x, algorithm="giac")

[Out] -1/2\*B\*c^(3/2)\*log((sqrt(c)\*x - sqrt(c\*x^2 + b))^2)\*sgn(x) + 2/15\*(30\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^8\*B\*b\*c^(3/2)\*sgn(x) + 15\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^8\*A\*c^(5/2)\*sgn(x) - 90\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^6\*B\*b^2\*c^(3/2)\*sgn(x) + 110\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*B\*b^3\*c^(3/2)\*sgn(x) + 30\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^4\*A\*b^2\*c^(5/2)\*sgn(x) - 70\*(sqrt(c)\*x - sqrt(c\*x^2 + b))^2\*B\*b^4\*c^(3/2)\*sgn(x) + 20\*B\*b^5\*c^(3/2)\*sgn(x) + 3\*A\*b^4\*c^(5/2)\*sgn(x))/((sqrt(c)\*x - sqrt(c\*x^2 + b))^2 - b)^5

**maple** [A] time = 0.06, size = 153, normalized size = 1.47

$$\frac{\left(c x^4+b x^2\right)^{\frac{3}{2}}\left(-15 B b^2 c^2 x^5 \ln \left(\sqrt{c} x+\sqrt{c x^2+b}\right)-15 \sqrt{c x^2+b} B b c^{\frac{5}{2}} x^6-10\left(c x^2+b\right)^{\frac{3}{2}} B c^{\frac{5}{2}} x^6+10\left(c x^2+b\right)^{\frac{5}{2}} B c^{\frac{3}{2}} x^4+5\left(c x^2+b\right)^{\frac{5}{2}} B b \sqrt{c} x^2+3\left(c x^2+b\right)^{\frac{5}{2}} A b \sqrt{c}\right)}{15\left(c x^2+b\right)^{\frac{3}{2}} b^2 \sqrt{c} x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^9,x)

[Out] -1/15\*(c\*x^4+b\*x^2)^(3/2)\*(-10\*B\*(c\*x^2+b)^(3/2)\*c^(5/2)\*x^6+10\*B\*(c\*x^2+b)^(5/2)\*c^(3/2)\*x^4-15\*B\*(c\*x^2+b)^(1/2)\*c^(5/2)\*x^6\*b-15\*B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*x^5\*b^2\*c^2+5\*(c\*x^2+b)^(5/2)\*B\*b\*c^(1/2)\*x^2+3\*(c\*x^2+b)^(5/2)\*A\*b\*c^(1/2))/x^8/(c\*x^2+b)^(3/2)/b^2/c^(1/2)

**maxima** [B] time = 1.38, size = 177, normalized size = 1.70

$$\frac{1}{6}\left(3 c^{\frac{3}{2}} \log \left(2 c x^2+b+2 \sqrt{c x^4+b x^2} \sqrt{c}\right)-\frac{7 \sqrt{c x^4+b x^2} c}{x^2}-\frac{\sqrt{c x^4+b x^2} b}{x^4}-\frac{\left(c x^4+b x^2\right)^{\frac{3}{2}}}{x^6}\right) B-\frac{1}{10} A\left(\frac{2 \sqrt{c x^4+b x^2} c^2}{b x^2}-\frac{\sqrt{c x^4+b x^2} c}{x^4}-\frac{3 \sqrt{c x^4+b x^2} b}{x^6}+\frac{5\left(c x^4+b x^2\right)^{\frac{3}{2}}}{x^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] 1/6\*(3\*c^(3/2)\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 7\*sqrt(c\*x^4 + b\*x^2)\*c/x^2 - sqrt(c\*x^4 + b\*x^2)\*b/x^4 - (c\*x^4 + b\*x^2)^(3/2)/x^6)\*B - 1/10\*A\*(2\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b\*x^2) - sqrt(c\*x^4 + b\*x^2)\*c/x^4 - 3\*sqrt(c\*x^4 + b\*x^2)\*b/x^6 + 5\*(c\*x^4 + b\*x^2)^(3/2)/x^8)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B x^2 + A) (c x^4 + b x^2)^{3/2}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^9,x)

[Out] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**9, x)`

[Out] `Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**9, x)`

$$3.114 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx$$

**Optimal.** Leaf size=61

$$-\frac{(bx^2 + cx^4)^{5/2} (7bB - 2Ac)}{35b^2x^{10}} - \frac{A (bx^2 + cx^4)^{5/2}}{7bx^{12}}$$

**Rubi [A]** time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2034, 792, 650}

$$-\frac{(bx^2 + cx^4)^{5/2} (7bB - 2Ac)}{35b^2x^{10}} - \frac{A (bx^2 + cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^11,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(5/2))/(7\*b\*x^12) - ((7\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(35\*b^2\*x^10)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps



$+ b))^{6A}b^{2c^{7/2}}\text{sgn}(x) + 77*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^{4B}b^{4c^{5/2}}\text{sgn}(x) + 28*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^{4A}b^{3c^{7/2}}\text{sgn}(x) - 14*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^{2B}b^{5c^{5/2}}\text{sgn}(x) + 14*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^{2A}b^{4c^{7/2}}\text{sgn}(x) + 7*B*b^{6c^{5/2}}\text{sgn}(x) - 2*A*b^{5c^{7/2}}\text{sgn}(x))/((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b))^{2 - b})^7$

**maple [A]** time = 0.05, size = 48, normalized size = 0.79

$$\frac{(cx^2 + b)(-2Acx^2 + 7Bbx^2 + 5Ab)(cx^4 + bx^2)^{\frac{3}{2}}}{35b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x)`

[Out]  $-1/35*(c*x^2+b)*(-2*A*c*x^2+7*B*b*x^2+5*A*b)*(c*x^4+b*x^2)^{(3/2)}/b^2/x^{10}$

**maxima [B]** time = 1.56, size = 193, normalized size = 3.16

$$-\frac{1}{10}B\left(\frac{2\sqrt{cx^4+bx^2}c^2}{bx^2} - \frac{\sqrt{cx^4+bx^2}c}{x^4} - \frac{3\sqrt{cx^4+bx^2}b}{x^6} + \frac{5(cx^4+bx^2)^{\frac{3}{2}}}{x^8}\right) + \frac{1}{140}A\left(\frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^2} - \frac{4\sqrt{cx^4+bx^2}c^2}{bx^4} + \frac{3\sqrt{cx^4+bx^2}c}{x^6} + \frac{15\sqrt{cx^4+bx^2}b}{x^8} - \frac{35(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")`

[Out]  $-1/10*B*(2*\text{sqrt}(c*x^4 + b*x^2)*c^2/(b*x^2) - \text{sqrt}(c*x^4 + b*x^2)*c/x^4 - 3*\text{sqrt}(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^{(3/2)}/x^8) + 1/140*A*(8*\text{sqrt}(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*\text{sqrt}(c*x^4 + b*x^2)*c^2/(b*x^4) + 3*\text{sqrt}(c*x^4 + b*x^2)*c/x^6 + 15*\text{sqrt}(c*x^4 + b*x^2)*b/x^8 - 35*(c*x^4 + b*x^2)^{(3/2)}/x^{10})$

**mupad [B]** time = 1.03, size = 156, normalized size = 2.56

$$\frac{2Ac^3\sqrt{cx^4+bx^2}}{35b^2x^2} - \frac{8Ac\sqrt{cx^4+bx^2}}{35x^6} - \frac{Bb\sqrt{cx^4+bx^2}}{5x^6} - \frac{2Bc\sqrt{cx^4+bx^2}}{5x^4} - \frac{Ac^2\sqrt{cx^4+bx^2}}{35bx^4} - \frac{Ab\sqrt{cx^4+bx^2}}{7x^8} - \frac{Bc^2\sqrt{cx^4+bx^2}}{5bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x)`

[Out]  $(2*A*c^3*(b*x^2 + c*x^4)^{(1/2)})/(35*b^2*x^2) - (8*A*c*(b*x^2 + c*x^4)^{(1/2)})/(35*x^6) - (B*b*(b*x^2 + c*x^4)^{(1/2)})/(5*x^6) - (2*B*c*(b*x^2 + c*x^4)^{(1/2)})/(5*x^4) - (A*c^2*(b*x^2 + c*x^4)^{(1/2)})/(35*b*x^4) - (A*b*(b*x^2 + c*x^4)^{(1/2)})/(7*x^8) - (B*c^2*(b*x^2 + c*x^4)^{(1/2)})/(5*b*x^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**11,x)`

[Out] `Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**11, x)`



$$3.115 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}} dx$$

**Optimal.** Leaf size=96

$$\frac{2c(bx^2+cx^4)^{5/2}(9bB-4Ac)}{315b^3x^{10}} - \frac{(bx^2+cx^4)^{5/2}(9bB-4Ac)}{63b^2x^{12}} - \frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

**Rubi [A]** time = 0.23, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{2c(bx^2+cx^4)^{5/2}(9bB-4Ac)}{315b^3x^{10}} - \frac{(bx^2+cx^4)^{5/2}(9bB-4Ac)}{63b^2x^{12}} - \frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^13,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(5/2))/(9\*b\*x^14) - ((9\*b\*B - 4\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(63\*b^2\*x^12) + (2\*c\*(9\*b\*B - 4\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(315\*b^3\*x^10)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{13}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^7} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{9bx^{14}} + \frac{\left(-7(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^6} dx, x \right)}{9b} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB - 4Ac)(bx^2 + cx^4)^{5/2}}{63b^2x^{12}} - \frac{(c(9bB - 4Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^6} dx, x \right)}{63b^2} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB - 4Ac)(bx^2 + cx^4)^{5/2}}{63b^2x^{12}} + \frac{2c(9bB - 4Ac)(bx^2 + cx^4)^{5/2}}{315b^3x^{10}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.69

$$\frac{(x^2(b + cx^2))^{5/2} (A(-35b^2 + 20bcx^2 - 8c^2x^4) + 9bBx^2(2cx^2 - 5b))}{315b^3x^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^13, x]

[Out] ((x^2\*(b + c\*x^2))^(5/2)\*(9\*b\*B\*x^2\*(-5\*b + 2\*c\*x^2) + A\*(-35\*b^2 + 20\*b\*c\*x^2 - 8\*c^2\*x^4)))/(315\*b^3\*x^14)

**IntegrateAlgebraic [A]** time = 0.45, size = 114, normalized size = 1.19

$$\frac{\sqrt{bx^2 + cx^4} (-35Ab^4 - 50Ab^3cx^2 - 3Ab^2c^2x^4 + 4Abc^3x^6 - 8Ac^4x^8 - 45b^4Bx^2 - 72b^3Bcx^4 - 9b^2Bc^2x^6 + 18bBc^3x^8)}{315b^3x^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^13, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-35\*A\*b^4 - 45\*b^4\*B\*x^2 - 50\*A\*b^3\*c\*x^2 - 72\*b^3\*B\*c\*x^4 - 3\*A\*b^2\*c^2\*x^4 - 9\*b^2\*B\*c^2\*x^6 + 4\*A\*b\*c^3\*x^6 + 18\*b\*B\*c^3\*x^8 - 8\*A\*c^4\*x^8))/(315\*b^3\*x^10)

**fricas [A]** time = 0.47, size = 109, normalized size = 1.14

$$\frac{(2(9Bbc^3 - 4Ac^4)x^8 - (9Bb^2c^2 - 4Abc^3)x^6 - 35Ab^4 - 3(24Bb^3c + Ab^2c^2)x^4 - 5(9Bb^4 + 10Ab^3c)x^2)\sqrt{cx^4 + bx^2}}{315b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^13,x, algorithm="fricas")

[Out] 1/315\*(2\*(9\*B\*b\*c^3 - 4\*A\*c^4)\*x^8 - (9\*B\*b^2\*c^2 - 4\*A\*b\*c^3)\*x^6 - 35\*A\*b^4 - 3\*(24\*B\*b^3\*c + A\*b^2\*c^2)\*x^4 - 5\*(9\*B\*b^4 + 10\*A\*b^3\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^3\*x^10)

**giac [B]** time = 2.86, size = 430, normalized size = 4.48

([In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^13,x, algorithm="fricas") [Out] 1/315\*(2\*(9\*B\*b\*c^3 - 4\*A\*c^4)\*x^8 - (9\*B\*b^2\*c^2 - 4\*A\*b\*c^3)\*x^6 - 35\*A\*b^4 - 3\*(24\*B\*b^3\*c + A\*b^2\*c^2)\*x^4 - 5\*(9\*B\*b^4 + 10\*A\*b^3\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^3\*x^10)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^13,x, algorithm="giac")

[Out]  $4/315*(315*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*B*c^{(7/2)}*\text{sgn}(x) - 315*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*B*b*c^{(7/2)}*\text{sgn}(x) + 840*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*B*b^2*c^{(7/2)}*\text{sgn}(x) + 1260*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*A*b*c^{(9/2)}*\text{sgn}(x) - 819*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*B*b^3*c^{(7/2)}*\text{sgn}(x) + 1764*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*A*b^2*c^{(9/2)}*\text{sgn}(x) + 441*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*B*b^4*c^{(7/2)}*\text{sgn}(x) + 504*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*A*b^3*c^{(9/2)}*\text{sgn}(x) - 9*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*B*b^5*c^{(7/2)}*\text{sgn}(x) + 144*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*A*b^4*c^{(9/2)}*\text{sgn}(x) + 81*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*B*b^6*c^{(7/2)}*\text{sgn}(x) - 36*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*A*b^5*c^{(9/2)}*\text{sgn}(x) - 9*B*b^7*c^{(7/2)}*\text{sgn}(x) + 4*A*b^6*c^{(9/2)}*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + b})^2 - b)^9$

**maple [A]** time = 0.05, size = 70, normalized size = 0.73

$$\frac{(cx^2 + b)(8Ac^2x^4 - 18Bbcx^4 - 20Abcx^2 + 45Bb^2x^2 + 35b^2A)(cx^4 + bx^2)^{\frac{3}{2}}}{315b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^13,x)

[Out]  $-1/315*(c*x^2+b)*(8*A*c^2*x^4-18*B*b*c*x^4-20*A*b*c*x^2+45*B*b^2*x^2+35*A*b^2)*(c*x^4+b*x^2)^(3/2)/b^3/x^{12}$

**maxima [B]** time = 1.53, size = 241, normalized size = 2.51

$$\frac{1}{140} \left( \frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^2} - \frac{4\sqrt{cx^4+bx^2}c^2}{bx^4} + \frac{3\sqrt{cx^4+bx^2}c}{x^6} + \frac{15\sqrt{cx^4+bx^2}b}{x^8} - \frac{35(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}} \right) - \frac{1}{630} A \left( \frac{16\sqrt{cx^4+bx^2}c^4}{b^3x^2} - \frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^4} + \frac{6\sqrt{cx^4+bx^2}c^2}{bx^6} - \frac{5\sqrt{cx^4+bx^2}c}{x^8} - \frac{35\sqrt{cx^4+bx^2}b}{x^{10}} + \frac{105(cx^4+bx^2)^{\frac{3}{2}}}{x^{12}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^13,x, algorithm="maxima")

[Out]  $1/140*B*(8*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^2) - 4*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^4) + 3*\sqrt{c*x^4 + b*x^2}*c/x^6 + 15*\sqrt{c*x^4 + b*x^2}*b/x^8 - 35*(c*x^4 + b*x^2)^(3/2)/x^{10}) - 1/630*A*(16*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^2) - 8*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^4) + 6*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^6) - 5*\sqrt{c*x^4 + b*x^2}*c/x^8 - 35*\sqrt{c*x^4 + b*x^2}*b/x^{10} + 105*(c*x^4 + b*x^2)^(3/2)/x^{12})$

**mupad [B]** time = 1.44, size = 206, normalized size = 2.15

$$\frac{4Ac^3\sqrt{cx^4+bx^2}}{315b^2x^4} - \frac{10Ac\sqrt{cx^4+bx^2}}{63x^8} - \frac{Bb\sqrt{cx^4+bx^2}}{7x^8} - \frac{8Bc\sqrt{cx^4+bx^2}}{35x^6} - \frac{Ac^2\sqrt{cx^4+bx^2}}{105bx^6} - \frac{Ab\sqrt{cx^4+bx^2}}{9x^{10}} - \frac{8Ac^4\sqrt{cx^4+bx^2}}{315b^3x^2} - \frac{Bc^2\sqrt{cx^4+bx^2}}{35bx^4} + \frac{2Bc^3\sqrt{cx^4+bx^2}}{35b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^13,x)

[Out]  $(4*A*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*A*c*(b*x^2 + c*x^4)^(1/2))/(63*x^8) - (B*b*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*B*c*(b*x^2 + c*x^4)^(1/2))/(35*x^6) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (A*b*(b*x^2 + c*x^4)^(1/2))/(9*x^{10}) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**13, x)
```

$$3.116 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} dx$$

**Optimal.** Leaf size=133

$$\frac{8c^2 (bx^2 + cx^4)^{5/2} (11bB - 6Ac)}{3465b^4x^{10}} + \frac{4c (bx^2 + cx^4)^{5/2} (11bB - 6Ac)}{693b^3x^{12}} - \frac{(bx^2 + cx^4)^{5/2} (11bB - 6Ac)}{99b^2x^{14}} - \frac{A (bx^2 + cx^4)^{5/2}}{11bx^{16}}$$

**Rubi [A]** time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{8c^2 (bx^2 + cx^4)^{5/2} (11bB - 6Ac)}{3465b^4x^{10}} + \frac{4c (bx^2 + cx^4)^{5/2} (11bB - 6Ac)}{693b^3x^{12}} - \frac{(bx^2 + cx^4)^{5/2} (11bB - 6Ac)}{99b^2x^{14}} - \frac{A (bx^2 + cx^4)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^15,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(5/2))/(11\*b\*x^16) - ((11\*b\*B - 6\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(99\*b^2\*x^14) + (4\*c\*(11\*b\*B - 6\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(693\*b^3\*x^12) - (8\*c^2\*(11\*b\*B - 6\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(3465\*b^4\*x^10)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((f\_.) + (g\_.)\*(x\_.))\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{15}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^8} dx, x, x^2 \right) \\
 &= -\frac{A(bx^2 + cx^4)^{5/2}}{11bx^{16}} + \frac{(-8(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^7} dx, x \right)}{11b} \\
 &= -\frac{A(bx^2 + cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB - 6Ac)(bx^2 + cx^4)^{5/2}}{99b^2x^{14}} - \frac{(2c(11bB - 6Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^7} dx, x \right)}{99b^2} \\
 &= -\frac{A(bx^2 + cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB - 6Ac)(bx^2 + cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB - 6Ac)(bx^2 + cx^4)^{5/2}}{693b^3x^{12}} \\
 &= -\frac{A(bx^2 + cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB - 6Ac)(bx^2 + cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB - 6Ac)(bx^2 + cx^4)^{5/2}}{693b^3x^{12}}
 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 89, normalized size = 0.67

$$\frac{(x^2(b + cx^2))^{5/2} (3A(105b^3 - 70b^2cx^2 + 40bc^2x^4 - 16c^3x^6) + 11bBx^2(35b^2 - 20bcx^2 + 8c^2x^4))}{3465b^4x^{16}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^15, x]

[Out] -1/3465\*((x^2\*(b + c\*x^2))^(5/2)\*(11\*b\*B\*x^2\*(35\*b^2 - 20\*b\*c\*x^2 + 8\*c^2\*x^4) + 3\*A\*(105\*b^3 - 70\*b^2\*c\*x^2 + 40\*b\*c^2\*x^4 - 16\*c^3\*x^6)))/(b^4\*x^16)

**IntegrateAlgebraic [A]** time = 0.48, size = 138, normalized size = 1.04

$$\frac{\sqrt{bx^2 + cx^4} (-315Ab^5 - 420Ab^4cx^2 - 15Ab^3c^2x^4 + 18Ab^2c^3x^6 - 24Abc^4x^8 + 48Ac^5x^{10} - 385b^5Bx^2 - 550b^4Bcx^4 - 33b^3Bc^2x^6 + 44b^2Bc^3x^8 - 88bBc^4x^{10})}{3465b^4x^{12}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^15, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-315\*A\*b^5 - 385\*b^5\*B\*x^2 - 420\*A\*b^4\*c\*x^2 - 550\*b^4\*B\*c\*x^4 - 15\*A\*b^3\*c^2\*x^4 - 33\*b^3\*B\*c^2\*x^6 + 18\*A\*b^2\*c^3\*x^6 + 44\*b^2\*B\*c^3\*x^8 - 24\*A\*b\*c^4\*x^8 - 88\*b\*B\*c^4\*x^10 + 48\*A\*c^5\*x^10))/(3465\*b^4\*x^12)

**fricas [A]** time = 0.52, size = 134, normalized size = 1.01

$$\frac{(8(11Bbc^4 - 6Ac^5)x^{10} - 4(11Bb^2c^3 - 6Abc^4)x^8 + 3(11Bb^3c^2 - 6Ab^2c^3)x^6 + 315Ab^5 + 5(110Bb^4c + 3Ab^3c^2)x^4 + 35(11Bb^5 + 12Ab^4c)x^2)\sqrt{cx^4 + bx^2}}{3465b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^15,x, algorithm="fricas")

[Out] -1/3465\*(8\*(11\*B\*b\*c^4 - 6\*A\*c^5)\*x^10 - 4\*(11\*B\*b^2\*c^3 - 6\*A\*b\*c^4)\*x^8 + 3\*(11\*B\*b^3\*c^2 - 6\*A\*b^2\*c^3)\*x^6 + 315\*A\*b^5 + 5\*(110\*B\*b^4\*c + 3\*A\*b^3\*c^2)\*x^4 + 35\*(11\*B\*b^5 + 12\*A\*b^4\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^4\*x^12)

**giac [B]** time = 3.23, size = 490, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^15,x, algorithm="giac")

[Out]  $16/3465*(2310*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{16}*B*c^{(9/2)}*\text{sgn}(x) - 1155*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*B*b*c^{(9/2)}*\text{sgn}(x) + 6930*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*A*c^{(11/2)}*\text{sgn}(x) + 231*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*B*b^2*c^{(9/2)}*\text{sgn}(x) + 12474*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*A*b*c^{(11/2)}*\text{sgn}(x) - 4851*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*B*b^3*c^{(9/2)}*\text{sgn}(x) + 15246*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*A*b^2*c^{(11/2)}*\text{sgn}(x) + 2475*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*B*b^4*c^{(9/2)}*\text{sgn}(x) + 4950*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*A*b^3*c^{(11/2)}*\text{sgn}(x) + 495*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*B*b^5*c^{(9/2)}*\text{sgn}(x) + 990*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*A*b^4*c^{(11/2)}*\text{sgn}(x) + 605*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*B*b^6*c^{(9/2)}*\text{sgn}(x) - 330*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*A*b^5*c^{(11/2)}*\text{sgn}(x) - 121*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*B*b^7*c^{(9/2)}*\text{sgn}(x) + 66*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*A*b^6*c^{(11/2)}*\text{sgn}(x) + 11*B*b^8*c^{(9/2)}*\text{sgn}(x) - 6*A*b^7*c^{(11/2)}*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + b})^2 - b)^{11}$

**maple [A]** time = 0.05, size = 94, normalized size = 0.71

$$\frac{(c x^2 + b)(-48 A c^3 x^6 + 88 B b c^2 x^6 + 120 A b c^2 x^4 - 220 B b^2 c x^4 - 210 A b^2 c x^2 + 385 B b^3 x^2 + 315 A b^3)(c x^4 + b x^2)^{\frac{3}{2}}}{3465 b^4 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^15,x)

[Out]  $-1/3465*(c*x^2+b)*(-48*A*c^3*x^6+88*B*b*c^2*x^6+120*A*b*c^2*x^4-220*B*b^2*c*x^4-210*A*b^2*c*x^2+385*B*b^3*x^2+315*A*b^3)*(c*x^4+b*x^2)^(3/2)/x^14/b^4$

**maxima [B]** time = 1.58, size = 289, normalized size = 2.17

$$-\frac{1}{630} B \left( \frac{16 \sqrt{c x^4 + b x^2} c^4}{b^3 x^2} - \frac{8 \sqrt{c x^4 + b x^2} c^3}{b^2 x^4} + \frac{6 \sqrt{c x^4 + b x^2} c^2}{b x^6} - \frac{5 \sqrt{c x^4 + b x^2} c}{x^8} - \frac{35 \sqrt{c x^4 + b x^2} b}{x^{10}} + \frac{105 (c x^4 + b x^2)^{\frac{3}{2}}}{x^{12}} \right) + \frac{1}{9240} A \left( \frac{128 \sqrt{c x^4 + b x^2} c^5}{b^4 x^2} - \frac{64 \sqrt{c x^4 + b x^2} c^4}{b^3 x^4} + \frac{48 \sqrt{c x^4 + b x^2} c^3}{b^2 x^6} - \frac{40 \sqrt{c x^4 + b x^2} c^2}{b x^8} + \frac{35 \sqrt{c x^4 + b x^2} c}{x^{10}} + \frac{315 \sqrt{c x^4 + b x^2} b}{x^{12}} - \frac{1155 (c x^4 + b x^2)^{\frac{3}{2}}}{x^{14}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^15,x, algorithm="maxima")

[Out]  $-1/630*B*(16*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^2) - 8*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^4) + 6*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^6) - 5*\sqrt{c*x^4 + b*x^2}*c/x^8 - 35*\sqrt{c*x^4 + b*x^2}*b/x^{10} + 105*(c*x^4 + b*x^2)^(3/2)/x^{12}) + 1/9240*A*(128*\sqrt{c*x^4 + b*x^2}*c^5/(b^4*x^2) - 64*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^4) + 48*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^6) - 40*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^8) + 35*\sqrt{c*x^4 + b*x^2}*c/x^{10} + 315*\sqrt{c*x^4 + b*x^2}*b/x^{12} - 1155*(c*x^4 + b*x^2)^(3/2)/x^{14})$

**mapad [B]** time = 1.92, size = 256, normalized size = 1.92

$$\frac{2 A c^5 \sqrt{c x^4 + b x^2}}{385 b^2 x^6} - \frac{4 A c^4 \sqrt{c x^4 + b x^2}}{33 x^{10}} - \frac{B b \sqrt{c x^4 + b x^2}}{9 x^{10}} - \frac{10 B c \sqrt{c x^4 + b x^2}}{63 x^8} - \frac{A c^2 \sqrt{c x^4 + b x^2}}{231 b x^8} - \frac{A b \sqrt{c x^4 + b x^2}}{11 x^{12}} - \frac{8 A c^4 \sqrt{c x^4 + b x^2}}{1155 b^3 x^4} + \frac{16 A c^5 \sqrt{c x^4 + b x^2}}{1155 b^4 x^2} - \frac{B c^2 \sqrt{c x^4 + b x^2}}{105 b x^6} + \frac{4 B c^3 \sqrt{c x^4 + b x^2}}{315 b^2 x^4} - \frac{8 B c^4 \sqrt{c x^4 + b x^2}}{315 b^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^15,x)

[Out]  $(2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*A*c*(b*x^2 + c*x^4)^(1/2))/(33*x^{10}) - (B*b*(b*x^2 + c*x^4)^(1/2))/(9*x^{10}) - (10*B*c*(b*x^2 + c*x^4)^(1/2))/(63*x^8) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (A*b*(b*x^2 + c*x^4)^(1/2))/(11*x^{12}) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4) + (16*A*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) + (4*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (8*B*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{15}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*15,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2)/x\*\*15, x)



$$3.117 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} dx$$

**Optimal.** Leaf size=170

$$\frac{16c^3 (bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{15015b^5x^{10}} - \frac{8c^2 (bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{3003b^4x^{12}} + \frac{2c (bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{429b^3x^{14}} - \frac{(bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{143b^2x^{16}} - \frac{A (bx^2 + cx^4)^{5/2}}{13bx^{18}}$$

**Rubi [A]** time = 0.32, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{16c^3 (bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{15015b^5x^{10}} - \frac{8c^2 (bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{3003b^4x^{12}} + \frac{2c (bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{429b^3x^{14}} - \frac{(bx^2 + cx^4)^{5/2} (13bB - 8Ac)}{143b^2x^{16}} - \frac{A (bx^2 + cx^4)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^17,x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(5/2))/(13\*b\*x^18) - ((13\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(143\*b^2\*x^16) + (2\*c\*(13\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(429\*b^3\*x^14) - (8\*c^2\*(13\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(3003\*b^4\*x^12) + (16\*c^3\*(13\*b\*B - 8\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(15015\*b^5\*x^10)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{17}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^9} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{13bx^{18}} + \frac{(-9(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x^8} dx, x \right)}{13b} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{143b^2x^{16}} - \frac{(3c(13bB - 8Ac)) \text{Subst} \left( \int \frac{(bx + cx^2)^{3/2}}{x^8} dx, x \right)}{143b^2} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{14}} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{14}} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB - 8Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{14}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 95, normalized size = 0.56

$$\frac{\sqrt{x^2(b + cx^2)} \left( \left( \frac{cx^3}{b} + x \right)^2 (105b^3 - 70b^2cx^2 + 40bc^2x^4 - 16c^3x^6) (8Ac - 13bB) - 1155Ab^2(b + cx^2)^2 \right)}{15015b^3x^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^17, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-1155\*A\*b^2\*(b + c\*x^2)^2 + (-13\*b\*B + 8\*A\*c)\*(x + (c\*x^3)/b)^2\*(105\*b^3 - 70\*b^2\*c\*x^2 + 40\*b\*c^2\*x^4 - 16\*c^3\*x^6)))/(15015\*b^3\*x^14)

**IntegrateAlgebraic [A]** time = 0.52, size = 162, normalized size = 0.95

$$\frac{\sqrt{bx^2 + cx^4} (-1155Ab^6 - 1470Ab^5cx^2 - 35Ab^4c^2x^4 + 40Ab^3c^3x^6 - 48Ab^2c^4x^8 + 64Abc^5x^{10} - 128Ac^6x^{12} - 1365b^6Bx^2 - 1820b^5Bcx^4 - 65b^4Bc^2x^6 + 78b^3Bc^3x^8 - 104b^2Bc^4x^{10} + 208bBc^5x^{12})}{15015b^5x^{14}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^17, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-1155\*A\*b^6 - 1365\*b^6\*B\*x^2 - 1470\*A\*b^5\*c\*x^2 - 1820\*b^5\*B\*c\*x^4 - 35\*A\*b^4\*c^2\*x^4 - 65\*b^4\*B\*c^2\*x^6 + 40\*A\*b^3\*c^3\*x^6 + 78\*b^3\*B\*c^3\*x^8 - 48\*A\*b^2\*c^4\*x^8 - 104\*b^2\*B\*c^4\*x^{10} + 64\*A\*b\*c^5\*x^{10} + 208\*b\*B\*c^5\*x^{12} - 128\*A\*c^6\*x^{12}))/((15015\*b^5\*x^{14}))

**fricas [A]** time = 0.61, size = 157, normalized size = 0.92

$$\frac{(16(13Bb^5 - 8Ac^6)x^{12} - 8(13Bb^2c^4 - 8Abc^5)x^{10} + 6(13Bb^3c^3 - 8Ab^2c^4)x^8 - 1155Ab^6 - 5(13Bb^4c^2 - 8Ab^3c^3)x^6 - 35(52Bb^5c + Ab^4c^2)x^4 - 105(13Bb^6 + 14Ab^5c)x^2)\sqrt{cx^4 + bx^2}}{15015b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^17,x, algorithm="fricas")

[Out] 1/15015\*(16\*(13\*B\*b\*c^5 - 8\*A\*c^6)\*x^12 - 8\*(13\*B\*b^2\*c^4 - 8\*A\*b\*c^5)\*x^10 + 6\*(13\*B\*b^3\*c^3 - 8\*A\*b^2\*c^4)\*x^8 - 1155\*A\*b^6 - 5\*(13\*B\*b^4\*c^2 - 8\*A\*

$$b^3*c^3*x^6 - 35*(52*B*b^5*c + A*b^4*c^2)*x^4 - 105*(13*B*b^6 + 14*A*b^5*c)*x^2)*\sqrt{c*x^4 + b*x^2}/(b^5*x^{14})$$

**giac [B]** time = 4.41, size = 550, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^17,x, algorithm="giac")

[Out]  $\frac{32}{15015}*(15015*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{18}*B*c^{(11/2)}*\text{sgn}(x) - 3003*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{16}*B*b*c^{(11/2)}*\text{sgn}(x) + 48048*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{16}*A*c^{(13/2)}*\text{sgn}(x) - 6006*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*B*b^2*c^{(11/2)}*\text{sgn}(x) + 96096*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*A*b*c^{(13/2)}*\text{sgn}(x) - 28314*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*B*b^3*c^{(11/2)}*\text{sgn}(x) + 109824*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*A*b^2*c^{(13/2)}*\text{sgn}(x) + 13728*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*B*b^4*c^{(11/2)}*\text{sgn}(x) + 37752*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*A*b^3*c^{(13/2)}*\text{sgn}(x) + 5720*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*B*b^5*c^{(11/2)}*\text{sgn}(x) + 5720*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*A*b^4*c^{(13/2)}*\text{sgn}(x) + 3718*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*B*b^6*c^{(11/2)}*\text{sgn}(x) - 2288*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*A*b^5*c^{(13/2)}*\text{sgn}(x) - 1014*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*B*b^7*c^{(11/2)}*\text{sgn}(x) + 624*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*A*b^6*c^{(13/2)}*\text{sgn}(x) + 169*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*B*b^8*c^{(11/2)}*\text{sgn}(x) - 104*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*A*b^7*c^{(13/2)}*\text{sgn}(x) - 13*B*b^9*c^{(11/2)}*\text{sgn}(x) + 8*A*b^8*c^{(13/2)}*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + b})^2 - b)^{13}$

**maple [A]** time = 0.04, size = 118, normalized size = 0.69

$$\frac{(cx^2 + b)(128Ac^4x^8 - 208Bbc^3x^8 - 320Abc^3x^6 + 520Bb^2c^2x^6 + 560Ab^2c^2x^4 - 910Bb^3cx^4 - 840Ab^3cx^2 + 1365Bb^4x^2 + 1155Ab^4)(cx^4 + bx^2)^{\frac{3}{2}}}{15015b^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^17,x)

[Out]  $-1/15015*(c*x^2+b)*(128*A*c^4*x^8-208*B*b*c^3*x^8-320*A*b*c^3*x^6+520*B*b^2*c^2*x^6+560*A*b^2*c^2*x^4-910*B*b^3*c*x^4-840*A*b^3*c*x^2+1365*B*b^4*x^2+1155*A*b^4)*(c*x^4+b*x^2)^(3/2)/b^5/x^{16}$

**maxima [B]** time = 1.62, size = 337, normalized size = 1.98

$$\frac{1}{9240} \left( \frac{128\sqrt{cx^4+bx^2}}{b^4x^8} - \frac{64\sqrt{cx^4+bx^2}}{b^3x^6} + \frac{48\sqrt{cx^4+bx^2}}{b^2x^4} - \frac{40\sqrt{cx^4+bx^2}}{bx^2} + \frac{35\sqrt{cx^4+bx^2}}{x^{10}} + \frac{315\sqrt{cx^4+bx^2}}{x^{12}} - \frac{1155(cx^4+bx^2)^{\frac{3}{2}}}{x^{14}} \right) \frac{1}{30030} \left( \frac{256\sqrt{cx^4+bx^2}}{b^5x^2} - \frac{128\sqrt{cx^4+bx^2}}{b^4x^4} + \frac{96\sqrt{cx^4+bx^2}}{b^3x^6} - \frac{80\sqrt{cx^4+bx^2}}{b^2x^8} + \frac{70\sqrt{cx^4+bx^2}}{bx^{10}} - \frac{63\sqrt{cx^4+bx^2}}{x^{12}} + \frac{693\sqrt{cx^4+bx^2}}{x^{14}} - \frac{3003(cx^4+bx^2)^{\frac{3}{2}}}{x^{16}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^17,x, algorithm="maxima")

[Out]  $\frac{1}{9240}B*(128*\sqrt{c*x^4 + b*x^2}*c^5/(b^4*x^2) - 64*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^4) + 48*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^6) - 40*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^8) + 35*\sqrt{c*x^4 + b*x^2}*c/x^{10} + 315*\sqrt{c*x^4 + b*x^2}*b/x^{12} - 1155*(c*x^4 + b*x^2)^{(3/2)}/x^{14}) - \frac{1}{30030}A*(256*\sqrt{c*x^4 + b*x^2}*c^6/(b^5*x^2) - 128*\sqrt{c*x^4 + b*x^2}*c^5/(b^4*x^4) + 96*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^6) - 80*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^8) + 70*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^{10}) - 63*\sqrt{c*x^4 + b*x^2}*c/x^{12} - 693*\sqrt{c*x^4 + b*x^2}*b/x^{14} + 3003*(c*x^4 + b*x^2)^{(3/2)}/x^{16})$

**mupad [B]** time = 2.50, size = 306, normalized size = 1.80

$$\frac{8Ac^4\sqrt{cx^4+bx^2}}{3003b^2x^8} - \frac{14Ac^4\sqrt{cx^4+bx^2}}{143x^{12}} - \frac{Bb\sqrt{cx^4+bx^2}}{11x^{12}} - \frac{4Bc\sqrt{cx^4+bx^2}}{33x^{10}} - \frac{Ac^2\sqrt{cx^4+bx^2}}{429bx^{10}} - \frac{Ab\sqrt{cx^4+bx^2}}{13x^{14}} - \frac{16Ac^4\sqrt{cx^4+bx^2}}{5005b^3x^6} + \frac{64Ac^4\sqrt{cx^4+bx^2}}{15015b^4x^4} - \frac{128Ac^4\sqrt{cx^4+bx^2}}{15015b^5x^2} - \frac{Bc^2\sqrt{cx^4+bx^2}}{231b^2x^8} + \frac{2Bc^2\sqrt{cx^4+bx^2}}{385b^3x^6} - \frac{8Bc^2\sqrt{cx^4+bx^2}}{1155b^4x^4} + \frac{16Bc^2\sqrt{cx^4+bx^2}}{1155b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x)`

[Out]  $(8Ac^3(bx^2 + cx^4)^{1/2})/(3003b^2x^8) - (14Ac(bx^2 + cx^4)^{1/2})/(143x^{12}) - (Bb(bx^2 + cx^4)^{1/2})/(11x^{12}) - (4Bc(bx^2 + cx^4)^{1/2})/(33x^{10}) - (A^2c(bx^2 + cx^4)^{1/2})/(429bx^{10}) - (Ab(bx^2 + cx^4)^{1/2})/(13x^{14}) - (16A^2c^4(bx^2 + cx^4)^{1/2})/(5005b^3x^6) + (64A^2c^5(bx^2 + cx^4)^{1/2})/(15015b^4x^4) - (128A^2c^6(bx^2 + cx^4)^{1/2})/(15015b^5x^2) - (Bc^2(bx^2 + cx^4)^{1/2})/(231bx^8) + (2Bc^3(bx^2 + cx^4)^{1/2})/(385b^2x^6) - (8Bc^4(bx^2 + cx^4)^{1/2})/(1155b^3x^4) + (16Bc^5(bx^2 + cx^4)^{1/2})/(1155b^4x^2)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{17}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**17,x)`

[Out] `Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**17, x)`

$$3.118 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} dx$$

**Optimal.** Leaf size=207

$$-\frac{128c^4(bx^2+cx^4)^{5/2}(3bB-2Ac)}{45045b^6x^{10}} + \frac{64c^3(bx^2+cx^4)^{5/2}(3bB-2Ac)}{9009b^5x^{12}} - \frac{16c^2(bx^2+cx^4)^{5/2}(3bB-2Ac)}{1287b^4x^{14}} + \frac{8c(bx^2+cx^4)^{5/2}(3bB-2Ac)}{15bx^{20}}$$

**Rubi [A]** time = 0.35, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$-\frac{128c^4(bx^2+cx^4)^{5/2}(3bB-2Ac)}{45045b^6x^{10}} + \frac{64c^3(bx^2+cx^4)^{5/2}(3bB-2Ac)}{9009b^5x^{12}} - \frac{16c^2(bx^2+cx^4)^{5/2}(3bB-2Ac)}{1287b^4x^{14}} + \frac{8c(bx^2+cx^4)^{5/2}(3bB-2Ac)}{429b^3x^{16}} - \frac{(bx^2+cx^4)^{5/2}(3bB-2Ac)}{39b^2x^{18}} - \frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^19, x]

[Out] -(A\*(b\*x^2 + c\*x^4)^(5/2))/(15\*b\*x^20) - ((3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(39\*b^2\*x^18) + (8\*c\*(3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(429\*b^3\*x^16) - (16\*c^2\*(3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(1287\*b^4\*x^14) + (64\*c^3\*(3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(9009\*b^5\*x^12) - (128\*c^4\*(3\*b\*B - 2\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(45045\*b^6\*x^10)

Rule 650

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

Rule 658

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

Rule 792

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((f\_.) + (g\_.)\*(x\_.))\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{19}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(A + Bx)(bx + cx^2)^{3/2}}{x^{10}} dx, x, x^2 \right) \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{15bx^{20}} + \frac{(-10(-bB + Ac) + \frac{5}{2}(-bB + 2Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^9} dx, \right)}{15b} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{39b^2x^{18}} - \frac{(4c(3bB - 2Ac)) \text{Subst} \left( \int \frac{(bx+cx^2)^{3/2}}{x^9} dx, \right)}{39b^2} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{16}} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{16}} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{16}} \\
&= -\frac{A(bx^2 + cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB - 2Ac)(bx^2 + cx^4)^{5/2}}{429b^3x^{16}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 89, normalized size = 0.43

$$\frac{(x^2(b + cx^2))^{5/2} (-3003Ab^5 - x^2(1155b^4 - 840b^3cx^2 + 560b^2c^2x^4 - 320bc^3x^6 + 128c^4x^8)(3bB - 2Ac))}{45045b^6x^{20}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^19, x]

[Out] ((x^2\*(b + c\*x^2))^(5/2)\*(-3003\*A\*b^5 - (3\*b\*B - 2\*A\*c)\*x^2\*(1155\*b^4 - 840\*b^3\*c\*x^2 + 560\*b^2\*c^2\*x^4 - 320\*b\*c^3\*x^6 + 128\*c^4\*x^8)))/(45045\*b^6\*x^20)

**IntegrateAlgebraic [A]** time = 0.56, size = 186, normalized size = 0.90

$$\frac{\sqrt{bx^2 + cx^4} (-3003Ab^7 - 3696Ab^6cx^2 - 63Ab^5c^2x^4 + 70Ab^4c^3x^6 - 80Ab^3c^4x^8 + 96Ab^2c^5x^{10} - 128Abc^6x^{12} + 256Ac^7x^{14} - 3465b^7Bx^2 - 4410b^6Bcx^4 - 105b^5Bc^2x^6 + 120b^4Bc^3x^8 - 144b^3Bc^4x^{10} + 192b^2Bc^5x^{12} - 384bBc^6x^{14})}{45045b^6x^{16}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^19, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-3003\*A\*b^7 - 3465\*b^7\*B\*x^2 - 3696\*A\*b^6\*c\*x^2 - 4410\*b^6\*B\*c\*x^4 - 63\*A\*b^5\*c^2\*x^4 - 105\*b^5\*B\*c^2\*x^6 + 70\*A\*b^4\*c^3\*x^6 + 120\*b^4\*B\*c^3\*x^8 - 80\*A\*b^3\*c^4\*x^8 - 144\*b^3\*B\*c^4\*x^10 + 96\*A\*b^2\*c^5\*x^10 + 192\*b^2\*B\*c^5\*x^12 - 128\*A\*b\*c^6\*x^12 - 384\*b\*B\*c^6\*x^14 + 256\*A\*c^7\*x^14))/(45045\*b^6\*x^16)

**fricas [A]** time = 0.75, size = 181, normalized size = 0.87

$$\frac{(128(3Bb^6 - 2Ac^7)x^{14} - 64(3Bb^5c^2 - 2Abc^6)x^{12} + 48(3Bb^4c^3 - 2Ab^2c^5)x^{10} - 40(3Bb^3c^4 - 2Ab^2c^4)x^8 + 3003Ab^7 + 35(3Bb^5c^2 - 2Ab^4c^3)x^6 + 63(70Bb^6c + Ab^5c^2)x^4 + 231(15Bb^7 + 16Ab^6c)x^2)\sqrt{cx^4 + bx^2}}{45045b^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^19,x, algorithm="fricas")

[Out] 
$$-1/45045*(128*(3*B*b*c^6 - 2*A*c^7)*x^{14} - 64*(3*B*b^2*c^5 - 2*A*b*c^6)*x^{12} + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^{10} - 40*(3*B*b^4*c^3 - 2*A*b^3*c^4)*x^8 + 3003*A*b^7 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^6 + 63*(70*B*b^6*c + A*b^5*c^2)*x^4 + 231*(15*B*b^7 + 16*A*b^6*c)*x^2)*\sqrt{c*x^4 + b*x^2}/(b^6*x^{16})$$

**giac** [B] time = 5.43, size = 582, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^19,x, algorithm="giac")

[Out] 
$$256/45045*(18018*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{20}*B*c^{(13/2)}*\text{sgn}(x) + 60060*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{18}*A*c^{(15/2)}*\text{sgn}(x) - 12870*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{16}*B*b^2*c^{(13/2)}*\text{sgn}(x) + 128700*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{16}*A*b*c^{(15/2)}*\text{sgn}(x) - 32175*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*B*b^3*c^{(13/2)}*\text{sgn}(x) + 141570*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{14}*A*b^2*c^{(15/2)}*\text{sgn}(x) + 15015*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*B*b^4*c^{(13/2)}*\text{sgn}(x) + 50050*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{12}*A*b^3*c^{(15/2)}*\text{sgn}(x) + 9009*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*B*b^5*c^{(13/2)}*\text{sgn}(x) + 6006*(\sqrt{c}*x - \sqrt{c*x^2 + b})^{10}*A*b^4*c^{(15/2)}*\text{sgn}(x) + 4095*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*B*b^6*c^{(13/2)}*\text{sgn}(x) - 2730*(\sqrt{c}*x - \sqrt{c*x^2 + b})^8*A*b^5*c^{(15/2)}*\text{sgn}(x) - 1365*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*B*b^7*c^{(13/2)}*\text{sgn}(x) + 910*(\sqrt{c}*x - \sqrt{c*x^2 + b})^6*A*b^6*c^{(15/2)}*\text{sgn}(x) + 315*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*B*b^8*c^{(13/2)}*\text{sgn}(x) - 210*(\sqrt{c}*x - \sqrt{c*x^2 + b})^4*A*b^7*c^{(15/2)}*\text{sgn}(x) - 45*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*B*b^9*c^{(13/2)}*\text{sgn}(x) + 30*(\sqrt{c}*x - \sqrt{c*x^2 + b})^2*A*b^8*c^{(15/2)}*\text{sgn}(x) + 3*B*b^10*c^{(13/2)}*\text{sgn}(x) - 2*A*b^9*c^{(15/2)}*\text{sgn}(x))/((\sqrt{c}*x - \sqrt{c*x^2 + b})^2 - b)^{15}$$

**maple** [A] time = 0.05, size = 142, normalized size = 0.69

$$\frac{(c x^2 + b)(-256 A c^5 x^{10} + 384 B b c^4 x^{10} + 640 A b c^4 x^8 - 960 B b^2 c^3 x^8 - 1120 A b^2 c^3 x^6 + 1680 B b^3 c^2 x^6 + 1680 A b^3 c^2 x^4 - 2520 B b^4 c x^4 - 2310 A b^4 c x^2 + 3465 B b^5 x^2 + 3003 A b^5)(c x^4 + b x^2)^{\frac{3}{2}}}{45045 b^6 x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^19,x)

[Out] 
$$-1/45045*(c*x^2+b)*(-256*A*c^5*x^{10}+384*B*b*c^4*x^{10}+640*A*b*c^4*x^8-960*B*b^2*c^3*x^8-1120*A*b^2*c^3*x^6+1680*B*b^3*c^2*x^6+1680*A*b^3*c^2*x^4-2520*B*b^4*c*x^4-2310*A*b^4*c*x^2+3465*B*b^5*x^2+3003*A*b^5)*(c*x^4+b*x^2)^(3/2)/x^{18}/b^6$$

**maxima** [B] time = 1.65, size = 385, normalized size = 1.86

$$\frac{1}{30030} \left( \frac{256 \sqrt{c^4 + b^2} c^5}{b^{12}} - \frac{128 \sqrt{c^4 + b^2} c^4}{b^{12}} - \frac{96 \sqrt{c^4 + b^2} c^4}{b^{12}} - \frac{80 \sqrt{c^4 + b^2} c^3}{b^{10}} - \frac{70 \sqrt{c^4 + b^2} c^3}{b^{10}} - \frac{63 \sqrt{c^4 + b^2} c^3}{b^{10}} - \frac{693 \sqrt{c^4 + b^2} c^2}{b^{10}} - \frac{3003 (c^4 + b^2)^{\frac{3}{2}}}{b^{10}} \right) \frac{1}{180180} \left( \frac{1024 \sqrt{c^4 + b^2} c^5}{b^{12}} - \frac{512 \sqrt{c^4 + b^2} c^4}{b^{12}} - \frac{384 \sqrt{c^4 + b^2} c^4}{b^{12}} - \frac{320 \sqrt{c^4 + b^2} c^3}{b^{10}} - \frac{280 \sqrt{c^4 + b^2} c^3}{b^{10}} - \frac{252 \sqrt{c^4 + b^2} c^3}{b^{10}} - \frac{231 \sqrt{c^4 + b^2} c^2}{b^{10}} - \frac{3003 \sqrt{c^4 + b^2} c^2}{b^{10}} - \frac{15015 (c^4 + b^2)^{\frac{3}{2}}}{b^{10}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^19,x, algorithm="maxima")

[Out] 
$$-1/30030*B*(256*\sqrt{c*x^4 + b*x^2}*c^6/(b^5*x^2) - 128*\sqrt{c*x^4 + b*x^2}*c^5/(b^4*x^4) + 96*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^6) - 80*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^8) + 70*\sqrt{c*x^4 + b*x^2}*c^2/(b*x^{10}) - 63*\sqrt{c*x^4 + b*x^2}*c/x^{12} - 693*\sqrt{c*x^4 + b*x^2}*b/x^{14} + 3003*(c*x^4 + b*x^2)^{(3/2)}/x^{16}) + 1/180180*A*(1024*\sqrt{c*x^4 + b*x^2}*c^7/(b^6*x^2) - 512*\sqrt{c*x^4 + b*x^2}*c^6/(b^5*x^4) + 384*\sqrt{c*x^4 + b*x^2}*c^5/(b^4*x^6) - 320*\sqrt{c*x^4 + b*x^2}*c^4/(b^3*x^8) + 280*\sqrt{c*x^4 + b*x^2}*c^3/(b^2*x^{10}) - 252*$$

\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b\*x^12) + 231\*sqrt(c\*x^4 + b\*x^2)\*c/x^14 + 3003\*sqrt(c\*x^4 + b\*x^2)\*b/x^16 - 15015\*(c\*x^4 + b\*x^2)^(3/2)/x^18)

mupad [B] time = 3.21, size = 356, normalized size = 1.72

$$\frac{2A^2\sqrt{cx^4+b^2}}{1287b^2x^{10}} - \frac{16Ac\sqrt{cx^4+b^2}}{195x^{14}} - \frac{Bb\sqrt{cx^4+b^2}}{13x^{14}} - \frac{148c\sqrt{cx^4+b^2}}{143x^{12}} - \frac{A^2\sqrt{cx^4+b^2}}{715bx^{12}} - \frac{Ab\sqrt{cx^4+b^2}}{15x^{16}} - \frac{16A^2\sqrt{cx^4+b^2}}{9009b^3x^8} - \frac{32A^2\sqrt{cx^4+b^2}}{15015b^4x^6} - \frac{128A^2\sqrt{cx^4+b^2}}{45045b^5x^4} - \frac{256A^2\sqrt{cx^4+b^2}}{45045b^6x^2} - \frac{Bc^2\sqrt{cx^4+b^2}}{429bx^{10}} - \frac{8Bc^2\sqrt{cx^4+b^2}}{3003b^2x^8} - \frac{16Bc^2\sqrt{cx^4+b^2}}{5005b^3x^6} - \frac{64Bc^2\sqrt{cx^4+b^2}}{15015b^4x^4} - \frac{128Bc^2\sqrt{cx^4+b^2}}{15015b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^19,x)

[Out] (2\*A\*c^3\*(b\*x^2 + c\*x^4)^(1/2))/(1287\*b^2\*x^10) - (16\*A\*c\*(b\*x^2 + c\*x^4)^(1/2))/(195\*x^14) - (B\*b\*(b\*x^2 + c\*x^4)^(1/2))/(13\*x^14) - (14\*B\*c\*(b\*x^2 + c\*x^4)^(1/2))/(143\*x^12) - (A\*c^2\*(b\*x^2 + c\*x^4)^(1/2))/(715\*b\*x^12) - (A\*b\*(b\*x^2 + c\*x^4)^(1/2))/(15\*x^16) - (16\*A\*c^4\*(b\*x^2 + c\*x^4)^(1/2))/(9009\*b^3\*x^8) + (32\*A\*c^5\*(b\*x^2 + c\*x^4)^(1/2))/(15015\*b^4\*x^6) - (128\*A\*c^6\*(b\*x^2 + c\*x^4)^(1/2))/(45045\*b^5\*x^4) + (256\*A\*c^7\*(b\*x^2 + c\*x^4)^(1/2))/(45045\*b^6\*x^2) - (B\*c^2\*(b\*x^2 + c\*x^4)^(1/2))/(429\*b\*x^10) + (8\*B\*c^3\*(b\*x^2 + c\*x^4)^(1/2))/(3003\*b^2\*x^8) - (16\*B\*c^4\*(b\*x^2 + c\*x^4)^(1/2))/(5005\*b^3\*x^6) + (64\*B\*c^5\*(b\*x^2 + c\*x^4)^(1/2))/(15015\*b^4\*x^4) - (128\*B\*c^6\*(b\*x^2 + c\*x^4)^(1/2))/(15015\*b^5\*x^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*19,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2)/x\*\*19, x)



$$3.119 \quad \int x^4 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$$

**Optimal.** Leaf size=168

$$\frac{16b^3 (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{15015c^5x^5} - \frac{8b^2 (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{3003c^4x^3} + \frac{2b (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{429c^3x} - \frac{x (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{143c^2} + \frac{Bx^3 (bx^2 + cx^4)^{5/2}}{13c}$$

**Rubi [A]** time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2039, 2016, 2002, 2014}

$$\frac{16b^3 (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{15015c^5x^5} - \frac{8b^2 (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{3003c^4x^3} + \frac{2b (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{429c^3x} - \frac{x (bx^2 + cx^4)^{5/2} (8bB - 13Ac)}{143c^2} + \frac{Bx^3 (bx^2 + cx^4)^{5/2}}{13c}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x]

[Out] (16\*b^3\*(8\*b\*B - 13\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(15015\*c^5\*x^5) - (8\*b^2\*(8\*b\*B - 13\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(3003\*c^4\*x^3) + (2\*b\*(8\*b\*B - 13\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(429\*c^3\*x) - ((8\*b\*B - 13\*A\*c)\*x\*(b\*x^2 + c\*x^4)^(5/2))/(143\*c^2) + (B\*x^3\*(b\*x^2 + c\*x^4)^(5/2))/(13\*c)

Rule 2002

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a\*x^j + b\*x^n)^(p + 1)/(a\*(j\*p + 1)\*x^(j - 1)), x] - Dist[(b\*(n\*p + n - j + 1))/(a\*(j\*p + 1)), Int[x^(n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n\*p + n - j + 1)/(n - j)], 0] && NeQ[j\*p + 1, 0]

Rule 2014

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := -Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(n - j)\*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n\*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegerQ[j, n] || GtQ[c, 0])

Rule 2039

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j + n)))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

Rubi steps

$$\begin{aligned}
\int x^4 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx &= \frac{Bx^3 (bx^2 + cx^4)^{5/2}}{13c} - \frac{(8bB - 13Ac) \int x^4 (bx^2 + cx^4)^{3/2} dx}{13c} \\
&= -\frac{(8bB - 13Ac)x (bx^2 + cx^4)^{5/2}}{143c^2} + \frac{Bx^3 (bx^2 + cx^4)^{5/2}}{13c} + \frac{(6b(8bB - 13Ac)) \int x^4 (bx^2 + cx^4)^{3/2} dx}{143c^2} \\
&= \frac{2b(8bB - 13Ac) (bx^2 + cx^4)^{5/2}}{429c^3x} - \frac{(8bB - 13Ac)x (bx^2 + cx^4)^{5/2}}{143c^2} + \frac{Bx^3 (bx^2 + cx^4)^{5/2}}{13c} \\
&= -\frac{8b^2(8bB - 13Ac) (bx^2 + cx^4)^{5/2}}{3003c^4x^3} + \frac{2b(8bB - 13Ac) (bx^2 + cx^4)^{5/2}}{429c^3x} - \frac{(8bB - 13Ac)x (bx^2 + cx^4)^{5/2}}{143c^2} \\
&= \frac{16b^3(8bB - 13Ac) (bx^2 + cx^4)^{5/2}}{15015c^5x^5} - \frac{8b^2(8bB - 13Ac) (bx^2 + cx^4)^{5/2}}{3003c^4x^3} + \frac{2b(8bB - 13Ac)x (bx^2 + cx^4)^{5/2}}{143c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 113, normalized size = 0.67

$$\frac{x(b + cx^2)^3 (-16b^3c(13A + 20Bx^2) + 40b^2c^2x^2(13A + 14Bx^2) - 70bc^3x^4(13A + 12Bx^2) + 105c^4x^6(13A + 11Bx^2) + 128b^4B)}{15015c^5\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(b + c\*x^2)^3\*(128\*b^4\*B + 105\*c^4\*x^6\*(13\*A + 11\*B\*x^2) - 70\*b\*c^3\*x^4\*(13\*A + 12\*B\*x^2) + 40\*b^2\*c^2\*x^2\*(13\*A + 14\*B\*x^2) - 16\*b^3\*c\*(13\*A + 20\*B\*x^2)))/(15015\*c^5\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.66, size = 111, normalized size = 0.66

$$\frac{(bx^2 + cx^4)^{5/2} (-208Ab^3c + 520Ab^2c^2x^2 - 910Abc^3x^4 + 1365Ac^4x^6 + 128b^4B - 320b^3Bcx^2 + 560b^2Bc^2x^4 - 840bBc^3x^6 + 1155Bc^4x^8)}{15015c^5x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((b\*x^2 + c\*x^4)^(5/2)\*(128\*b^4\*B - 208\*A\*b^3\*c - 320\*b^3\*B\*c\*x^2 + 520\*A\*b^2\*c^2\*x^2 + 560\*b^2\*B\*c^2\*x^4 - 910\*A\*b\*c^3\*x^4 - 840\*b\*B\*c^3\*x^6 + 1365\*A\*c^4\*x^6 + 1155\*B\*c^4\*x^8))/(15015\*c^5\*x^5)

**fricas [A]** time = 0.41, size = 154, normalized size = 0.92

$$\frac{(1155Bc^6x^{12} + 105(14Bbc^5 + 13Ac^6)x^{10} + 35(Bb^2c^4 + 52Abc^5)x^8 + 128Bb^6 - 208Ab^5c - 5(8Bb^3c^3 - 13Ab^2c^4)x^6 + 6(8Bb^4c^2 - 13Ab^3c^3)x^4 - 8(8Bb^5c - 13Ab^4c^2)x^2)\sqrt{cx^4 + bx^2}}{15015c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/15015\*(1155\*B\*c^6\*x^12 + 105\*(14\*B\*b\*c^5 + 13\*A\*c^6)\*x^10 + 35\*(B\*b^2\*c^4 + 52\*A\*b\*c^5)\*x^8 + 128\*B\*b^6 - 208\*A\*b^5\*c - 5\*(8\*B\*b^3\*c^3 - 13\*A\*b^2\*c^4)\*x^6 + 6\*(8\*B\*b^4\*c^2 - 13\*A\*b^3\*c^3)\*x^4 - 8\*(8\*B\*b^5\*c - 13\*A\*b^4\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^5\*x)

**giac [A]** time = 0.34, size = 175, normalized size = 1.04

$$\frac{16(8Bb^3c^3 - 13Ab^2c^4)\operatorname{sgn}(x)}{15015c^5} + \frac{1155(c^2 + b)^{\frac{11}{2}}B\operatorname{sgn}(x) - 5460(c^2 + b)^{\frac{11}{2}}Bb\operatorname{sgn}(x) + 10010(c^2 + b)^{\frac{9}{2}}Bb^2\operatorname{sgn}(x) - 8580(c^2 + b)^{\frac{7}{2}}Bb^3\operatorname{sgn}(x) + 3003(c^2 + b)^{\frac{5}{2}}Bb^4\operatorname{sgn}(x) + 1365(c^2 + b)^{\frac{3}{2}}Ac\operatorname{sgn}(x) - 5005(c^2 + b)^{\frac{3}{2}}Abc\operatorname{sgn}(x) + 6435(c^2 + b)^{\frac{1}{2}}Ab^2c\operatorname{sgn}(x) - 3003(c^2 + b)^{\frac{1}{2}}Ab^3c\operatorname{sgn}(x)}{15015c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] 
$$-16/15015*(8*B*b^{13/2} - 13*A*b^{11/2}*c)*\text{sgn}(x)/c^5 + 1/15015*(1155*(c*x^2 + b)^{13/2}*B*\text{sgn}(x) - 5460*(c*x^2 + b)^{11/2}*B*b*\text{sgn}(x) + 10010*(c*x^2 + b)^{9/2}*B*b^2*\text{sgn}(x) - 8580*(c*x^2 + b)^{7/2}*B*b^3*\text{sgn}(x) + 3003*(c*x^2 + b)^{5/2}*B*b^4*\text{sgn}(x) + 1365*(c*x^2 + b)^{11/2}*A*c*\text{sgn}(x) - 5005*(c*x^2 + b)^{9/2}*A*b*c*\text{sgn}(x) + 6435*(c*x^2 + b)^{7/2}*A*b^2*c*\text{sgn}(x) - 3003*(c*x^2 + b)^{5/2}*A*b^3*c*\text{sgn}(x))/c^5$$

**maple [A]** time = 0.05, size = 115, normalized size = 0.68

$$\frac{(cx^2 + b)(-1155Bx^8c^4 - 1365A^4c^6 + 840Bbc^3x^6 + 910Abc^3x^4 - 560Bb^2c^2x^4 - 520Ab^2c^2x^2 + 320Bb^3cx^2 + 208Ab^3c - 128Bb^4)(cx^4 + bx^2)^{\frac{3}{2}}}{15015c^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x)

[Out] 
$$-1/15015*(c*x^2+b)*(-1155*B*c^4*x^8-1365*A*c^4*x^6+840*B*b*c^3*x^6+910*A*b*c^3*x^4-560*B*b^2*c^2*x^4-520*A*b^2*c^2*x^2+320*B*b^3*c*x^2+208*A*b^3*c-128*B*b^4)*(c*x^4+b*x^2)^{3/2}/c^5/x^3$$

**maxima [A]** time = 1.55, size = 150, normalized size = 0.89

$$\frac{(105c^5x^{10} + 140bc^4x^8 + 5b^2c^3x^6 - 6b^3c^2x^4 + 8b^4cx^2 - 16b^5)\sqrt{cx^2 + b}A}{1155c^4} + \frac{(1155c^6x^{12} + 1470bc^5x^{10} + 35b^2c^4x^8 - 40b^3c^3x^6 + 48b^4c^2x^4 - 64b^5cx^2 + 128b^6)\sqrt{cx^2 + b}B}{15015c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 
$$1/1155*(105*c^5*x^{10} + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*\text{sqrt}(c*x^2 + b)*A/c^4 + 1/15015*(1155*c^6*x^{12} + 1470*b*c^5*x^{10} + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*\text{sqrt}(c*x^2 + b)*B/c^5$$

**mupad [B]** time = 0.35, size = 143, normalized size = 0.85

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{128Bb^6 - 208Ab^5c}{15015c^5} + \frac{x^{10}(1365Ac^6 + 1470Bbc^5)}{15015c^5} + \frac{Bcx^{12}}{13} + \frac{b^2x^6(13Ac - 8Bb)}{3003c^2} - \frac{2b^3x^4(13Ac - 8Bb)}{5005c^3} + \frac{8b^4x^2(13Ac - 8Bb)}{15015c^4} + \frac{bx^8(52Ac + Bb)}{429c} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x)

[Out] 
$$((b*x^2 + c*x^4)^{1/2}*((128*B*b^6 - 208*A*b^5*c)/(15015*c^5) + (x^{10}*(1365*A*c^6 + 1470*B*b*c^5))/(15015*c^5) + (B*c*x^{12})/13 + (b^2*x^6*(13*A*c - 8*B*b))/(3003*c^2) - (2*b^3*x^4*(13*A*c - 8*B*b))/(5005*c^3) + (8*b^4*x^2*(13*A*c - 8*B*b))/(15015*c^4) + (b*x^8*(52*A*c + B*b))/(429*c)))/x$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (x^2 (b + cx^2))^{\frac{3}{2}} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*\*4\*(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2), x)

$$3.120 \quad \int x^2 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$$

Optimal. Leaf size=131

$$-\frac{8b^2 (bx^2 + cx^4)^{5/2} (6bB - 11Ac)}{3465c^4x^5} + \frac{4b (bx^2 + cx^4)^{5/2} (6bB - 11Ac)}{693c^3x^3} - \frac{(bx^2 + cx^4)^{5/2} (6bB - 11Ac)}{99c^2x} + \frac{Bx (bx^2 + cx^4)^5}{11c}$$

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2039, 2016, 2002, 2014}

$$-\frac{8b^2 (bx^2 + cx^4)^{5/2} (6bB - 11Ac)}{3465c^4x^5} - \frac{(bx^2 + cx^4)^{5/2} (6bB - 11Ac)}{99c^2x} + \frac{4b (bx^2 + cx^4)^{5/2} (6bB - 11Ac)}{693c^3x^3} + \frac{Bx (bx^2 + cx^4)^{5/2}}{11c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x]

[Out] (-8\*b^2\*(6\*b\*B - 11\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(3465\*c^4\*x^5) + (4\*b\*(6\*b\*B - 11\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(693\*c^3\*x^3) - ((6\*b\*B - 11\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(99\*c^2\*x) + (B\*x\*(b\*x^2 + c\*x^4)^(5/2))/(11\*c)

#### Rule 2002

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a\*x^j + b\*x^n)^(p + 1)/(a\*(j\*p + 1)\*x^(j - 1)), x] - Dist[(b\*(n\*p + n - j + 1))/(a\*(j\*p + 1)), Int[x^(n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n\*p + n - j + 1)/(n - j)], 0] && NeQ[j\*p + 1, 0]

#### Rule 2014

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := -Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(n - j)\*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n\*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

#### Rule 2016

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

#### Rule 2039

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m)\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned}
\int x^2 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx &= \frac{Bx (bx^2 + cx^4)^{5/2}}{11c} - \frac{(6bB - 11Ac) \int x^2 (bx^2 + cx^4)^{3/2} dx}{11c} \\
&= -\frac{(6bB - 11Ac) (bx^2 + cx^4)^{5/2}}{99c^2x} + \frac{Bx (bx^2 + cx^4)^{5/2}}{11c} + \frac{(4b(6bB - 11Ac)) \int}{99c^2} \\
&= \frac{4b(6bB - 11Ac) (bx^2 + cx^4)^{5/2}}{693c^3x^3} - \frac{(6bB - 11Ac) (bx^2 + cx^4)^{5/2}}{99c^2x} + \frac{Bx (bx^2 + cx^4)^{5/2}}{11c} \\
&= -\frac{8b^2(6bB - 11Ac) (bx^2 + cx^4)^{5/2}}{3465c^4x^5} + \frac{4b(6bB - 11Ac) (bx^2 + cx^4)^{5/2}}{693c^3x^3} - \frac{(6bB - 11Ac) (bx^2 + cx^4)^{5/2}}{99c^2x}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 92, normalized size = 0.70

$$\frac{x(b + cx^2)^3 (8b^2c(11A + 15Bx^2) - 10bc^2x^2(22A + 21Bx^2) + 35c^3x^4(11A + 9Bx^2) - 48b^3B)}{3465c^4 \sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(b + c\*x^2)^3\*(-48\*b^3\*B + 35\*c^3\*x^4\*(11\*A + 9\*B\*x^2) + 8\*b^2\*c\*(11\*A + 15\*B\*x^2) - 10\*b\*c^2\*x^2\*(22\*A + 21\*B\*x^2)))/(3465\*c^4\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.56, size = 87, normalized size = 0.66

$$\frac{(bx^2 + cx^4)^{5/2} (88Ab^2c - 220Abc^2x^2 + 385Ac^3x^4 - 48b^3B + 120b^2Bcx^2 - 210bBc^2x^4 + 315Bc^3x^6)}{3465c^4x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((b\*x^2 + c\*x^4)^(5/2)\*(-48\*b^3\*B + 88\*A\*b^2\*c + 120\*b^2\*B\*c\*x^2 - 220\*A\*b\*c^2\*x^2 - 210\*b\*B\*c^2\*x^4 + 385\*A\*c^3\*x^4 + 315\*B\*c^3\*x^6))/(3465\*c^4\*x^5)

**fricas [A]** time = 0.42, size = 131, normalized size = 1.00

$$\frac{(315Bc^5x^{10} + 35(12Bbc^4 + 11Ac^5)x^8 + 5(3Bb^2c^3 + 110Abc^4)x^6 - 48Bb^5 + 88Ab^4c - 3(6Bb^3c^2 - 11Ab^2c^3)x^4 + 4(6Bb^4c - 11Ab^3c^2)x^2)\sqrt{cx^4 + bx^2}}{3465c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/3465\*(315\*B\*c^5\*x^10 + 35\*(12\*B\*b\*c^4 + 11\*A\*c^5)\*x^8 + 5\*(3\*B\*b^2\*c^3 + 110\*A\*b\*c^4)\*x^6 - 48\*B\*b^5 + 88\*A\*b^4\*c - 3\*(6\*B\*b^3\*c^2 - 11\*A\*b^2\*c^3)\*x^4 + 4\*(6\*B\*b^4\*c - 11\*A\*b^3\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^4\*x)

**giac [A]** time = 0.21, size = 140, normalized size = 1.07

$$\frac{8(6Bb^{\frac{11}{2}} - 11Ab^{\frac{9}{2}}c)\operatorname{sgn}(x)}{3465c^4} + \frac{315(cx^2 + b)^{\frac{11}{2}}B\operatorname{sgn}(x) - 1155(cx^2 + b)^{\frac{9}{2}}Bb\operatorname{sgn}(x) + 1485(cx^2 + b)^{\frac{7}{2}}Bb^2\operatorname{sgn}(x) - 693(cx^2 + b)^{\frac{5}{2}}Bb^3\operatorname{sgn}(x) + 385(cx^2 + b)^{\frac{3}{2}}Ac\operatorname{sgn}(x) - 990(cx^2 + b)^{\frac{1}{2}}Abc\operatorname{sgn}(x) + 693(cx^2 + b)^{\frac{1}{2}}Ab^2c\operatorname{sgn}(x)}{3465c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="giac")

[Out]  $8/3465*(6*B*b^{(11/2)} - 11*A*b^{(9/2)}*c)*\text{sgn}(x)/c^4 + 1/3465*(315*(c*x^2 + b)^{(11/2)}*B*\text{sgn}(x) - 1155*(c*x^2 + b)^{(9/2)}*B*b*\text{sgn}(x) + 1485*(c*x^2 + b)^{(7/2)}*B*b^2*\text{sgn}(x) - 693*(c*x^2 + b)^{(5/2)}*B*b^3*\text{sgn}(x) + 385*(c*x^2 + b)^{(9/2)}*A*c*\text{sgn}(x) - 990*(c*x^2 + b)^{(7/2)}*A*b*c*\text{sgn}(x) + 693*(c*x^2 + b)^{(5/2)}*A*b^2*c*\text{sgn}(x))/c^4$

**maple [A]** time = 0.05, size = 91, normalized size = 0.69

$$\frac{(cx^2 + b)(315Bc^3x^6 + 385Ac^3x^4 - 210Bbc^2x^4 - 220Abc^2x^2 + 120Bb^2cx^2 + 88Ab^2c - 48Bb^3)(cx^4 + bx^2)^{\frac{3}{2}}}{3465c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)`

[Out]  $1/3465*(c*x^2+b)*(315*B*c^3*x^6+385*A*c^3*x^4-210*B*b*c^2*x^4-220*A*b*c^2*x^2+120*B*b^2*c*x^2+88*A*b^2*c-48*B*b^3)*(c*x^4+b*x^2)^{(3/2)}/c^4/x^3$

**maxima [A]** time = 1.53, size = 128, normalized size = 0.98

$$\frac{(35c^4x^8 + 50bc^3x^6 + 3b^2c^2x^4 - 4b^3cx^2 + 8b^4)\sqrt{cx^2 + b}A}{315c^3} + \frac{(105c^5x^{10} + 140bc^4x^8 + 5b^2c^3x^6 - 6b^3c^2x^4 + 8b^4cx^2 - 16b^5)\sqrt{cx^2 + b}B}{1155c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")`

[Out]  $1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*\text{sqrt}(c*x^2 + b)*A/c^3 + 1/1155*(105*c^5*x^{10} + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*\text{sqrt}(c*x^2 + b)*B/c^4$

**mupad [B]** time = 0.29, size = 124, normalized size = 0.95

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{x^8(385Ac^5 + 420Bbc^4)}{3465c^4} - \frac{48Bb^5 - 88Ab^4c}{3465c^4} + \frac{Bcx^{10}}{11} + \frac{b^2x^4(11Ac - 6Bb)}{1155c^2} - \frac{4b^3x^2(11Ac - 6Bb)}{3465c^3} + \frac{bx^6(110Ac + 3Bb)}{693c} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)`

[Out]  $((b*x^2 + c*x^4)^{(1/2)}*((x^8*(385*A*c^5 + 420*B*b*c^4))/(3465*c^4) - (48*B*b^5 - 88*A*b^4*c)/(3465*c^4) + (B*c*x^{10})/11 + (b^2*x^4*(11*A*c - 6*B*b))/(1155*c^2) - (4*b^3*x^2*(11*A*c - 6*B*b))/(3465*c^3) + (b*x^6*(110*A*c + 3*B*b))/(693*c)))/x$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (x^2 (b + cx^2))^{\frac{3}{2}} (A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)`

[Out] `Integral(x**2*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)`

$$3.121 \quad \int (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$$

Optimal. Leaf size=96

$$\frac{2b(bx^2 + cx^4)^{5/2} (4bB - 9Ac)}{315c^3x^5} - \frac{(bx^2 + cx^4)^{5/2} (4bB - 9Ac)}{63c^2x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{9cx}$$

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {1145, 2002, 2014}

$$-\frac{(bx^2 + cx^4)^{5/2} (4bB - 9Ac)}{63c^2x^3} + \frac{2b(bx^2 + cx^4)^{5/2} (4bB - 9Ac)}{315c^3x^5} + \frac{B(bx^2 + cx^4)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (2\*b\*(4\*b\*B - 9\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(315\*c^3\*x^5) - ((4\*b\*B - 9\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(63\*c^2\*x^3) + (B\*(b\*x^2 + c\*x^4)^(5/2))/(9\*c\*x)

#### Rule 1145

Int[((d\_) + (e\_.)\*(x\_)^2)\*((b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> Simp[(e\*(b\*x^2 + c\*x^4)^(p + 1))/(c\*(4\*p + 3)\*x), x] - Dist[(b\*e\*(2\*p + 1) - c\*d\*(4\*p + 3))/(c\*(4\*p + 3)), Int[(b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{b, c, d, e, p}, x] && !IntegerQ[p] && NeQ[4\*p + 3, 0] && NeQ[b\*e\*(2\*p + 1) - c\*d\*(4\*p + 3), 0]

#### Rule 2002

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a\*x^j + b\*x^n)^(p + 1)/(a\*(j\*p + 1)\*x^(j - 1)), x] - Dist[(b\*(n\*p + n - j + 1))/(a\*(j\*p + 1)), Int[x^(n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n\*p + n - j + 1)/(n - j)], 0] && NeQ[j\*p + 1, 0]

#### Rule 2014

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> -Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(n - j)\*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n\*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

#### Rubi steps

$$\begin{aligned} \int (A + Bx^2) (bx^2 + cx^4)^{3/2} dx &= \frac{B(bx^2 + cx^4)^{5/2}}{9cx} - \frac{(4bB - 9Ac) \int (bx^2 + cx^4)^{3/2} dx}{9c} \\ &= -\frac{(4bB - 9Ac)(bx^2 + cx^4)^{5/2}}{63c^2x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{9cx} + \frac{(2b(4bB - 9Ac)) \int \frac{(bx^2 + cx^4)}{x^2}}{63c^2} \\ &= \frac{2b(4bB - 9Ac)(bx^2 + cx^4)^{5/2}}{315c^3x^5} - \frac{(4bB - 9Ac)(bx^2 + cx^4)^{5/2}}{63c^2x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{9cx} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 71, normalized size = 0.74

$$\frac{x(b + cx^2)^3(-2bc(9A + 10Bx^2) + 5c^2x^2(9A + 7Bx^2) + 8b^2B)}{315c^3\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(b + c\*x^2)^3\*(8\*b^2\*B + 5\*c^2\*x^2\*(9\*A + 7\*B\*x^2) - 2\*b\*c\*(9\*A + 10\*B\*x^2)))/(315\*c^3\*sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.55, size = 63, normalized size = 0.66

$$\frac{(bx^2 + cx^4)^{5/2}(-18Abc + 45Ac^2x^2 + 8b^2B - 20bBcx^2 + 35Bc^2x^4)}{315c^3x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((b\*x^2 + c\*x^4)^(5/2)\*(8\*b^2\*B - 18\*A\*b\*c - 20\*b\*B\*c\*x^2 + 45\*A\*c^2\*x^2 + 35\*B\*c^2\*x^4))/(315\*c^3\*x^5)

**fricas [A]** time = 0.42, size = 106, normalized size = 1.10

$$\frac{(35Bc^4x^8 + 5(10Bbc^3 + 9Ac^4)x^6 + 8Bb^4 - 18Ab^3c + 3(Bb^2c^2 + 24Abc^3)x^4 - (4Bb^3c - 9Ab^2c^2)x^2)\sqrt{cx^4 + bx^2}}{315c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/315\*(35\*B\*c^4\*x^8 + 5\*(10\*B\*b\*c^3 + 9\*A\*c^4)\*x^6 + 8\*B\*b^4 - 18\*A\*b^3\*c + 3\*(B\*b^2\*c^2 + 24\*A\*b\*c^3)\*x^4 - (4\*B\*b^3\*c - 9\*A\*b^2\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^3\*x)

**giac [A]** time = 0.17, size = 105, normalized size = 1.09

$$\frac{2(4Bb^2 - 9Ab^2c)\operatorname{sgn}(x)}{315c^3} + \frac{35(cx^2 + b)^2B\operatorname{sgn}(x) - 90(cx^2 + b)^2Bb\operatorname{sgn}(x) + 63(cx^2 + b)^2Bb^2\operatorname{sgn}(x) + 45(cx^2 + b)^2Ac\operatorname{sgn}(x) - 63(cx^2 + b)^2Abc\operatorname{sgn}(x)}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x, algorithm="giac")

[Out] -2/315\*(4\*B\*b^(9/2) - 9\*A\*b^(7/2)\*c)\*sgn(x)/c^3 + 1/315\*(35\*(c\*x^2 + b)^(9/2)\*B\*sgn(x) - 90\*(c\*x^2 + b)^(7/2)\*B\*b\*sgn(x) + 63\*(c\*x^2 + b)^(5/2)\*B\*b^2\*sgn(x) + 45\*(c\*x^2 + b)^(7/2)\*A\*c\*sgn(x) - 63\*(c\*x^2 + b)^(5/2)\*A\*b\*c\*sgn(x))/c^3

**maple [A]** time = 0.05, size = 67, normalized size = 0.70

$$\frac{(cx^2 + b)(-35Bc^2x^4 - 45Ac^2x^2 + 20Bbcx^2 + 18Abc - 8Bb^2)(cx^4 + bx^2)^{3/2}}{315c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2), x)

[Out] -1/315\*(c\*x^2+b)\*(-35\*B\*c^2\*x^4-45\*A\*c^2\*x^2+20\*B\*b\*c\*x^2+18\*A\*b\*c-8\*B\*b^2)\*(c\*x^4+b\*x^2)^(3/2)/c^3/x^3



**maxima [A]** time = 1.56, size = 105, normalized size = 1.09

$$\frac{(5c^3x^6 + 8bc^2x^4 + b^2cx^2 - 2b^3)\sqrt{cx^2 + b}A}{35c^2} + \frac{(35c^4x^8 + 50bc^3x^6 + 3b^2c^2x^4 - 4b^3cx^2 + 8b^4)\sqrt{cx^2 + b}B}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/35\*(5\*c^3\*x^6 + 8\*b\*c^2\*x^4 + b^2\*c\*x^2 - 2\*b^3)\*sqrt(c\*x^2 + b)\*A/c^2 + 1/315\*(35\*c^4\*x^8 + 50\*b\*c^3\*x^6 + 3\*b^2\*c^2\*x^4 - 4\*b^3\*c\*x^2 + 8\*b^4)\*sqrt(c\*x^2 + b)\*B/c^3

**mupad [B]** time = 0.26, size = 103, normalized size = 1.07

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{8Bb^4 - 18Ab^3c}{315c^3} + \frac{x^6(45Ac^4 + 50Bbc^3)}{315c^3} + \frac{Bcx^8}{9} + \frac{b^2x^2(9Ac - 4Bb)}{315c^2} + \frac{bx^4(24Ac + Bb)}{105c} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2),x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*((8\*B\*b^4 - 18\*A\*b^3\*c)/(315\*c^3) + (x^6\*(45\*A\*c^4 + 50\*B\*b\*c^3))/(315\*c^3) + (B\*c\*x^8)/9 + (b^2\*x^2\*(9\*A\*c - 4\*B\*b))/(315\*c^2) + (b\*x^4\*(24\*A\*c + B\*b))/(105\*c)))/x

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2), x)

$$3.122 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx$$

**Optimal.** Leaf size=61

$$\frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(bx^2+cx^4)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

**Rubi [A]** time = 0.16, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2039, 2014}

$$\frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(bx^2+cx^4)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^2,x]

[Out] -((2\*b\*B - 7\*A\*c)\*(b\*x^2 + c\*x^4)^(5/2))/(35\*c^2\*x^5) + (B\*(b\*x^2 + c\*x^4)^(5/2))/(7\*c\*x^3)

**Rule 2014**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := -Simp[(c^(j-1)\*(c\*x)^(m-j+1)\*(a\*x^j + b\*x^n)^(p+1))/(a\*(n-j)\*(p+1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n\*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

**Rule 2039**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(b\*(m+n+p\*(j+n)+1)), x] - Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(b\*(m+n+p\*(j+n)+1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m+n+p\*(j+n)+1, 0] && (GtQ[e, 0] || IntegerQ[j])

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx &= \frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(2bB-7Ac) \int \frac{(bx^2+cx^4)^{3/2}}{x^2} dx}{7c} \\ &= -\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5} + \frac{B(bx^2+cx^4)^{5/2}}{7cx^3} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 48, normalized size = 0.79

$$\frac{x(b+cx^2)^3(7Ac-2bB+5Bcx^2)}{35c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^2,x]

[Out]  $(x*(b + c*x^2)^3*(-2*b*B + 7*A*c + 5*B*c*x^2))/(35*c^2*\text{Sqrt}[x^2*(b + c*x^2)])$

**IntegrateAlgebraic** [A] time = 0.56, size = 41, normalized size = 0.67

$$\frac{(bx^2 + cx^4)^{5/2} (7Ac - 2bB + 5Bcx^2)}{35c^2x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^2,x]

[Out]  $((-2*b*B + 7*A*c + 5*B*c*x^2)*(b*x^2 + c*x^4)^{5/2})/(35*c^2*x^5)$

**fricas** [A] time = 0.42, size = 80, normalized size = 1.31

$$\frac{(5Bc^3x^6 + (8Bbc^2 + 7Ac^3)x^4 - 2Bb^3 + 7Ab^2c + (Bb^2c + 14Abc^2)x^2)\sqrt{cx^4 + bx^2}}{35c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out]  $1/35*(5*B*c^3*x^6 + (8*B*b*c^2 + 7*A*c^3)*x^4 - 2*B*b^3 + 7*A*b^2*c + (B*b^2*c + 14*A*b*c^2)*x^2)*\text{sqrt}(c*x^4 + b*x^2)/(c^2*x)$

**giac** [A] time = 0.21, size = 72, normalized size = 1.18

$$\frac{(2Bb^{\frac{7}{2}} - 7Ab^{\frac{5}{2}}c)\text{sgn}(x)}{35c^2} + \frac{5(cx^2 + b)^{\frac{7}{2}}B\text{sgn}(x) - 7(cx^2 + b)^{\frac{5}{2}}Bb\text{sgn}(x) + 7(cx^2 + b)^{\frac{5}{2}}Ac\text{sgn}(x)}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^2,x, algorithm="giac")

[Out]  $1/35*(2*B*b^{7/2} - 7*A*b^{5/2}*c)*\text{sgn}(x)/c^2 + 1/35*(5*(c*x^2 + b)^{7/2}*B*\text{sgn}(x) - 7*(c*x^2 + b)^{5/2}*B*b*\text{sgn}(x) + 7*(c*x^2 + b)^{5/2}*A*c*\text{sgn}(x))/c^2$

**maple** [A] time = 0.05, size = 45, normalized size = 0.74

$$\frac{(cx^2 + b)(5Bcx^2 + 7Ac - 2bB)(cx^4 + bx^2)^{\frac{3}{2}}}{35c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^2,x)

[Out]  $1/35*(c*x^2+b)*(5*B*c*x^2+7*A*c-2*B*b)*(c*x^4+b*x^2)^{3/2}/c^2/x^3$

**maxima** [A] time = 1.55, size = 80, normalized size = 1.31

$$\frac{(c^2x^4 + 2bcx^2 + b^2)\sqrt{cx^2 + b}A}{5c} + \frac{(5c^3x^6 + 8bc^2x^4 + b^2cx^2 - 2b^3)\sqrt{cx^2 + b}B}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^2,x, algorithm="maxima")

[Out]  $1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*\text{sqrt}(c*x^2 + b)*A/c + 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*\text{sqrt}(c*x^2 + b)*B/c^2$

mupad [B] time = 0.23, size = 83, normalized size = 1.36

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{x^4(7Ac^3 + 8Bbc^2)}{35c^2} - \frac{2Bb^3 - 7Ab^2c}{35c^2} + \frac{Bcx^6}{7} + \frac{bx^2(14Ac + Bb)}{35c} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^2,x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*((x^4\*(7\*A\*c^3 + 8\*B\*b\*c^2))/(35\*c^2) - (2\*B\*b^3 - 7\*A\*b^2\*c)/(35\*c^2) + (B\*c\*x^6)/7 + (b\*x^2\*(14\*A\*c + B\*b))/(35\*c)))/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*2,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*3/2\*(A + B\*x\*\*2)/x\*\*2, x)

$$3.123 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} dx$$

**Optimal.** Leaf size=102

$$-Ab^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5}$$

**Rubi [A]** time = 0.21, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2039, 2021, 2008, 206}

$$-Ab^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^4,x]

[Out] (A\*b\*Sqrt[b\*x^2 + c\*x^4])/x + (A\*(b\*x^2 + c\*x^4)^(3/2))/(3\*x^3) + (B\*(b\*x^2 + c\*x^4)^(5/2))/(5\*c\*x^5) - A\*b^(3/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2021

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a\*x^j + b\*x^n)^p)/(c\*(m + n\*p + 1)), x] + Dist[(a\*(n - j)\*p)/(c^j\*(m + n\*p + 1)), Int[(c\*x)^(m + j)\*(a\*x^j + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n\*p + 1, 0]

#### Rule 2039

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^4} dx &= \frac{B(bx^2 + cx^4)^{5/2}}{5cx^5} + A \int \frac{(bx^2 + cx^4)^{3/2}}{x^4} dx \\
&= \frac{A(bx^2 + cx^4)^{3/2}}{3x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{5cx^5} + (Ab) \int \frac{\sqrt{bx^2 + cx^4}}{x^2} dx \\
&= \frac{Ab\sqrt{bx^2 + cx^4}}{x} + \frac{A(bx^2 + cx^4)^{3/2}}{3x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{5cx^5} + (Ab^2) \int \frac{1}{\sqrt{bx^2 + cx^4}} dx \\
&= \frac{Ab\sqrt{bx^2 + cx^4}}{x} + \frac{A(bx^2 + cx^4)^{3/2}}{3x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{5cx^5} - (Ab^2) \operatorname{Subst} \left( \int \frac{1}{1 - bu} du \right) \\
&= \frac{Ab\sqrt{bx^2 + cx^4}}{x} + \frac{A(bx^2 + cx^4)^{3/2}}{3x^3} + \frac{B(bx^2 + cx^4)^{5/2}}{5cx^5} - Ab^{3/2} \tanh^{-1} \left( \frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 109, normalized size = 1.07

$$\frac{(x^2(b + cx^2))^{3/2} \left( -15Ab^{3/2}c \tanh^{-1} \left( \frac{\sqrt{b+cx^2}}{\sqrt{b}} \right) + 5Ac(b + cx^2)^{3/2} + 15Abc\sqrt{b + cx^2} + 3B(b + cx^2)^{5/2} \right)}{15cx^3(b + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^4, x]

[Out] ((x^2\*(b + c\*x^2))^(3/2)\*(15\*A\*b\*c\*Sqrt[b + c\*x^2] + 5\*A\*c\*(b + c\*x^2)^(3/2) + 3\*B\*(b + c\*x^2)^(5/2) - 15\*A\*b^(3/2)\*c\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]])/(15\*c\*x^3\*(b + c\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.68, size = 95, normalized size = 0.93

$$\frac{\sqrt{bx^2 + cx^4} (20Abc + 5Ac^2x^2 + 3b^2B + 6bBcx^2 + 3Bc^2x^4)}{15cx} - Ab^{3/2} \tanh^{-1} \left( \frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^4, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(3\*b^2\*B + 20\*A\*b\*c + 6\*b\*B\*c\*x^2 + 5\*A\*c^2\*x^2 + 3\*B\*c^2\*x^4))/(15\*c\*x) - A\*b^(3/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]]

**fricas [A]** time = 0.43, size = 206, normalized size = 2.02

$$\left[ \frac{15Ab^2cx \log \left( \frac{-cx^3 + 2bx^2 - 2\sqrt{cx^4 + bx^2}\sqrt{b}}{x^3} \right) + 2(3Bc^2x^4 + 3Bb^2 + 20Abc + (6Bbc + 5Ac^2)x^2)\sqrt{cx^4 + bx^2}}{30cx}, \frac{15A\sqrt{-b}bcx \arctan \left( \frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx} \right) + (3Bc^2x^4 + 3Bb^2 + 20Abc + (6Bbc + 5Ac^2)x^2)\sqrt{cx^4 + bx^2}}{15cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^4, x, algorithm="fricas")

[Out] [1/30\*(15\*A\*b^(3/2)\*c\*x\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*(3\*B\*c^2\*x^4 + 3\*B\*b^2 + 20\*A\*b\*c + (6\*B\*b\*c + 5\*A\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c\*x), 1/15\*(15\*A\*sqrt(-b)\*b\*c\*x\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + (3\*B\*c^2\*x^4 + 3\*B\*b^2 + 20\*A\*b\*c + (6\*B\*b\*c + 5\*A\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(c\*x)]

**giac [A]** time = 0.18, size = 140, normalized size = 1.37

$$\frac{Ab^2 \arctan \left( \frac{\sqrt{cx^2 + b}}{\sqrt{-b}} \right) \operatorname{sgn}(x)}{\sqrt{-b}} - \frac{(15Ab^2c \arctan \left( \frac{\sqrt{b}}{\sqrt{-b}} \right) + 3B\sqrt{-b}b^{\frac{5}{2}} + 20A\sqrt{-b}b^{\frac{3}{2}}) \operatorname{sgn}(x)}{15\sqrt{-b}c} + \frac{3(cx^2 + b)^{\frac{5}{2}}Bc^4 \operatorname{sgn}(x) + 5(cx^2 + b)^{\frac{3}{2}}Ac^5 \operatorname{sgn}(x) + 15\sqrt{cx^2 + b}Abc^5 \operatorname{sgn}(x)}{15c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^4,x, algorithm="giac")

[Out] A\*b^2\*arctan(sqrt(c\*x^2 + b)/sqrt(-b))\*sgn(x)/sqrt(-b) - 1/15\*(15\*A\*b^2\*c\*arctan(sqrt(b)/sqrt(-b)) + 3\*B\*sqrt(-b)\*b^(5/2) + 20\*A\*sqrt(-b)\*b^(3/2)\*c)\*sgn(x)/(sqrt(-b)\*c) + 1/15\*(3\*(c\*x^2 + b)^(5/2)\*B\*c^4\*sgn(x) + 5\*(c\*x^2 + b)^(3/2)\*A\*c^5\*sgn(x) + 15\*sqrt(c\*x^2 + b)\*A\*b\*c^5\*sgn(x))/c^5

**maple** [A] time = 0.05, size = 99, normalized size = 0.97

$$\frac{(cx^4 + bx^2)^{\frac{3}{2}} \left( 15Ab^{\frac{3}{2}}c \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 15\sqrt{cx^2+b}Abc - 5(cx^2 + b)^{\frac{3}{2}}Ac - 3(cx^2 + b)^{\frac{5}{2}}B \right)}{15(cx^2 + b)^{\frac{3}{2}}cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^4,x)

[Out] -1/15\*(c\*x^4+b\*x^2)^(3/2)\*(-3\*B\*(c\*x^2+b)^(5/2)+15\*A\*b^(3/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*c-5\*A\*(c\*x^2+b)^(3/2)\*c-15\*A\*(c\*x^2+b)^(1/2)\*b\*c)/x^3/(c\*x^2+b)^(3/2)/c

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}(Bx^2 + A)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate((c\*x^4 + b\*x^2)^(3/2)\*(B\*x^2 + A)/x^4, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^4,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*4,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\* (3/2)\*(A + B\*x\*\*2)/x\*\*4, x)

$$3.124 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} dx$$

**Optimal.** Leaf size=133

$$\frac{\sqrt{bx^2+cx^4}(3Ac+2bB)}{2x} - \frac{1}{2}\sqrt{b}(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{(bx^2+cx^4)^{3/2}(3Ac+2bB)}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

**Rubi [A]** time = 0.22, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2038, 2021, 2008, 206}

$$\frac{(bx^2+cx^4)^{3/2}(3Ac+2bB)}{6bx^3} + \frac{\sqrt{bx^2+cx^4}(3Ac+2bB)}{2x} - \frac{1}{2}\sqrt{b}(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^6,x]

[Out] ((2\*b\*B + 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(2\*x) + ((2\*b\*B + 3\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(6\*b\*x^3) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(2\*b\*x^7) - (Sqrt[b]\*(2\*b\*B + 3\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/2

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2008

Int[1/Sqrt[(a\_)\*(x\_)^2 + (b\_)\*(x\_)^(n\_)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 2021

Int[((c\_)\*(x\_)^(m\_))\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+n\*p+1)), x] + Dist[(a\*(n-j)\*p)/(c^j\*(m+n\*p+1)), Int[(c\*x)^(m+j)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m+n\*p+1, 0]

Rule 2038

Int[((e\_)\*(x\_)^(m\_))\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(jn\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m+j\*p, -1] || (IntegersQ[m-1/2, p-1/2] && LtQ[p, 0] && LtQ[m, -(n\*p)-1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m+j\*p+1, 0] && NeQ[m-n+j\*p+1, 0]

Rubi steps



$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^6} dx &= -\frac{A(bx^2 + cx^4)^{5/2}}{2bx^7} - \frac{(-2bB - 3Ac) \int \frac{(bx^2 + cx^4)^{3/2}}{x^4} dx}{2b} \\
&= \frac{(2bB + 3Ac)(bx^2 + cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2 + cx^4)^{5/2}}{2bx^7} - \frac{1}{2}(-2bB - 3Ac) \int \frac{\sqrt{bx^2 + cx^4}}{x^2} dx \\
&= \frac{(2bB + 3Ac)\sqrt{bx^2 + cx^4}}{2x} + \frac{(2bB + 3Ac)(bx^2 + cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2 + cx^4)^{5/2}}{2bx^7} + \dots \\
&= \frac{(2bB + 3Ac)\sqrt{bx^2 + cx^4}}{2x} + \frac{(2bB + 3Ac)(bx^2 + cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2 + cx^4)^{5/2}}{2bx^7} + \dots \\
&= \frac{(2bB + 3Ac)\sqrt{bx^2 + cx^4}}{2x} + \frac{(2bB + 3Ac)(bx^2 + cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2 + cx^4)^{5/2}}{2bx^7} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 109, normalized size = 0.82

$$\frac{\sqrt{x^2(b + cx^2)} \left( \sqrt{b + cx^2} (-3Ab + 6Acx^2 + 8bBx^2 + 2Bcx^4) - 3\sqrt{b} x^2 (3Ac + 2bB) \tanh^{-1} \left( \frac{\sqrt{b+cx^2}}{\sqrt{b}} \right) \right)}{6x^3 \sqrt{b + cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^6, x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(Sqrt[b + c\*x^2]\*(-3\*A\*b + 8\*b\*B\*x^2 + 6\*A\*c\*x^2 + 2\*B\*c\*x^4) - 3\*Sqrt[b]\*(2\*b\*B + 3\*A\*c)\*x^2\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]])/(6\*x^3\*Sqrt[b + c\*x^2])

**IntegrateAlgebraic [A]** time = 0.80, size = 94, normalized size = 0.71

$$\frac{1}{2} \left( -3A\sqrt{b}c - 2b^{3/2}B \right) \tanh^{-1} \left( \frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}} \right) + \frac{\sqrt{bx^2 + cx^4} (-3Ab + 6Acx^2 + 8bBx^2 + 2Bcx^4)}{6x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^6, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-3\*A\*b + 8\*b\*B\*x^2 + 6\*A\*c\*x^2 + 2\*B\*c\*x^4))/(6\*x^3) + ((-2\*b^(3/2)\*B - 3\*A\*Sqrt[b]\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/2

**fricas [A]** time = 0.44, size = 195, normalized size = 1.47

$$\left[ \frac{3(2Bb + 3Ac)\sqrt{b}x^3 \log\left(-\frac{cx^3 + 2bx - 2\sqrt{cx^4 + bx^2}\sqrt{b}}{x^3}\right) + 2(2Bcx^4 + 2(4Bb + 3Ac)x^2 - 3Ab)\sqrt{cx^4 + bx^2}}{12x^3}, \frac{3(2Bb + 3Ac)\sqrt{-b}x^3 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx}\right) + (2Bcx^4 + 2(4Bb + 3Ac)x^2 - 3Ab)\sqrt{cx^4 + bx^2}}{6x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^6,x, algorithm="fricas")

[Out] [1/12\*(3\*(2\*B\*b + 3\*A\*c)\*sqrt(b)\*x^3\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(b))/x^3) + 2\*(2\*B\*c\*x^4 + 2\*(4\*B\*b + 3\*A\*c)\*x^2 - 3\*A\*b)\*sqrt(c\*x^4 + b\*x^2))/x^3, 1/6\*(3\*(2\*B\*b + 3\*A\*c)\*sqrt(-b)\*x^3\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + (2\*B\*c\*x^4 + 2\*(4\*B\*b + 3\*A\*c)\*x^2 - 3\*A\*b)\*sqrt(c\*x^4 + b\*x^2))/x^3]

**giac** [A] time = 0.28, size = 115, normalized size = 0.86

$$\frac{2(cx^2 + b)^{\frac{3}{2}} Bc \operatorname{sgn}(x) + 6\sqrt{cx^2 + b} Bbc \operatorname{sgn}(x) + 6\sqrt{cx^2 + b} Ac^2 \operatorname{sgn}(x) - \frac{3\sqrt{cx^2 + b} Abc \operatorname{sgn}(x)}{x^2} + \frac{3(2Bb^2c \operatorname{sgn}(x) + 3Abc^2 \operatorname{sgn}(x)) \arctan\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^6,x, algorithm="giac")

[Out] 1/6\*(2\*(c\*x^2 + b)^(3/2)\*B\*c\*sgn(x) + 6\*sqrt(c\*x^2 + b)\*B\*b\*c\*sgn(x) + 6\*sqrt(c\*x^2 + b)\*A\*c^2\*sgn(x) - 3\*sqrt(c\*x^2 + b)\*A\*b\*c\*sgn(x)/x^2 + 3\*(2\*B\*b^2\*c\*sgn(x) + 3\*A\*b\*c^2\*sgn(x))\*arctan(sqrt(c\*x^2 + b)/sqrt(-b))/sqrt(-b)/c

**maple** [A] time = 0.06, size = 172, normalized size = 1.29

$$\frac{(cx^4 + bx^2)^{\frac{3}{2}} \left( 9Ab^{\frac{3}{2}}c^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) + 6Bb^{\frac{5}{2}}x^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 9\sqrt{cx^2+b} Abcx^2 - 6\sqrt{cx^2+b} Bb^2x^2 - 3(cx^2 + b)^{\frac{3}{2}} Acx^2 - 2(cx^2 + b)^{\frac{3}{2}} Bbx^2 + 3(cx^2 + b)^{\frac{5}{2}} A \right)}{6(cx^2 + b)^{\frac{3}{2}} bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^6,x)

[Out] -1/6\*(c\*x^4+b\*x^2)^(3/2)\*(9\*A\*b^(3/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^2\*c-3\*(c\*x^2+b)^(3/2)\*A\*c\*x^2+6\*B\*b^(5/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^2-2\*(c\*x^2+b)^(3/2)\*B\*b\*x^2+3\*A\*(c\*x^2+b)^(5/2)-9\*A\*(c\*x^2+b)^(1/2)\*x^2\*b\*c-6\*B\*(c\*x^2+b)^(1/2)\*x^2\*b^2)/x^5/(c\*x^2+b)^(3/2)/b

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}} (Bx^2 + A)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^6,x, algorithm="maxima")

[Out] integrate((c\*x^4 + b\*x^2)^(3/2)\*(B\*x^2 + A)/x^6, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^6,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}} (A + Bx^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*6,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2)/x\*\*6, x)

$$3.125 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8} dx$$

**Optimal.** Leaf size=135

$$\frac{3c\sqrt{bx^2+cx^4}(Ac+4bB)}{8bx} - \frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}} - \frac{(bx^2+cx^4)^{3/2}(Ac+4bB)}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

**Rubi [A]** time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2038, 2020, 2021, 2008, 206}

$$\frac{(bx^2+cx^4)^{3/2}(Ac+4bB)}{8bx^5} + \frac{3c\sqrt{bx^2+cx^4}(Ac+4bB)}{8bx} - \frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^8, x]

[Out] (3\*c\*(4\*b\*B + A\*c)\*Sqrt[b\*x^2 + c\*x^4]/(8\*b\*x) - ((4\*b\*B + A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(8\*b\*x^5) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(4\*b\*x^9) - (3\*c\*(4\*b\*B + A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]]/(8\*Sqrt[b])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2020

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+j\*p+1)), x] - Dist[(b\*p\*(n-j))/(c^n\*(m+j\*p+1)), Int[(c\*x)^(m+n)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j\*p+1, 0]

#### Rule 2021

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+n\*p+1)), x] + Dist[(a\*(n-j)\*p)/(c^j\*(m+n\*p+1)), Int[(c\*x)^(m+j)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m+n\*p+1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m+j\*p, -1]

|| (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1]) && (G  
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1,  
0]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^8} dx &= -\frac{A(bx^2 + cx^4)^{5/2}}{4bx^9} - \frac{(-4bB - Ac) \int \frac{(bx^2 + cx^4)^{3/2}}{x^6} dx}{4b} \\ &= -\frac{(4bB + Ac)(bx^2 + cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2 + cx^4)^{5/2}}{4bx^9} + \frac{(3c(4bB + Ac)) \int \frac{\sqrt{bx^2 + cx^4}}{x^2} dx}{8b} \\ &= \frac{3c(4bB + Ac)\sqrt{bx^2 + cx^4}}{8bx} - \frac{(4bB + Ac)(bx^2 + cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2 + cx^4)^{5/2}}{4bx^9} + \frac{1}{8} \int \frac{\sqrt{bx^2 + cx^4}}{x} dx \\ &= \frac{3c(4bB + Ac)\sqrt{bx^2 + cx^4}}{8bx} - \frac{(4bB + Ac)(bx^2 + cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2 + cx^4)^{5/2}}{4bx^9} - \frac{1}{8} \int \frac{\sqrt{bx^2 + cx^4}}{x} dx \\ &= \frac{3c(4bB + Ac)\sqrt{bx^2 + cx^4}}{8bx} - \frac{(4bB + Ac)(bx^2 + cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2 + cx^4)^{5/2}}{4bx^9} - \frac{3c}{8} \int \frac{\sqrt{bx^2 + cx^4}}{x} dx \end{aligned}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.47

$$\frac{(x^2(b + cx^2))^{5/2} \left( cx^4(Ac + 4bB) {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; \frac{cx^2}{b} + 1\right) - 5Ab^2 \right)}{20b^3x^9}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^8, x]

[Out] ((x^2\*(b + c\*x^2))^(5/2)\*(-5\*A\*b^2 + c\*(4\*b\*B + A\*c)\*x^4\*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c\*x^2)/b]))/(20\*b^3\*x^9)

IntegrateAlgebraic [A] time = 0.88, size = 92, normalized size = 0.68

$$\frac{\sqrt{bx^2 + cx^4} (-2Ab - 5Acx^2 - 4bBx^2 + 8Bcx^4)}{8x^5} - \frac{3(Ac^2 + 4bBc) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^8, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-2\*A\*b - 4\*b\*B\*x^2 - 5\*A\*c\*x^2 + 8\*B\*c\*x^4))/(8\*x^5) - (3\*(4\*b\*B\*c + A\*c^2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(8\*Sqrt[b])

fricas [A] time = 0.45, size = 217, normalized size = 1.61

$$\left[ \frac{3(4Bbc + Ac^2)\sqrt{b}x^5 \log\left(\frac{-cx^3 + 2bx - 2\sqrt{cx^4 + bx^2}\sqrt{b}}{x^3}\right) + 2(8Bbcx^4 - 2Ab^2 - (4Bb^2 + 5Abc)x^2)\sqrt{cx^4 + bx^2}}{16bx^5}, \frac{3(4Bbc + Ac^2)\sqrt{-b}x^5 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx}\right) + (8Bbcx^4 - 2Ab^2 - (4Bb^2 + 5Abc)x^2)\sqrt{cx^4 + bx^2}}{8bx^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^8,x, algorithm="fricas")

[Out] [1/16\*(3\*(4\*B\*b\*c + A\*c^2)\*sqrt(b)\*x^5\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*(8\*B\*b\*c\*x^4 - 2\*A\*b^2 - (4\*B\*b^2 + 5\*A\*b\*c)\*x^2)

$\sqrt{c x^4 + b x^2} / (b x^5), 1/8 * (3 * (4 * B * b * c + A * c^2) * \sqrt{-b} * x^5 * \arctan(\sqrt{c x^4 + b x^2} * \sqrt{-b} / (c x^3 + b x)) + (8 * B * b * c * x^4 - 2 * A * b^2 - (4 * B * b^2 + 5 * A * b * c) * x^2) * \sqrt{c x^4 + b x^2} / (b x^5)]$

**giac** [A] time = 0.26, size = 145, normalized size = 1.07

$$\frac{8 \sqrt{c x^2 + b} B c^2 \operatorname{sgn}(x) + \frac{3(4 B b c^2 \operatorname{sgn}(x) + A c^3 \operatorname{sgn}(x)) \arctan\left(\frac{\sqrt{c x^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4(c x^2 + b)^{\frac{3}{2}} B b c^2 \operatorname{sgn}(x) - 4 \sqrt{c x^2 + b} B b^2 c^2 \operatorname{sgn}(x) + 5(c x^2 + b)^{\frac{3}{2}} A c^3 \operatorname{sgn}(x) - 3 \sqrt{c x^2 + b} A b c^3 \operatorname{sgn}(x)}{c^2 x^4}}{8 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^8,x, algorithm="giac")

[Out]  $1/8 * (8 * \sqrt{c x^2 + b} * B * c^2 * \operatorname{sgn}(x) + 3 * (4 * B * b * c^2 * \operatorname{sgn}(x) + A * c^3 * \operatorname{sgn}(x)) * \arctan(\sqrt{c x^2 + b} / \sqrt{-b}) / \sqrt{-b} - (4 * (c x^2 + b)^{(3/2)} * B * b * c^2 * \operatorname{sgn}(x) - 4 * \sqrt{c x^2 + b} * B * b^2 * c^2 * \operatorname{sgn}(x) + 5 * (c x^2 + b)^{(3/2)} * A * c^3 * \operatorname{sgn}(x) - 3 * \sqrt{c x^2 + b} * A * b * c^3 * \operatorname{sgn}(x)) / (c^2 * x^4)) / c$

**maple** [A] time = 0.06, size = 213, normalized size = 1.58

$$\frac{(c x^4 + b x^2)^{\frac{3}{2}} \left( 3 A b^{\frac{3}{2}} c^2 x^4 \ln\left(\frac{2 b + 2 \sqrt{c x^2 + b} \sqrt{b}}{x}\right) + 12 B b^{\frac{5}{2}} c x^4 \ln\left(\frac{2 b + 2 \sqrt{c x^2 + b} \sqrt{b}}{x}\right) - 3 \sqrt{c x^2 + b} A b c^2 x^4 - 12 \sqrt{c x^2 + b} B b^2 c x^4 - (c x^2 + b)^{\frac{3}{2}} A c^2 x^4 - 4 (c x^2 + b)^{\frac{3}{2}} B b c x^4 + (c x^2 + b)^{\frac{3}{2}} A c x^2 + 4 (c x^2 + b)^{\frac{5}{2}} B b x^2 + 2 (c x^2 + b)^{\frac{5}{2}} A b \right)}{8 (c x^2 + b)^{\frac{3}{2}} b^2 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^8,x)

[Out]  $-1/8 * (c x^4 + b x^2)^{(3/2)} * (-A * (c x^2 + b)^{(3/2)} * x^4 * c^2 + 3 * A * b^{(3/2)} * \ln(2 * (b + (c x^2 + b)^{(1/2)} * b^{(1/2)}) / x) * x^4 * c^2 - 4 * B * (c x^2 + b)^{(3/2)} * x^4 * b * c + 12 * B * b^{(5/2)} * \ln(2 * (b + (c x^2 + b)^{(1/2)} * b^{(1/2)}) / x) * x^4 * c + A * (c x^2 + b)^{(5/2)} * x^2 * c - 3 * A * (c x^2 + b)^{(1/2)} * x^4 * b * c^2 + 4 * B * (c x^2 + b)^{(5/2)} * x^2 * b - 12 * B * (c x^2 + b)^{(1/2)} * x^4 * b^2 * c + 2 * A * (c x^2 + b)^{(5/2)} * b) / x^7 / (c x^2 + b)^{(3/2)} / b^2$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c x^4 + b x^2)^{\frac{3}{2}} (B x^2 + A)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^8,x, algorithm="maxima")

[Out] integrate((c\*x^4 + b\*x^2)^(3/2)\*(B\*x^2 + A)/x^8, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B x^2 + A) (c x^4 + b x^2)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^8,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^8, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 (b + c x^2))^{\frac{3}{2}} (A + B x^2)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*8,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\* (3/2)\*(A + B\*x\*\*2)/x\*\*8, x)

$$3.126 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}} dx$$

**Optimal.** Leaf size=140

$$\frac{c^2(6bB - Ac) \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}} - \frac{(bx^2 + cx^4)^{3/2} (6bB - Ac)}{24bx^7} - \frac{c\sqrt{bx^2 + cx^4} (6bB - Ac)}{16bx^3} - \frac{A (bx^2 + cx^4)^{5/2}}{6bx^{11}}$$

**Rubi [A]** time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2038, 2020, 2008, 206}

$$\frac{c^2(6bB - Ac) \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}} - \frac{(bx^2 + cx^4)^{3/2} (6bB - Ac)}{24bx^7} - \frac{c\sqrt{bx^2 + cx^4} (6bB - Ac)}{16bx^3} - \frac{A (bx^2 + cx^4)^{5/2}}{6bx^{11}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^10,x]

[Out] -(c\*(6\*b\*B - A\*c)\*Sqrt[b\*x^2 + c\*x^4])/((16\*b\*x^3) - ((6\*b\*B - A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(24\*b\*x^7) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(6\*b\*x^11) - (c^2\*(6\*b\*B - A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(16\*b^(3/2)))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 2020

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p/(c\*(m+j\*p+1)), x] - Dist[(b\*p\*(n-j))/(c^n\*(m+j\*p+1)), Int[(c\*x)^(m+n)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j\*p+1, 0]

Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m+j\*p, -1] || (IntegersQ[m-1/2, p-1/2] && LtQ[p, 0] && LtQ[m, -(n\*p)-1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m+j\*p+1, 0] && NeQ[m-n+j\*p+1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{10}} dx &= -\frac{A(bx^2 + cx^4)^{5/2}}{6bx^{11}} - \frac{(-6bB + Ac) \int \frac{(bx^2 + cx^4)^{3/2}}{x^8} dx}{6b} \\
&= -\frac{(6bB - Ac)(bx^2 + cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2 + cx^4)^{5/2}}{6bx^{11}} + \frac{(c(6bB - Ac)) \int \frac{\sqrt{bx^2 + cx^4}}{x^4} dx}{8b} \\
&= -\frac{c(6bB - Ac)\sqrt{bx^2 + cx^4}}{16bx^3} - \frac{(6bB - Ac)(bx^2 + cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2 + cx^4)^{5/2}}{6bx^{11}} + \\
&= -\frac{c(6bB - Ac)\sqrt{bx^2 + cx^4}}{16bx^3} - \frac{(6bB - Ac)(bx^2 + cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2 + cx^4)^{5/2}}{6bx^{11}} \\
&= -\frac{c(6bB - Ac)\sqrt{bx^2 + cx^4}}{16bx^3} - \frac{(6bB - Ac)(bx^2 + cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2 + cx^4)^{5/2}}{6bx^{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 121, normalized size = 0.86

$$\frac{(b + cx^2)(A(8b^2 + 14bcx^2 + 3c^2x^4) + 6bBx^2(2b + 5cx^2)) + 3c^2x^6\sqrt{\frac{cx^2}{b} + 1}(6bB - Ac)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b} + 1}\right)}{48bx^5\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^10, x]

[Out] -1/48\*((b + c\*x^2)\*(6\*b\*B\*x^2\*(2\*b + 5\*c\*x^2) + A\*(8\*b^2 + 14\*b\*c\*x^2 + 3\*c^2\*x^4)) + 3\*c^2\*(6\*b\*B - A\*c)\*x^6\*Sqrt[1 + (c\*x^2)/b]\*ArcTanh[Sqrt[1 + (c\*x^2)/b]])/(b\*x^5\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 1.04, size = 112, normalized size = 0.80

$$\frac{(Ac^3 - 6bBc^2)\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2 + cx^4}}\right) + \sqrt{bx^2 + cx^4}(-8Ab^2 - 14Abcx^2 - 3Ac^2x^4 - 12b^2Bx^2 - 30bBcx^4)}{16b^{3/2}48bx^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^10, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-8\*A\*b^2 - 12\*b^2\*B\*x^2 - 14\*A\*b\*c\*x^2 - 30\*b\*B\*c\*x^4 - 3\*A\*c^2\*x^4))/(48\*b\*x^7) + ((-6\*b\*B\*c^2 + A\*c^3)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(16\*b^(3/2))

**fricas [A]** time = 0.44, size = 250, normalized size = 1.79

$$\frac{3(6Bbc^2 - Ac^3)\sqrt{bx}\log\left(\frac{cx^2 + 2\sqrt{cx^2 + bx^2}\sqrt{b}}{b}\right) + 2(3(10Bb^2c + Abc^2)x^4 + 8Ab^3 + 2(6Bb^3 + 7Ab^2c)x^2)\sqrt{cx^2 + bx^2} - 3(6Bbc^2 - Ac^3)\sqrt{-bx^2}\arctan\left(\frac{\sqrt{cx^2 + bx^2}\sqrt{bx}}{cx^2 + bx}\right) - (3(10Bb^2c + Abc^2)x^4 + 8Ab^3 + 2(6Bb^3 + 7Ab^2c)x^2)\sqrt{cx^2 + bx^2}}{96b^2x^748b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^10,x, algorithm="fricas")

[Out] [-1/96\*(3\*(6\*B\*b\*c^2 - A\*c^3)\*sqrt(b)\*x^7\*log(-(c\*x^3 + 2\*b\*x + 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*(3\*(10\*B\*b^2\*c + A\*b\*c^2)\*x^4 + 8\*A\*b^3 + 2\*(6\*B\*b^3 + 7\*A\*b^2\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^2\*x^7), 1/48\*(3\*(6\*B\*b\*c^2 - A\*c^3)\*sqrt(-b)\*x^7\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) - (3\*(10\*B\*b^2\*c + A\*b\*c^2)\*x^4 + 8\*A\*b^3 + 2\*(6\*B\*b^3 + 7\*A\*b^2\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^2\*x^7)]

**giac** [A] time = 0.27, size = 175, normalized size = 1.25

$$\frac{3(6Bbc^3\operatorname{sgn}(x)-Ac^4\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)-30(cx^2+b)^{\frac{5}{2}}Bbc^3\operatorname{sgn}(x)-48(cx^2+b)^{\frac{3}{2}}Bb^2c^3\operatorname{sgn}(x)+18\sqrt{cx^2+b}Bb^3c^3\operatorname{sgn}(x)+3(cx^2+b)^{\frac{5}{2}}Ac^4\operatorname{sgn}(x)+8(cx^2+b)^{\frac{3}{2}}Abc^4\operatorname{sgn}(x)-3\sqrt{cx^2+b}Ab^2c^4\operatorname{sgn}(x)}{\sqrt{-b} \cdot 48c \cdot bc^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^10,x, algorithm="giac")

[Out] 1/48\*(3\*(6\*B\*b\*c^3\*sgn(x) - A\*c^4\*sgn(x))\*arctan(sqrt(c\*x^2 + b)/sqrt(-b))/(sqrt(-b)\*b) - (30\*(c\*x^2 + b)^(5/2)\*B\*b\*c^3\*sgn(x) - 48\*(c\*x^2 + b)^(3/2)\*B\*b^2\*c^3\*sgn(x) + 18\*sqrt(c\*x^2 + b)\*B\*b^3\*c^3\*sgn(x) + 3\*(c\*x^2 + b)^(5/2)\*A\*c^4\*sgn(x) + 8\*(c\*x^2 + b)^(3/2)\*A\*b\*c^4\*sgn(x) - 3\*sqrt(c\*x^2 + b)\*A\*b^2\*c^4\*sgn(x))/(b\*c^3\*x^6))/c

**maple** [B] time = 0.08, size = 259, normalized size = 1.85

$$\frac{(cx^4+bx^2)^{\frac{3}{2}}\left(3Ab^{\frac{3}{2}}c^3x^4\ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right)-18Bb^{\frac{5}{2}}c^3x^4\ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right)-3\sqrt{cx^2+b}Abc^3x^6+18\sqrt{cx^2+b}Bb^2c^3x^6-(cx^2+b)^{\frac{5}{2}}Ac^3x^6+6(cx^2+b)^{\frac{3}{2}}Bb^2c^3x^6+(cx^2+b)^{\frac{5}{2}}Ac^2x^4-6(cx^2+b)^{\frac{3}{2}}Bbcx^4+2(cx^2+b)^{\frac{5}{2}}Abcx^2-12(cx^2+b)^{\frac{3}{2}}Bb^2x^2-8(cx^2+b)^{\frac{5}{2}}Ab^2\right)}{48(cx^2+b)^{\frac{3}{2}}b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^10,x)

[Out] 1/48\*(c\*x^4+b\*x^2)^(3/2)\*(-A\*(c\*x^2+b)^(3/2)\*x^6\*c^3+3\*A\*b^(3/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^6\*c^3+6\*B\*(c\*x^2+b)^(3/2)\*x^6\*b\*c^2-18\*B\*b^(5/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^6\*c^2+A\*(c\*x^2+b)^(5/2)\*x^4\*c^2-3\*A\*(c\*x^2+b)^(1/2)\*x^6\*b\*c^3-6\*B\*(c\*x^2+b)^(5/2)\*x^4\*b\*c+18\*B\*(c\*x^2+b)^(1/2)\*x^6\*b^2\*c^2+2\*A\*(c\*x^2+b)^(5/2)\*x^2\*b\*c-12\*B\*(c\*x^2+b)^(5/2)\*x^2\*b^2-8\*A\*(c\*x^2+b)^(5/2)\*b^2)/x^9/(c\*x^2+b)^(3/2)/b^3

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}(Bx^2 + A)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^10,x, algorithm="maxima")

[Out] integrate((c\*x^4 + b\*x^2)^(3/2)\*(B\*x^2 + A)/x^10, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^10,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^10, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*10,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2)/x\*\*10, x)



$$3.127 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}} dx$$

**Optimal.** Leaf size=177

$$\frac{c^3(8bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{bx^2+cx^4}}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}} - \frac{c^2\sqrt{bx^2+cx^4}(8bB - 3Ac)}{128b^2x^3} - \frac{(bx^2+cx^4)^{3/2}(8bB - 3Ac)}{48bx^9} - \frac{c\sqrt{bx^2+cx^4}(8bB - 3Ac)}{64bx^5}$$

**Rubi [A]** time = 0.28, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2038, 2020, 2025, 2008, 206}

$$-\frac{c^2\sqrt{bx^2+cx^4}(8bB - 3Ac)}{128b^2x^3} + \frac{c^3(8bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{bx^2+cx^4}}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}} - \frac{c\sqrt{bx^2+cx^4}(8bB - 3Ac)}{64bx^5} - \frac{(bx^2+cx^4)^{3/2}(8bB - 3Ac)}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^12,x]

[Out] -(c\*(8\*b\*B - 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(64\*b\*x^5) - (c^2\*(8\*b\*B - 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(128\*b^2\*x^3) - ((8\*b\*B - 3\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(48\*b\*x^9) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(8\*b\*x^13) + (c^3\*(8\*b\*B - 3\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(128\*b^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2020

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a\*x^j + b\*x^n)^p)/(c\*(m + j\*p + 1)), x] - Dist[(b\*p\*(n - j))/(c^n\*(m + j\*p + 1)), Int[(c\*x)^(m + n)\*(a\*x^j + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j\*p + 1, 0]

#### Rule 2025

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j\*p + 1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*(m + j\*p + 1)), x] + Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(a\*e^n\*(m + j\*p + 1)), Int[(e\*x)^(m + n)\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +

n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m + j\*p, -1] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{12}} dx &= -\frac{A(bx^2 + cx^4)^{5/2}}{8bx^{13}} - \frac{(-8bB + 3Ac) \int \frac{(bx^2 + cx^4)^{3/2}}{x^{10}} dx}{8b} \\ &= -\frac{(8bB - 3Ac)(bx^2 + cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2 + cx^4)^{5/2}}{8bx^{13}} + \frac{(c(8bB - 3Ac)) \int \frac{\sqrt{bx^2 + cx^4}}{x^6} dx}{16b} \\ &= -\frac{c(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{64bx^5} - \frac{(8bB - 3Ac)(bx^2 + cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2 + cx^4)^{5/2}}{8bx^{13}} + \\ &= -\frac{c(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{64bx^5} - \frac{c^2(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{128b^2x^3} - \frac{(8bB - 3Ac)(bx^2 + cx^4)^{5/2}}{48bx^9} \\ &= -\frac{c(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{64bx^5} - \frac{c^2(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{128b^2x^3} - \frac{(8bB - 3Ac)(bx^2 + cx^4)^{5/2}}{48bx^9} \\ &= -\frac{c(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{64bx^5} - \frac{c^2(8bB - 3Ac)\sqrt{bx^2 + cx^4}}{128b^2x^3} - \frac{(8bB - 3Ac)(bx^2 + cx^4)^{5/2}}{48bx^9} \end{aligned}$$

**Mathematica [C]** time = 0.04, size = 66, normalized size = 0.37

$$\frac{(x^2(b + cx^2))^{5/2} \left( c^3 x^8 (8bB - 3Ac) {}_2F_1\left(\frac{5}{2}, 4; \frac{7}{2}; \frac{cx^2}{b} + 1\right) - 5Ab^4 \right)}{40b^5 x^{13}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^12,x]

[Out] ((x^2\*(b + c\*x^2))^(5/2)\*(-5\*A\*b^4 + c^3\*(8\*b\*B - 3\*A\*c)\*x^8\*Hypergeometric2F1[5/2, 4, 7/2, 1 + (c\*x^2)/b]))/(40\*b^5\*x^13)

**IntegrateAlgebraic [A]** time = 1.24, size = 137, normalized size = 0.77

$$\frac{(8bBc^3 - 3Ac^4) \tanh^{-1}\left(\frac{\sqrt{bx^2 + cx^4}}{\sqrt{bx^2 + cx^4}}\right)}{128b^5/2} + \frac{\sqrt{bx^2 + cx^4} (-48Ab^3 - 72Ab^2cx^2 - 6Abc^2x^4 + 9Ac^3x^6 - 64b^3Bx^2 - 112b^2Bcx^4 - 24bBc^2x^6)}{384b^2x^9}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^12,x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-48\*A\*b^3 - 64\*b^3\*B\*x^2 - 72\*A\*b^2\*c\*x^2 - 112\*b^2\*B\*c\*x^4 - 6\*A\*b\*c^2\*x^4 - 24\*b\*B\*c^2\*x^6 + 9\*A\*c^3\*x^6))/(384\*b^2\*x^9) + ((8\*b\*B\*c^3 - 3\*A\*c^4)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(128\*b^(5/2))

**fricas [A]** time = 0.46, size = 299, normalized size = 1.69

$$\frac{3(8Bbc^3 - 3Ac^4)\sqrt{bx^2 + cx^4} \log\left(\frac{c^2x^2 + 2\sqrt{bx^2 + cx^4}\sqrt{bx^2 + cx^4}}{c^2x^2 + 2\sqrt{bx^2 + cx^4}\sqrt{bx^2 + cx^4}}\right) + 2(3(8Bb^2c^2 - 3Abc^3)x^4 + 48Ab^4 + 2(56Bb^2c + 3Ab^2c^2)x^4 + 8(8Bb^4 + 9Ab^2c^2))\sqrt{bx^2 + cx^4} - 3(8Bbc^3 - 3Ac^4)\sqrt{-b}x^3 \arctan\left(\frac{\sqrt{bx^2 + cx^4}\sqrt{bx^2 + cx^4}}{c^2x^2 + 2\sqrt{bx^2 + cx^4}\sqrt{bx^2 + cx^4}}\right) + (3(8Bb^2c^2 - 3Abc^3)x^4 + 48Ab^4 + 2(56Bb^2c + 3Ab^2c^2)x^4 + 8(8Bb^4 + 9Ab^2c^2))\sqrt{bx^2 + cx^4}}{768b^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^12,x, algorithm="fricas")

[Out]  $[-1/768*(3*(8*B*b*c^3 - 3*A*c^4)*\sqrt{b})*x^9*\log(-(c*x^3 + 2*b*x - 2*\sqrt{c*x^4 + b*x^2})*\sqrt{b})/x^3) + 2*(3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*\sqrt{c*x^4 + b*x^2})/(b^3*x^9), -1/384*(3*(8*B*b*c^3 - 3*A*c^4)*\sqrt{-b})*x^9*\arctan(\sqrt{c*x^4 + b*x^2}*\sqrt{-b}/(c*x^3 + b*x)) + (3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*\sqrt{c*x^4 + b*x^2})/(b^3*x^9)]$

**giac** [A] time = 0.32, size = 214, normalized size = 1.21

$$\frac{3(8Bb^4\operatorname{sgn}(x)-3Ac^5\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-c}}\right)+24(cx^2+b)^{7/2}Bbc^4\operatorname{sgn}(x)+40(cx^2+b)^{5/2}Bb^2c^4\operatorname{sgn}(x)-88(cx^2+b)^{3/2}Bb^3c^4\operatorname{sgn}(x)+24\sqrt{cx^2+b}Bb^4c^4\operatorname{sgn}(x)-9(cx^2+b)^{7/2}Ac^5\operatorname{sgn}(x)+33(cx^2+b)^{5/2}Abc^5\operatorname{sgn}(x)+33(cx^2+b)^{3/2}Ab^2c^5\operatorname{sgn}(x)-9\sqrt{cx^2+b}Ab^3c^5\operatorname{sgn}(x)}{\sqrt{-b}b^2} + \frac{24(cx^2+b)^{7/2}Bbc^4\operatorname{sgn}(x)+40(cx^2+b)^{5/2}Bb^2c^4\operatorname{sgn}(x)-88(cx^2+b)^{3/2}Bb^3c^4\operatorname{sgn}(x)+24\sqrt{cx^2+b}Bb^4c^4\operatorname{sgn}(x)-9(cx^2+b)^{7/2}Ac^5\operatorname{sgn}(x)+33(cx^2+b)^{5/2}Abc^5\operatorname{sgn}(x)+33(cx^2+b)^{3/2}Ab^2c^5\operatorname{sgn}(x)-9\sqrt{cx^2+b}Ab^3c^5\operatorname{sgn}(x)}{384c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")`

[Out]  $-1/384*(3*(8*B*b*c^4*\operatorname{sgn}(x) - 3*A*c^5*\operatorname{sgn}(x))*\arctan(\sqrt{c*x^2 + b}/\sqrt{-b})/(\sqrt{-b}*b^2) + (24*(c*x^2 + b)^{(7/2)}*B*b*c^4*\operatorname{sgn}(x) + 40*(c*x^2 + b)^{(5/2)}*B*b^2*c^4*\operatorname{sgn}(x) - 88*(c*x^2 + b)^{(3/2)}*B*b^3*c^4*\operatorname{sgn}(x) + 24*\sqrt{c*x^2 + b}*B*b^4*c^4*\operatorname{sgn}(x) - 9*(c*x^2 + b)^{(7/2)}*A*c^5*\operatorname{sgn}(x) + 33*(c*x^2 + b)^{(5/2)}*A*b*c^5*\operatorname{sgn}(x) + 33*(c*x^2 + b)^{(3/2)}*A*b^2*c^5*\operatorname{sgn}(x) - 9*\sqrt{c*x^2 + b}*A*b^3*c^5*\operatorname{sgn}(x))/b^2*c^4*x^8)/c$

**maple** [A] time = 0.08, size = 302, normalized size = 1.71

$$\frac{(c^4 + b^2)^{3/2} \left( 9AB^3c^4 \ln\left(\frac{2b+2\sqrt{c^2+b}}{c}\right) - 24B^2c^4 \ln\left(\frac{2b+2\sqrt{c^2+b}}{c}\right) - 9\sqrt{c^2+b}Ab^3c^4 + 24\sqrt{c^2+b}Bb^2c^4 - 3(c^2+b)^{3/2}Ac^4 + 8(c^2+b)^{3/2}Bbc^4 + 3(c^2+b)^{3/2}Ac^4 - 8(c^2+b)^{3/2}Bbc^4 + 6(c^2+b)^{3/2}Ab^2c^4 - 16(c^2+b)^{3/2}Bb^2c^4 - 24(c^2+b)^{3/2}Ab^2c^4 + 64(c^2+b)^{3/2}Bb^2c^4 + 48(c^2+b)^{3/2}Ab^3 \right)}{384(c^2+b)^{3/2}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x)`

[Out]  $-1/384*(c*x^4+b*x^2)^{(3/2)}*(-3*A*(c*x^2+b)^{(3/2)}*x^8*c^4+9*A*b^{(3/2)}*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)})/x)*x^8*c^4+8*B*(c*x^2+b)^{(3/2)}*x^8*b*c^3-24*B*b^{(5/2)}*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)})/x)*x^8*c^3+3*A*(c*x^2+b)^{(5/2)}*x^6*c^3-9*A*(c*x^2+b)^{(1/2)}*x^8*b*c^4-8*B*(c*x^2+b)^{(5/2)}*x^6*b*c^2+24*B*(c*x^2+b)^{(1/2)}*x^8*b^2*c^3+6*A*(c*x^2+b)^{(5/2)}*x^4*b*c^2-16*B*(c*x^2+b)^{(5/2)}*x^4*b^2*c-24*A*(c*x^2+b)^{(5/2)}*x^2*b^2*c+64*B*(c*x^2+b)^{(5/2)}*x^2*b^3+48*A*(c*x^2+b)^{(5/2)}*b^3)/x^{11}/(c*x^2+b)^{(3/2)}/b^4$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2)^2 (Bx^2 + A)}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^12, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{3/2}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x)`

[Out] `int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*12,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2)/x\*\*12, x)

$$3.128 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx$$

**Optimal.** Leaf size=214

$$\frac{3c^4(2bB - Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}} + \frac{3c^3\sqrt{bx^2+cx^4}(2bB - Ac)}{256b^3x^3} - \frac{c^2\sqrt{bx^2+cx^4}(2bB - Ac)}{128b^2x^5} - \frac{(bx^2+cx^4)^{3/2}(2bB - Ac)}{16bx^{11}}$$

**Rubi [A]** time = 0.34, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2038, 2020, 2025, 2008, 206}

$$\frac{3c^3\sqrt{bx^2+cx^4}(2bB - Ac)}{256b^3x^3} - \frac{c^2\sqrt{bx^2+cx^4}(2bB - Ac)}{128b^2x^5} - \frac{3c^4(2bB - Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}} - \frac{c\sqrt{bx^2+cx^4}(2bB - Ac)}{32bx^7} - \frac{(bx^2+cx^4)^{3/2}(2bB - Ac)}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^14, x]

[Out] -(c\*(2\*b\*B - A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(32\*b\*x^7) - (c^2\*(2\*b\*B - A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(128\*b^2\*x^5) + (3\*c^3\*(2\*b\*B - A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(256\*b^3\*x^3) - ((2\*b\*B - A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(16\*b\*x^11) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(10\*b\*x^15) - (3\*c^4\*(2\*b\*B - A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(256\*b^(7/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 2020

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+j\*p+1)), x] - Dist[(b\*p\*(n-j))/(c^n\*(m+j\*p+1)), Int[(c\*x)^(m+n)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j\*p+1, 0]

Rule 2025

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j-1)\*(c\*x)^(m-j+1)\*(a\*x^j + b\*x^n)^(p+1))/(a\*(m+j\*p+1)), x] - Dist[(b\*(m+n\*p+n-j+1))/(a\*c^(n-j)\*(m+j\*p+1)), Int[(c\*x)^(m+n-j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m+j\*p+1, 0]

Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j

+ b\*x^(j + n))^p, x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m + j\*p, -1] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1, 0]

Rubi steps

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{14}} dx = -\frac{A(bx^2 + cx^4)^{5/2}}{10bx^{15}} - \frac{(-10bB + 5Ac) \int \frac{(bx^2 + cx^4)^{3/2}}{x^{12}} dx}{10b}$$

$$= -\frac{(2bB - Ac)(bx^2 + cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2 + cx^4)^{5/2}}{10bx^{15}} + \frac{(3c(2bB - Ac)) \int \frac{\sqrt{bx^2 + cx^4}}{x^8} dx}{16b}$$

$$= -\frac{c(2bB - Ac)\sqrt{bx^2 + cx^4}}{32bx^7} - \frac{(2bB - Ac)(bx^2 + cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2 + cx^4)^{5/2}}{10bx^{15}} + \frac{c^2(2bB - Ac)\sqrt{bx^2 + cx^4}}{128b^2x^5} - \frac{(2bB - Ac)(bx^2 + cx^4)^{3/2}}{16bx^{11}}$$

$$= -\frac{c(2bB - Ac)\sqrt{bx^2 + cx^4}}{32bx^7} - \frac{c^2(2bB - Ac)\sqrt{bx^2 + cx^4}}{128b^2x^5} + \frac{3c^3(2bB - Ac)\sqrt{bx^2 + cx^4}}{256b^3x^3} - \frac{(2bB - Ac)(bx^2 + cx^4)^{3/2}}{16bx^{11}}$$

$$= -\frac{c(2bB - Ac)\sqrt{bx^2 + cx^4}}{32bx^7} - \frac{c^2(2bB - Ac)\sqrt{bx^2 + cx^4}}{128b^2x^5} + \frac{3c^3(2bB - Ac)\sqrt{bx^2 + cx^4}}{256b^3x^3} - \frac{(2bB - Ac)(bx^2 + cx^4)^{3/2}}{16bx^{11}}$$

$$= -\frac{c(2bB - Ac)\sqrt{bx^2 + cx^4}}{32bx^7} - \frac{c^2(2bB - Ac)\sqrt{bx^2 + cx^4}}{128b^2x^5} + \frac{3c^3(2bB - Ac)\sqrt{bx^2 + cx^4}}{256b^3x^3} - \frac{(2bB - Ac)(bx^2 + cx^4)^{3/2}}{16bx^{11}}$$

**Mathematica [C]** time = 0.04, size = 65, normalized size = 0.30

$$\frac{(x^2(b + cx^2))^{5/2} \left( Ab^5 + c^4x^{10}(2bB - Ac) {}_2F_1\left(\frac{5}{2}, 5; \frac{7}{2}; \frac{cx^2}{b} + 1\right) \right)}{10b^6x^{15}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^14, x]

[Out] -1/10\*((x^2\*(b + c\*x^2))^(5/2)\*(A\*b^5 + c^4\*(2\*b\*B - A\*c)\*x^10\*Hypergeometric2F1[5/2, 5, 7/2, 1 + (c\*x^2)/b]))/(b^6\*x^15)

**IntegrateAlgebraic [A]** time = 1.50, size = 161, normalized size = 0.75

$$\frac{\sqrt{bx^2 + cx^4} (-128Ab^4 - 176Ab^3cx^2 - 8Ab^2c^2x^4 + 10Abc^3x^6 - 15Ac^4x^8 - 160b^4Bx^2 - 240b^3Bcx^4 - 20b^2Bc^2x^6 + 30bBc^3x^8)}{1280b^3x^{11}} - \frac{3(2bBc^4 - Ac^5) \tanh^{-1}\left(\frac{\sqrt{bx^2 + cx^4}}{\sqrt{bx^2 + cx^4}}\right)}{256b^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^14, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-128\*A\*b^4 - 160\*b^4\*B\*x^2 - 176\*A\*b^3\*c\*x^2 - 240\*b^3\*B\*c\*x^4 - 8\*A\*b^2\*c^2\*x^4 - 20\*b^2\*B\*c^2\*x^6 + 10\*A\*b\*c^3\*x^6 + 30\*b\*B\*c^3\*x^8 - 15\*A\*c^4\*x^8))/(1280\*b^3\*x^11) - (3\*(2\*b\*B\*c^4 - A\*c^5)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(256\*b^(7/2))

**fricas [A]** time = 0.51, size = 345, normalized size = 1.61

$$\frac{15(2Bb^4 - Ac^5)\sqrt{bx^2 + cx^4} \operatorname{arctanh}\left(\frac{\sqrt{bx^2 + cx^4}}{\sqrt{bx^2 + cx^4}}\right) - 2(15(2Bb^2c^2 - Ab^2c^2)x^4 - 128Ab^4 - 8(30Bb^4 + Ab^2c^2)x^4 - 16(10Bb^4 + 11Ab^2c^2)\sqrt{cx^4 + b^2} - 15(2Bb^4 - Ac^5)\sqrt{bx^2 + cx^4} \operatorname{arctanh}\left(\frac{\sqrt{bx^2 + cx^4}}{\sqrt{bx^2 + cx^4}}\right) + (15(2Bb^2c^2 - Ab^2c^2)x^4 - 128Ab^4 - 8(30Bb^4 + Ab^2c^2)x^4 - 16(10Bb^4 + 11Ab^2c^2)\sqrt{cx^4 + b^2})\sqrt{bx^2 + cx^4}}{1280b^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^14,x, algorithm="fricas")

[Out] [-1/2560\*(15\*(2\*B\*b\*c^4 - A\*c^5)\*sqrt(b)\*x^11\*log(-(c\*x^3 + 2\*b\*x + 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) - 2\*(15\*(2\*B\*b^2\*c^3 - A\*b\*c^4)\*x^8 - 10\*(2\*B\*b^3\*c^2 - A\*b^2\*c^3)\*x^6 - 128\*A\*b^5 - 8\*(30\*B\*b^4\*c + A\*b^3\*c^2)\*x^4 - 16\*(10\*B\*b^5 + 11\*A\*b^4\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(b^4\*x^11), 1/1280\*(15\*(2\*B\*b\*c^4 - A\*c^5)\*sqrt(-b)\*x^11\*arctan(sqrt(c\*x^4 + b\*x^2))\*sqrt(-b)/(c\*x^3 + b\*x)) + (15\*(2\*B\*b^2\*c^3 - A\*b\*c^4)\*x^8 - 10\*(2\*B\*b^3\*c^2 - A\*b^2\*c^3)\*x^6 - 128\*A\*b^5 - 8\*(30\*B\*b^4\*c + A\*b^3\*c^2)\*x^4 - 16\*(10\*B\*b^5 + 11\*A\*b^4\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(b^4\*x^11)]

**giac** [A] time = 0.31, size = 234, normalized size = 1.09

$$\frac{15(2Bbc^5\operatorname{sgn}(x) - Ac^6\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^4+b}}{\sqrt{-b}}\right) + 30(cx^2+b)^{\frac{9}{2}}Bbc^5\operatorname{sgn}(x) - 140(cx^2+b)^{\frac{7}{2}}Bb^2c^5\operatorname{sgn}(x) + 140(cx^2+b)^{\frac{5}{2}}Bb^3c^5\operatorname{sgn}(x) - 30\sqrt{cx^2+b}Bb^4c^5\operatorname{sgn}(x) - 15(cx^2+b)^{\frac{9}{2}}Ac^6\operatorname{sgn}(x) + 70(cx^2+b)^{\frac{7}{2}}Ab^2c^6\operatorname{sgn}(x) - 128(cx^2+b)^{\frac{5}{2}}Ab^3c^6\operatorname{sgn}(x) - 70(cx^2+b)^{\frac{3}{2}}Ab^4c^6\operatorname{sgn}(x) + 15\sqrt{cx^2+b}Ab^5c^6\operatorname{sgn}(x)}{\sqrt{-b}b^3}}{1280c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^14,x, algorithm="giac")

[Out] 1/1280\*(15\*(2\*B\*b\*c^5\*sgn(x) - A\*c^6\*sgn(x))\*arctan(sqrt(c\*x^2 + b)/sqrt(-b))/sqrt(-b)\*b^3) + (30\*(c\*x^2 + b)^(9/2)\*B\*b\*c^5\*sgn(x) - 140\*(c\*x^2 + b)^(7/2)\*B\*b^2\*c^5\*sgn(x) + 140\*(c\*x^2 + b)^(3/2)\*B\*b^4\*c^5\*sgn(x) - 30\*sqrt(c\*x^2 + b)\*B\*b^5\*c^5\*sgn(x) - 15\*(c\*x^2 + b)^(9/2)\*A\*c^6\*sgn(x) + 70\*(c\*x^2 + b)^(7/2)\*A\*b\*c^6\*sgn(x) - 128\*(c\*x^2 + b)^(5/2)\*A\*b^2\*c^6\*sgn(x) - 70\*(c\*x^2 + b)^(3/2)\*A\*b^3\*c^6\*sgn(x) + 15\*sqrt(c\*x^2 + b)\*A\*b^4\*c^6\*sgn(x))/(b^3\*c^5\*x^10))/c

**maple** [A] time = 0.09, size = 344, normalized size = 1.61

$$\frac{(c^2 + b)^{\frac{3}{2}} \left( 15A^2 b^2 c^2 \ln\left(\frac{2\sqrt{cx^2+b}}{\sqrt{-b}}\right) - 30Bb^2 c^2 \ln\left(\frac{2\sqrt{cx^2+b}}{\sqrt{-b}}\right) - 15\sqrt{cx^2+b} Bb^2 c^2 + 30\sqrt{cx^2+b} Bb^2 c^2 - 5(c^2 + b)^{\frac{3}{2}} A c^6 + 10(c^2 + b)^{\frac{3}{2}} B b c^5 + 5(c^2 + b)^{\frac{3}{2}} A c^6 - 10(c^2 + b)^{\frac{3}{2}} B b c^5 + 10(c^2 + b)^{\frac{3}{2}} A b^2 c^6 - 20(c^2 + b)^{\frac{3}{2}} B b^2 c^6 - 40(c^2 + b)^{\frac{3}{2}} A b^3 c^6 + 80(c^2 + b)^{\frac{3}{2}} B b^3 c^6 + 80(c^2 + b)^{\frac{3}{2}} A b^4 c^6 - 160(c^2 + b)^{\frac{3}{2}} B b^4 c^6 - 128(c^2 + b)^{\frac{3}{2}} A b^5 c^6 \right)}{1280(c^2 + b)^{\frac{3}{2}} b^3 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^14,x)

[Out] 1/1280\*(c\*x^4+b\*x^2)^(3/2)\*(-5\*A\*(c\*x^2+b)^(3/2)\*x^10\*c^5+15\*A\*b^(3/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^10\*c^5+10\*B\*(c\*x^2+b)^(3/2)\*x^10\*b\*c^4-30\*B\*b^(5/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^10\*c^4+5\*A\*(c\*x^2+b)^(5/2)\*x^8\*c^4-15\*A\*(c\*x^2+b)^(1/2)\*x^10\*b\*c^5-10\*B\*(c\*x^2+b)^(5/2)\*x^8\*b\*c^3+30\*B\*(c\*x^2+b)^(1/2)\*x^10\*b^2\*c^4+10\*A\*(c\*x^2+b)^(5/2)\*x^6\*b\*c^3-20\*B\*(c\*x^2+b)^(5/2)\*x^6\*b^2\*c^2-40\*A\*(c\*x^2+b)^(5/2)\*x^4\*b^2\*c^2+80\*B\*(c\*x^2+b)^(5/2)\*x^4\*b^3\*c+80\*A\*(c\*x^2+b)^(5/2)\*x^2\*b^3\*c-160\*B\*(c\*x^2+b)^(5/2)\*x^2\*b^4-128\*A\*(c\*x^2+b)^(5/2)\*b^4)/x^13/(c\*x^2+b)^(3/2)/b^5

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}} (Bx^2 + A)}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^14,x, algorithm="maxima")

[Out] integrate((c\*x^4 + b\*x^2)^(3/2)\*(B\*x^2 + A)/x^14, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{\frac{3}{2}}}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14, x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**14, x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**14, x)
```



$$3.129 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}} dx$$

**Optimal.** Leaf size=251

$$\frac{c^5(12bB - 7Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}} - \frac{c^4\sqrt{bx^2+cx^4}(12bB - 7Ac)}{1024b^4x^3} + \frac{c^3\sqrt{bx^2+cx^4}(12bB - 7Ac)}{1536b^3x^5} - \frac{c^2\sqrt{bx^2+cx^4}}{1920b}$$

**Rubi [A]** time = 0.39, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2038, 2020, 2025, 2008, 206}

$$-\frac{c^4\sqrt{bx^2+cx^4}(12bB-7Ac)}{1024b^4x^3} + \frac{c^3\sqrt{bx^2+cx^4}(12bB-7Ac)}{1536b^3x^5} - \frac{c^2\sqrt{bx^2+cx^4}(12bB-7Ac)}{1920b^2x^7} + \frac{c^5(12bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}} - \frac{c\sqrt{bx^2+cx^4}(12bB-7Ac)}{320bx^9} - \frac{(bx^2+cx^4)^{3/2}(12bB-7Ac)}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^16, x]

[Out] -(c\*(12\*b\*B - 7\*A\*c)\*Sqrt[b\*x^2 + c\*x^4]/(320\*b\*x^9) - (c^2\*(12\*b\*B - 7\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(1920\*b^2\*x^7) + (c^3\*(12\*b\*B - 7\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(1536\*b^3\*x^5) - (c^4\*(12\*b\*B - 7\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(1024\*b^4\*x^3) - ((12\*b\*B - 7\*A\*c)\*(b\*x^2 + c\*x^4)^(3/2))/(120\*b\*x^13) - (A\*(b\*x^2 + c\*x^4)^(5/2))/(12\*b\*x^17) + (c^5\*(12\*b\*B - 7\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(1024\*b^(9/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2020

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a\*x^j + b\*x^n)^p)/(c\*(m+j\*p+1)), x] - Dist[(b\*p\*(n-j))/(c^n\*(m+j\*p+1)), Int[(c\*x)^(m+n)\*(a\*x^j + b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j\*p+1, 0]

#### Rule 2025

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(c^(j-1)\*(c\*x)^(m-j+1)\*(a\*x^j + b\*x^n)^(p+1))/(a\*(m+j\*p+1)), x] - Dist[(b\*(m+n\*p+n-j+1))/(a\*c^(n-j)\*(m+j\*p+1)), Int[(c\*x)^(m+n-j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m+j\*p+1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j

+ b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m + j\*p, -1] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1, 0]

Rubi steps

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{16}} dx = -\frac{A(bx^2 + cx^4)^{5/2}}{12bx^{17}} - \frac{(-12bB + 7Ac) \int \frac{(bx^2 + cx^4)^{3/2}}{x^{14}} dx}{12b}$$

$$= -\frac{(12bB - 7Ac)(bx^2 + cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2 + cx^4)^{5/2}}{12bx^{17}} + \frac{(c(12bB - 7Ac)) \int \frac{\sqrt{bx^2 + cx^4}}{x^{10}} dx}{40b}$$

$$= -\frac{c(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{320bx^9} - \frac{(12bB - 7Ac)(bx^2 + cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2 + cx^4)^{5/2}}{12bx^{17}}$$

$$= -\frac{c(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{320bx^9} - \frac{c^2(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1920b^2x^7} - \frac{(12bB - 7Ac)(bx^2 + cx^4)^{3/2}}{120bx^{13}}$$

$$= -\frac{c(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{320bx^9} - \frac{c^2(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1920b^2x^7} + \frac{c^3(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1536b^3x^5}$$

$$= -\frac{c(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{320bx^9} - \frac{c^2(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1920b^2x^7} + \frac{c^3(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1536b^3x^5}$$

$$= -\frac{c(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{320bx^9} - \frac{c^2(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1920b^2x^7} + \frac{c^3(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1536b^3x^5}$$

$$= -\frac{c(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{320bx^9} - \frac{c^2(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1920b^2x^7} + \frac{c^3(12bB - 7Ac)\sqrt{bx^2 + cx^4}}{1536b^3x^5}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.26

$$\frac{(x^2(b + cx^2))^{5/2} \left( c^5 x^{12} (12bB - 7Ac) {}_2F_1\left(\frac{5}{2}, 6; \frac{7}{2}; \frac{cx^2}{b} + 1\right) - 5Ab^6 \right)}{60b^7 x^{17}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^16, x]

[Out] ((x^2\*(b + c\*x^2))^(5/2)\*(-5\*A\*b^6 + c^5\*(12\*b\*B - 7\*A\*c)\*x^12\*Hypergeometric2F1[5/2, 6, 7/2, 1 + (c\*x^2)/b]))/(60\*b^7\*x^17)

IntegrateAlgebraic [A] time = 1.84, size = 185, normalized size = 0.74

$$\frac{(12bBc^5 - 7Ac^6) \tanh^{-1}\left(\frac{\sqrt{bx^2 + cx^4}}{\sqrt{bx^2 + cx^4}}\right) + \sqrt{bx^2 + cx^4} (-1280Ab^5 - 1664Ab^4cx^2 - 48Ab^3c^2x^4 + 56Ab^2c^3x^6 - 70Abc^4x^8 + 105Ac^5x^{10} - 1536b^5Bx^2 - 2112b^4Bcx^4 - 96b^3Bc^2x^6 + 120b^2Bc^3x^8 - 180bBc^4x^{10})}{1024b^{9/2} + 15360b^4x^{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^16, x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-1280\*A\*b^5 - 1536\*b^5\*B\*x^2 - 1664\*A\*b^4\*c\*x^2 - 2112\*b^4\*B\*c\*x^4 - 48\*A\*b^3\*c^2\*x^4 - 96\*b^3\*B\*c^2\*x^6 + 56\*A\*b^2\*c^3\*x^6 + 120\*b^2\*B\*c^3\*x^8 - 70\*A\*b\*c^4\*x^8 - 180\*b\*B\*c^4\*x^10 + 105\*A\*c^5\*x^10))/(15360\*b^4\*x^13) + ((12\*b\*B\*c^5 - 7\*A\*c^6)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(1024\*b^(9/2))

**fricas** [A] time = 0.58, size = 393, normalized size = 1.57

$$\frac{15(12Bb^2 - 7A^2)\sqrt{c^2+b} \arctan\left(\frac{\sqrt{c^2+b}\sqrt{cx^4+bx^2}}{\sqrt{c^2+b}}\right) + 2(15(12Bb^2 - 7A^2)c^{10} - 10(12Bb^2 - 7A^2)c^8 + 1280Ab^6 + 8(12Bb^2 - 7A^2)c^6 + 48(44Bb^2 + A^2)c^4 + 128(12Bb^2 + 13A^2)c^2)\sqrt{c^2+b}}{30720c^{12}} + \frac{15(12Bb^2 - 7A^2)\sqrt{c^2+b} \arctan\left(\frac{\sqrt{c^2+b}}{\sqrt{c^2+b}}\right) + (15(12Bb^2 - 7A^2)c^{10} - 10(12Bb^2 - 7A^2)c^8 + 1280Ab^6 + 8(12Bb^2 - 7A^2)c^6 + 48(44Bb^2 + A^2)c^4 + 128(12Bb^2 + 13A^2)c^2)\sqrt{c^2+b}}{15360c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^16,x, algorithm="fricas")

[Out] [-1/30720\*(15\*(12\*B\*b\*c^5 - 7\*A\*c^6)\*sqrt(b)\*x^13\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*(15\*(12\*B\*b^2\*c^4 - 7\*A\*b\*c^5)\*x^10 - 10\*(12\*B\*b^3\*c^3 - 7\*A\*b^2\*c^4)\*x^8 + 1280\*A\*b^6 + 8\*(12\*B\*b^4\*c^2 - 7\*A\*b^3\*c^3)\*x^6 + 48\*(44\*B\*b^5\*c + A\*b^4\*c^2)\*x^4 + 128\*(12\*B\*b^6 + 13\*A\*b^5\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(b^5\*x^13), -1/15360\*(15\*(12\*B\*b\*c^5 - 7\*A\*c^6)\*sqrt(-b)\*x^13\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + (15\*(12\*B\*b^2\*c^4 - 7\*A\*b\*c^5)\*x^10 - 10\*(12\*B\*b^3\*c^3 - 7\*A\*b^2\*c^4)\*x^8 + 1280\*A\*b^6 + 8\*(12\*B\*b^4\*c^2 - 7\*A\*b^3\*c^3)\*x^6 + 48\*(44\*B\*b^5\*c + A\*b^4\*c^2)\*x^4 + 128\*(12\*B\*b^6 + 13\*A\*b^5\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(b^5\*x^13)]

**giac** [A] time = 0.37, size = 294, normalized size = 1.17

$$\frac{15(12Bb^2 - 7A^2)\sqrt{c^2+b} \arctan\left(\frac{\sqrt{c^2+b}}{\sqrt{c^2+b}}\right) + 180(c^2+b)^{11/2}Bb^2\sqrt{c^2+b} - 1020(c^2+b)^{9/2}Bb^2\sqrt{c^2+b} + 2376(c^2+b)^{7/2}Bb^2\sqrt{c^2+b} - 696(c^2+b)^{5/2}Bb^2\sqrt{c^2+b} - 1020(c^2+b)^{3/2}Bb^2\sqrt{c^2+b} + 180\sqrt{c^2+b}Bb^2\sqrt{c^2+b} - 105(c^2+b)^{11/2}A^2\sqrt{c^2+b} + 595(c^2+b)^{9/2}A^2\sqrt{c^2+b} - 1386(c^2+b)^{7/2}A^2\sqrt{c^2+b} + 1686(c^2+b)^{5/2}A^2\sqrt{c^2+b} - 105\sqrt{c^2+b}A^2\sqrt{c^2+b}}{\sqrt{c^2+b}^{12}} + \frac{15360c}{\sqrt{c^2+b}^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^16,x, algorithm="giac")

[Out] -1/15360\*(15\*(12\*B\*b\*c^6\*sgn(x) - 7\*A\*c^7\*sgn(x))\*arctan(sqrt(c\*x^2 + b)/sqrt(-b))/(sqrt(-b)\*b^4) + (180\*(c\*x^2 + b)^(11/2)\*B\*b\*c^6\*sgn(x) - 1020\*(c\*x^2 + b)^(9/2)\*B\*b^2\*c^6\*sgn(x) + 2376\*(c\*x^2 + b)^(7/2)\*B\*b^3\*c^6\*sgn(x) - 696\*(c\*x^2 + b)^(5/2)\*B\*b^4\*c^6\*sgn(x) - 1020\*(c\*x^2 + b)^(3/2)\*B\*b^5\*c^6\*sgn(x) + 180\*sqrt(c\*x^2 + b)\*B\*b^6\*c^6\*sgn(x) - 105\*(c\*x^2 + b)^(11/2)\*A\*c^7\*sgn(x) + 595\*(c\*x^2 + b)^(9/2)\*A\*b\*c^7\*sgn(x) - 1386\*(c\*x^2 + b)^(7/2)\*A\*b^2\*c^7\*sgn(x) + 1686\*(c\*x^2 + b)^(5/2)\*A\*b^3\*c^7\*sgn(x) + 595\*(c\*x^2 + b)^(3/2)\*A\*b^4\*c^7\*sgn(x) - 105\*sqrt(c\*x^2 + b)\*A\*b^5\*c^7\*sgn(x))/(b^4\*c^6\*x^12))/c

**maple** [A] time = 0.15, size = 386, normalized size = 1.54

$$\frac{(c^2 + b)^{11/2} B b^2 \sqrt{c^2 + b} - 1020 (c^2 + b)^{9/2} B b^2 \sqrt{c^2 + b} + 2376 (c^2 + b)^{7/2} B b^2 \sqrt{c^2 + b} - 696 (c^2 + b)^{5/2} B b^2 \sqrt{c^2 + b} - 1020 (c^2 + b)^{3/2} B b^2 \sqrt{c^2 + b} + 180 \sqrt{c^2 + b} B b^2 \sqrt{c^2 + b} - 105 (c^2 + b)^{11/2} A^2 \sqrt{c^2 + b} + 595 (c^2 + b)^{9/2} A^2 \sqrt{c^2 + b} - 1386 (c^2 + b)^{7/2} A^2 \sqrt{c^2 + b} + 1686 (c^2 + b)^{5/2} A^2 \sqrt{c^2 + b} - 105 \sqrt{c^2 + b} A^2 \sqrt{c^2 + b}}{\sqrt{c^2 + b}^{12}} + \frac{15360 c}{\sqrt{c^2 + b}^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^16,x)

[Out] -1/15360\*(c\*x^4+b\*x^2)^(3/2)\*(-35\*A\*(c\*x^2+b)^(3/2)\*x^12\*c^6+105\*A\*b^(3/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^12\*c^6+60\*B\*(c\*x^2+b)^(3/2)\*x^12\*b\*c^5-180\*B\*b^(5/2)\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^12\*c^5+35\*A\*(c\*x^2+b)^(5/2)\*x^10\*c^5-105\*A\*(c\*x^2+b)^(1/2)\*x^12\*b\*c^6-60\*B\*(c\*x^2+b)^(5/2)\*x^10\*b\*c^4+180\*B\*(c\*x^2+b)^(1/2)\*x^12\*b^2\*c^5+70\*A\*(c\*x^2+b)^(5/2)\*x^8\*b\*c^4-120\*B\*(c\*x^2+b)^(5/2)\*x^8\*b^2\*c^3-280\*A\*(c\*x^2+b)^(5/2)\*x^6\*b^2\*c^3+480\*B\*(c\*x^2+b)^(5/2)\*x^6\*b^3\*c^2+560\*A\*(c\*x^2+b)^(5/2)\*x^4\*b^3\*c^2-960\*B\*(c\*x^2+b)^(5/2)\*x^4\*b^4\*c-896\*A\*(c\*x^2+b)^(5/2)\*x^2\*b^4\*c+1536\*B\*(c\*x^2+b)^(5/2)\*x^2\*b^5+1280\*A\*(c\*x^2+b)^(5/2)\*b^5)/x^15/(c\*x^2+b)^(3/2)/b^6

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}(Bx^2 + A)}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^(3/2)/x^16,x, algorithm="maxima")

[Out] integrate((c\*x^4 + b\*x^2)^(3/2)\*(B\*x^2 + A)/x^16, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(Bx^2 + A)(cx^4 + bx^2)^{3/2}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^16,x)

[Out] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^(3/2))/x^16, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b + cx^2))^{\frac{3}{2}}(A + Bx^2)}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*(3/2)/x\*\*16,x)

[Out] Integral((x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)\*(A + B\*x\*\*2)/x\*\*16, x)

$$3.130 \quad \int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=176

$$\frac{5b^3(7bB - 8Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}} - \frac{5b^2\sqrt{bx^2+cx^4}(7bB - 8Ac)}{128c^4} + \frac{5bx^2\sqrt{bx^2+cx^4}(7bB - 8Ac)}{192c^3} - \frac{x^4\sqrt{bx^2+cx^4}}{48c^2}$$

**Rubi [A]** time = 0.33, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 794, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{bx^2+cx^4}(7bB - 8Ac)}{128c^4} + \frac{5b^3(7bB - 8Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}} - \frac{x^4\sqrt{bx^2+cx^4}(7bB - 8Ac)}{48c^2} + \frac{5bx^2\sqrt{bx^2+cx^4}(7bB - 8Ac)}{192c^3} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c}$$

Antiderivative was successfully verified.

[In] Int[(x^7\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (-5\*b^2\*(7\*b\*B - 8\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(128\*c^4) + (5\*b\*(7\*b\*B - 8\*A\*c)\*x^2\*Sqrt[b\*x^2 + c\*x^4])/(192\*c^3) - ((7\*b\*B - 8\*A\*c)\*x^4\*Sqrt[b\*x^2 + c\*x^4])/(48\*c^2) + (B\*x^6\*Sqrt[b\*x^2 + c\*x^4])/(8\*c) + (5\*b^3\*(7\*b\*B - 8\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(128\*c^(9/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 794

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(c\*e\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[m + 2\*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])

Rule 2034

```
Int[(x_)^(m_)*((b_)*(x_)^(k_) + (a_)*(x_)^(j_))^(p_)*((c_) + (d_)*(x_)
^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^7 (A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^3 (A + Bx)}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\ &= \frac{Bx^6 \sqrt{bx^2 + cx^4}}{8c} + \frac{\left( 3(-bB + Ac) + \frac{1}{2}(-bB + 2Ac) \right) \text{Subst} \left( \int \frac{x^3}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{8c} \\ &= -\frac{(7bB - 8Ac)x^4 \sqrt{bx^2 + cx^4}}{48c^2} + \frac{Bx^6 \sqrt{bx^2 + cx^4}}{8c} + \frac{(5b(7bB - 8Ac)) \text{Subst} \left( \int \frac{x^2}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{96c^2} \\ &= \frac{5b(7bB - 8Ac)x^2 \sqrt{bx^2 + cx^4}}{192c^3} - \frac{(7bB - 8Ac)x^4 \sqrt{bx^2 + cx^4}}{48c^2} + \frac{Bx^6 \sqrt{bx^2 + cx^4}}{8c} - \frac{(5b^2(7bB - 8Ac)) \sqrt{bx^2 + cx^4}}{48c^2} \\ &= -\frac{5b^2(7bB - 8Ac) \sqrt{bx^2 + cx^4}}{128c^4} + \frac{5b(7bB - 8Ac)x^2 \sqrt{bx^2 + cx^4}}{192c^3} - \frac{(7bB - 8Ac)x^4 \sqrt{bx^2 + cx^4}}{48c^2} \\ &= -\frac{5b^2(7bB - 8Ac) \sqrt{bx^2 + cx^4}}{128c^4} + \frac{5b(7bB - 8Ac)x^2 \sqrt{bx^2 + cx^4}}{192c^3} - \frac{(7bB - 8Ac)x^4 \sqrt{bx^2 + cx^4}}{48c^2} \\ &= -\frac{5b^2(7bB - 8Ac) \sqrt{bx^2 + cx^4}}{128c^4} + \frac{5b(7bB - 8Ac)x^2 \sqrt{bx^2 + cx^4}}{192c^3} - \frac{(7bB - 8Ac)x^4 \sqrt{bx^2 + cx^4}}{48c^2} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 145, normalized size = 0.82

$$\frac{x \left( 15b^3 \sqrt{b + cx^2} (7bB - 8Ac) \tanh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b + cx^2}} \right) - \sqrt{c} x (b + cx^2) (-10b^2c(12A + 7Bx^2) + 8bc^2x^2(10A + 7Bx^2) - 16c^3x^4(4A + 3Bx^2) + 105b^3B) \right)}{384c^{9/2} \sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (x\*(-(Sqrt[c]\*x\*(b + c\*x^2)\*(105\*b^3\*B - 16\*c^3\*x^4\*(4\*A + 3\*B\*x^2) + 8\*b\*c^2\*x^2\*(10\*A + 7\*B\*x^2) - 10\*b^2\*c\*(12\*A + 7\*B\*x^2))) + 15\*b^3\*(7\*b\*B - 8\*A\*c)\*Sqrt[b + c\*x^2]\*ArcTanh[(Sqrt[c]\*x)/Sqrt[b + c\*x^2]])/(384\*c^(9/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.57, size = 139, normalized size = 0.79

$$\frac{\sqrt{bx^2 + cx^4} (120Ab^2c - 80Abc^2x^2 + 64Ac^3x^4 - 105b^3B + 70b^2Bcx^2 - 56bBc^2x^4 + 48Bc^3x^6)}{384c^4} - \frac{5(7b^4B - 8Ab^3c) \log(-2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2)}{256c^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-105\*b^3\*B + 120\*A\*b^2\*c + 70\*b^2\*B\*c\*x^2 - 80\*A\*b\*c^2\*x^2 - 56\*b\*B\*c^2\*x^4 + 64\*A\*c^3\*x^4 + 48\*B\*c^3\*x^6))/(384\*c^4) - (5\*(7\*b^4\*B - 8\*A\*b^3\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(256\*c^(9/2))

**fricas** [A] time = 0.46, size = 275, normalized size = 1.56

$$\frac{15(7Bb^4 - 8Ab^3c)\sqrt{c}\log(-2cx^2 - b + 2\sqrt{cx^4 + bx^2}\sqrt{c}) - 2(48Bc^4e - 105Bb^3c + 120Ab^2c^2 - 8(7Bbc^3 - 8Ac^4)^4 + 10(7Bb^2c^2 - 8Abc^3)x^2)\sqrt{cx^4 + bx^2}}{768c^5} - \frac{15(7Bb^4 - 8Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (48Bc^4e - 105Bb^3c + 120Ab^2c^2 - 8(7Bbc^3 - 8Ac^4)^4 + 10(7Bb^2c^2 - 8Abc^3)x^2)\sqrt{cx^4 + bx^2}}{384c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/768\*(15\*(7\*B\*b^4 - 8\*A\*b^3\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(48\*B\*c^4\*x^6 - 105\*B\*b^3\*c + 120\*A\*b^2\*c^2 - 8\*(7\*B\*b\*c^3 - 8\*A\*c^4)\*x^4 + 10\*(7\*B\*b^2\*c^2 - 8\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^5, -1/384\*(15\*(7\*B\*b^4 - 8\*A\*b^3\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) - (48\*B\*c^4\*x^6 - 105\*B\*b^3\*c + 120\*A\*b^2\*c^2 - 8\*(7\*B\*b\*c^3 - 8\*A\*c^4)\*x^4 + 10\*(7\*B\*b^2\*c^2 - 8\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/c^5]

**giac** [A] time = 0.22, size = 149, normalized size = 0.85

$$\frac{1}{384}\sqrt{cx^4 + bx^2}\left(2\left(4\left(\frac{6Bx^2}{c} - \frac{7Bbc^2 - 8Ac^3}{c^4}\right)x^2 + \frac{5(7Bb^2c - 8Abc^2)}{c^4}\right)x^2 - \frac{15(7Bb^3 - 8Ab^2c)}{c^4}\right) - \frac{5(7Bb^4 - 8Ab^3c)\log\left(-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right)}{256c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] 1/384\*sqrt(c\*x^4 + b\*x^2)\*(2\*(4\*(6\*B\*x^2/c - (7\*B\*b\*c^2 - 8\*A\*c^3)/c^4)\*x^2 + 5\*(7\*B\*b^2\*c - 8\*A\*b\*c^2)/c^4)\*x^2 - 15\*(7\*B\*b^3 - 8\*A\*b^2\*c)/c^4 - 5/2\*56\*(7\*B\*b^4 - 8\*A\*b^3\*c)\*log(abs(-2\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))\*sqrt(c) - b))/c^(9/2)

**maple** [A] time = 0.06, size = 211, normalized size = 1.20

$$\frac{\sqrt{cx^2 + b}\left(48\sqrt{cx^2 + b}Bc^2x^7 + 64\sqrt{cx^2 + b}Ac^2x^5 - 56\sqrt{cx^2 + b}Bbc^2x^3 - 80\sqrt{cx^2 + b}Abc^2x^3 + 70\sqrt{cx^2 + b}Bb^2c^2x^3 - 120Ab^2c^2\ln(\sqrt{c}x + \sqrt{cx^2 + b}) + 105Bb^4c\ln(\sqrt{c}x + \sqrt{cx^2 + b}) + 120\sqrt{cx^2 + b}Ab^2c^2x - 105\sqrt{cx^2 + b}Bb^2c^2x\right)x}{384\sqrt{cx^4 + bx^2}c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x)

[Out] 1/384\*x\*(c\*x^2+b)^(1/2)\*(48\*B\*(c\*x^2+b)^(1/2)\*c^(9/2)\*x^7+64\*A\*(c\*x^2+b)^(1/2)\*c^(9/2)\*x^5-56\*B\*(c\*x^2+b)^(1/2)\*c^(7/2)\*x^5\*b-80\*A\*(c\*x^2+b)^(1/2)\*c^(7/2)\*x^3\*b+70\*B\*(c\*x^2+b)^(1/2)\*c^(5/2)\*x^3\*b^2+120\*A\*(c\*x^2+b)^(1/2)\*c^(5/2)\*x\*b^2-105\*B\*(c\*x^2+b)^(1/2)\*c^(3/2)\*x\*b^3-120\*A\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^3\*c^2+105\*B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^4\*c)/(c\*x^4+b\*x^2)^(1/2)/c^(11/2)

**maxima** [A] time = 1.38, size = 231, normalized size = 1.31

$$\frac{1}{96}\left(\frac{16\sqrt{cx^4 + bx^2}x^4}{c} - \frac{20\sqrt{cx^4 + bx^2}bx^2}{c^2} - \frac{15b^3\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{7}{2}}} + \frac{30\sqrt{cx^4 + bx^2}b^2}{c^3}\right)A + \frac{1}{768}\left(\frac{96\sqrt{cx^4 + bx^2}x^6}{c} - \frac{112\sqrt{cx^4 + bx^2}bx^4}{c^2} + \frac{140\sqrt{cx^4 + bx^2}b^2x^2}{c^3} + \frac{105b^4\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{9}{2}}} - \frac{210\sqrt{cx^4 + bx^2}b^3}{c^4}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/96\*(16\*sqrt(c\*x^4 + b\*x^2)\*x^4/c - 20\*sqrt(c\*x^4 + b\*x^2)\*b\*x^2/c^2 - 15\*b^3\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(7/2) + 30\*sqrt(c\*x^4 + b\*x^2)\*b^2/c^3)\*A + 1/768\*(96\*sqrt(c\*x^4 + b\*x^2)\*x^6/c - 112\*sqrt(c\*x^4 + b\*x^2)\*b\*x^4/c^2 + 140\*sqrt(c\*x^4 + b\*x^2)\*b^2\*x^2/c^3 + 105\*b^4\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(9/2) - 210\*sqrt(c\*x^4 + b\*x^2)\*b^3/c^4)\*B

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7 (Bx^2 + A)}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)`

[Out] `int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^2)}{\sqrt{x^2 (b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(1/2), x)`

[Out] `Integral(x**7*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)`



$$3.131 \quad \int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=139

$$\frac{b^2(5bB - 6Ac) \tanh^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}} + \frac{b\sqrt{bx^2+cx^4}(5bB - 6Ac)}{16c^3} - \frac{x^2\sqrt{bx^2+cx^4}(5bB - 6Ac)}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

**Rubi [A]** time = 0.27, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 794, 670, 640, 620, 206}

$$\frac{b^2(5bB - 6Ac) \tanh^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}} - \frac{x^2\sqrt{bx^2+cx^4}(5bB - 6Ac)}{24c^2} + \frac{b\sqrt{bx^2+cx^4}(5bB - 6Ac)}{16c^3} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (b\*(5\*b\*B - 6\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(16\*c^3) - ((5\*b\*B - 6\*A\*c)\*x^2\*Sqrt[b\*x^2 + c\*x^4])/(24\*c^2) + (B\*x^4\*Sqrt[b\*x^2 + c\*x^4])/(6\*c) - (b^2\*(5\*b\*B - 6\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(16\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 794

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(c\*e\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[m + 2\*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])

## Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_.))^(p_.)*((c_.) + (d_.)*(x_)
^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

## Rubi steps

$$\begin{aligned} \int \frac{x^5 (A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2(A + Bx)}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\ &= \frac{Bx^4 \sqrt{bx^2 + cx^4}}{6c} + \frac{\left(2(-bB + Ac) + \frac{1}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{x^2}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{6c} \\ &= -\frac{(5bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^2} + \frac{Bx^4 \sqrt{bx^2 + cx^4}}{6c} + \frac{(b(5bB - 6Ac)) \text{Subst} \left( \int \frac{x}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{16c^2} \\ &= \frac{b(5bB - 6Ac) \sqrt{bx^2 + cx^4}}{16c^3} - \frac{(5bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^2} + \frac{Bx^4 \sqrt{bx^2 + cx^4}}{6c} - \frac{(b^2(5bB - 6Ac)) \text{Subst} \left( \int \frac{1}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{16c^2} \\ &= \frac{b(5bB - 6Ac) \sqrt{bx^2 + cx^4}}{16c^3} - \frac{(5bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^2} + \frac{Bx^4 \sqrt{bx^2 + cx^4}}{6c} - \frac{(b^2(5bB - 6Ac)) \text{Subst} \left( \int \frac{1}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{16c^2} \\ &= \frac{b(5bB - 6Ac) \sqrt{bx^2 + cx^4}}{16c^3} - \frac{(5bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^2} + \frac{Bx^4 \sqrt{bx^2 + cx^4}}{6c} - \frac{b^2(5bB - 6Ac)}{16c^2} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 123, normalized size = 0.88

$$\frac{x \left( \sqrt{c} x (b + cx^2) (-2bc(9A + 5Bx^2) + 4c^2 x^2 (3A + 2Bx^2) + 15b^2 B) - 3b^2 \sqrt{b + cx^2} (5bB - 6Ac) \tanh^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b + cx^2}} \right) \right)}{48c^{7/2} \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (x\*(Sqrt[c]\*x\*(b + c\*x^2)\*(15\*b^2\*B + 4\*c^2\*x^2\*(3\*A + 2\*B\*x^2) - 2\*b\*c\*(9\*A + 5\*B\*x^2)) - 3\*b^2\*(5\*b\*B - 6\*A\*c)\*Sqrt[b + c\*x^2]\*ArcTanh[(Sqrt[c]\*x)/Sqrt[b + c\*x^2]])/(48\*c^(7/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.58, size = 115, normalized size = 0.83

$$\frac{\sqrt{bx^2 + cx^4} (-18Abc + 12Ac^2x^2 + 15b^2B - 10bBcx^2 + 8Bc^2x^4)}{48c^3} + \frac{(5b^3B - 6Ab^2c) \log(-2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2)}{32c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(15\*b^2\*B - 18\*A\*b\*c - 10\*b\*B\*c\*x^2 + 12\*A\*c^2\*x^2 + 8\*B\*c^2\*x^4))/(48\*c^3) + ((5\*b^3\*B - 6\*A\*b^2\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(32\*c^(7/2))

**fricas [A]** time = 0.45, size = 226, normalized size = 1.63

$$\frac{3(5Bb^3 - 6Ab^2c)\sqrt{c} \log(-2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c}) - 2(8Bc^3x^4 + 15Bb^2c - 18Abc^2 - 2(5Bbc^2 - 6Ac^2)x^2)\sqrt{cx^4 + bx^2} - 3(5Bb^3 - 6Ab^2c)\sqrt{c} \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) + (8Bc^3x^4 + 15Bb^2c - 18Abc^2 - 2(5Bbc^2 - 6Ac^2)x^2)\sqrt{cx^4 + bx^2}}{96c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out]  $[-1/96*(3*(5*B*b^3 - 6*A*b^2*c)*\sqrt{c}*\log(-2*c*x^2 - b - 2*\sqrt{c*x^4 + b*x^2}*\sqrt{c}) - 2*(8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*\sqrt{c*x^4 + b*x^2})/c^4, 1/48*(3*(5*B*b^3 - 6*A*b^2*c)*\sqrt{c}*(-c)*\arctan(\sqrt{c*x^4 + b*x^2}*\sqrt{-c}/(c*x^2 + b)) + (8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*\sqrt{c*x^4 + b*x^2})/c^4]$

**giac** [A] time = 0.25, size = 119, normalized size = 0.86

$$\frac{1}{48}\sqrt{cx^4+bx^2}\left(2\left(\frac{4Bx^2}{c}-\frac{5Bbc-6Ac^2}{c^3}\right)x^2+\frac{3(5Bb^2-6Abc)}{c^3}\right)+\frac{(5Bb^3-6Ab^2c)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{32c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $1/48*\sqrt{c*x^4 + b*x^2}*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 + 3*(5*B*b^2 - 6*A*b*c)/c^3) + 1/32*(5*B*b^3 - 6*A*b^2*c)*\log(\text{abs}(-2*(\sqrt{c})*x^2 - \sqrt{c*x^4 + b*x^2})*\sqrt{c} - b))/c^{(7/2)}$

**maple** [A] time = 0.06, size = 169, normalized size = 1.22

$$\frac{\sqrt{cx^2+b}\left(8\sqrt{cx^2+b}Bc^{\frac{7}{2}}x^5+12\sqrt{cx^2+b}Ac^{\frac{7}{2}}x^3-10\sqrt{cx^2+b}Bbc^{\frac{5}{2}}x^3+18Ab^2c^2\ln\left(\sqrt{c}x+\sqrt{cx^2+b}\right)-15Bb^3c\ln\left(\sqrt{c}x+\sqrt{cx^2+b}\right)-18\sqrt{cx^2+b}Abc^{\frac{5}{2}}x+15\sqrt{cx^2+b}Bb^2c^{\frac{3}{2}}x\right)}{48\sqrt{cx^4+bx^2}c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $1/48*x*(c*x^2+b)^{(1/2)}*(8*B*(c*x^2+b)^{(1/2)}*c^{(7/2)}*x^5+12*A*(c*x^2+b)^{(1/2)}*c^{(7/2)}*x^3-10*B*(c*x^2+b)^{(1/2)}*c^{(5/2)}*x^3*b-18*A*(c*x^2+b)^{(1/2)}*c^{(5/2)}*x*b+15*B*(c*x^2+b)^{(1/2)}*c^{(3/2)}*x*b^2+18*A*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*b^2*c^2-15*B*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*b^3*c)/(c*x^4+b*x^2)^{(1/2)}/c^{(9/2)}$

**maxima** [A] time = 1.45, size = 183, normalized size = 1.32

$$\frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}x^2}{c}+\frac{3b^2\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^4+bx^2}b}{c^2}\right)_A+\frac{1}{96}\left(\frac{16\sqrt{cx^4+bx^2}x^4}{c}-\frac{20\sqrt{cx^4+bx^2}bx^2}{c^2}-\frac{15b^3\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}\right)_B+\frac{30\sqrt{cx^4+bx^2}b^2}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $1/16*(4*\sqrt{c*x^4 + b*x^2})*x^2/c + 3*b^2*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2}*\sqrt{c})/c^{(5/2)} - 6*\sqrt{c*x^4 + b*x^2}*b/c^2)*A + 1/96*(16*\sqrt{c*x^4 + b*x^2})*x^4/c - 20*\sqrt{c*x^4 + b*x^2}*b*x^2/c^2 - 15*b^3*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2}*\sqrt{c})/c^{(7/2)} + 30*\sqrt{c*x^4 + b*x^2}*b^2/c^3)*B$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5 (Bx^2 + A)}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2),x)

[Out] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^2)}{\sqrt{x^2 (b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*5\*(A + B\*x\*\*2)/sqrt(x\*\*2\*(b + c\*x\*\*2)), x)

$$3.132 \quad \int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=83

$$\frac{b(3bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}} - \frac{\sqrt{bx^2 + cx^4} (-4Ac + 3bB - 2Bcx^2)}{8c^2}$$

**Rubi [A]** time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 779, 620, 206}

$$\frac{b(3bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}} - \frac{\sqrt{bx^2 + cx^4} (-4Ac + 3bB - 2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] -((3\*b\*B - 4\*A\*c - 2\*B\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(8\*c^2) + (b\*(3\*b\*B - 4\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(8\*c^(5/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^p)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x(A + Bx)}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{(3bB - 4Ac - 2Bcx^2) \sqrt{bx^2 + cx^4}}{8c^2} + \frac{(b(3bB - 4Ac)) \text{Subst} \left( \int \frac{1}{\sqrt{bx+cx^2}} dx, x, x^2 \right)}{16c^2} \\
&= -\frac{(3bB - 4Ac - 2Bcx^2) \sqrt{bx^2 + cx^4}}{8c^2} + \frac{(b(3bB - 4Ac)) \text{Subst} \left( \int \frac{1}{1-cx^2} dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}} \right)}{8c^2} \\
&= -\frac{(3bB - 4Ac - 2Bcx^2) \sqrt{bx^2 + cx^4}}{8c^2} + \frac{b(3bB - 4Ac) \tanh^{-1} \left( \frac{\sqrt{c} x^2}{\sqrt{bx^2+cx^4}} \right)}{8c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 97, normalized size = 1.17

$$\frac{x \left( \sqrt{c} x (b + cx^2) (4Ac - 3bB + 2Bcx^2) + b\sqrt{b + cx^2} (3bB - 4Ac) \tanh^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b+cx^2}} \right) \right)}{8c^{5/2} \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (x\*(Sqrt[c]\*x\*(b + c\*x^2)\*(-3\*b\*B + 4\*A\*c + 2\*B\*c\*x^2) + b\*(3\*b\*B - 4\*A\*c)\*Sqrt[b + c\*x^2]\*ArcTanh[(Sqrt[c]\*x)/Sqrt[b + c\*x^2]])/(8\*c^(5/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.47, size = 91, normalized size = 1.10

$$\frac{(4Abc - 3b^2B) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{16c^{5/2}} + \frac{\sqrt{bx^2 + cx^4} (4Ac - 3bB + 2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] ((-3\*b\*B + 4\*A\*c + 2\*B\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(8\*c^2) + ((-3\*b^2\*B + 4\*A\*b\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(16\*c^(5/2))

**fricas [A]** time = 0.43, size = 177, normalized size = 2.13

$$\left[ \frac{(3Bb^2 - 4Abc)\sqrt{c} \log \left( -2cx^2 - b + 2\sqrt{cx^4 + bx^2} \sqrt{c} \right) - 2(2Bc^2x^2 - 3Bbc + 4Ac^2)\sqrt{cx^4 + bx^2}}{16c^3}, \frac{(3Bb^2 - 4Abc)\sqrt{-c} \arctan \left( \frac{\sqrt{cx^4 + bx^2} \sqrt{-c}}{cx^2 + b} \right) - (2Bc^2x^2 - 3Bbc + 4Ac^2)\sqrt{cx^4 + bx^2}}{8c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/16\*((3\*B\*b^2 - 4\*A\*b\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b + 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(c) - 2\*(2\*B\*c^2\*x^2 - 3\*B\*b\*c + 4\*A\*c^2)\*sqrt(c\*x^4 + b\*x^2))/c^3, -1/8\*((3\*B\*b^2 - 4\*A\*b\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) - (2\*B\*c^2\*x^2 - 3\*B\*b\*c + 4\*A\*c^2)\*sqrt(c\*x^4 + b\*x^2))/c^3]

**giac [A]** time = 0.22, size = 91, normalized size = 1.10

$$\frac{1}{8} \sqrt{cx^4 + bx^2} \left( \frac{2Bx^2}{c} - \frac{3Bb - 4Ac}{c^2} \right) - \frac{(3Bb^2 - 4Abc) \log \left( \left| -2 \left( \sqrt{c} x^2 - \sqrt{cx^4 + bx^2} \right) \sqrt{c} - b \right| \right)}{16c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] 1/8\*sqrt(c\*x^4 + b\*x^2)\*(2\*B\*x^2/c - (3\*B\*b - 4\*A\*c)/c^2) - 1/16\*(3\*B\*b^2 - 4\*A\*b\*c)\*log(abs(-2\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))\*sqrt(c) - b))/c^(5/2)

**maple** [A] time = 0.05, size = 127, normalized size = 1.53

$$\frac{\sqrt{cx^2+b} \left( 2\sqrt{cx^2+b} Bc^{\frac{5}{2}}x^3 - 4Abc^2 \ln\left(\sqrt{c}x + \sqrt{cx^2+b}\right) + 3Bb^2c \ln\left(\sqrt{c}x + \sqrt{cx^2+b}\right) + 4\sqrt{cx^2+b} Ac^{\frac{5}{2}}x - 3\sqrt{cx^2+b} Bbc^{\frac{3}{2}}x \right)}{8\sqrt{cx^4+bx^2}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x)

[Out] 1/8\*x\*(c\*x^2+b)^(1/2)\*(2\*B\*c^(5/2)\*(c\*x^2+b)^(1/2)\*x^3+4\*A\*c^(5/2)\*(c\*x^2+b)^(1/2)\*x-3\*B\*c^(3/2)\*(c\*x^2+b)^(1/2)\*x\*b-4\*A\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b\*c^2+3\*B\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^2\*c)/(c\*x^4+b\*x^2)^(1/2)/c^(7/2)

**maxima** [A] time = 1.46, size = 134, normalized size = 1.61

$$\frac{1}{16} \left( \frac{4\sqrt{cx^4+bx^2}x^2}{c} + \frac{3b^2 \log(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c})}{c^{\frac{5}{2}}} - \frac{6\sqrt{cx^4+bx^2}b}{c^2} \right) B - \frac{1}{4} A \left( \frac{b \log(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c})}{c^{\frac{3}{2}}} - \frac{2\sqrt{cx^4+bx^2}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/16\*(4\*sqrt(c\*x^4 + b\*x^2)\*x^2/c + 3\*b^2\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(5/2) - 6\*sqrt(c\*x^4 + b\*x^2)\*b/c^2)\*B - 1/4\*A\*(b\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(3/2) - 2\*sqrt(c\*x^4 + b\*x^2)/c)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 (Bx^2 + A)}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2),x)

[Out] int((x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2),x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{\sqrt{x^2 (b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*3\*(A + B\*x\*\*2)/sqrt(x\*\*2\*(b + c\*x\*\*2)),x)

$$3.133 \quad \int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=66

$$\frac{B\sqrt{bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

**Rubi [A]** time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2034, 640, 620, 206}

$$\frac{B\sqrt{bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (B\*Sqrt[b\*x^2 + c\*x^4])/(2\*c) - ((b\*B - 2\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(2\*c^(3/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps



$$\begin{aligned}
\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{\sqrt{bx+cx^2}} dx, x, x^2 \right) \\
&= \frac{B\sqrt{bx^2+cx^4}}{2c} + \frac{(-bB+2Ac) \text{Subst} \left( \int \frac{1}{\sqrt{bx+cx^2}} dx, x, x^2 \right)}{4c} \\
&= \frac{B\sqrt{bx^2+cx^4}}{2c} + \frac{(-bB+2Ac) \text{Subst} \left( \int \frac{1}{1-cx^2} dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}} \right)}{2c} \\
&= \frac{B\sqrt{bx^2+cx^4}}{2c} - \frac{(bB-2Ac) \tanh^{-1} \left( \frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}} \right)}{2c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 81, normalized size = 1.23

$$\frac{x \left( B\sqrt{c}x(b+cx^2) - \sqrt{b+cx^2}(bB-2Ac) \tanh^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b+cx^2}} \right) \right)}{2c^{3/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (x\*(B\*Sqrt[c]\*x\*(b + c\*x^2) - (b\*B - 2\*A\*c)\*Sqrt[b + c\*x^2]\*ArcTanh[(Sqrt[c]\*x)/Sqrt[b + c\*x^2]])/(2\*c^(3/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.38, size = 76, normalized size = 1.15

$$\frac{(bB-2Ac) \log \left( -2c^{3/2}\sqrt{bx^2+cx^4} + bc + 2c^2x^2 \right)}{4c^{3/2}} + \frac{B\sqrt{bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (B\*Sqrt[b\*x^2 + c\*x^4])/(2\*c) + ((b\*B - 2\*A\*c)\*Log[b\*c + 2\*c^2\*x^2 - 2\*c^(3/2)\*Sqrt[b\*x^2 + c\*x^4]])/(4\*c^(3/2))

**fricas [A]** time = 0.43, size = 131, normalized size = 1.98

$$\left[ \frac{2\sqrt{cx^4+bx^2}Bc - (Bb-2Ac)\sqrt{c} \log \left( -2cx^2 - b - 2\sqrt{cx^4+bx^2}\sqrt{c} \right)}{4c^2}, \frac{\sqrt{cx^4+bx^2}Bc + (Bb-2Ac)\sqrt{-c} \arctan \left( \frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b} \right)}{2c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(c\*x^4 + b\*x^2)\*B\*c - (B\*b - 2\*A\*c)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)))/c^2, 1/2\*(sqrt(c\*x^4 + b\*x^2)\*B\*c + (B\*b - 2\*A\*c)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)))/c^2]

**giac [A]** time = 0.21, size = 67, normalized size = 1.02

$$\frac{\sqrt{cx^4+bx^2}B}{2c} + \frac{(Bb-2Ac) \log \left( \left| -2 \left( \sqrt{c}x^2 - \sqrt{cx^4+bx^2} \right) \sqrt{c} - b \right| \right)}{4c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{2}\sqrt{c*x^4 + b*x^2}*B/c + \frac{1}{4}*(B*b - 2*A*c)*\log(\text{abs}(-2*(\sqrt{c})*x^2 - \sqrt{c*x^4 + b*x^2}))*\sqrt{c} - b)/c^{(3/2)}$

**maple** [A] time = 0.06, size = 88, normalized size = 1.33

$$\frac{\sqrt{cx^2 + b} \left( 2A c^2 \ln \left( \sqrt{c} x + \sqrt{cx^2 + b} \right) - Bbc \ln \left( \sqrt{c} x + \sqrt{cx^2 + b} \right) + \sqrt{cx^2 + b} B c^{\frac{3}{2}} x \right)}{2\sqrt{cx^4 + bx^2} c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $\frac{1}{2}*x*(c*x^2+b)^{(1/2)}*(B*c^{(3/2)}*(c*x^2+b)^{(1/2)}*x+2*A*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)}))*c^2-B*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*b*c)/(c*x^4+b*x^2)^{(1/2)}/c^{(5/2)}$

**maxima** [A] time = 1.44, size = 88, normalized size = 1.33

$$-\frac{1}{4}B \left( \frac{b \log \left( 2cx^2 + b + 2\sqrt{cx^4 + bx^2} \sqrt{c} \right)}{c^{\frac{3}{2}}} - \frac{2\sqrt{cx^4 + bx^2}}{c} \right) + \frac{A \log \left( 2cx^2 + b + 2\sqrt{cx^4 + bx^2} \sqrt{c} \right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $-\frac{1}{4}*B*(b*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2})*\sqrt{c})/c^{(3/2)} - 2*\sqrt{c*x^4 + b*x^2}/c + \frac{1}{2}*A*\log(2*c*x^2 + b + 2*\sqrt{c*x^4 + b*x^2})*\sqrt{c})/\sqrt{c}$

**mapad** [B] time = 0.81, size = 89, normalized size = 1.35

$$\frac{B \sqrt{cx^4 + bx^2}}{2c} + \frac{A \ln \left( \frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2} \right)}{2\sqrt{c}} - \frac{B b \ln \left( \frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2} \right)}{4c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2),x)

[Out]  $\frac{(B*(b*x^2 + c*x^4)^{(1/2)})/(2*c) + (A*\log((b/2 + c*x^2)/c^{(1/2)} + (b*x^2 + c*x^4)^{(1/2)}))/(2*c^{(1/2)}) - (B*b*\log((b/2 + c*x^2)/c^{(1/2)} + (b*x^2 + c*x^4)^{(1/2)}))/(4*c^{(3/2)})}$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{\sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*(A + B\*x\*\*2)/sqrt(x\*\*2\*(b + c\*x\*\*2)), x)

$$3.134 \quad \int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=57

$$\frac{B \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}} - \frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

**Rubi [A]** time = 0.15, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 620, 206}

$$\frac{B \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}} - \frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] -((A\*Sqrt[b\*x^2 + c\*x^4])/(b\*x^2)) + (B\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/Sqrt[c]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^p, x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^p\*((c\_) + (d\_.)\*(x\_)^(n\_.))^q, x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x\sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x\sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{bx^2} + \frac{1}{2}B \text{Subst} \left( \int \frac{1}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{bx^2} + B \text{Subst} \left( \int \frac{1}{1 - cx^2} dx, x, \frac{x^2}{\sqrt{bx^2 + cx^4}} \right) \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{bx^2} + \frac{B \tanh^{-1} \left( \frac{\sqrt{c}x^2}{\sqrt{bx^2 + cx^4}} \right)}{\sqrt{c}}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 74, normalized size = 1.30

$$\frac{bBx\sqrt{b + cx^2} \tanh^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b+cx^2}} \right) - A\sqrt{c} (b + cx^2)}{b\sqrt{c} \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out]  $(-(A\sqrt{c}(b + cx^2)) + bBx\sqrt{b + cx^2} \text{ArcTanh}[(\sqrt{c}x)/\sqrt{b + cx^2}]) / (b\sqrt{c}\sqrt{x^2(b + cx^2)})$

**IntegrateAlgebraic** [A] time = 0.32, size = 66, normalized size = 1.16

$$-\frac{A\sqrt{bx^2 + cx^4}}{bx^2} - \frac{B \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out]  $-((A\sqrt{bx^2 + cx^4})/(bx^2)) - (B\text{Log}[b + 2cx^2 - 2\sqrt{c}\sqrt{bx^2 + cx^4}]) / (2\sqrt{c})$

**fricas** [A] time = 0.43, size = 136, normalized size = 2.39

$$\left[ \frac{Bb\sqrt{c}x^2 \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c} \right) - 2\sqrt{cx^4 + bx^2}Ac}{2bcx^2}, -\frac{Bb\sqrt{-c}x^2 \arctan \left( \frac{\sqrt{cx^4 + bx^2}\sqrt{-c}}{cx^2 + b} \right) + \sqrt{cx^4 + bx^2}Ac}{bcx^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out]  $[1/2*(B*b*\sqrt{c})x^2*\log(-2cx^2 - b - 2*\sqrt{cx^4 + bx^2}*\sqrt{c}) - 2*\sqrt{cx^4 + bx^2}*A*c)/(b*c*x^2), -(B*b*\sqrt{-c})x^2*\arctan(\sqrt{cx^4 + bx^2}*\sqrt{-c}/(cx^2 + b)) + \sqrt{cx^4 + bx^2}*A*c)/(b*c*x^2)]$

**giac** [A] time = 0.22, size = 66, normalized size = 1.16

$$-\frac{B \log \left( \left| 2 \left( \sqrt{c}x^2 - \sqrt{cx^4 + bx^2} \right) \sqrt{c} + b \right| \right)}{2\sqrt{c}} + \frac{A}{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $-1/2*B*\log(\text{abs}(2*(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2))*\text{sqrt}(c) + b))/\text{sqrt}(c) + A/(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2))$

**maple [A]** time = 0.06, size = 67, normalized size = 1.18

$$\frac{\sqrt{cx^2 + b} \left( -Bbx \ln \left( \sqrt{c} x + \sqrt{cx^2 + b} \right) + \sqrt{cx^2 + b} A\sqrt{c} \right)}{\sqrt{cx^4 + bx^2} b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $-(c*x^2+b)^{(1/2)}*(-B*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*x*b+A*(c*x^2+b)^{(1/2)}*c^{(1/2)})/(c*x^4+b*x^2)^{(1/2)}/c^{(1/2)}/b$

**maxima [A]** time = 1.41, size = 56, normalized size = 0.98

$$\frac{B \log \left( 2cx^2 + b + 2\sqrt{cx^4 + bx^2} \sqrt{c} \right)}{2\sqrt{c}} - \frac{\sqrt{cx^4 + bx^2} A}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $1/2*B*\log(2*c*x^2 + b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c))/\text{sqrt}(c) - \text{sqrt}(c*x^4 + b*x^2)*A/(b*x^2)$

**mupad [B]** time = 0.51, size = 57, normalized size = 1.00

$$\frac{B \ln \left( \frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2} \right)}{2\sqrt{c}} - \frac{A \sqrt{cx^4 + bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^(1/2)),x)

[Out]  $(B*\log((b/2 + c*x^2)/c^{(1/2)} + (b*x^2 + c*x^4)^{(1/2)}))/(2*c^{(1/2)}) - (A*(b*x^2 + c*x^4)^{(1/2)})/(b*x^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x\sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*sqrt(x\*\*2\*(b + c\*x\*\*2))), x)

$$3.135 \quad \int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}} dx$$

Optimal. Leaf size=61

$$-\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{3b^2x^2} - \frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

**Rubi [A]** time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2034, 792, 650}

$$-\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{3b^2x^2} - \frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^3\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] -(A\*Sqrt[b\*x^2 + c\*x^4])/(3\*b\*x^4) - ((3\*b\*B - 2\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(3\*b^2\*x^2)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned} \int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{x^2\sqrt{bx+cx^2}} dx, x, x^2 \right) \\ &= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} + \frac{\left(-2(-bB+Ac) + \frac{1}{2}(-bB+2Ac)\right) \text{Subst} \left( \int \frac{1}{x\sqrt{bx+cx^2}} dx, x, x^2 \right)}{3b} \\ &= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 43, normalized size = 0.70

$$-\frac{\sqrt{x^2(b+cx^2)}(A(b-2cx^2)+3bBx^2)}{3b^2x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^3\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] -1/3\*(Sqrt[x^2\*(b + c\*x^2)]\*(3\*b\*B\*x^2 + A\*(b - 2\*c\*x^2)))/(b^2\*x^4)

**IntegrateAlgebraic [A]** time = 0.32, size = 44, normalized size = 0.72

$$\frac{\sqrt{bx^2 + cx^4}(-Ab + 2Acx^2 - 3bBx^2)}{3b^2x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^3\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] ((-(A\*b) - 3\*b\*B\*x^2 + 2\*A\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(3\*b^2\*x^4)

**fricas [A]** time = 0.41, size = 38, normalized size = 0.62

$$-\frac{\sqrt{cx^4 + bx^2}((3Bb - 2Ac)x^2 + Ab)}{3b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] -1/3\*sqrt(c\*x^4 + b\*x^2)\*((3\*B\*b - 2\*A\*c)\*x^2 + A\*b)/(b^2\*x^4)

**giac [A]** time = 0.20, size = 88, normalized size = 1.44

$$\frac{3\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)^2 B + 3\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)A\sqrt{c} + Ab}{3\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out] 1/3\*(3\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))^2\*B + 3\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))\*A\*sqrt(c) + A\*b)/(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))^3

**maple [A]** time = 0.05, size = 47, normalized size = 0.77

$$-\frac{(cx^2 + b)(-2Acx^2 + 3Bbx^2 + Ab)}{3\sqrt{cx^4 + bx^2}b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(1/2), x)

[Out] -1/3\*(c\*x^2+b)\*(-2\*A\*c\*x^2+3\*B\*b\*x^2+A\*b)/x^2/b^2/(c\*x^4+b\*x^2)^(1/2)

**maxima [A]** time = 1.47, size = 70, normalized size = 1.15

$$\frac{1}{3}A\left(\frac{2\sqrt{cx^4 + bx^2}c}{b^2x^2} - \frac{\sqrt{cx^4 + bx^2}}{bx^4}\right) - \frac{\sqrt{cx^4 + bx^2}B}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/3\*A\*(2\*sqrt(c\*x^4 + b\*x^2)\*c/(b^2\*x^2) - sqrt(c\*x^4 + b\*x^2)/(b\*x^4)) - s  
qrt(c\*x^4 + b\*x^2)\*B/(b\*x^2)

mupad [B] time = 0.20, size = 39, normalized size = 0.64

$$\frac{\sqrt{cx^4 + bx^2} (Ab - 2Acx^2 + 3Bbx^2)}{3b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)^(1/2)),x)

[Out] -((b\*x^2 + c\*x^4)^(1/2)\*(A\*b - 2\*A\*c\*x^2 + 3\*B\*b\*x^2))/(3\*b^2\*x^4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3 \sqrt{x^2 (b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*3/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*3\*sqrt(x\*\*2\*(b + c\*x\*\*2))), x)



$$3.136 \quad \int \frac{A+Bx^2}{x^5 \sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=96

$$\frac{2c\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^3x^2} - \frac{\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^2x^4} - \frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

**Rubi [A]** time = 0.21, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{2c\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^3x^2} - \frac{\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^2x^4} - \frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^5\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] -(A\*Sqrt[b\*x^2 + c\*x^4])/(5\*b\*x^6) - ((5\*b\*B - 4\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(15\*b^2\*x^4) + (2\*c\*(5\*b\*B - 4\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(15\*b^3\*x^2)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^5 \sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^3 \sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{5bx^6} + \frac{\left(-3(-bB + Ac) + \frac{1}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{1}{x^2 \sqrt{bx + cx^2}} dx, x, x^2 \right)}{5b} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{5bx^6} - \frac{(5bB - 4Ac)\sqrt{bx^2 + cx^4}}{15b^2x^4} - \frac{(c(5bB - 4Ac)) \text{Subst} \left( \int \frac{1}{x \sqrt{bx + cx^2}} dx, x, x^2 \right)}{15b^2} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{5bx^6} - \frac{(5bB - 4Ac)\sqrt{bx^2 + cx^4}}{15b^2x^4} + \frac{2c(5bB - 4Ac)\sqrt{bx^2 + cx^4}}{15b^3x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 64, normalized size = 0.67

$$\frac{\sqrt{x^2(b + cx^2)} \left( A(-3b^2 + 4bcx^2 - 8c^2x^4) - 5bBx^2(b - 2cx^2) \right)}{15b^3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^5\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-5\*b\*B\*x^2\*(b - 2\*c\*x^2) + A\*(-3\*b^2 + 4\*b\*c\*x^2 - 8\*c^2\*x^4)))/(15\*b^3\*x^6)

**IntegrateAlgebraic [A]** time = 0.34, size = 66, normalized size = 0.69

$$\frac{\sqrt{bx^2 + cx^4} \left( -3Ab^2 + 4Abcx^2 - 8Ac^2x^4 - 5b^2Bx^2 + 10bBcx^4 \right)}{15b^3x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^5\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-3\*A\*b^2 - 5\*b^2\*B\*x^2 + 4\*A\*b\*c\*x^2 + 10\*b\*B\*c\*x^4 - 8\*A\*c^2\*x^4))/(15\*b^3\*x^6)

**fricas [A]** time = 0.41, size = 62, normalized size = 0.65

$$\frac{\left( 2(5Bbc - 4Ac^2)x^4 - 3Ab^2 - (5Bb^2 - 4Abc)x^2 \right) \sqrt{cx^4 + bx^2}}{15b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/15\*(2\*(5\*B\*b\*c - 4\*A\*c^2)\*x^4 - 3\*A\*b^2 - (5\*B\*b^2 - 4\*A\*b\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^3\*x^6)

**giac [A]** time = 0.21, size = 153, normalized size = 1.59

$$\frac{15 \left( \sqrt{cx^2 - \sqrt{cx^4 + bx^2}} \right)^3 B \sqrt{c} + 5 \left( \sqrt{cx^2 - \sqrt{cx^4 + bx^2}} \right)^2 B b + 20 \left( \sqrt{cx^2 - \sqrt{cx^4 + bx^2}} \right)^2 A c + 15 \left( \sqrt{cx^2 - \sqrt{cx^4 + bx^2}} \right) A b \sqrt{c} + 3 A b^2}{15 \left( \sqrt{cx^2 - \sqrt{cx^4 + bx^2}} \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] 1/15\*(15\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))^3\*B\*sqrt(c) + 5\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))^2\*B\*b + 20\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))^2\*A\*c

+ 15\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))\*A\*b\*sqrt(c) + 3\*A\*b^2)/(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))^5

**maple [A]** time = 0.05, size = 70, normalized size = 0.73

$$\frac{(cx^2 + b)(8Ac^2x^4 - 10Bbcx^4 - 4Abcx^2 + 5Bb^2x^2 + 3b^2A)}{15\sqrt{cx^4 + bx^2}b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(1/2), x)

[Out] -1/15\*(c\*x^2+b)\*(8\*A\*c^2\*x^4-10\*B\*b\*c\*x^4-4\*A\*b\*c\*x^2+5\*B\*b^2\*x^2+3\*A\*b^2)/x^4/b^3/(c\*x^4+b\*x^2)^(1/2)

**maxima [A]** time = 1.49, size = 119, normalized size = 1.24

$$\frac{1}{3}B\left(\frac{2\sqrt{cx^4 + bx^2}c}{b^2x^2} - \frac{\sqrt{cx^4 + bx^2}}{bx^4}\right) - \frac{1}{15}A\left(\frac{8\sqrt{cx^4 + bx^2}c^2}{b^3x^2} - \frac{4\sqrt{cx^4 + bx^2}c}{b^2x^4} + \frac{3\sqrt{cx^4 + bx^2}}{bx^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(1/2), x, algorithm="maxima")

[Out] 1/3\*B\*(2\*sqrt(c\*x^4 + b\*x^2)\*c/(b^2\*x^2) - sqrt(c\*x^4 + b\*x^2)/(b\*x^4)) - 1/15\*A\*(8\*sqrt(c\*x^4 + b\*x^2)\*c^2/(b^3\*x^2) - 4\*sqrt(c\*x^4 + b\*x^2)\*c/(b^2\*x^4) + 3\*sqrt(c\*x^4 + b\*x^2)/(b\*x^6))

**mupad [B]** time = 0.25, size = 62, normalized size = 0.65

$$\frac{\sqrt{cx^4 + bx^2} (5Bb^2x^2 + 3Ab^2 - 10Bbcx^4 - 4Abcx^2 + 8Ac^2x^4)}{15b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)^(1/2)), x)

[Out] -((b\*x^2 + c\*x^4)^(1/2)\*(3\*A\*b^2 + 5\*B\*b^2\*x^2 + 8\*A\*c^2\*x^4 - 4\*A\*b\*c\*x^2 - 10\*B\*b\*c\*x^4))/(15\*b^3\*x^6)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5 \sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*5/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2), x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*5\*sqrt(x\*\*2\*(b + c\*x\*\*2))), x)

$$3.137 \quad \int \frac{A+Bx^2}{x^7 \sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=133

$$-\frac{8c^2 \sqrt{bx^2+cx^4} (7bB-6Ac)}{105b^4x^2} + \frac{4c \sqrt{bx^2+cx^4} (7bB-6Ac)}{105b^3x^4} - \frac{\sqrt{bx^2+cx^4} (7bB-6Ac)}{35b^2x^6} - \frac{A \sqrt{bx^2+cx^4}}{7bx^8}$$

**Rubi [A]** time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2 \sqrt{bx^2+cx^4} (7bB-6Ac)}{105b^4x^2} + \frac{4c \sqrt{bx^2+cx^4} (7bB-6Ac)}{105b^3x^4} - \frac{\sqrt{bx^2+cx^4} (7bB-6Ac)}{35b^2x^6} - \frac{A \sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^7\*sqrt[b\*x^2 + c\*x^4]),x]

[Out] -(A\*sqrt[b\*x^2 + c\*x^4])/(7\*b\*x^8) - ((7\*b\*B - 6\*A\*c)\*sqrt[b\*x^2 + c\*x^4])/(35\*b^2\*x^6) + (4\*c\*(7\*b\*B - 6\*A\*c)\*sqrt[b\*x^2 + c\*x^4])/(105\*b^3\*x^4) - (8\*c^2\*(7\*b\*B - 6\*A\*c)\*sqrt[b\*x^2 + c\*x^4])/(105\*b^4\*x^2)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^7 \sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^4 \sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{7bx^8} + \frac{\left(-4(-bB + Ac) + \frac{1}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{1}{x^3 \sqrt{bx + cx^2}} dx, x, x^2 \right)}{7b} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{7bx^8} - \frac{(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{35b^2x^6} - \frac{(2c(7bB - 6Ac)) \text{Subst} \left( \int \frac{1}{x^2 \sqrt{bx + cx^2}} dx, x, x^2 \right)}{35b^2} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{7bx^8} - \frac{(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{35b^2x^6} + \frac{4c(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{105b^3x^4} + \frac{(4c^2(7bB - 6Ac)) \text{Subst} \left( \int \frac{1}{x \sqrt{bx + cx^2}} dx, x, x^2 \right)}{105b^3x^4} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{7bx^8} - \frac{(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{35b^2x^6} + \frac{4c(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{105b^3x^4} - \frac{8c^2(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{105b^3x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 89, normalized size = 0.67

$$-\frac{\sqrt{x^2(b + cx^2)} \left( 3A(5b^3 - 6b^2cx^2 + 8bc^2x^4 - 16c^3x^6) + 7bBx^2(3b^2 - 4bcx^2 + 8c^2x^4) \right)}{105b^4x^8}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^7\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] -1/105\*(Sqrt[x^2\*(b + c\*x^2)]\*(7\*b\*B\*x^2\*(3\*b^2 - 4\*b\*c\*x^2 + 8\*c^2\*x^4) + 3\*A\*(5\*b^3 - 6\*b^2\*c\*x^2 + 8\*b\*c^2\*x^4 - 16\*c^3\*x^6)))/(b^4\*x^8)

**IntegrateAlgebraic [A]** time = 0.37, size = 90, normalized size = 0.68

$$\frac{\sqrt{bx^2 + cx^4} \left( -15Ab^3 + 18Ab^2cx^2 - 24Abc^2x^4 + 48Ac^3x^6 - 21b^3Bx^2 + 28b^2Bcx^4 - 56bBc^2x^6 \right)}{105b^4x^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^7\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-15\*A\*b^3 - 21\*b^3\*B\*x^2 + 18\*A\*b^2\*c\*x^2 + 28\*b^2\*B\*c\*x^4 - 24\*A\*b\*c^2\*x^4 - 56\*b\*B\*c^2\*x^6 + 48\*A\*c^3\*x^6))/(105\*b^4\*x^8)

**fricas [A]** time = 0.42, size = 86, normalized size = 0.65

$$\frac{\left( 8(7Bbc^2 - 6Ac^3)x^6 - 4(7Bb^2c - 6Abc^2)x^4 + 15Ab^3 + 3(7Bb^3 - 6Ab^2c)x^2 \right) \sqrt{cx^4 + bx^2}}{105b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^7/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] -1/105\*(8\*(7\*B\*b\*c^2 - 6\*A\*c^3)\*x^6 - 4\*(7\*B\*b^2\*c - 6\*A\*b\*c^2)\*x^4 + 15\*A\*b^3 + 3\*(7\*B\*b^3 - 6\*A\*b^2\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^4\*x^8)

**giac [A]** time = 0.31, size = 219, normalized size = 1.65

$$\frac{140(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^4 Bc + 105(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^3 Bb\sqrt{c} + 210(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^3 Ac^{\frac{3}{2}} + 21(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^2 Bb^2 + 252(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^2 Abc + 105(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^2 Ab^2\sqrt{c} + 15Ab^3}{105(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^7/(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out]  $1/105*(140*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2})^4*B*c + 105*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2})^3*B*b*\sqrt{c} + 210*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2})^2*B*b^2 + 252*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2})^2*A*b*c + 105*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2})^2)*A*b^2*\sqrt{c} + 15*A*b^3)/(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2})^7$

**maple [A]** time = 0.05, size = 94, normalized size = 0.71

$$\frac{(cx^2 + b)(-48Ac^3x^6 + 56Bbc^2x^6 + 24Abc^2x^4 - 28Bb^2cx^4 - 18Ab^2cx^2 + 21Bb^3x^2 + 15Ab^3)}{105\sqrt{cx^4 + bx^2}b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x)`

[Out]  $-1/105*(c*x^2+b)*(-48*A*c^3*x^6+56*B*b*c^2*x^6+24*A*b*c^2*x^4-28*B*b^2*c*x^4-18*A*b^2*c*x^2+21*B*b^3*x^2+15*A*b^3)/x^6/b^4/(c*x^4+b*x^2)^(1/2)$

**maxima [A]** time = 1.49, size = 167, normalized size = 1.26

$$-\frac{1}{15}B\left(\frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^2} - \frac{4\sqrt{cx^4+bx^2}c}{b^2x^4} + \frac{3\sqrt{cx^4+bx^2}}{bx^6}\right) + \frac{1}{35}A\left(\frac{16\sqrt{cx^4+bx^2}c^3}{b^4x^2} - \frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^4} + \frac{6\sqrt{cx^4+bx^2}c}{b^2x^6} - \frac{5\sqrt{cx^4+bx^2}}{bx^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")`

[Out]  $-1/15*B*(8*\sqrt{c*x^4 + b*x^2}*c^2/(b^3*x^2) - 4*\sqrt{c*x^4 + b*x^2}*c/(b^2*x^4) + 3*\sqrt{c*x^4 + b*x^2}/(b*x^6)) + 1/35*A*(16*\sqrt{c*x^4 + b*x^2}*c^3/(b^4*x^2) - 8*\sqrt{c*x^4 + b*x^2}*c^2/(b^3*x^4) + 6*\sqrt{c*x^4 + b*x^2}*c/(b^2*x^6) - 5*\sqrt{c*x^4 + b*x^2}/(b*x^8))$

**mupad [B]** time = 0.30, size = 121, normalized size = 0.91

$$\frac{(6Ac - 7Bb)\sqrt{cx^4 + bx^2}}{35b^2x^6} - \frac{A\sqrt{cx^4 + bx^2}}{7bx^8} - \frac{(24Ac^2 - 28Bbc)\sqrt{cx^4 + bx^2}}{105b^3x^4} + \frac{(48Ac^3 - 56Bbc^2)\sqrt{cx^4 + bx^2}}{105b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(x^7*(b*x^2 + c*x^4)^(1/2)),x)`

[Out]  $((6*A*c - 7*B*b)*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (A*(b*x^2 + c*x^4)^(1/2))/(7*b*x^8) - ((24*A*c^2 - 28*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x^4) + ((48*A*c^3 - 56*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(105*b^4*x^2)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^7 \sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)/x**7/(c*x**4+b*x**2)**(1/2),x)`

[Out] `Integral((A + B*x**2)/(x**7*sqrt(x**2*(b + c*x**2))), x)`

$$3.138 \quad \int \frac{A+Bx^2}{x^9 \sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=170

$$\frac{16c^3 \sqrt{bx^2 + cx^4} (9bB - 8Ac)}{315b^5 x^2} - \frac{8c^2 \sqrt{bx^2 + cx^4} (9bB - 8Ac)}{315b^4 x^4} + \frac{2c \sqrt{bx^2 + cx^4} (9bB - 8Ac)}{105b^3 x^6} - \frac{\sqrt{bx^2 + cx^4} (9bB - 8Ac)}{63b^2 x^8}$$

**Rubi [A]** time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 650}

$$\frac{16c^3 \sqrt{bx^2 + cx^4} (9bB - 8Ac)}{315b^5 x^2} - \frac{8c^2 \sqrt{bx^2 + cx^4} (9bB - 8Ac)}{315b^4 x^4} + \frac{2c \sqrt{bx^2 + cx^4} (9bB - 8Ac)}{105b^3 x^6} - \frac{\sqrt{bx^2 + cx^4} (9bB - 8Ac)}{63b^2 x^8} - \frac{A \sqrt{bx^2 + cx^4}}{9bx^{10}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^9\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] -(A\*Sqrt[b\*x^2 + c\*x^4])/((9\*b\*x^10) - ((9\*b\*B - 8\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(63\*b^2\*x^8) + (2\*c\*(9\*b\*B - 8\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(105\*b^3\*x^6) - (8\*c^2\*(9\*b\*B - 8\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(315\*b^4\*x^4) + (16\*c^3\*(9\*b\*B - 8\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(315\*b^5\*x^2)

#### Rule 650

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(2\*c\*d - b\*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^9 \sqrt{bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^5 \sqrt{bx + cx^2}} dx, x, x^2 \right) \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{9bx^{10}} + \frac{\left(-5(-bB + Ac) + \frac{1}{2}(-bB + 2Ac)\right) \text{Subst} \left( \int \frac{1}{x^4 \sqrt{bx + cx^2}} dx, x, x^2 \right)}{9b} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{9bx^{10}} - \frac{(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{63b^2x^8} - \frac{(c(9bB - 8Ac)) \text{Subst} \left( \int \frac{1}{x^3 \sqrt{bx + cx^2}} dx, x, x^2 \right)}{21b^2} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{9bx^{10}} - \frac{(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{63b^2x^8} + \frac{2c(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{105b^3x^6} + \frac{(4c^2(9bB - 8Ac)) \text{Subst} \left( \int \frac{1}{x^2 \sqrt{bx + cx^2}} dx, x, x^2 \right)}{315b^3} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{9bx^{10}} - \frac{(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{63b^2x^8} + \frac{2c(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{105b^3x^6} - \frac{8c^2(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{315b^3} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{9bx^{10}} - \frac{(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{63b^2x^8} + \frac{2c(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{105b^3x^6} - \frac{8c^2(9bB - 8Ac)\sqrt{bx^2 + cx^4}}{315b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 94, normalized size = 0.55

$$\frac{x^2 \left( \frac{cx^2}{b} + 1 \right) (5b^3 - 6b^2cx^2 + 8bc^2x^4 - 16c^3x^6) (8Ac - 9bB) - 35Ab^3 (b + cx^2)}{315b^4x^8 \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^9\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] (-35\*A\*b^3\*(b + c\*x^2) + (-9\*b\*B + 8\*A\*c)\*x^2\*(1 + (c\*x^2)/b)\*(5\*b^3 - 6\*b^2\*c\*x^2 + 8\*b\*c^2\*x^4 - 16\*c^3\*x^6))/(315\*b^4\*x^8\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.40, size = 114, normalized size = 0.67

$$\frac{\sqrt{bx^2 + cx^4} (-35Ab^4 + 40Ab^3cx^2 - 48Ab^2c^2x^4 + 64Abc^3x^6 - 128Ac^4x^8 - 45b^4Bx^2 + 54b^3Bcx^4 - 72b^2Bc^2x^6 + 144bBc^3x^8)}{315b^5x^{10}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^9\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-35\*A\*b^4 - 45\*b^4\*B\*x^2 + 40\*A\*b^3\*c\*x^2 + 54\*b^3\*B\*c\*x^4 - 48\*A\*b^2\*c^2\*x^4 - 72\*b^2\*B\*c^2\*x^6 + 64\*A\*b\*c^3\*x^6 + 144\*b\*B\*c^3\*x^8 - 128\*A\*c^4\*x^8))/(315\*b^5\*x^10)

**fricas [A]** time = 0.47, size = 110, normalized size = 0.65

$$\frac{(16(9Bbc^3 - 8Ac^4)x^8 - 8(9Bb^2c^2 - 8Abc^3)x^6 - 35Ab^4 + 6(9Bb^3c - 8Ab^2c^2)x^4 - 5(9Bb^4 - 8Ab^3c)x^2)\sqrt{cx^4 + bx^2}}{315b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^9/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/315\*(16\*(9\*B\*b\*c^3 - 8\*A\*c^4)\*x^8 - 8\*(9\*B\*b^2\*c^2 - 8\*A\*b\*c^3)\*x^6 - 35\*A\*b^4 + 6\*(9\*B\*b^3\*c - 8\*A\*b^2\*c^2)\*x^4 - 5\*(9\*B\*b^4 - 8\*A\*b^3\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^5\*x^10)

**giac [A]** time = 0.27, size = 287, normalized size = 1.69

$$\frac{630(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^5 Bc^3 + 756(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^4 Bbc + 1008(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^3 Ac^2 + 315(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^2 Bb^2\sqrt{c} + 1680(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})Abc^3 + 45(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^2 Bb^3 + 1080(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})Ab^2c + 315(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})Ab^3\sqrt{c} + 35Ab^4}{315(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}})^9}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^9/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{315}*(630*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^5*B*c^{(3/2)} + 756*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^4*A*c^2 + 315*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^3*B*b^2*\sqrt{c} + 1680*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^3*A*b*c^{(3/2)} + 45*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^2*B*b^3 + 1080*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^2*A*b^2*c + 315*(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})*A*b^3*\sqrt{c} + 35*A*b^4)/(\sqrt{c})x^2 - \sqrt{cx^4 + bx^2})^9$

**maple** [A] time = 0.05, size = 118, normalized size = 0.69

$$\frac{(cx^2 + b)(128Ac^4x^8 - 144Bbc^3x^8 - 64Abc^3x^6 + 72Bb^2c^2x^6 + 48Ab^2c^2x^4 - 54Bb^3cx^4 - 40Ab^3cx^2 + 45Bb^4x^2 + 35Ab^4)}{315\sqrt{cx^4 + bx^2}b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^9/(c\*x^4+b\*x^2)^(1/2),x)

[Out]  $-\frac{1}{315}*(cx^2+b)*(128*A*c^4*x^8-144*B*b*c^3*x^8-64*A*b*c^3*x^6+72*B*b^2*c^2*x^6+48*A*b^2*c^2*x^4-54*B*b^3*c*x^4-40*A*b^3*c*x^2+45*B*b^4*x^2+35*A*b^4)/x^8/b^5/(cx^4+bx^2)^{(1/2)}$

**maxima** [A] time = 1.54, size = 215, normalized size = 1.26

$$\frac{1}{35}B\left(\frac{16\sqrt{cx^4+bx^2}c^3}{b^4x^2} - \frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^4} + \frac{6\sqrt{cx^4+bx^2}c}{b^2x^6} - \frac{5\sqrt{cx^4+bx^2}}{bx^8}\right) - \frac{1}{315}A\left(\frac{128\sqrt{cx^4+bx^2}c^4}{b^5x^2} - \frac{64\sqrt{cx^4+bx^2}c^3}{b^4x^4} + \frac{48\sqrt{cx^4+bx^2}c^2}{b^3x^6} - \frac{40\sqrt{cx^4+bx^2}c}{b^2x^8} + \frac{35\sqrt{cx^4+bx^2}}{bx^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^9/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{35}B*(16*\sqrt{cx^4 + bx^2}*c^3/(b^4*x^2) - 8*\sqrt{cx^4 + bx^2}*c^2/(b^3*x^4) + 6*\sqrt{cx^4 + bx^2}*c/(b^2*x^6) - 5*\sqrt{cx^4 + bx^2}/(b*x^8)) - \frac{1}{315}A*(128*\sqrt{cx^4 + bx^2}*c^4/(b^5*x^2) - 64*\sqrt{cx^4 + bx^2}*c^3/(b^4*x^4) + 48*\sqrt{cx^4 + bx^2}*c^2/(b^3*x^6) - 40*\sqrt{cx^4 + bx^2}*c/(b^2*x^8) + 35*\sqrt{cx^4 + bx^2}/(b*x^{10}))$

**mupad** [B] time = 0.32, size = 156, normalized size = 0.92

$$\frac{(8Ac - 9Bb)\sqrt{cx^4 + bx^2}}{63b^2x^8} - \frac{A\sqrt{cx^4 + bx^2}}{9bx^{10}} - \frac{(16Ac^2 - 18Bbc)\sqrt{cx^4 + bx^2}}{105b^3x^6} + \frac{(64Ac^3 - 72Bbc^2)\sqrt{cx^4 + bx^2}}{315b^4x^4} - \frac{(128Ac^4 - 144Bbc^3)\sqrt{cx^4 + bx^2}}{315b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^9\*(b\*x^2 + c\*x^4)^(1/2)),x)

[Out]  $((8*A*c - 9*B*b)*(b*x^2 + c*x^4)^{(1/2)})/(63*b^2*x^8) - (A*(b*x^2 + c*x^4)^{(1/2)})/(9*b*x^{10}) - ((16*A*c^2 - 18*B*b*c)*(b*x^2 + c*x^4)^{(1/2)})/(105*b^3*x^6) + ((64*A*c^3 - 72*B*b*c^2)*(b*x^2 + c*x^4)^{(1/2)})/(315*b^4*x^4) - ((128*A*c^4 - 144*B*b*c^3)*(b*x^2 + c*x^4)^{(1/2)})/(315*b^5*x^2)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^9 \sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*9/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*9\*sqrt(x\*\*2\*(b + c\*x\*\*2))), x)

$$3.139 \quad \int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=131

$$-\frac{8b^2\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^4x} + \frac{4bx\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^3} - \frac{x^3\sqrt{bx^2+cx^4}(6bB-7Ac)}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

**Rubi [A]** time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2039, 2016, 1588}

$$-\frac{8b^2\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^4x} - \frac{x^3\sqrt{bx^2+cx^4}(6bB-7Ac)}{35c^2} + \frac{4bx\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^3} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

[In] Int[(x^6\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4],x]

[Out] (-8\*b^2\*(6\*b\*B - 7\*A\*c)\*Sqrt[b\*x^2 + c\*x^4]/(105\*c^4\*x) + (4\*b\*(6\*b\*B - 7\*A\*c)\*x\*Sqrt[b\*x^2 + c\*x^4]/(105\*c^3) - ((6\*b\*B - 7\*A\*c)\*x^3\*Sqrt[b\*x^2 + c\*x^4]/(35\*c^2) + (B\*x^5\*Sqrt[b\*x^2 + c\*x^4]/(7\*c)

#### Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

#### Rule 2016

Int[((c\_)\*(x\_))^(m\_)\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

#### Rule 2039

Int[((e\_)\*(x\_))^(m\_)\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(jn\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^m\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned}
\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx &= \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} - \frac{(6bB-7Ac) \int \frac{x^6}{\sqrt{bx^2+cx^4}} dx}{7c} \\
&= -\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} + \frac{(4b(6bB-7Ac)) \int \frac{x^4}{\sqrt{bx^2+cx^4}} dx}{35c^2} \\
&= \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} - \frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} - \frac{(8b^2(6bB-7Ac))}{105c^4x} \\
&= -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x} + \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} - \frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 85, normalized size = 0.65

$$\frac{\sqrt{x^2(b+cx^2)}(8b^2c(7A+3Bx^2)-2bc^2x^2(14A+9Bx^2)+3c^3x^4(7A+5Bx^2)-48b^3B)}{105c^4x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[x^2\*(b + c\*x^2)]\*(-48\*b^3\*B + 8\*b^2\*c\*(7\*A + 3\*B\*x^2) + 3\*c^3\*x^4\*(7\*A + 5\*B\*x^2) - 2\*b\*c^2\*x^2\*(14\*A + 9\*B\*x^2)))/(105\*c^4\*x)

**IntegrateAlgebraic [A]** time = 0.11, size = 87, normalized size = 0.66

$$\frac{\sqrt{bx^2+cx^4}(56Ab^2c-28Abc^2x^2+21Ac^3x^4-48b^3B+24b^2Bcx^2-18bBc^2x^4+15Bc^3x^6)}{105c^4x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-48\*b^3\*B + 56\*A\*b^2\*c + 24\*b^2\*B\*c\*x^2 - 28\*A\*b\*c^2\*x^2 - 18\*b\*B\*c^2\*x^4 + 21\*A\*c^3\*x^4 + 15\*B\*c^3\*x^6))/(105\*c^4\*x)

**fricas [A]** time = 0.42, size = 83, normalized size = 0.63

$$\frac{(15Bc^3x^6 - 3(6Bbc^2 - 7Ac^3)x^4 - 48Bb^3 + 56Ab^2c + 4(6Bb^2c - 7Abc^2)x^2)\sqrt{cx^4 + bx^2}}{105c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/105\*(15\*B\*c^3\*x^6 - 3\*(6\*B\*b\*c^2 - 7\*A\*c^3)\*x^4 - 48\*B\*b^3 + 56\*A\*b^2\*c + 4\*(6\*B\*b^2\*c - 7\*A\*b\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^4\*x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^6}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out] integrate((B\*x^2 + A)\*x^6/sqrt(c\*x^4 + b\*x^2), x)

**maple** [A] time = 0.05, size = 89, normalized size = 0.68

$$\frac{(cx^2 + b)(15Bc^3x^6 + 21Ac^3x^4 - 18Bbc^2x^4 - 28Abc^2x^2 + 24Bb^2cx^2 + 56Ab^2c - 48Bb^3)x}{105\sqrt{cx^4 + bx^2}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x)

[Out] 1/105\*(c\*x^2+b)\*(15\*B\*c^3\*x^6+21\*A\*c^3\*x^4-18\*B\*b\*c^2\*x^4-28\*A\*b\*c^2\*x^2+24\*B\*b^2\*c\*x^2+56\*A\*b^2\*c-48\*B\*b^3)\*x/c^4/(c\*x^4+b\*x^2)^(1/2)

**maxima** [A] time = 1.53, size = 106, normalized size = 0.81

$$\frac{(3c^3x^6 - bc^2x^4 + 4b^2cx^2 + 8b^3)A}{15\sqrt{cx^2 + bc^3}} + \frac{(5c^4x^8 - bc^3x^6 + 2b^2c^2x^4 - 8b^3cx^2 - 16b^4)B}{35\sqrt{cx^2 + bc^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/15\*(3\*c^3\*x^6 - b\*c^2\*x^4 + 4\*b^2\*c\*x^2 + 8\*b^3)\*A/(sqrt(c\*x^2 + b)\*c^3) + 1/35\*(5\*c^4\*x^8 - b\*c^3\*x^6 + 2\*b^2\*c^2\*x^4 - 8\*b^3\*c\*x^2 - 16\*b^4)\*B/(sqrt(c\*x^2 + b)\*c^4)

**mupad** [B] time = 0.26, size = 87, normalized size = 0.66

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{48Bb^3 - 56Ab^2c}{105c^4} - \frac{Bx^6}{7c} - \frac{x^4(21Ac^3 - 18Bbc^2)}{105c^4} + \frac{4bx^2(7Ac - 6Bb)}{105c^3} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2),x)

[Out] -((b\*x^2 + c\*x^4)^(1/2)\*((48\*B\*b^3 - 56\*A\*b^2\*c)/(105\*c^4) - (B\*x^6)/(7\*c) - (x^4\*(21\*A\*c^3 - 18\*B\*b\*c^2))/(105\*c^4) + (4\*b\*x^2\*(7\*A\*c - 6\*B\*b))/(105\*c^3)))/x

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(A + Bx^2)}{\sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*6\*(A + B\*x\*\*2)/sqrt(x\*\*2\*(b + c\*x\*\*2)), x)

$$3.140 \quad \int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=94

$$\frac{2b\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^3x} - \frac{x\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

**Rubi [A]** time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2039, 2016, 1588}

$$-\frac{x\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^2} + \frac{2b\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^3x} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (2\*b\*(4\*b\*B - 5\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(15\*c^3\*x) - ((4\*b\*B - 5\*A\*c)\*x\*Sqrt[b\*x^2 + c\*x^4])/(15\*c^2) + (B\*x^3\*Sqrt[b\*x^2 + c\*x^4])/(5\*c)

Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2016

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2039

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j + n)))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

Rubi steps

$$\begin{aligned} \int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx &= \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} - \frac{(4bB-5Ac) \int \frac{x^4}{\sqrt{bx^2+cx^4}} dx}{5c} \\ &= -\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} + \frac{(2b(4bB-5Ac)) \int \frac{x^2}{\sqrt{bx^2+cx^4}} dx}{15c^2} \\ &= \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x} - \frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 63, normalized size = 0.67

$$\frac{\sqrt{x^2(b+cx^2)}(-2bc(5A+2Bx^2)+c^2x^2(5A+3Bx^2)+8b^2B)}{15c^3x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(A+B\*x^2))/Sqrt[b\*x^2+c\*x^4],x]

[Out] (Sqrt[x^2\*(b+c\*x^2)]\*(8\*b^2\*B-2\*b\*c\*(5\*A+2\*B\*x^2)+c^2\*x^2\*(5\*A+3\*B\*x^2)))/(15\*c^3\*x)

**IntegrateAlgebraic** [A] time = 0.09, size = 63, normalized size = 0.67

$$\frac{\sqrt{bx^2+cx^4}(-10Abc+5Ac^2x^2+8b^2B-4bBcx^2+3Bc^2x^4)}{15c^3x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4\*(A+B\*x^2))/Sqrt[b\*x^2+c\*x^4],x]

[Out] (Sqrt[b\*x^2+c\*x^4]\*(8\*b^2\*B-10\*A\*b\*c-4\*b\*B\*c\*x^2+5\*A\*c^2\*x^2+3\*B\*c^2\*x^4))/(15\*c^3\*x)

**fricas** [A] time = 0.41, size = 59, normalized size = 0.63

$$\frac{(3Bc^2x^4+8Bb^2-10Abc-(4Bbc-5Ac^2)x^2)\sqrt{cx^4+bx^2}}{15c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/15\*(3\*B\*c^2\*x^4+8\*B\*b^2-10\*A\*b\*c-(4\*B\*b\*c-5\*A\*c^2)\*x^2)\*sqrt(c\*x^4+b\*x^2)/(c^3\*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2+A)x^4}{\sqrt{cx^4+bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] integrate((B\*x^2+A)\*x^4/sqrt(c\*x^4+b\*x^2),x)

**maple** [A] time = 0.05, size = 65, normalized size = 0.69

$$\frac{(cx^2+b)(-3Bc^2x^4-5Ac^2x^2+4Bbcx^2+10Abc-8Bb^2)x}{15\sqrt{cx^4+bx^2}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x)

[Out] -1/15\*(c\*x^2+b)\*(-3\*B\*c^2\*x^4-5\*A\*c^2\*x^2+4\*B\*b\*c\*x^2+10\*A\*b\*c-8\*B\*b^2)\*x/c^3/(c\*x^4+b\*x^2)^(1/2)

**maxima** [A] time = 1.54, size = 83, normalized size = 0.88

$$\frac{(c^2x^4-bcx^2-2b^2)A}{3\sqrt{cx^2+bc^2}} + \frac{(3c^3x^6-bc^2x^4+4b^2cx^2+8b^3)B}{15\sqrt{cx^2+bc^2}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/3\*(c^2\*x^4 - b\*c\*x^2 - 2\*b^2)\*A/(sqrt(c\*x^2 + b)\*c^2) + 1/15\*(3\*c^3\*x^6 - b\*c^2\*x^4 + 4\*b^2\*c\*x^2 + 8\*b^3)\*B/(sqrt(c\*x^2 + b)\*c^3)

mupad [B] time = 0.23, size = 64, normalized size = 0.68

$$\frac{\sqrt{cx^4 + bx^2} \left( \frac{8Bb^2 - 10Abc}{15c^3} + \frac{x^2(5Ac^2 - 4Bbc)}{15c^3} + \frac{Bx^4}{5c} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(1/2),x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*((8\*B\*b^2 - 10\*A\*b\*c)/(15\*c^3) + (x^2\*(5\*A\*c^2 - 4\*B\*b\*c))/(15\*c^3) + (B\*x^4)/(5\*c)))/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(A + Bx^2)}{\sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*4\*(A + B\*x\*\*2)/sqrt(x\*\*2\*(b + c\*x\*\*2)), x)

$$3.141 \quad \int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=59

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{3c^2x}$$

**Rubi [A]** time = 0.14, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2039, 1588}

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{3c^2x}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4],x]

[Out] -((2\*b\*B - 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(3\*c^2\*x) + (B\*x\*Sqrt[b\*x^2 + c\*x^4])/(3\*c)

**Rule 1588**

Int[(Pp\_)\*(Qq\_)^(m\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1)]/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

**Rule 2039**

Int[((e\_)\*(x\_))^(m\_.)\*((a\_)\*(x\_)^(j\_.) + (b\_)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^(m\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

**Rubi steps**

$$\begin{aligned} \int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx &= \frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{(2bB-3Ac) \int \frac{x^2}{\sqrt{bx^2+cx^4}} dx}{3c} \\ &= -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x} + \frac{Bx\sqrt{bx^2+cx^4}}{3c} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 40, normalized size = 0.68

$$\frac{\sqrt{x^2(b+cx^2)}(3Ac-2bB+Bcx^2)}{3c^2x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(A + B\*x^2))/Sqrt[b\*x^2 + c\*x^4],x]



[Out]  $(\text{Sqrt}[x^2*(b + c*x^2)]*(-2*b*B + 3*A*c + B*c*x^2))/(3*c^2*x)$

**IntegrateAlgebraic** [A] time = 0.07, size = 40, normalized size = 0.68

$$\frac{\sqrt{bx^2 + cx^4} (3Ac - 2bB + Bcx^2)}{3c^2x}$$

Antiderivative was successfully verified.

[In] `IntegrateAlgebraic[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]`

[Out]  $((-2*b*B + 3*A*c + B*c*x^2)*\text{Sqrt}[b*x^2 + c*x^4])/(3*c^2*x)$

**fricas** [A] time = 0.41, size = 36, normalized size = 0.61

$$\frac{\sqrt{cx^4 + bx^2} (Bcx^2 - 2Bb + 3Ac)}{3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")`

[Out]  $1/3*\text{sqrt}(c*x^4 + b*x^2)*(B*c*x^2 - 2*B*b + 3*A*c)/(c^2*x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^2}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")`

[Out] `integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2), x)`

**maple** [A] time = 0.05, size = 42, normalized size = 0.71

$$\frac{(cx^2 + b)(Bcx^2 + 3Ac - 2bB)x}{3\sqrt{cx^4 + bx^2} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)`

[Out]  $1/3*(c*x^2+b)*(B*c*x^2+3*A*c-2*B*b)*x/c^2/(c*x^4+b*x^2)^(1/2)$

**maxima** [A] time = 1.46, size = 50, normalized size = 0.85

$$\frac{\sqrt{cx^2 + b} A}{c} + \frac{(c^2x^4 - bcx^2 - 2b^2)B}{3\sqrt{cx^2 + b} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")`

[Out]  $\text{sqrt}(c*x^2 + b)*A/c + 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*B/(\text{sqrt}(c*x^2 + b)*c^2)$

**mupad** [B] time = 0.19, size = 41, normalized size = 0.69

$$\frac{\left(\frac{3Ac-2Bb}{3c^2} + \frac{Bx^2}{3c}\right) \sqrt{cx^4 + bx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)`

[Out] `((3*A*c - 2*B*b)/(3*c^2) + (B*x^2)/(3*c))*(b*x^2 + c*x^4)^(1/2)/x`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{\sqrt{x^2 (b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(1/2), x)`

[Out] `Integral(x**2*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)`

$$3.142 \quad \int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} dx$$

Optimal. Leaf size=55

$$\frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {1145, 2008, 206}

$$\frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/Sqrt[b\*x^2 + c\*x^4], x]

[Out] (B\*Sqrt[b\*x^2 + c\*x^4])/(c\*x) - (A\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/Sqrt[b]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1145

Int[((d\_) + (e\_.)\*(x\_)^2)\*((b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> Simp[(e\*(b\*x^2 + c\*x^4)^(p + 1))/(c\*(4\*p + 3)\*x), x] - Dist[(b\*e\*(2\*p + 1) - c\*d\*(4\*p + 3))/(c\*(4\*p + 3)), Int[(b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{b, c, d, e, p}, x] && !IntegerQ[p] && NeQ[4\*p + 3, 0] && NeQ[b\*e\*(2\*p + 1) - c\*d\*(4\*p + 3), 0]

Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rubi steps

$$\begin{aligned} \int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} dx &= \frac{B\sqrt{bx^2+cx^4}}{cx} + A \int \frac{1}{\sqrt{bx^2+cx^4}} dx \\ &= \frac{B\sqrt{bx^2+cx^4}}{cx} - A \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\ &= \frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 73, normalized size = 1.33

$$\frac{x \left( \sqrt{b} B (b + cx^2) - Ac \sqrt{b + cx^2} \tanh^{-1} \left( \frac{\sqrt{b+cx^2}}{\sqrt{b}} \right) \right)}{\sqrt{b} c \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/Sqrt[b\*x^2 + c\*x^4],x]

[Out] (x\*(Sqrt[b]\*B\*(b + c\*x^2) - A\*c\*Sqrt[b + c\*x^2]\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]]))/(Sqrt[b]\*c\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic** [A] time = 0.11, size = 55, normalized size = 1.00

$$\frac{B\sqrt{bx^2 + cx^4}}{cx} - \frac{A \tanh^{-1} \left( \frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}} \right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/Sqrt[b\*x^2 + c\*x^4],x]

[Out] (B\*Sqrt[b\*x^2 + c\*x^4])/(c\*x) - (A\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/Sqrt[b]

**fricas** [A] time = 0.43, size = 138, normalized size = 2.51

$$\left[ \frac{A\sqrt{b} cx \log \left( -\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3} \right) + 2\sqrt{cx^4+bx^2} Bb}{2bcx}, \frac{A\sqrt{-b} cx \arctan \left( \frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx} \right) + \sqrt{cx^4+bx^2} Bb}{bcx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2\*(A\*sqrt(b)\*c\*x\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*B\*b)/(b\*c\*x), (A\*sqrt(-b)\*c\*x\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + sqrt(c\*x^4 + b\*x^2)\*B\*b)/(b\*c\*x)]

**giac** [A] time = 0.33, size = 60, normalized size = 1.09

$$\frac{A \log \left( \left( \sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x} \right)^2 \right)}{2\sqrt{b}} - \frac{2B\sqrt{b}}{\left( \sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x} \right)^2 - c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] 1/2\*A\*log((sqrt(c + b/x^2) - sqrt(b)/x)^2)/sqrt(b) - 2\*B\*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)

**maple** [A] time = 0.06, size = 72, normalized size = 1.31

$$\frac{\sqrt{cx^2 + b} \left( Ac \ln \left( \frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x} \right) - \sqrt{cx^2 + b} B\sqrt{b} \right) x}{\sqrt{cx^4 + bx^2} \sqrt{b} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)`

[Out] `-x*(c*x^2+b)^(1/2)*(A*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*c-B*(c*x^2+b)^(1/2)*b^(1/2))/(c*x^4+b*x^2)^(1/2)/c/b^(1/2)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2),x)`

[Out] `int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{\sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)`

[Out] `Integral((A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)`

$$3.143 \quad \int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}} dx$$

Optimal. Leaf size=68

$$-\frac{(2bB - Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} - \frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

**Rubi [A]** time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2038, 2008, 206}

$$-\frac{(2bB - Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} - \frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^2\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] -(A\*Sqrt[b\*x^2 + c\*x^4])/(2\*b\*x^3) - ((2\*b\*B - A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(2\*b^(3/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2008

Int[1/Sqrt[(a\_)\*(x\_)^2 + (b\_)\*(x\_)^(n\_)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 2038

Int[((e\_)\*(x\_)^(m\_))\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(jn\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*(m + j\*p + 1)), x] + Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(a\*e^n\*(m + j\*p + 1)), Int[(e\*x)^(m + n)\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m + j\*p, -1] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^2\sqrt{bx^2+cx^4}} dx &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} - \frac{(-2bB + Ac) \int \frac{1}{\sqrt{bx^2+cx^4}} dx}{2b} \\ &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} + \frac{(-2bB + Ac) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b} \\ &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} - \frac{(2bB - Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 87, normalized size = 1.28

$$\frac{x\sqrt{b+cx^2} \left( -\frac{2\left(bB-\frac{Ac}{2}\right)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{A\sqrt{b+cx^2}}{bx^2} \right)}{2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^2\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] (x\*Sqrt[b + c\*x^2]\*(-(A\*Sqrt[b + c\*x^2])/(b\*x^2)) - (2\*(b\*B - (A\*c)/2)\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]]/b^(3/2)))/(2\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.16, size = 67, normalized size = 0.99

$$\frac{(Ac - 2bB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} - \frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^2\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] -1/2\*(A\*Sqrt[b\*x^2 + c\*x^4])/(b\*x^3) + ((-2\*b\*B + A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(2\*b^(3/2))

**fricas [A]** time = 0.44, size = 152, normalized size = 2.24

$$\left[ \frac{(2Bb - Ac)\sqrt{b}x^3 \log\left(-\frac{cx^3+2bx+2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4+bx^2}Ab}{4b^2x^3}, \frac{(2Bb - Ac)\sqrt{-b}x^3 \arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4+bx^2}Ab}{2b^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/4\*((2\*B\*b - A\*c)\*sqrt(b)\*x^3\*log(-(c\*x^3 + 2\*b\*x + 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*A\*b)/(b^2\*x^3), 1/2\*((2\*B\*b - A\*c)\*sqrt(-b)\*x^3\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) - sqrt(c\*x^4 + b\*x^2)\*A\*b)/(b^2\*x^3)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^(1/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,2]%%},0,%%{1,[0,2,4]%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,2]%%},0,%%{1,[0,2,4]%%}] at parameters values [66.1769613782,93,91]-4\*A/8/b/x\*sqrt(b\*(1/x)^2+c)+2\*(-A\*c+2\*B\*b)/4/b/sqrt(b)\*ln(abs(sqrt(b\*(1/x)^2+c)-sqrt(b)/x))

**maple [A]** time = 0.06, size = 105, normalized size = 1.54

$$\frac{\sqrt{cx^2+b} \left( -Abcx^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) + 2Bb^2x^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) + \sqrt{cx^2+b} Ab^{\frac{3}{2}} \right)}{2\sqrt{cx^4+bx^2} b^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x)`

[Out]  $-1/2/x*(c*x^2+b)^{(1/2)}*(2*B*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)})/x)*x^2*b^2-A*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)})/x)*x^2*b*c+A*b^{(3/2)}*(c*x^2+b)^{(1/2)})/(c*x^4+b*x^2)^{(1/2)}/b^{(5/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Bx^2 + A}{x^2 \sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)),x)`

[Out] `int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2 \sqrt{x^2 (b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(1/2),x)`

[Out] `Integral((A + B*x**2)/(x**2*sqrt(x**2*(b + c*x**2))), x)`



$$3.144 \quad \int \frac{A+Bx^2}{x^4 \sqrt{bx^2+cx^4}} dx$$

**Optimal.** Leaf size=103

$$\frac{c(4bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}} - \frac{\sqrt{bx^2 + cx^4}(4bB - 3Ac)}{8b^2x^3} - \frac{A\sqrt{bx^2 + cx^4}}{4bx^5}$$

**Rubi [A]** time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2038, 2025, 2008, 206}

$$-\frac{\sqrt{bx^2 + cx^4}(4bB - 3Ac)}{8b^2x^3} + \frac{c(4bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}} - \frac{A\sqrt{bx^2 + cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^4\*Sqrt[b\*x^2 + c\*x^4]),x]

[Out] -(A\*Sqrt[b\*x^2 + c\*x^4])/(4\*b\*x^5) - ((4\*b\*B - 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(8\*b^2\*x^3) + (c\*(4\*b\*B - 3\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(8\*b^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2025

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j\*p + 1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*(m + j\*p + 1)), x] + Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(a\*e^n\*(m + j\*p + 1)), Int[(e\*x)^(m + n)\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m + j\*p, -1] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^4 \sqrt{bx^2 + cx^4}} dx &= -\frac{A\sqrt{bx^2 + cx^4}}{4bx^5} - \frac{(-4bB + 3Ac) \int \frac{1}{x^2 \sqrt{bx^2 + cx^4}} dx}{4b} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{4bx^5} - \frac{(4bB - 3Ac)\sqrt{bx^2 + cx^4}}{8b^2x^3} - \frac{(c(4bB - 3Ac)) \int \frac{1}{\sqrt{bx^2 + cx^4}} dx}{8b^2} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{4bx^5} - \frac{(4bB - 3Ac)\sqrt{bx^2 + cx^4}}{8b^2x^3} + \frac{(c(4bB - 3Ac)) \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2 + cx^4}}\right)}{8b^2} \\
&= -\frac{A\sqrt{bx^2 + cx^4}}{4bx^5} - \frac{(4bB - 3Ac)\sqrt{bx^2 + cx^4}}{8b^2x^3} + \frac{c(4bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}}\right)}{8b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 104, normalized size = 1.01

$$\frac{\sqrt{x^2(b + cx^2)} \left( b\sqrt{\frac{cx^2}{b} + 1} (2Ab - 3Acx^2 + 4bBx^2) + cx^4(3Ac - 4bB) \tanh^{-1}\left(\sqrt{\frac{cx^2}{b} + 1}\right) \right)}{8b^3x^5\sqrt{\frac{cx^2}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^4\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] -1/8\*(Sqrt[x^2\*(b + c\*x^2)]\*(b\*(2\*A\*b + 4\*b\*B\*x^2 - 3\*A\*c\*x^2)\*Sqrt[1 + (c\*x^2)/b] + c\*(-4\*b\*B + 3\*A\*c)\*x^4\*ArcTanh[Sqrt[1 + (c\*x^2)/b]])/(b^3\*x^5\*Sqrt[1 + (c\*x^2)/b])

**IntegrateAlgebraic [A]** time = 0.22, size = 89, normalized size = 0.86

$$\frac{(4bBc - 3Ac^2) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}}\right)}{8b^{5/2}} + \frac{\sqrt{bx^2 + cx^4} (-2Ab + 3Acx^2 - 4bBx^2)}{8b^2x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^4\*Sqrt[b\*x^2 + c\*x^4]), x]

[Out] ((-2\*A\*b - 4\*b\*B\*x^2 + 3\*A\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(8\*b^2\*x^5) + ((4\*b\*B\*c - 3\*A\*c^2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(8\*b^(5/2))

**fricas [A]** time = 0.44, size = 199, normalized size = 1.93

$$\left[ \frac{(4Bbc - 3Ac^2)\sqrt{b}x^5 \log\left(\frac{cx^3 + 2bx - 2\sqrt{cx^4 + bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4 + bx^2}(2Ab^2 + (4Bb^2 - 3Abc)x^2)}{16b^3x^5}, \frac{(4Bbc - 3Ac^2)\sqrt{-b}x^5 \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx}\right) + \sqrt{cx^4 + bx^2}(2Ab^2 + (4Bb^2 - 3Abc)x^2)}{8b^3x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^4/(c\*x^4+b\*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/16\*((4\*B\*b\*c - 3\*A\*c^2)\*sqrt(b)\*x^5\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*(2\*A\*b^2 + (4\*B\*b^2 - 3\*A\*b\*c)\*x^2))/(b^3\*x^5), -1/8\*((4\*B\*b\*c - 3\*A\*c^2)\*sqrt(-b)\*x^5\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) + sqrt(c\*x^4 + b\*x^2)\*(2\*A\*b^2 + (4\*B\*b^2 - 3\*A\*b\*c)\*x^2))/(b^3\*x^5)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^4/(c\*x^4+b\*x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,2]%%},0,%%{1,[0,2,4]%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,2]%%},0,%%{1,[0,2,4]%%}] at parameters values [66.1769613782,93,91]2\*(-4\*b^2\*A/32/b^3/x/x-(8\*b^2\*B-6\*b\*A\*c)/32/b^3)/x\*sqrt(b\*(1/x)^2+c)+2\*(3\*A\*c^2-4\*B\*b\*c)/16/b^2/sqrt(b)\*ln(abs(sqrt(b\*(1/x)^2+c)-sqrt(b)/x))

**maple** [A] time = 0.06, size = 146, normalized size = 1.42

$$\frac{\sqrt{cx^2+b} \left( 3Ab^2c^2x^4 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 4Bb^2cx^4 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 3\sqrt{cx^2+b} Ab^{\frac{3}{2}}cx^2 + 4\sqrt{cx^2+b} Bb^{\frac{5}{2}}x^2 + 2\sqrt{cx^2+b} Ab^{\frac{5}{2}} \right)}{8\sqrt{cx^2+b} b^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^4/(c\*x^4+b\*x^2)^(1/2),x)

[Out] -1/8\*(c\*x^2+b)^(1/2)\*(3\*A\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^4\*b\*c^2+4\*B\*(c\*x^2+b)^(1/2)\*b^(5/2)\*x^2-4\*B\*ln(2\*(b+(c\*x^2+b)^(1/2)\*b^(1/2))/x)\*x^4\*b^2\*c-3\*A\*(c\*x^2+b)^(1/2)\*b^(3/2)\*x^2\*c+2\*A\*(c\*x^2+b)^(1/2)\*b^(5/2))/x^3/(c\*x^4+b\*x^2)^(1/2)/b^(7/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^4/(c\*x^4+b\*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate((B\*x^2 + A)/(sqrt(c\*x^4 + b\*x^2)\*x^4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Bx^2 + A}{x^4 \sqrt{cx^4 + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)^(1/2)),x)

[Out] int((A + B\*x^2)/(x^4\*(b\*x^2 + c\*x^4)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^4 \sqrt{x^2(b + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*4/(c\*x\*\*4+b\*x\*\*2)\*\*(1/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*4\*sqrt(x\*\*2\*(b + c\*x\*\*2))), x)

$$3.145 \quad \int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=184

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}} + \frac{5b\sqrt{bx^2+cx^4}(7bB-6Ac)}{16c^4} - \frac{5x^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{24c^3} + \frac{x^4\sqrt{bx^2+cx^4}(7bB-6Ac)}{6bc^2}$$

**Rubi [A]** time = 0.33, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 788, 670, 640, 620, 206}

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}} + \frac{x^4\sqrt{bx^2+cx^4}(7bB-6Ac)}{6bc^2} - \frac{5x^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{24c^3} + \frac{5b\sqrt{bx^2+cx^4}(7bB-6Ac)}{16c^4} - \frac{x^8(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^8)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) + (5\*b\*(7\*b\*B - 6\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(16\*c^4) - (5\*(7\*b\*B - 6\*A\*c)\*x^2\*Sqrt[b\*x^2 + c\*x^4])/(24\*c^3) + ((7\*b\*B - 6\*A\*c)\*x^4\*Sqrt[b\*x^2 + c\*x^4])/(6\*b\*c^2) - (5\*b^2\*(7\*b\*B - 6\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(16\*c^(9/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

Rule 788

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((g\*(c\*d - b\*e) + c\*e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(p + 1)\*(2\*c\*d - b\*e)), x] - Dist[(e\*(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g)))/(c\*(p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ

[p, -1] && GtQ[m, 0]

### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

### Rubi steps

$$\begin{aligned} \int \frac{x^9 (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^4 (A + Bx)}{(bx + cx^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{(bB - Ac)x^8}{bc\sqrt{bx^2 + cx^4}} + \frac{1}{2} \left( -\frac{6A}{b} + \frac{7B}{c} \right) \text{Subst} \left( \int \frac{x^3}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\ &= -\frac{(bB - Ac)x^8}{bc\sqrt{bx^2 + cx^4}} + \frac{(7bB - 6Ac)x^4 \sqrt{bx^2 + cx^4}}{6bc^2} - \frac{(5(7bB - 6Ac)) \text{Subst} \left( \int \frac{x^2}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{12c^2} \\ &= -\frac{(bB - Ac)x^8}{bc\sqrt{bx^2 + cx^4}} - \frac{5(7bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^3} + \frac{(7bB - 6Ac)x^4 \sqrt{bx^2 + cx^4}}{6bc^2} + \frac{(5b(7bB - 6Ac)) \text{Subst} \left( \int \frac{x}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{12c^2} \\ &= -\frac{(bB - Ac)x^8}{bc\sqrt{bx^2 + cx^4}} + \frac{5b(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{16c^4} - \frac{5(7bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^3} + \frac{(7bB - 6Ac)x^4 \sqrt{bx^2 + cx^4}}{6bc^2} \\ &= -\frac{(bB - Ac)x^8}{bc\sqrt{bx^2 + cx^4}} + \frac{5b(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{16c^4} - \frac{5(7bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^3} + \frac{(7bB - 6Ac)x^4 \sqrt{bx^2 + cx^4}}{6bc^2} \\ &= -\frac{(bB - Ac)x^8}{bc\sqrt{bx^2 + cx^4}} + \frac{5b(7bB - 6Ac)\sqrt{bx^2 + cx^4}}{16c^4} - \frac{5(7bB - 6Ac)x^2 \sqrt{bx^2 + cx^4}}{24c^3} + \frac{(7bB - 6Ac)x^4 \sqrt{bx^2 + cx^4}}{6bc^2} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 136, normalized size = 0.74

$$\frac{x \left( \sqrt{c} x (b^2 (35Bcx^2 - 90Ac) - 2bc^2 x^2 (15A + 7Bx^2) + 4c^3 x^4 (3A + 2Bx^2) + 105b^3 B) - 15b^{5/2} \sqrt{\frac{cx^2}{b} + 1} (7bB - 6Ac) \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{b}} \right) \right)}{48c^{9/2} \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(Sqrt[c]\*x\*(105\*b^3\*B + 4\*c^3\*x^4\*(3\*A + 2\*B\*x^2) - 2\*b\*c^2\*x^2\*(15\*A + 7\*B\*x^2) + b^2\*(-90\*A\*c + 35\*B\*c\*x^2)) - 15\*b^(5/2)\*(7\*b\*B - 6\*A\*c)\*Sqrt[1 + (c\*x^2)/b]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(48\*c^(9/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.67, size = 148, normalized size = 0.80

$$\frac{5(7b^3B - 6Ab^2c) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{32c^{9/2}} + \frac{\sqrt{bx^2 + cx^4} (-90Ab^2c - 30Abc^2x^2 + 12Ac^3x^4 + 105b^3B + 35b^2Bcx^2 - 14bBc^2x^4 + 8Bc^3x^6)}{48c^4 (b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2),x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(105\*b^3\*B - 90\*A\*b^2\*c + 35\*b^2\*B\*c\*x^2 - 30\*A\*b\*c^2\*x^2 - 14\*b\*B\*c^2\*x^4 + 12\*A\*c^3\*x^4 + 8\*B\*c^3\*x^6))/(48\*c^4\*(b + c\*x^2)) + (5\*(7\*b^3\*B - 6\*A\*b^2\*c)\*Log[b + 2\*c\*x^2 - 2\*Sqrt[c]\*Sqrt[b\*x^2 + c\*x^4]])/(32\*c^(9/2))

**fricas** [A] time = 0.44, size = 340, normalized size = 1.85

$$\frac{15(7Bb^4 - 6Ab^3c + 7Bb^2c^2 - 6Ab^2c^2)\sqrt{c}\log\left(\frac{-2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c}}{\sqrt{cx^4 + bx^2}}\right) - 2(8Bb^4 + 105Bb^3c - 90Ab^2c^2 - 2(7Bb^3 - 6Ab^2c)x^4 + 5(7Bb^2c^2 - 6Ab^2c^2)\sqrt{cx^4 + bx^2})}{96(c^2 + b^2)} - \frac{15(7Bb^4 - 6Ab^3c + 7Bb^2c^2 - 6Ab^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4 + bx^2}}{\sqrt{-c}}\right) + (8Bb^4 + 105Bb^3c - 90Ab^2c^2 - 2(7Bb^3 - 6Ab^2c)x^4 + 5(7Bb^2c^2 - 6Ab^2c^2)\sqrt{cx^4 + bx^2})}{48(c^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/96\*(15\*(7\*B\*b^4 - 6\*A\*b^3\*c + (7\*B\*b^3\*c - 6\*A\*b^2\*c^2)\*x^2)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(8\*B\*c^4\*x^6 + 105\*B\*b^3\*c - 90\*A\*b^2\*c^2 - 2\*(7\*B\*b\*c^3 - 6\*A\*c^4)\*x^4 + 5\*(7\*B\*b^2\*c^2 - 6\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(c^6\*x^2 + b\*c^5), 1/48\*(15\*(7\*B\*b^4 - 6\*A\*b^3\*c + (7\*B\*b^3\*c - 6\*A\*b^2\*c^2)\*x^2)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + (8\*B\*c^4\*x^6 + 105\*B\*b^3\*c - 90\*A\*b^2\*c^2 - 2\*(7\*B\*b\*c^3 - 6\*A\*c^4)\*x^4 + 5\*(7\*B\*b^2\*c^2 - 6\*A\*b\*c^3)\*x^2)\*sqrt(c\*x^4 + b\*x^2))/(c^6\*x^2 + b\*c^5)]

**giac** [A] time = 0.31, size = 176, normalized size = 0.96

$$\frac{1}{48}\sqrt{cx^4 + bx^2}\left(2x^2\left(\frac{4Bx^2}{c^2} - \frac{11Bbc^{10} - 6Ac^{11}}{c^{13}}\right) + \frac{3(19Bb^2c^9 - 14Abc^{10})}{c^{13}}\right) + \frac{5(7Bb^3 - 6Ab^2c)\log\left(\left|-2\left(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}}\sqrt{c} - b\right)\right|\right)}{32c^{\frac{9}{2}}} + \frac{Bb^4 - Ab^3c}{\left(\left(\sqrt{cx^2 - \sqrt{cx^4 + bx^2}}\right)c + b\sqrt{c}\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] 1/48\*sqrt(c\*x^4 + b\*x^2)\*(2\*x^2\*(4\*B\*x^2/c^2 - (11\*B\*b\*c^10 - 6\*A\*c^11)/c^13) + 3\*(19\*B\*b^2\*c^9 - 14\*A\*b\*c^10)/c^13) + 5/32\*(7\*B\*b^3 - 6\*A\*b^2\*c)\*log(abs(-2\*(sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))\*sqrt(c) - b))/c^(9/2) + (B\*b^4 - A\*b^3\*c)/(((sqrt(c)\*x^2 - sqrt(c\*x^4 + b\*x^2))\*c + b\*sqrt(c))\*c^4)

**maple** [A] time = 0.06, size = 166, normalized size = 0.90

$$\frac{(cx^2 + b)\left(8Bc^{\frac{9}{2}}x^7 + 12Ac^{\frac{9}{2}}x^5 - 14Bbc^{\frac{7}{2}}x^3 - 30Abc^{\frac{7}{2}}x^3 + 35Bb^2c^{\frac{5}{2}}x^3 - 90Ab^2c^{\frac{5}{2}}x + 105Bb^3c^{\frac{3}{2}}x + 90\sqrt{cx^2 + b}Ab^2c^2\ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right) - 105\sqrt{cx^2 + b}Bb^3c\ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right)\right)x^3}{48(c^4 + b^2)^{\frac{3}{2}}c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out] 1/48\*x^3\*(c\*x^2+b)\*(8\*B\*c^(9/2)\*x^7+12\*A\*c^(9/2)\*x^5-14\*B\*c^(7/2)\*x^5\*b-30\*A\*c^(7/2)\*x^3\*b+35\*B\*c^(5/2)\*x^3\*b^2-90\*A\*c^(5/2)\*x\*b^2+105\*B\*c^(3/2)\*x\*b^3+90\*A\*(c\*x^2+b)^(1/2)\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^2\*c^2-105\*B\*(c\*x^2+b)^(1/2)\*ln(c^(1/2)\*x+(c\*x^2+b)^(1/2))\*b^3\*c)/(c\*x^4+b\*x^2)^(3/2)/c^(11/2)

**maxima** [A] time = 1.49, size = 237, normalized size = 1.29

$$\frac{1}{16}\left(\frac{4x^6}{\sqrt{cx^4 + bx^2}} - \frac{10bx^4}{\sqrt{cx^4 + bx^2}c^2} - \frac{30b^2x^2}{\sqrt{cx^4 + bx^2}c^3} + \frac{15b^2\log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^2}\right)A + \frac{1}{96}\left(\frac{16x^8}{\sqrt{cx^4 + bx^2}c} - \frac{28bx^6}{\sqrt{cx^4 + bx^2}c^2} + \frac{70b^2x^4}{\sqrt{cx^4 + bx^2}c^3} + \frac{210b^3x^2}{\sqrt{cx^4 + bx^2}c^4} - \frac{105b^3\log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^{\frac{9}{2}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/16\*(4\*x^6/(sqrt(c\*x^4 + b\*x^2)\*c) - 10\*b\*x^4/(sqrt(c\*x^4 + b\*x^2)\*c^2) - 30\*b^2\*x^2/(sqrt(c\*x^4 + b\*x^2)\*c^3) + 15\*b^2\*log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c))/c^(7/2))\*A + 1/96\*(16\*x^8/(sqrt(c\*x^4 + b\*x^2)\*c) - 28\*

$b*x^6/(\text{sqrt}(c*x^4 + b*x^2)*c^2) + 70*b^2*x^4/(\text{sqrt}(c*x^4 + b*x^2)*c^3) + 210*b^3*x^2/(\text{sqrt}(c*x^4 + b*x^2)*c^4) - 105*b^3*\log(2*c*x^2 + b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c))/c^{(9/2)})*B$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9 (Bx^2 + A)}{(cx^4 + bx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x)

[Out] int((x^9\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 (A + Bx^2)}{(x^2 (b + cx^2))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*9\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2), x)

[Out] Integral(x\*\*9\*(A + B\*x\*\*2)/(x\*\*2\*(b + c\*x\*\*2))\*\*3/2, x)

$$3.146 \quad \int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=147

$$\frac{3b(5bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}} - \frac{3\sqrt{bx^2+cx^4}(5bB - 4Ac)}{8c^3} + \frac{x^2\sqrt{bx^2+cx^4}(5bB - 4Ac)}{4bc^2} - \frac{x^6(bB - Ac)}{bc\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.28, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2034, 788, 670, 640, 620, 206}

$$\frac{x^2\sqrt{bx^2+cx^4}(5bB - 4Ac)}{4bc^2} - \frac{3\sqrt{bx^2+cx^4}(5bB - 4Ac)}{8c^3} + \frac{3b(5bB - 4Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}} - \frac{x^6(bB - Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^6)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) - (3\*(5\*b\*B - 4\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(8\*c^3) + ((5\*b\*B - 4\*A\*c)\*x^2\*Sqrt[b\*x^2 + c\*x^4])/(4\*b\*c^2) + (3\*b\*(5\*b\*B - 4\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(8\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 670

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[((m + p)\*(2\*c\*d - b\*e))/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 1] && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

#### Rule 788

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((g\*(c\*d - b\*e) + c\*e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(p + 1)\*(2\*c\*d - b\*e)), x] - Dist[(e\*(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g)))/(c\*(p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ



[p, -1] && GtQ[m, 0]

### Rule 2034

Int[(x\_)^(m\_)\*((b\_)\*(x\_)^(k\_) + (a\_)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

### Rubi steps

$$\begin{aligned} \int \frac{x^7 (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^3 (A + Bx)}{(bx + cx^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{(bB - Ac)x^6}{bc\sqrt{bx^2 + cx^4}} + \frac{1}{2} \left( -\frac{4A}{b} + \frac{5B}{c} \right) \text{Subst} \left( \int \frac{x^2}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\ &= -\frac{(bB - Ac)x^6}{bc\sqrt{bx^2 + cx^4}} + \frac{(5bB - 4Ac)x^2\sqrt{bx^2 + cx^4}}{4bc^2} - \frac{(3(5bB - 4Ac)) \text{Subst} \left( \int \frac{x}{\sqrt{bx + cx^2}} dx, x \right)}{8c^2} \\ &= -\frac{(bB - Ac)x^6}{bc\sqrt{bx^2 + cx^4}} - \frac{3(5bB - 4Ac)\sqrt{bx^2 + cx^4}}{8c^3} + \frac{(5bB - 4Ac)x^2\sqrt{bx^2 + cx^4}}{4bc^2} + \frac{(3b(5bB - 4Ac)) \text{Subst} \left( \int \frac{x}{\sqrt{bx + cx^2}} dx, x \right)}{8c^2} \\ &= -\frac{(bB - Ac)x^6}{bc\sqrt{bx^2 + cx^4}} - \frac{3(5bB - 4Ac)\sqrt{bx^2 + cx^4}}{8c^3} + \frac{(5bB - 4Ac)x^2\sqrt{bx^2 + cx^4}}{4bc^2} + \frac{(3b(5bB - 4Ac)) \text{Subst} \left( \int \frac{x}{\sqrt{bx + cx^2}} dx, x \right)}{8c^2} \\ &= -\frac{(bB - Ac)x^6}{bc\sqrt{bx^2 + cx^4}} - \frac{3(5bB - 4Ac)\sqrt{bx^2 + cx^4}}{8c^3} + \frac{(5bB - 4Ac)x^2\sqrt{bx^2 + cx^4}}{4bc^2} + \frac{3b(5bB - 4Ac)}{8c^2} \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 113, normalized size = 0.77

$$\frac{x \left( 3b^{3/2} \sqrt{\frac{cx^2}{b}} + 1 (5bB - 4Ac) \sinh^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right) + \sqrt{c}x (bc(12A - 5Bx^2) + 2c^2x^2(2A + Bx^2) - 15b^2B) \right)}{8c^{7/2} \sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(Sqrt[c]\*x\*(-15\*b^2\*B + b\*c\*(12\*A - 5\*B\*x^2) + 2\*c^2\*x^2\*(2\*A + B\*x^2)) + 3\*b^(3/2)\*(5\*b\*B - 4\*A\*c)\*Sqrt[1 + (c\*x^2)/b]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(8\*c^(7/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.58, size = 122, normalized size = 0.83

$$\frac{\sqrt{bx^2 + cx^4} (12Abc + 4Ac^2x^2 - 15b^2B - 5bBcx^2 + 2Bc^2x^4)}{8c^3(b + cx^2)} - \frac{3(5b^2B - 4Abc) \log \left( -2\sqrt{c} \sqrt{bx^2 + cx^4} + b + 2cx^2 \right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out]  $(\text{Sqrt}[b*x^2 + c*x^4]*(-15*b^2*B + 12*A*b*c - 5*b*B*c*x^2 + 4*A*c^2*x^2 + 2*B*c^2*x^4))/(8*c^3*(b + c*x^2)) - (3*(5*b^2*B - 4*A*b*c)*\text{Log}[b + 2*c*x^2 - 2*\text{Sqrt}[c]*\text{Sqrt}[b*x^2 + c*x^4]])/(16*c^{(7/2)})$

**fricas** [A] time = 0.44, size = 289, normalized size = 1.97

$$\frac{3(5Bb^2 - 4Ab^2c + (5Bb^2c - 4Abc^2)x^2)\sqrt{c}\log(-2cx^2 - b + 2\sqrt{cx^4 + bx^2}\sqrt{c}) - 2(2Bc^3x^4 - 15Bb^2c + 12Abc^2 - (5Bbc^2 - 4Ac^3)x^2)\sqrt{cx^4 + bx^2} - 3(5Bb^3 - 4Ab^2c + (5Bb^2c - 4Abc^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (2Bc^3x^4 - 15Bb^2c + 12Abc^2 - (5Bbc^2 - 4Ac^3)x^2)\sqrt{cx^4 + bx^2}}{16(c^3x^2 + bc^4)} - \frac{3(5Bb^3 - 4Ab^2c + (5Bb^2c - 4Abc^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{c}}{cx^2 + b}\right) - (2Bc^3x^4 - 15Bb^2c + 12Abc^2 - (5Bbc^2 - 4Ac^3)x^2)\sqrt{cx^4 + bx^2}}{8(c^3x^2 + bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")`

[Out]  $[-1/16*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*\text{sqrt}(c)*\text{log}(-2*c*x^2 - b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c)) - 2*(2*B*c^3*x^4 - 15*B*b^2*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*\text{sqrt}(c*x^4 + b*x^2))/(c^5*x^2 + b*c^4), -1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*\text{sqrt}(-c)*\text{arctan}(\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(-c)/(c*x^2 + b)) - (2*B*c^3*x^4 - 15*B*b^2*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*\text{sqrt}(c*x^4 + b*x^2))/(c^5*x^2 + b*c^4)]$

**giac** [A] time = 0.28, size = 147, normalized size = 1.00

$$\frac{1}{8}\sqrt{cx^4 + bx^2}\left(\frac{2Bx^2}{c^2} - \frac{7Bbc^5 - 4Ac^6}{c^8}\right) - \frac{3(5Bb^2 - 4Abc)\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{16c^{\frac{7}{2}}} - \frac{Bb^3 - Ab^2c}{\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)c + b\sqrt{c}\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

[Out]  $1/8*\text{sqrt}(c*x^4 + b*x^2)*(2*B*x^2/c^2 - (7*B*b*c^5 - 4*A*c^6)/c^8) - 3/16*(5*B*b^2 - 4*A*b*c)*\text{log}(\text{abs}(-2*(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2))*\text{sqrt}(c) - b))/c^{(7/2)} - (B*b^3 - A*b^2*c)/(((\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2))*c + b*\text{sqrt}(c))*c^3)$

**maple** [A] time = 0.06, size = 140, normalized size = 0.95

$$\frac{(cx^2 + b)\left(-2Bc^{\frac{7}{2}}x^5 - 4Ac^{\frac{7}{2}}x^3 + 5Bbc^{\frac{5}{2}}x^3 - 12Abc^{\frac{5}{2}}x + 15Bb^2c^{\frac{3}{2}}x + 12\sqrt{cx^2 + b}Abc^2\ln(\sqrt{c}x + \sqrt{cx^2 + b}) - 15\sqrt{cx^2 + b}Bb^2c\ln(\sqrt{c}x + \sqrt{cx^2 + b})\right)x^3}{8(cx^4 + bx^2)^{\frac{3}{2}}c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)`

[Out]  $-1/8*x^3*(c*x^2+b)*(-2*B*c^{(7/2)}*x^5-4*A*c^{(7/2)}*x^3+5*B*c^{(5/2)}*x^3*b-12*A*c^{(5/2)}*x*b+15*B*c^{(3/2)}*x*b^2+12*A*(c*x^2+b)^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*b*c^2-15*B*(c*x^2+b)^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+b)^{(1/2)})*b^2*c)/(c*x^4+b*x^2)^{(3/2)}/c^{(9/2)}$

**maxima** [A] time = 1.54, size = 187, normalized size = 1.27

$$\frac{1}{4}\left(\frac{2x^4}{\sqrt{cx^4 + bx^2}c} + \frac{6bx^2}{\sqrt{cx^4 + bx^2}c^2} - \frac{3b\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{5}{2}}}\right)A + \frac{1}{16}\left(\frac{4x^6}{\sqrt{cx^4 + bx^2}c} - \frac{10bx^4}{\sqrt{cx^4 + bx^2}c^2} - \frac{30b^2x^2}{\sqrt{cx^4 + bx^2}c^3} + \frac{15b^2\log(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c})}{c^{\frac{7}{2}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")`

[Out]  $1/4*(2*x^4/(\text{sqrt}(c*x^4 + b*x^2)*c) + 6*b*x^2/(\text{sqrt}(c*x^4 + b*x^2)*c^2) - 3*b*\text{log}(2*c*x^2 + b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c))/c^{(5/2)})*A + 1/16*(4*x^6/(\text{sqrt}(c*x^4 + b*x^2)*c) - 10*b*x^4/(\text{sqrt}(c*x^4 + b*x^2)*c^2) - 30*b^2*x^2/(\text{sqrt}(c*x^4 + b*x^2)*c^3) + 15*b^2*\text{log}(2*c*x^2 + b + 2*\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(c))/c^{(7/2)})*B$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7 (Bx^2 + A)}{(cx^4 + bx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x)

[Out] int((x^7\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^2)}{(x^2 (b + cx^2))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2), x)

[Out] Integral(x\*\*7\*(A + B\*x\*\*2)/(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2), x)

$$3.147 \quad \int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=112

$$-\frac{(3bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}} + \frac{\sqrt{bx^2 + cx^4} (3bB - 2Ac)}{2bc^2} - \frac{x^4(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

**Rubi [A]** time = 0.24, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2034, 788, 640, 620, 206}

$$\frac{\sqrt{bx^2 + cx^4} (3bB - 2Ac)}{2bc^2} - \frac{(3bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}} - \frac{x^4(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^4)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) + ((3\*b\*B - 2\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(2\*b\*c^2) - ((3\*b\*B - 2\*A\*c)\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/(2\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 788

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((g\*(c\*d - b\*e) + c\*e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(p + 1)\*(2\*c\*d - b\*e)), x] - Dist[(e\*(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g)))/(c\*(p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +

1)/n]] && NeQ[n^2, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2 (A + Bx)}{(bx + cx^2)^{3/2}} dx, x, x^2 \right) \\
 &= -\frac{(bB - Ac)x^4}{bc\sqrt{bx^2 + cx^4}} + \frac{1}{2} \left( -\frac{2A}{b} + \frac{3B}{c} \right) \text{Subst} \left( \int \frac{x}{\sqrt{bx + cx^2}} dx, x, x^2 \right) \\
 &= -\frac{(bB - Ac)x^4}{bc\sqrt{bx^2 + cx^4}} + \frac{(3bB - 2Ac)\sqrt{bx^2 + cx^4}}{2bc^2} - \frac{(3bB - 2Ac) \text{Subst} \left( \int \frac{1}{\sqrt{bx + cx^2}} dx, x, x^2 \right)}{4c^2} \\
 &= -\frac{(bB - Ac)x^4}{bc\sqrt{bx^2 + cx^4}} + \frac{(3bB - 2Ac)\sqrt{bx^2 + cx^4}}{2bc^2} - \frac{(3bB - 2Ac) \text{Subst} \left( \int \frac{1}{1 - cx^2} dx, x, \frac{x^2}{\sqrt{bx^2 + cx^4}} \right)}{2c^2} \\
 &= -\frac{(bB - Ac)x^4}{bc\sqrt{bx^2 + cx^4}} + \frac{(3bB - 2Ac)\sqrt{bx^2 + cx^4}}{2bc^2} - \frac{(3bB - 2Ac) \tanh^{-1} \left( \frac{\sqrt{c}x^2}{\sqrt{bx^2 + cx^4}} \right)}{2c^{5/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 91, normalized size = 0.81

$$\frac{x \left( \sqrt{c}x (-2Ac + 3bB + Bcx^2) - \sqrt{b} \sqrt{\frac{cx^2}{b} + 1} (3bB - 2Ac) \sinh^{-1} \left( \frac{\sqrt{c}x}{\sqrt{b}} \right) \right)}{2c^{5/2} \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(Sqrt[c]\*x\*(3\*b\*B - 2\*A\*c + B\*c\*x^2) - Sqrt[b]\*(3\*b\*B - 2\*A\*c)\*Sqrt[1 + (c\*x^2)/b]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]])/(2\*c^(5/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.54, size = 102, normalized size = 0.91

$$\frac{\sqrt{bx^2 + cx^4} (-2Ac + 3bB + Bcx^2)}{2c^2 (b + cx^2)} + \frac{(3bB - 2Ac) \log \left( -2c^{5/2} \sqrt{bx^2 + cx^4} + bc^2 + 2c^3 x^2 \right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((3\*b\*B - 2\*A\*c + B\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(2\*c^2\*(b + c\*x^2)) + ((3\*b\*B - 2\*A\*c)\*Log[b\*c^2 + 2\*c^3\*x^2 - 2\*c^(5/2)\*Sqrt[b\*x^2 + c\*x^4]])/(4\*c^(5/2))

**fricas [A]** time = 0.43, size = 230, normalized size = 2.05

$$\left[ \frac{(3Bb^2 - 2Abc + (3Bbc - 2Ac^2)x^2)\sqrt{c} \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c} \right) - 2(Bc^2x^2 + 3Bbc - 2Ac^2)\sqrt{cx^4 + bx^2} (3Bb^2 - 2Abc + (3Bbc - 2Ac^2)x^2)\sqrt{-c} \arctan \left( \frac{\sqrt{cx^4 + bx^2}\sqrt{-c}}{cx^2 + b} \right) + (Bc^2x^2 + 3Bbc - 2Ac^2)\sqrt{cx^4 + bx^2}}{4(c^4x^2 + bc^3)}, \frac{(3Bb^2 - 2Abc + (3Bbc - 2Ac^2)x^2)\sqrt{c} \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2}\sqrt{c} \right) - 2(Bc^2x^2 + 3Bbc - 2Ac^2)\sqrt{cx^4 + bx^2} (3Bb^2 - 2Abc + (3Bbc - 2Ac^2)x^2)\sqrt{-c} \arctan \left( \frac{\sqrt{cx^4 + bx^2}\sqrt{-c}}{cx^2 + b} \right) + (Bc^2x^2 + 3Bbc - 2Ac^2)\sqrt{cx^4 + bx^2}}{2(c^4x^2 + bc^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/4\*((3\*B\*b^2 - 2\*A\*b\*c + (3\*B\*b\*c - 2\*A\*c^2)\*x^2)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*(B\*c^2\*x^2 + 3\*B\*b\*c - 2\*A\*c^2)\*sqr

$t(cx^4 + bx^2)/(c^4x^2 + b^3c^3), 1/2*((3Bb^2 - 2A*bc + (3B*bc - 2A*c^2)*x^2)*\sqrt{-c}*\arctan(\sqrt{cx^4 + bx^2}*\sqrt{-c}/(cx^2 + b)) + (B*c^2*x^2 + 3B*bc - 2A*c^2)*\sqrt{cx^4 + bx^2})/(c^4x^2 + b^3c^3]$

**giac** [A] time = 0.26, size = 116, normalized size = 1.04

$$\frac{\sqrt{cx^4 + bx^2}B}{2c^2} + \frac{(3Bb - 2Ac)\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{4c^{\frac{5}{2}}} + \frac{Bb^2 - Abc}{\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)c + b\sqrt{c}\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out]  $1/2*\sqrt{cx^4 + bx^2}*B/c^2 + 1/4*(3*B*b - 2*A*c)*\log(\text{abs}(-2*(\sqrt{c}*x^2 - \sqrt{cx^4 + bx^2})*\sqrt{c} - b))/c^{(5/2)} + (B*b^2 - A*b*c)/(((\sqrt{c}*x^2 - \sqrt{cx^4 + bx^2})*c + b*\sqrt{c})*c^2)$

**maple** [A] time = 0.06, size = 115, normalized size = 1.03

$$\frac{(cx^2 + b)\left(-Bc^{\frac{5}{2}}x^3 + 2Ac^{\frac{5}{2}}x - 3Bbc^{\frac{3}{2}}x - 2\sqrt{cx^2 + b}Ac^2\ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right) + 3\sqrt{cx^2 + b}Bbc\ln\left(\sqrt{c}x + \sqrt{cx^2 + b}\right)\right)x^3}{2\left(cx^4 + bx^2\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out]  $-1/2*x^3*(cx^2+b)*(-B*c^{(5/2)}*x^3+2*A*c^{(5/2)}*x-3*B*c^{(3/2)}*x*b-2*A*\ln(c^{(1/2)}*x+(cx^2+b)^{(1/2)})*(cx^2+b)^{(1/2)}*c^2+3*B*\ln(c^{(1/2)}*x+(cx^2+b)^{(1/2)}))*(cx^2+b)^{(1/2)}*b*c)/(cx^4+b*x^2)^{(3/2)}/c^{(7/2)}$

**maxima** [A] time = 1.51, size = 138, normalized size = 1.23

$$\frac{1}{4}\left(\frac{2x^4}{\sqrt{cx^4 + bx^2}c} + \frac{6bx^2}{\sqrt{cx^4 + bx^2}c^2} - \frac{3b\log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)B - \frac{1}{2}A\left(\frac{2x^2}{\sqrt{cx^4 + bx^2}c} - \frac{\log\left(2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out]  $1/4*(2*x^4/(\sqrt{cx^4 + bx^2})*c) + 6*b*x^2/(\sqrt{cx^4 + bx^2})*c^2) - 3*b*\log(2*c*x^2 + b + 2*\sqrt{cx^4 + bx^2}*\sqrt{c})/c^{(5/2)})*B - 1/2*A*(2*x^2/(\sqrt{cx^4 + bx^2})*c) - \log(2*c*x^2 + b + 2*\sqrt{cx^4 + bx^2}*\sqrt{c})/c^{(3/2)})$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5 (Bx^2 + A)}{(cx^4 + bx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2),x)

[Out] int((x^5\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^2)}{(x^2 (b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

$$3.148 \quad \int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=67

$$\frac{B \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}} - \frac{x^2(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

**Rubi [A]** time = 0.18, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 777, 620, 206}

$$\frac{B \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}} - \frac{x^2(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^2)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) + (B\*ArcTanh[(Sqrt[c]\*x^2)/Sqrt[b\*x^2 + c\*x^4]])/c^(3/2)

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 620

Int[1/Sqrt[(b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(1 - c\*x^2), x], x, x/Sqrt[b\*x + c\*x^2]], x] /; FreeQ[{b, c}, x]

#### Rule 777

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((2\*a\*c\*(e\*f + d\*g) - b\*(c\*d\*f + a\*e\*g) - (b^2\*e\*g - b\*c\*(e\*f + d\*g) + 2\*c\*(c\*d\*f - a\*e\*g))\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(c\*(p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps



$$\begin{aligned}
\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B \text{Subst} \left( \int \frac{1}{\sqrt{bx+cx^2}} dx, x, x^2 \right)}{2c} \\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B \text{Subst} \left( \int \frac{1}{1-cx^2} dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}} \right)}{c} \\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B \tanh^{-1} \left( \frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}} \right)}{c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 75, normalized size = 1.12

$$\frac{x \left( \sqrt{c} x (Ac - bB) + b^{3/2} B \sqrt{\frac{cx^2}{b} + 1} \sinh^{-1} \left( \frac{\sqrt{c} x}{\sqrt{b}} \right) \right)}{bc^{3/2} \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(Sqrt[c]\*(-(b\*B) + A\*c)\*x + b^(3/2)\*B\*Sqrt[1 + (c\*x^2)/b]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[b]]))/(b\*c^(3/2)\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.46, size = 85, normalized size = 1.27

$$\frac{\sqrt{bx^2+cx^4}(Ac-bB)}{bc(b+cx^2)} - \frac{B \log \left( -2c^{3/2} \sqrt{bx^2+cx^4} + bc + 2c^2x^2 \right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((-(b\*B) + A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(b\*c\*(b + c\*x^2)) - (B\*Log[b\*c + 2\*c^2\*x^2 - 2\*c^(3/2)\*Sqrt[b\*x^2 + c\*x^4]])/(2\*c^(3/2))

**fricas [A]** time = 0.43, size = 188, normalized size = 2.81

$$\left[ \frac{(Bbcx^2 + Bb^2)\sqrt{c} \log \left( -2cx^2 - b - 2\sqrt{cx^4 + bx^2} \sqrt{c} \right) - 2\sqrt{cx^4 + bx^2} (Bbc - Ac^2)}{2(bc^3x^2 + b^2c^2)}, -\frac{(Bbcx^2 + Bb^2)\sqrt{-c} \arctan \left( \frac{\sqrt{cx^4 + bx^2} \sqrt{-c}}{cx^2 + b} \right) + \sqrt{cx^4 + bx^2} (Bbc - Ac^2)}{bc^3x^2 + b^2c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] [1/2\*((B\*b\*c\*x^2 + B\*b^2)\*sqrt(c)\*log(-2\*c\*x^2 - b - 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(c)) - 2\*sqrt(c\*x^4 + b\*x^2)\*(B\*b\*c - A\*c^2))/(b\*c^3\*x^2 + b^2\*c^2), -(B\*b\*c\*x^2 + B\*b^2)\*sqrt(-c)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-c)/(c\*x^2 + b)) + sqrt(c\*x^4 + b\*x^2)\*(B\*b\*c - A\*c^2)/(b\*c^3\*x^2 + b^2\*c^2)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error  
%{-%2, [1]} / %{{1, [2]}, [2, 0]} + %{{-%2, [1]}, 0}: [1, 0, %{-1, [1]}], [1, 1]} + %{{1, [1]}, [0, 2]} Error: Bad  
Argument Value

**maple** [A] time = 0.05, size = 75, normalized size = 1.12

$$\frac{(cx^2 + b) \left( Ac^{\frac{5}{2}}x - Bbc^{\frac{3}{2}}x + \sqrt{cx^2 + b} Bbc \ln \left( \sqrt{c}x + \sqrt{cx^2 + b} \right) \right) x^3}{(cx^4 + bx^2)^{\frac{3}{2}} bc^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out] x^3\*(c\*x^2+b)\*(A\*c^(5/2)\*x-B\*b\*c^(3/2)\*x+(c\*x^2+b)^(1/2)\*B\*b\*c\*ln(c^(1/2)\*x  
+(c\*x^2+b)^(1/2)))/(c\*x^4+b\*x^2)^(3/2)/c^(5/2)/b

**maxima** [A] time = 1.48, size = 79, normalized size = 1.18

$$-\frac{1}{2}B \left( \frac{2x^2}{\sqrt{cx^4 + bx^2}c} - \frac{\log \left( 2cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c} \right)}{c^{\frac{3}{2}}} \right) + \frac{Ax^2}{\sqrt{cx^4 + bx^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] -1/2\*B\*(2\*x^2/(sqrt(c\*x^4 + b\*x^2)\*c) - log(2\*c\*x^2 + b + 2\*sqrt(c\*x^4 + b\*x  
x^2)\*sqrt(c))/c^(3/2)) + A\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b)

**mupad** [B] time = 0.63, size = 78, normalized size = 1.16

$$\frac{B \ln \left( \frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2} \right)}{2c^{3/2}} + \frac{Ax^2}{b\sqrt{cx^4 + bx^2}} - \frac{Bx^2}{c\sqrt{cx^4 + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2),x)

[Out] (B\*log((b/2 + c\*x^2)/c^(1/2) + (b\*x^2 + c\*x^4)^(1/2)))/(2\*c^(3/2)) + (A\*x^2  
)/(b\*(b\*x^2 + c\*x^4)^(1/2)) - (B\*x^2)/(c\*(b\*x^2 + c\*x^4)^(1/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{(x^2 (b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*\*3\*(A + B\*x\*\*2)/(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2), x)

$$3.149 \quad \int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=37

$$-\frac{Ab - x^2(bB - 2Ac)}{b^2\sqrt{bx^2 + cx^4}}$$

**Rubi [A]** time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {2034, 636}

$$-\frac{Ab - x^2(bB - 2Ac)}{b^2\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -((A\*b - (b\*B - 2\*A\*c)\*x^2)/(b^2\*Sqrt[b\*x^2 + c\*x^4]))

Rule 636

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(3/2), x\_Symbol] :> Simp[(-2\*(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0]

Rule 2034

Int[(x\_)^(m\_.)\*((b\_.)\*(x\_)^(k\_.) + (a\_.)\*(x\_)^(j\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = \frac{1}{2} \text{Subst} \left( \int \frac{A+Bx}{(bx+cx^2)^{3/2}} dx, x, x^2 \right) \\ = -\frac{Ab - (bB - 2Ac)x^2}{b^2\sqrt{bx^2 + cx^4}}$$

**Mathematica [A]** time = 0.02, size = 37, normalized size = 1.00

$$\frac{bBx^2 - A(b + 2cx^2)}{b^2\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (b\*B\*x^2 - A\*(b + 2\*c\*x^2))/(b^2\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.39, size = 49, normalized size = 1.32

$$\frac{\sqrt{bx^2 + cx^4} (-Ab - 2Acx^2 + bBx^2)}{b^2x^2(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2),x]

[Out] ((-(A\*b) + b\*B\*x^2 - 2\*A\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(b^2\*x^2\*(b + c\*x^2))

**fricas [A]** time = 0.41, size = 49, normalized size = 1.32

$$\frac{\sqrt{cx^4 + bx^2} ((Bb - 2Ac)x^2 - Ab)}{b^2cx^4 + b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="fricas")

[Out] sqrt(c\*x^4 + b\*x^2)\*((B\*b - 2\*A\*c)\*x^2 - A\*b)/(b^2\*c\*x^4 + b^3\*x^2)

**giac [A]** time = 0.21, size = 36, normalized size = 0.97

$$\frac{\frac{(Bb-2Ac)x^2}{b^2} - \frac{A}{b}}{\sqrt{cx^4 + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] ((B\*b - 2\*A\*c)\*x^2/b^2 - A/b)/sqrt(c\*x^4 + b\*x^2)

**maple [A]** time = 0.05, size = 47, normalized size = 1.27

$$\frac{(cx^2 + b)(2Acx^2 - Bbx^2 + Ab)x^2}{(cx^4 + bx^2)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out] -(c\*x^2+b)\*x^2\*(2\*A\*c\*x^2-B\*b\*x^2+A\*b)/b^2/(c\*x^4+b\*x^2)^(3/2)

**maxima [A]** time = 1.49, size = 65, normalized size = 1.76

$$-A\left(\frac{2cx^2}{\sqrt{cx^4 + bx^2}b^2} + \frac{1}{\sqrt{cx^4 + bx^2}b}\right) + \frac{Bx^2}{\sqrt{cx^4 + bx^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] -A\*(2\*c\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^2) + 1/(sqrt(c\*x^4 + b\*x^2)\*b)) + B\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b)

**mapad [B]** time = 0.18, size = 53, normalized size = 1.43

$$\frac{\left(\frac{A}{b} - x^2\left(\frac{B}{b} - \frac{2Ac}{b^2}\right)\right)\sqrt{cx^4 + bx^2}}{x(cx^3 + bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)`

[Out] `-((A/b - x^2*(B/b - (2*A*c)/b^2))*(b*x^2 + c*x^4)^(1/2))/(x*(b*x + c*x^3))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{(x^2(b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(3/2), x)`

[Out] `Integral(x*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)`

$$3.150 \quad \int \frac{A+Bx^2}{x(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=66

$$-\frac{(b+2cx^2)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}} - \frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.16, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2034, 792, 613}

$$-\frac{(b+2cx^2)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}} - \frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] -A/(3\*b\*x^2\*Sqrt[b\*x^2 + c\*x^4]) - ((3\*b\*B - 4\*A\*c)\*(b + 2\*c\*x^2))/(3\*b^3\*Sqrt[b\*x^2 + c\*x^4])

#### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x(bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x(bx + cx^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{A}{3bx^2\sqrt{bx^2 + cx^4}} + \frac{\left(bB - Ac + \frac{1}{2}(bB - 2Ac)\right) \text{Subst} \left( \int \frac{1}{(bx+cx^2)^{3/2}} dx, x, x^2 \right)}{3b} \\ &= -\frac{A}{3bx^2\sqrt{bx^2 + cx^4}} - \frac{(3bB - 4Ac)(b + 2cx^2)}{3b^3\sqrt{bx^2 + cx^4}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 64, normalized size = 0.97

$$\frac{A(-b^2 + 4bcx^2 + 8c^2x^4) - 3bBx^2(b + 2cx^2)}{3b^3x^2\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (-3\*b\*B\*x^2\*(b + 2\*c\*x^2) + A\*(-b^2 + 4\*b\*c\*x^2 + 8\*c^2\*x^4))/(3\*b^3\*x^2\*sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.39, size = 75, normalized size = 1.14

$$\frac{\sqrt{bx^2 + cx^4}(-Ab^2 + 4Abcx^2 + 8Ac^2x^4 - 3b^2Bx^2 - 6bBcx^4)}{3b^3x^4(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (sqrt[b\*x^2 + c\*x^4]\*(-(A\*b^2) - 3\*b^2\*B\*x^2 + 4\*A\*b\*c\*x^2 - 6\*b\*B\*c\*x^4 + 8\*A\*c^2\*x^4))/(3\*b^3\*x^4\*(b + c\*x^2))

**fricas [A]** time = 0.42, size = 72, normalized size = 1.09

$$\frac{(2(3Bbc - 4Ac^2)x^4 + Ab^2 + (3Bb^2 - 4Abc)x^2)\sqrt{cx^4 + bx^2}}{3(b^3cx^6 + b^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] -1/3\*(2\*(3\*B\*b\*c - 4\*A\*c^2)\*x^4 + A\*b^2 + (3\*B\*b^2 - 4\*A\*b\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^3\*c\*x^6 + b^4\*x^4)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(3/2), x, algorithm="giac")

[Out] integrate((B\*x^2 + A)/((c\*x^4 + b\*x^2)^(3/2)\*x), x)

**maple** [A] time = 0.05, size = 66, normalized size = 1.00

$$\frac{(cx^2 + b)(-8Ac^2x^4 + 6Bbcx^4 - 4Abcx^2 + 3Bb^2x^2 + b^2A)}{3(cx^4 + bx^2)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(3/2), x)

[Out] -1/3\*(c\*x^2+b)\*(-8\*A\*c^2\*x^4+6\*B\*b\*c\*x^4-4\*A\*b\*c\*x^2+3\*B\*b^2\*x^2+A\*b^2)/b^3/(c\*x^4+b\*x^2)^(3/2)

**maxima** [A] time = 1.39, size = 112, normalized size = 1.70

$$-B\left(\frac{2cx^2}{\sqrt{cx^4 + bx^2}b^2} + \frac{1}{\sqrt{cx^4 + bx^2}b}\right) + \frac{1}{3}A\left(\frac{8c^2x^2}{\sqrt{cx^4 + bx^2}b^3} + \frac{4c}{\sqrt{cx^4 + bx^2}b^2} - \frac{1}{\sqrt{cx^4 + bx^2}bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x/(c\*x^4+b\*x^2)^(3/2), x, algorithm="maxima")

[Out] -B\*(2\*c\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^2) + 1/(sqrt(c\*x^4 + b\*x^2)\*b)) + 1/3\*A\*(8\*c^2\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^3) + 4\*c/(sqrt(c\*x^4 + b\*x^2)\*b^2) - 1/(sqrt(c\*x^4 + b\*x^2)\*b\*x^2))

**mupad** [B] time = 0.27, size = 70, normalized size = 1.06

$$\frac{\sqrt{cx^4 + bx^2} (3Bb^2x^2 + Ab^2 + 6Bbcx^4 - 4Abcx^2 - 8Ac^2x^4)}{3b^3x^4 (cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x\*(b\*x^2 + c\*x^4)^(3/2)), x)

[Out] -((b\*x^2 + c\*x^4)^(1/2)\*(A\*b^2 + 3\*B\*b^2\*x^2 - 8\*A\*c^2\*x^4 - 4\*A\*b\*c\*x^2 + 6\*B\*b\*c\*x^4))/(3\*b^3\*x^4\*(b + c\*x^2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(x^2(b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2), x)

[Out] Integral((A + B\*x\*\*2)/(x\*(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)), x)



$$3.151 \quad \int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=101

$$\frac{4c(b+2cx^2)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}} - \frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}} - \frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.22, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 613}

$$\frac{4c(b+2cx^2)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}} - \frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}} - \frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] -A/(5\*b\*x^4\*Sqrt[b\*x^2 + c\*x^4]) - (5\*b\*B - 6\*A\*c)/(15\*b^2\*x^2\*Sqrt[b\*x^2 + c\*x^4]) + (4\*c\*(5\*b\*B - 6\*A\*c)\*(b + 2\*c\*x^2))/(15\*b^4\*Sqrt[b\*x^2 + c\*x^4])

#### Rule 613

Int[((a\_.) + (b\_.)\*(x\_)) + (c\_.)\*(x\_)^2]^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_)) + (c\_.)\*(x\_)^2]^(p\_), x\_Symbol] := -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_)) + (c\_.)\*(x\_)^2]^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^(m\_)\*((b\_.)\*(x\_)^(k\_) + (a\_.)\*(x\_)^(j\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^3 (bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^2 (bx + cx^2)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{A}{5bx^4 \sqrt{bx^2 + cx^4}} + \frac{\left(\frac{1}{2}(bB - 2Ac) - 2(-bB + Ac)\right) \text{Subst} \left( \int \frac{1}{x(bx+cx^2)^{3/2}} dx, x, x^2 \right)}{5b} \\
&= -\frac{A}{5bx^4 \sqrt{bx^2 + cx^4}} - \frac{5bB - 6Ac}{15b^2 x^2 \sqrt{bx^2 + cx^4}} - \frac{(2c(5bB - 6Ac)) \text{Subst} \left( \int \frac{1}{(bx+cx^2)^{3/2}} dx, x, x^2 \right)}{15b^2} \\
&= -\frac{A}{5bx^4 \sqrt{bx^2 + cx^4}} - \frac{5bB - 6Ac}{15b^2 x^2 \sqrt{bx^2 + cx^4}} + \frac{4c(5bB - 6Ac)(b + 2cx^2)}{15b^4 \sqrt{bx^2 + cx^4}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 85, normalized size = 0.84

$$\frac{-3A(b^3 - 2b^2cx^2 + 8bc^2x^4 + 16c^3x^6) - 5bBx^2(b^2 - 4bcx^2 - 8c^2x^4)}{15b^4x^4\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (-5\*b\*B\*x^2\*(b^2 - 4\*b\*c\*x^2 - 8\*c^2\*x^4) - 3\*A\*(b^3 - 2\*b^2\*c\*x^2 + 8\*b\*c^2\*x^4 + 16\*c^3\*x^6))/(15\*b^4\*x^4\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.42, size = 99, normalized size = 0.98

$$\frac{\sqrt{bx^2 + cx^4} (-3Ab^3 + 6Ab^2cx^2 - 24Abc^2x^4 - 48Ac^3x^6 - 5b^3Bx^2 + 20b^2Bcx^4 + 40bBc^2x^6)}{15b^4x^6(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-3\*A\*b^3 - 5\*b^3\*B\*x^2 + 6\*A\*b^2\*c\*x^2 + 20\*b^2\*B\*c\*x^4 - 24\*A\*b\*c^2\*x^4 + 40\*b\*B\*c^2\*x^6 - 48\*A\*c^3\*x^6))/(15\*b^4\*x^6\*(b + c\*x^2))

**fricas [A]** time = 0.42, size = 98, normalized size = 0.97

$$\frac{(8(5Bbc^2 - 6Ac^3)x^6 + 4(5Bb^2c - 6Abc^2)x^4 - 3Ab^3 - (5Bb^3 - 6Ab^2c)x^2)\sqrt{cx^4 + bx^2}}{15(b^4cx^8 + b^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/15\*(8\*(5\*B\*b\*c^2 - 6\*A\*c^3)\*x^6 + 4\*(5\*B\*b^2\*c - 6\*A\*b\*c^2)\*x^4 - 3\*A\*b^3 - (5\*B\*b^3 - 6\*A\*b^2\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^4\*c\*x^8 + b^5\*x^6)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B\*x^2 + A)/((c\*x^4 + b\*x^2)^(3/2)\*x^3), x)

**maple [A]** time = 0.05, size = 94, normalized size = 0.93

$$\frac{(cx^2 + b)(48Ac^3x^6 - 40Bbc^2x^6 + 24Abc^2x^4 - 20Bb^2cx^4 - 6Ab^2cx^2 + 5Bb^3x^2 + 3Ab^3)}{15(cx^4 + bx^2)^{\frac{3}{2}}b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(3/2),x)

[Out] -1/15\*(c\*x^2+b)\*(48\*A\*c^3\*x^6-40\*B\*b\*c^2\*x^6+24\*A\*b\*c^2\*x^4-20\*B\*b^2\*c\*x^4-6\*A\*b^2\*c\*x^2+5\*B\*b^3\*x^2+3\*A\*b^3)/x^2/b^4/(c\*x^4+b\*x^2)^(3/2)

**maxima [A]** time = 1.50, size = 160, normalized size = 1.58

$$\frac{1}{3}B\left(\frac{8c^2x^2}{\sqrt{cx^4+bx^2}b^3} + \frac{4c}{\sqrt{cx^4+bx^2}b^2} - \frac{1}{\sqrt{cx^4+bx^2}bx^2}\right) - \frac{1}{5}A\left(\frac{16c^3x^2}{\sqrt{cx^4+bx^2}b^4} + \frac{8c^2}{\sqrt{cx^4+bx^2}b^3} - \frac{2c}{\sqrt{cx^4+bx^2}b^2x^2} + \frac{1}{\sqrt{cx^4+bx^2}bx^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^3/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/3\*B\*(8\*c^2\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^3) + 4\*c/(sqrt(c\*x^4 + b\*x^2)\*b^2) - 1/(sqrt(c\*x^4 + b\*x^2)\*b\*x^2)) - 1/5\*A\*(16\*c^3\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^4) + 8\*c^2/(sqrt(c\*x^4 + b\*x^2)\*b^3) - 2\*c/(sqrt(c\*x^4 + b\*x^2)\*b^2\*x^2) + 1/(sqrt(c\*x^4 + b\*x^2)\*b\*x^4))

**mupad [B]** time = 0.42, size = 95, normalized size = 0.94

$$\frac{\sqrt{cx^4 + bx^2} (5Bb^3x^2 + 3Ab^3 - 20Bb^2cx^4 - 6Ab^2cx^2 - 40Bbc^2x^6 + 24Abc^2x^4 + 48Ac^3x^6)}{15b^4x^6 (cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^3\*(b\*x^2 + c\*x^4)^(3/2)),x)

[Out] -((b\*x^2 + c\*x^4)^(1/2)\*(3\*A\*b^3 + 5\*B\*b^3\*x^2 + 48\*A\*c^3\*x^6 - 6\*A\*b^2\*c\*x^2 + 24\*A\*b\*c^2\*x^4 - 20\*B\*b^2\*c\*x^4 - 40\*B\*b\*c^2\*x^6))/(15\*b^4\*x^6\*(b + c\*x^2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3 (x^2 (b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*3/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*3\*(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)), x)

$$3.152 \quad \int \frac{A+Bx^2}{x^5(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=138

$$-\frac{8c^2(b+2cx^2)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} - \frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2034, 792, 658, 613}

$$-\frac{8c^2(b+2cx^2)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} - \frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)^(3/2)),x]

[Out] -A/(7\*b\*x^6\*Sqrt[b\*x^2 + c\*x^4]) - (7\*b\*B - 8\*A\*c)/(35\*b^2\*x^4\*Sqrt[b\*x^2 + c\*x^4]) + (2\*c\*(7\*b\*B - 8\*A\*c))/(35\*b^3\*x^2\*Sqrt[b\*x^2 + c\*x^4]) - (8\*c^2\*(7\*b\*B - 8\*A\*c)\*(b + 2\*c\*x^2))/(35\*b^5\*Sqrt[b\*x^2 + c\*x^4])

#### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 658

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^p, x\_Symbol] := -Simp[(e\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((m + p + 1)\*(2\*c\*d - b\*e)), x] + Dist[(c\*Simplify[m + 2\*p + 2])/((m + p + 1)\*(2\*c\*d - b\*e)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

#### Rule 792

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_) + (a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^p, x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

#### Rule 2034

Int[(x\_)^m\*((b\_.)\*(x\_)^k + (a\_.)\*(x\_)^j)^p\*((c\_) + (d\_.)\*(x\_)^n)^q, x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a\*x^Simplify[j/n] + b\*x^Simplify[k/n])^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^5 (bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{A + Bx}{x^3 (bx + cx^2)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{A}{7bx^6 \sqrt{bx^2 + cx^4}} + \frac{\left(\frac{1}{2}(bB - 2Ac) - 3(-bB + Ac)\right) \text{Subst} \left( \int \frac{1}{x^2 (bx + cx^2)^{3/2}} dx, x, x^2 \right)}{7b} \\
&= -\frac{A}{7bx^6 \sqrt{bx^2 + cx^4}} - \frac{7bB - 8Ac}{35b^2 x^4 \sqrt{bx^2 + cx^4}} - \frac{(3c(7bB - 8Ac)) \text{Subst} \left( \int \frac{1}{x (bx + cx^2)^{3/2}} dx, x, x^2 \right)}{35b^2} \\
&= -\frac{A}{7bx^6 \sqrt{bx^2 + cx^4}} - \frac{7bB - 8Ac}{35b^2 x^4 \sqrt{bx^2 + cx^4}} + \frac{2c(7bB - 8Ac)}{35b^3 x^2 \sqrt{bx^2 + cx^4}} + \frac{(4c^2(7bB - 8Ac)) \text{Subst} \left( \int \frac{1}{x^2 (bx + cx^2)^{3/2}} dx, x, x^2 \right)}{35b^2} \\
&= -\frac{A}{7bx^6 \sqrt{bx^2 + cx^4}} - \frac{7bB - 8Ac}{35b^2 x^4 \sqrt{bx^2 + cx^4}} + \frac{2c(7bB - 8Ac)}{35b^3 x^2 \sqrt{bx^2 + cx^4}} - \frac{8c^2(7bB - 8Ac)(b + cx^2)}{35b^5 \sqrt{bx^2 + cx^4}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 75, normalized size = 0.54

$$\frac{x^2 (b^3 - 2b^2 cx^2 + 8bc^2 x^4 + 16c^3 x^6) (8Ac - 7bB) - 5Ab^4}{35b^5 x^6 \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (-5\*A\*b^4 + (-7\*b\*B + 8\*A\*c)\*x^2\*(b^3 - 2\*b^2\*c\*x^2 + 8\*b\*c^2\*x^4 + 16\*c^3\*x^6))/(35\*b^5\*x^6\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.51, size = 123, normalized size = 0.89

$$\frac{\sqrt{bx^2 + cx^4} (-5Ab^4 + 8Ab^3cx^2 - 16Ab^2c^2x^4 + 64Abc^3x^6 + 128Ac^4x^8 - 7b^4Bx^2 + 14b^3Bcx^4 - 56b^2Bc^2x^6 - 112bBc^3x^8)}{35b^5x^8(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-5\*A\*b^4 - 7\*b^4\*B\*x^2 + 8\*A\*b^3\*c\*x^2 + 14\*b^3\*B\*c\*x^4 - 16\*A\*b^2\*c^2\*x^4 - 56\*b^2\*B\*c^2\*x^6 + 64\*A\*b\*c^3\*x^6 - 112\*b\*B\*c^3\*x^8 + 128\*A\*c^4\*x^8))/(35\*b^5\*x^8\*(b + c\*x^2))

**fricas [A]** time = 0.43, size = 121, normalized size = 0.88

$$\frac{(16(7Bbc^3 - 8Ac^4)x^8 + 8(7Bb^2c^2 - 8Abc^3)x^6 + 5Ab^4 - 2(7Bb^3c - 8Ab^2c^2)x^4 + (7Bb^4 - 8Ab^3c)x^2)\sqrt{cx^4 + bx^2}}{35(b^5cx^{10} + b^6x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] -1/35\*(16\*(7\*B\*b\*c^3 - 8\*A\*c^4)\*x^8 + 8\*(7\*B\*b^2\*c^2 - 8\*A\*b\*c^3)\*x^6 + 5\*A\*b^4 - 2\*(7\*B\*b^3\*c - 8\*A\*b^2\*c^2)\*x^4 + (7\*B\*b^4 - 8\*A\*b^3\*c)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(b^5\*c\*x^10 + b^6\*x^8)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B\*x^2 + A)/((c\*x^4 + b\*x^2)^(3/2)\*x^5), x)

**maple** [A] time = 0.05, size = 118, normalized size = 0.86

$$\frac{(cx^2 + b)(-128Ac^4x^8 + 112Bbc^3x^8 - 64Abc^3x^6 + 56Bb^2c^2x^6 + 16Ab^2c^2x^4 - 14Bb^3cx^4 - 8Ab^3cx^2 + 7Bb^4x^2 + 5Ab^4)}{35(cx^4 + bx^2)^{\frac{3}{2}}b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(3/2),x)

[Out] -1/35\*(c\*x^2+b)\*(-128\*A\*c^4\*x^8+112\*B\*b\*c^3\*x^8-64\*A\*b\*c^3\*x^6+56\*B\*b^2\*c^2\*x^6+16\*A\*b^2\*c^2\*x^4-14\*B\*b^3\*c\*x^4-8\*A\*b^3\*c\*x^2+7\*B\*b^4\*x^2+5\*A\*b^4)/x^4/b^5/(c\*x^4+b\*x^2)^(3/2)

**maxima** [A] time = 1.48, size = 208, normalized size = 1.51

$$\frac{1}{5}B\left(\frac{16c^3x^2}{\sqrt{cx^4+bx^2}b^4} + \frac{8c^2}{\sqrt{cx^4+bx^2}b^3} - \frac{2c}{\sqrt{cx^4+bx^2}b^2x^2} + \frac{1}{\sqrt{cx^4+bx^2}bx^4}\right) + \frac{1}{35}A\left(\frac{128c^4x^2}{\sqrt{cx^4+bx^2}b^5} + \frac{64c^3}{\sqrt{cx^4+bx^2}b^4} - \frac{16c^2}{\sqrt{cx^4+bx^2}b^3x^2} + \frac{8c}{\sqrt{cx^4+bx^2}b^2x^4} - \frac{5}{\sqrt{cx^4+bx^2}bx^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^5/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] -1/5\*B\*(16\*c^3\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^4) + 8\*c^2/(sqrt(c\*x^4 + b\*x^2)\*b^3) - 2\*c/(sqrt(c\*x^4 + b\*x^2)\*b^2\*x^2) + 1/(sqrt(c\*x^4 + b\*x^2)\*b\*x^4)) + 1/35\*A\*(128\*c^4\*x^2/(sqrt(c\*x^4 + b\*x^2)\*b^5) + 64\*c^3/(sqrt(c\*x^4 + b\*x^2)\*b^4) - 16\*c^2/(sqrt(c\*x^4 + b\*x^2)\*b^3\*x^2) + 8\*c/(sqrt(c\*x^4 + b\*x^2)\*b^2\*x^4) - 5/(sqrt(c\*x^4 + b\*x^2)\*b\*x^6))

**mupad** [B] time = 0.56, size = 173, normalized size = 1.25

$$\frac{(7Bb^2 - 13Abc)\sqrt{cx^4 + bx^2}}{35b^4x^6} - \frac{\left(x^2\left(\frac{58Ac^4 - 42Bbc^3}{35b^5} - \frac{2c^3(93Ac - 77Bb)}{35b^5}\right) - \frac{c^2(93Ac - 77Bb)}{35b^4}\right)\sqrt{cx^4 + bx^2}}{x^2(cx^2 + b)} - \frac{A\sqrt{cx^4 + bx^2}}{7b^2x^8} - \frac{c(29Ac - 21Bb)\sqrt{cx^4 + bx^2}}{35b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^5\*(b\*x^2 + c\*x^4)^(3/2)),x)

[Out] - ((7\*B\*b^2 - 13\*A\*b\*c)\*(b\*x^2 + c\*x^4)^(1/2))/(35\*b^4\*x^6) - ((x^2\*((58\*A\*c^4 - 42\*B\*b\*c^3)/(35\*b^5) - (2\*c^3\*(93\*A\*c - 77\*B\*b))/(35\*b^5)) - (c^2\*(93\*A\*c - 77\*B\*b))/(35\*b^4))\*(b\*x^2 + c\*x^4)^(1/2))/(x^2\*(b + c\*x^2)) - (A\*(b\*x^2 + c\*x^4)^(1/2))/(7\*b^2\*x^8) - (c\*(29\*A\*c - 21\*B\*b)\*(b\*x^2 + c\*x^4)^(1/2))/(35\*b^4\*x^4)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5 \left(x^2 (b + cx^2)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*5/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*5\*(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)), x)

$$3.153 \quad \int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=139

$$\frac{8b\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^4x} - \frac{4x\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^3} + \frac{x^3\sqrt{bx^2+cx^4}(6bB-5Ac)}{5bc^2} - \frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.25, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2037, 2016, 1588}

$$\frac{x^3\sqrt{bx^2+cx^4}(6bB-5Ac)}{5bc^2} - \frac{4x\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^3} + \frac{8b\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^4x} - \frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^7)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) + (8\*b\*(6\*b\*B - 5\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(15\*c^4\*x) - (4\*(6\*b\*B - 5\*A\*c)\*x\*Sqrt[b\*x^2 + c\*x^4])/(15\*c^3) + ((6\*b\*B - 5\*A\*c)\*x^3\*Sqrt[b\*x^2 + c\*x^4])/(5\*b\*c^2)

Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2016

Int[((c\_)\*(x\_)^(m\_.)\*((a\_)\*(x\_)^(j\_.) + (b\_)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2037

Int[((e\_)\*(x\_)^(m\_.)\*((a\_)\*(x\_)^(j\_.) + (b\_)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_.)), x\_Symbol] := -Simp[(e^(j - 1)\*(b\*c - a\*d)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(e^j\*(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1)))/(a\*b\*n\*(p + 1)), Int[(e\*x)^(m - j)\*(a\*x^j + b\*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx &= -\frac{(bB - Ac)x^7}{bc\sqrt{bx^2 + cx^4}} + \frac{(6bB - 5Ac) \int \frac{x^6}{\sqrt{bx^2 + cx^4}} dx}{bc} \\
&= -\frac{(bB - Ac)x^7}{bc\sqrt{bx^2 + cx^4}} + \frac{(6bB - 5Ac)x^3\sqrt{bx^2 + cx^4}}{5bc^2} - \frac{(4(6bB - 5Ac)) \int \frac{x^4}{\sqrt{bx^2 + cx^4}} dx}{5c^2} \\
&= -\frac{(bB - Ac)x^7}{bc\sqrt{bx^2 + cx^4}} - \frac{4(6bB - 5Ac)x\sqrt{bx^2 + cx^4}}{15c^3} + \frac{(6bB - 5Ac)x^3\sqrt{bx^2 + cx^4}}{5bc^2} + \frac{(8b(6bB - 5Ac)) \int \frac{x^2}{\sqrt{bx^2 + cx^4}} dx}{15c^3} \\
&= -\frac{(bB - Ac)x^7}{bc\sqrt{bx^2 + cx^4}} + \frac{8b(6bB - 5Ac)\sqrt{bx^2 + cx^4}}{15c^4x} - \frac{4(6bB - 5Ac)x\sqrt{bx^2 + cx^4}}{15c^3} + \frac{(6bB - 5Ac) \int \frac{x^0}{\sqrt{bx^2 + cx^4}} dx}{15c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 82, normalized size = 0.59

$$\frac{x(-8b^2c(5A - 3Bx^2) - 2bc^2x^2(10A + 3Bx^2) + c^3x^4(5A + 3Bx^2) + 48b^3B)}{15c^4\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(48\*b^3\*B - 8\*b^2\*c\*(5\*A - 3\*B\*x^2) + c^3\*x^4\*(5\*A + 3\*B\*x^2) - 2\*b\*c^2\*x^2\*(10\*A + 3\*B\*x^2)))/(15\*c^4\*sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 1.03, size = 96, normalized size = 0.69

$$\frac{\sqrt{bx^2 + cx^4}(-40Ab^2c - 20Abc^2x^2 + 5Ac^3x^4 + 48b^3B + 24b^2Bcx^2 - 6bBc^2x^4 + 3Bc^3x^6)}{15c^4x(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (sqrt[b\*x^2 + c\*x^4]\*(48\*b^3\*B - 40\*A\*b^2\*c + 24\*b^2\*B\*c\*x^2 - 20\*A\*b\*c^2\*x^2 - 6\*b\*B\*c^2\*x^4 + 5\*A\*c^3\*x^4 + 3\*B\*c^3\*x^6))/(15\*c^4\*x\*(b + c\*x^2))

**fricas [A]** time = 0.42, size = 93, normalized size = 0.67

$$\frac{(3Bc^3x^6 - (6Bbc^2 - 5Ac^3)x^4 + 48Bb^3 - 40Ab^2c + 4(6Bb^2c - 5Abc^2)x^2)\sqrt{cx^4 + bx^2}}{15(c^5x^3 + bc^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/15\*(3\*B\*c^3\*x^6 - (6\*B\*b\*c^2 - 5\*A\*c^3)\*x^4 + 48\*B\*b^3 - 40\*A\*b^2\*c + 4\*(6\*B\*b^2\*c - 5\*A\*b\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^5\*x^3 + b\*c^4\*x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^8}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B\*x^2 + A)\*x^8/(c\*x^4 + b\*x^2)^(3/2), x)

**maple [A]** time = 0.05, size = 91, normalized size = 0.65

$$\frac{(cx^2 + b)(-3Bc^3x^6 - 5Ac^3x^4 + 6Bbc^2x^4 + 20Abc^2x^2 - 24Bb^2cx^2 + 40Ab^2c - 48Bb^3)x^3}{15(cx^4 + bx^2)^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out] -1/15\*(c\*x^2+b)\*(-3\*B\*c^3\*x^6-5\*A\*c^3\*x^4+6\*B\*b\*c^2\*x^4+20\*A\*b\*c^2\*x^2-24\*B\*b^2\*c\*x^2+40\*A\*b^2\*c-48\*B\*b^3)\*x^3/c^4/(c\*x^4+b\*x^2)^(3/2)

**maxima [A]** time = 1.58, size = 82, normalized size = 0.59

$$\frac{(c^2x^4 - 4bcx^2 - 8b^2)A}{3\sqrt{cx^2 + b}c^3} + \frac{(c^3x^6 - 2bc^2x^4 + 8b^2cx^2 + 16b^3)B}{5\sqrt{cx^2 + b}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/3\*(c^2\*x^4 - 4\*b\*c\*x^2 - 8\*b^2)\*A/(sqrt(c\*x^2 + b)\*c^3) + 1/5\*(c^3\*x^6 - 2\*b\*c^2\*x^4 + 8\*b^2\*c\*x^2 + 16\*b^3)\*B/(sqrt(c\*x^2 + b)\*c^4)

**mupad [B]** time = 0.40, size = 92, normalized size = 0.66

$$\frac{\sqrt{cx^4 + bx^2} (48Bb^3 + 24Bb^2cx^2 - 40Ab^2c - 6Bbc^2x^4 - 20Abc^2x^2 + 3Bc^3x^6 + 5Ac^3x^4)}{15c^4x(cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2),x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*(48\*B\*b^3 + 5\*A\*c^3\*x^4 + 3\*B\*c^3\*x^6 - 40\*A\*b^2\*c - 20\*A\*b\*c^2\*x^2 + 24\*B\*b^2\*c\*x^2 - 6\*B\*b\*c^2\*x^4))/(15\*c^4\*x\*(b + c\*x^2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8(A + Bx^2)}{(x^2(b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*\*8\*(A + B\*x\*\*2)/(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2), x)

$$3.154 \quad \int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=104

$$-\frac{2\sqrt{bx^2+cx^4}(4bB-3Ac)}{3c^3x} + \frac{x\sqrt{bx^2+cx^4}(4bB-3Ac)}{3bc^2} - \frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2037, 2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}(4bB-3Ac)}{3bc^2} - \frac{2\sqrt{bx^2+cx^4}(4bB-3Ac)}{3c^3x} - \frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^5)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) - (2\*(4\*b\*B - 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(3\*c^3\*x) + ((4\*b\*B - 3\*A\*c)\*x\*Sqrt[b\*x^2 + c\*x^4])/(3\*b\*c^2)

Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2016

Int[((c\_)\*(x\_))^(m\_)\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(m + j\*p + 1)), x] - Dist[(b\*(m + n\*p + n - j + 1))/(a\*c^(n - j)\*(m + j\*p + 1)), Int[(c\*x)^(m + n - j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n\*p + n - j + 1)/(n - j)], 0] && NeQ[m + j\*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2037

Int[((e\_)\*(x\_))^(m\_)\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(jn\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := -Simp[(e^(j - 1)\*(b\*c - a\*d)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(e^j\*(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1)))/(a\*b\*n\*(p + 1)), Int[(e\*x)^(m - j)\*(a\*x^j + b\*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])

Rubi steps

$$\begin{aligned} \int \frac{x^6 (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx &= -\frac{(bB - Ac)x^5}{bc\sqrt{bx^2 + cx^4}} + \frac{(4bB - 3Ac) \int \frac{x^4}{\sqrt{bx^2 + cx^4}} dx}{bc} \\ &= -\frac{(bB - Ac)x^5}{bc\sqrt{bx^2 + cx^4}} + \frac{(4bB - 3Ac)x\sqrt{bx^2 + cx^4}}{3bc^2} - \frac{(2(4bB - 3Ac)) \int \frac{x^2}{\sqrt{bx^2 + cx^4}} dx}{3c^2} \\ &= -\frac{(bB - Ac)x^5}{bc\sqrt{bx^2 + cx^4}} - \frac{2(4bB - 3Ac)\sqrt{bx^2 + cx^4}}{3c^3x} + \frac{(4bB - 3Ac)x\sqrt{bx^2 + cx^4}}{3bc^2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.58

$$\frac{x(b(6Ac - 4Bcx^2) + c^2x^2(3A + Bx^2) - 8b^2B)}{3c^3\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(-8\*b^2\*B + c^2\*x^2\*(3\*A + B\*x^2) + b\*(6\*A\*c - 4\*B\*c\*x^2)))/(3\*c^3\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.85, size = 71, normalized size = 0.68

$$\frac{\sqrt{bx^2 + cx^4} (6Abc + 3Ac^2x^2 - 8b^2B - 4bBcx^2 + Bc^2x^4)}{3c^3x(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-8\*b^2\*B + 6\*A\*b\*c - 4\*b\*B\*c\*x^2 + 3\*A\*c^2\*x^2 + B\*c^2\*x^4))/(3\*c^3\*x\*(b + c\*x^2))

**fricas [A]** time = 0.40, size = 68, normalized size = 0.65

$$\frac{(Bc^2x^4 - 8Bb^2 + 6Abc - (4Bbc - 3Ac^2)x^2)\sqrt{cx^4 + bx^2}}{3(c^4x^3 + bc^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/3\*(B\*c^2\*x^4 - 8\*B\*b^2 + 6\*A\*b\*c - (4\*B\*b\*c - 3\*A\*c^2)\*x^2)\*sqrt(c\*x^4 + b\*x^2)/(c^4\*x^3 + b\*c^3\*x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^6}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="giac")

[Out] integrate((B\*x^2 + A)\*x^6/(c\*x^4 + b\*x^2)^(3/2), x)

**maple** [A] time = 0.05, size = 66, normalized size = 0.63

$$\frac{(cx^2 + b)(Bc^2x^4 + 3Ac^2x^2 - 4Bbcx^2 + 6Abc - 8Bb^2)x^3}{3(cx^4 + bx^2)^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out] 1/3\*(c\*x^2+b)\*(B\*c^2\*x^4+3\*A\*c^2\*x^2-4\*B\*b\*c\*x^2+6\*A\*b\*c-8\*B\*b^2)\*x^3/c^3/(c\*x^4+b\*x^2)^(3/2)

**maxima** [A] time = 1.56, size = 59, normalized size = 0.57

$$\frac{(cx^2 + 2b)A}{\sqrt{cx^2 + b}c^2} + \frac{(c^2x^4 - 4bcx^2 - 8b^2)B}{3\sqrt{cx^2 + b}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] (c\*x^2 + 2\*b)\*A/(sqrt(c\*x^2 + b)\*c^2) + 1/3\*(c^2\*x^4 - 4\*b\*c\*x^2 - 8\*b^2)\*B/(sqrt(c\*x^2 + b)\*c^3)

**mupad** [B] time = 0.28, size = 67, normalized size = 0.64

$$\frac{\sqrt{cx^4 + bx^2}(-8Bb^2 - 4Bbcx^2 + 6Abc + Bc^2x^4 + 3Ac^2x^2)}{3c^3x(cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2),x)

[Out] ((b\*x^2 + c\*x^4)^(1/2)\*(3\*A\*c^2\*x^2 - 8\*B\*b^2 + B\*c^2\*x^4 + 6\*A\*b\*c - 4\*B\*b\*c\*x^2))/(3\*c^3\*x\*(b + c\*x^2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(A + Bx^2)}{(x^2(b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*\*6\*(A + B\*x\*\*2)/(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2), x)

$$3.155 \quad \int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=69

$$\frac{\sqrt{bx^2 + cx^4}(2bB - Ac)}{bc^2x} - \frac{x^3(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

Rubi [A] time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2037, 1588}

$$\frac{\sqrt{bx^2 + cx^4}(2bB - Ac)}{bc^2x} - \frac{x^3(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x^3)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) + ((2\*b\*B - A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(b\*c^2\*x)

Rule 1588

Int[(Pp\_)\*(Qq\_)^(m\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q + 1)\*Qq^(m + 1))/((p + m\*q + 1)\*Coeff[Qq, x, q]), x] /; NeQ[p + m\*q + 1, 0] && EqQ[(p + m\*q + 1)\*Coeff[Qq, x, q]\*Pp, Coeff[Pp, x, p]\*x^(p - q)\*((p - q + 1)\*Qq + (m + 1)\*x\*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2037

Int[((e\_)\*(x\_))^(m\_.)\*((a\_)\*(x\_)^(j\_.) + (b\_)\*(x\_)^(jn\_.))^(p\_.)\*((c\_) + (d\_)\*(x\_)^(n\_.)), x\_Symbol] := -Simp[(e^(j - 1)\*(b\*c - a\*d)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(e^j\*(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1)))/(a\*b\*n\*(p + 1)), Int[(e\*x)^(m - j)\*(a\*x^j + b\*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])

Rubi steps

$$\begin{aligned} \int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx &= -\frac{(bB - Ac)x^3}{bc\sqrt{bx^2 + cx^4}} + \frac{(2bB - Ac) \int \frac{x^2}{\sqrt{bx^2+cx^4}} dx}{bc} \\ &= -\frac{(bB - Ac)x^3}{bc\sqrt{bx^2 + cx^4}} + \frac{(2bB - Ac)\sqrt{bx^2 + cx^4}}{bc^2x} \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.51

$$\frac{x(-Ac + 2bB + Bcx^2)}{c^2\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (x\*(2\*b\*B - A\*c + B\*c\*x^2))/(c^2\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.73, size = 46, normalized size = 0.67

$$\frac{\sqrt{bx^2 + cx^4} (-Ac + 2bB + Bcx^2)}{c^2x(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((2\*b\*B - A\*c + B\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/(c^2\*x\*(b + c\*x^2))

**fricas [A]** time = 0.41, size = 45, normalized size = 0.65

$$\frac{\sqrt{cx^4 + bx^2} (Bcx^2 + 2Bb - Ac)}{c^3x^3 + bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] sqrt(c\*x^4 + b\*x^2)\*(B\*c\*x^2 + 2\*B\*b - A\*c)/(c^3\*x^3 + b\*c^2\*x)

**giac [A]** time = 0.29, size = 60, normalized size = 0.87

$$-\frac{2B\sqrt{b}}{\left(\left(\sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x}\right)^2 - c\right)c} + \frac{Bb - Ac}{\sqrt{c + \frac{b}{x^2}}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="giac")

[Out] -2\*B\*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)\*c) + (B\*b - A\*c)/(sqrt(c + b/x^2)\*c^2\*x)

**maple [A]** time = 0.05, size = 44, normalized size = 0.64

$$\frac{(cx^2 + b)(-Bcx^2 + Ac - 2bB)x^3}{(cx^4 + bx^2)^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x)

[Out] -(c\*x^2+b)\*(-B\*c\*x^2+A\*c-2\*B\*b)\*x^3/c^2/(c\*x^4+b\*x^2)^(3/2)

**maxima [A]** time = 1.57, size = 39, normalized size = 0.57

$$\frac{(cx^2 + 2b)B}{\sqrt{cx^2 + b}c^2} - \frac{A}{\sqrt{cx^2 + b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="maxima")

[Out] (c\*x^2 + 2\*b)\*B/(sqrt(c\*x^2 + b)\*c^2) - A/(sqrt(c\*x^2 + b)\*c)

**mupad** [B] time = 0.21, size = 44, normalized size = 0.64

$$\frac{\sqrt{cx^4 + bx^2} (Bcx^2 - Ac + 2Bb)}{c^2 x (cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)`

[Out] `((b*x^2 + c*x^4)^(1/2)*(2*B*b - A*c + B*c*x^2))/(c^2*x*(b + c*x^2))`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^2)}{(x^2 (b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(3/2), x)`

[Out] `Integral(x**4*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)`

$$3.156 \quad \int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=64

$$-\frac{A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}} - \frac{x(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

**Rubi [A]** time = 0.14, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {2037, 2008, 206}

$$-\frac{A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}} - \frac{x(bB - Ac)}{bc\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -(((b\*B - A\*c)\*x)/(b\*c\*Sqrt[b\*x^2 + c\*x^4])) - (A\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/b^(3/2)

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2037

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := -Simp[(e^(j - 1)\*(b\*c - a\*d)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(e^j\*(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1)))/(a\*b\*n\*(p + 1)), Int[(e\*x)^(m - j)\*(a\*x^j + b\*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])

#### Rubi steps

$$\begin{aligned} \int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx &= -\frac{(bB - Ac)x}{bc\sqrt{bx^2 + cx^4}} + \frac{A \int \frac{1}{\sqrt{bx^2+cx^4}} dx}{b} \\ &= -\frac{(bB - Ac)x}{bc\sqrt{bx^2 + cx^4}} - \frac{A \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{b} \\ &= -\frac{(bB - Ac)x}{bc\sqrt{bx^2 + cx^4}} - \frac{A \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}} \end{aligned}$$



**Mathematica [A]** time = 0.03, size = 73, normalized size = 1.14

$$\frac{x \left( \sqrt{b} (bB - Ac) + Ac \sqrt{b + cx^2} \tanh^{-1} \left( \frac{\sqrt{b+cx^2}}{\sqrt{b}} \right) \right)}{b^{3/2} c \sqrt{x^2 (b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -((x\*(Sqrt[b]\*(b\*B - A\*c) + A\*c\*Sqrt[b + c\*x^2]\*ArcTanh[Sqrt[b + c\*x^2]/Sqrt[b]]))/(b^(3/2)\*c\*Sqrt[x^2\*(b + c\*x^2)])

**IntegrateAlgebraic [A]** time = 0.80, size = 74, normalized size = 1.16

$$\frac{\sqrt{bx^2 + cx^4} (Ac - bB)}{bcx (b + cx^2)} - \frac{A \tanh^{-1} \left( \frac{\sqrt{b} x}{\sqrt{bx^2 + cx^4}} \right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((-(b\*B) + A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(b\*c\*x\*(b + c\*x^2)) - (A\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/b^(3/2)

**fricas [A]** time = 0.43, size = 199, normalized size = 3.11

$$\left[ \frac{(Ac^2x^3 + Abcx)\sqrt{b} \log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) - 2\sqrt{cx^4+bx^2}(Bb^2 - Abc) (Ac^2x^3 + Abcx)\sqrt{-b} \arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4+bx^2}(Bb^2 - Abc)}{2(b^2c^2x^3 + b^3cx)}, \frac{(Ac^2x^3 + Abcx)\sqrt{-b} \arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4+bx^2}(Bb^2 - Abc)}{b^2c^2x^3 + b^3cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] [1/2\*((A\*c^2\*x^3 + A\*b\*c\*x)\*sqrt(b)\*log(-(c\*x^3 + 2\*b\*x - 2\*sqrt(c\*x^4 + b\*x^2))\*sqrt(b))/x^3) - 2\*sqrt(c\*x^4 + b\*x^2)\*(B\*b^2 - A\*b\*c))/(b^2\*c^2\*x^3 + b^3\*c\*x), ((A\*c^2\*x^3 + A\*b\*c\*x)\*sqrt(-b)\*arctan(sqrt(c\*x^4 + b\*x^2)\*sqrt(-b)/(c\*x^3 + b\*x)) - sqrt(c\*x^4 + b\*x^2)\*(B\*b^2 - A\*b\*c))/(b^2\*c^2\*x^3 + b^3\*c\*x)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,2]%%},0,%%{1,[0,2,4]%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,2]%%},0,%%{1,[0,2,4]%%}] at parameters values [66.1769613782,93,91]-2\*(2\*b\*B-2\*A\*c)/4/b/c/x\*sqrt(b\*(1/x)^2+c)/(b\*(1/x)^2+c)+2\*A/2/b/sqrt(b)\*ln(abs(sqrt(b\*(1/x)^2+c)-sqrt(b)/x))

**maple [A]** time = 0.06, size = 79, normalized size = 1.23

$$\frac{(cx^2 + b) \left( -\sqrt{cx^2 + b} \operatorname{Abc} \ln \left( \frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x} \right) + Ab^{\frac{3}{2}}c - Bb^{\frac{5}{2}} \right) x^3}{(cx^4 + bx^2)^{\frac{3}{2}} b^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)`

[Out]  $x^3*(c*x^2+b)*(A*b^{(3/2)}*c-A*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2))}/x)*(c*x^2+b)^{(1/2)}*b*c-B*b^{(5/2)})/(c*x^4+b*x^2)^{(3/2)}/c/b^{(5/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^2}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2)^(3/2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 (Bx^2 + A)}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)`

[Out] `int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{(x^2 (b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)`

[Out] `Integral(x**2*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)`

$$3.157 \quad \int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=142

$$-\frac{(2bB-3Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}} + \frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} - \frac{B}{3cx\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1145, 2006, 2025, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3} - \frac{(2bB-3Ac) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} - \frac{B}{3cx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] -B/(3\*c\*x\*Sqrt[b\*x^2 + c\*x^4]) - (2\*b\*B - 3\*A\*c)/(3\*b\*c\*x\*Sqrt[b\*x^2 + c\*x^4]) + ((2\*b\*B - 3\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(2\*b^2\*c\*x^3) - ((2\*b\*B - 3\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(2\*b^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 1145

Int[((d\_) + (e\_.)\*(x\_)^2)\*((b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> Simp[(e\*(b\*x^2 + c\*x^4)^(p+1))/(c\*(4\*p+3)\*x), x] - Dist[(b\*e\*(2\*p+1) - c\*d\*(4\*p+3))/(c\*(4\*p+3)), Int[(b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{b, c, d, e, p}, x] && !IntegerQ[p] && NeQ[4\*p+3, 0] && NeQ[b\*e\*(2\*p+1) - c\*d\*(4\*p+3), 0]

#### Rule 2006

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> -Simp[(a\*x^j + b\*x^n)^(p+1)/(a\*(n-j)\*(p+1)\*x^(j-1)), x] + Dist[(n\*p + n - j + 1)/(a\*(n-j)\*(p+1)), Int[(a\*x^j + b\*x^n)^(p+1)/x^j, x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Dist[2/(2-n), Subst[Int[1/(1-a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2025

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(c^(j-1)\*(c\*x)^(m-j+1)\*(a\*x^j + b\*x^n)^(p+1))/(a\*(m+j\*p+1)), x] - Dist[(b\*(m+n\*p+n-j+1))/(a\*c^(n-j)\*(m+j\*p+1)), Int[(c\*x)^(m+n-j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j\*p + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{(bx^2 + cx^4)^{3/2}} dx &= -\frac{B}{3cx\sqrt{bx^2 + cx^4}} + \frac{(-2bB + 3Ac) \int \frac{1}{(bx^2 + cx^4)^{3/2}} dx}{3c} \\
&= -\frac{B}{3cx\sqrt{bx^2 + cx^4}} - \frac{2bB - 3Ac}{3bcx\sqrt{bx^2 + cx^4}} + \frac{(-2bB + 3Ac) \int \frac{1}{x^2\sqrt{bx^2 + cx^4}} dx}{bc} \\
&= -\frac{B}{3cx\sqrt{bx^2 + cx^4}} - \frac{2bB - 3Ac}{3bcx\sqrt{bx^2 + cx^4}} + \frac{(2bB - 3Ac)\sqrt{bx^2 + cx^4}}{2b^2cx^3} + \frac{(2bB - 3Ac) \int \frac{1}{\sqrt{bx^2 + cx^4}} dx}{2b^2} \\
&= -\frac{B}{3cx\sqrt{bx^2 + cx^4}} - \frac{2bB - 3Ac}{3bcx\sqrt{bx^2 + cx^4}} + \frac{(2bB - 3Ac)\sqrt{bx^2 + cx^4}}{2b^2cx^3} - \frac{(2bB - 3Ac) \operatorname{Subst}\left(\int \frac{1}{\sqrt{bx^2 + cx^4}} dx\right)}{2b^2} \\
&= -\frac{B}{3cx\sqrt{bx^2 + cx^4}} - \frac{2bB - 3Ac}{3bcx\sqrt{bx^2 + cx^4}} + \frac{(2bB - 3Ac)\sqrt{bx^2 + cx^4}}{2b^2cx^3} - \frac{(2bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{bx^2 + cx^4}}{\sqrt{bx^2 + cx^4}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.43

$$\frac{x^2(2bB - 3Ac) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{cx^2}{b} + 1\right) - Ab}{2b^2x\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] (- (A\*b) + (2\*b\*B - 3\*A\*c)\*x^2\*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c\*x^2)/b])/ (2\*b^2\*x\*Sqrt[x^2\*(b + c\*x^2)])

IntegrateAlgebraic [A] time = 0.88, size = 95, normalized size = 0.67

$$\frac{(3Ac - 2bB) \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2 + cx^4}}\right)}{2b^{5/2}} + \frac{\sqrt{bx^2 + cx^4} (-Ab - 3Acx^2 + 2bBx^2)}{2b^2x^3(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(b\*x^2 + c\*x^4)^(3/2), x]

[Out] ((- (A\*b) + 2\*b\*B\*x^2 - 3\*A\*c\*x^2)\*Sqrt[b\*x^2 + c\*x^4])/ (2\*b^2\*x^3\*(b + c\*x^2)) + ((-2\*b\*B + 3\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/ (2\*b^(5/2))

fricas [A] time = 0.44, size = 260, normalized size = 1.83

$$\left[ \frac{((2Bbc - 3Ac^2)x^5 + (2Bb^2 - 3Abc)x^3)\sqrt{b} \log\left(\frac{-cx^3 + 2bx^2 + \sqrt{cx^4 + bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4 + bx^2}(Ab^2 - (2Bb^2 - 3Abc)x^2)}{4(b^3cx^5 + b^4x^3)}, \frac{((2Bbc - 3Ac^2)x^5 + (2Bb^2 - 3Abc)x^3)\sqrt{-b} \arctan\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{-b}}{cx^3 + bx}\right) - \sqrt{cx^4 + bx^2}(Ab^2 - (2Bb^2 - 3Abc)x^2)}{2(b^3cx^5 + b^4x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/4\*(((2\*B\*b\*c - 3\*A\*c^2)\*x^5 + (2\*B\*b^2 - 3\*A\*b\*c)\*x^3)\*sqrt(b)\*log(-(c\*x^3 + 2\*b\*x + 2\*sqrt(c\*x^4 + b\*x^2)\*sqrt(b))/x^3) + 2\*sqrt(c\*x^4 + b\*x^2)\*(A\*b^2 - (2\*B\*b^2 - 3\*A\*b\*c)\*x^2))/ (b^3\*c\*x^5 + b^4\*x^3), 1/2\*(((2\*B\*b\*c - 3

$*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*\text{sqrt}(-b)*\text{arctan}(\text{sqrt}(c*x^4 + b*x^2)*\text{sqrt}(-b)/(c*x^3 + b*x)) - \text{sqrt}(c*x^4 + b*x^2)*(A*b^2 - (2*B*b^2 - 3*A*b*c)*x^2))/(b^3*c*x^5 + b^4*x^3)]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

*sage0x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="giac")

[Out] sage0\*x

**maple** [A] time = 0.06, size = 129, normalized size = 0.91

$$\frac{(cx^2 + b) \left( -3\sqrt{cx^2 + b} Abcx^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) + 2\sqrt{cx^2 + b} Bb^2x^2 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) + 3Ab^{\frac{3}{2}}cx^2 - 2Bb^{\frac{5}{2}}x^2 + Ab^{\frac{5}{2}} \right) x}{2(cx^4 + bx^2)^{\frac{3}{2}} b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x)

[Out]  $-1/2*x*(c*x^2+b)*(3*A*b^{(3/2)}*x^2*c-3*A*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)}))/x) * (c*x^2+b)^{(1/2)}*x^2*b*c-2*B*b^{(5/2)}*x^2+2*B*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)}))/x) * (c*x^2+b)^{(1/2)}*x^2*b^2+A*b^{(5/2)})/(c*x^4+b*x^2)^{(3/2)}/b^{(7/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((B\*x^2 + A)/(c\*x^4 + b\*x^2)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(b\*x^2 + c\*x^4)^(3/2),x)

[Out] int((A + B\*x^2)/(b\*x^2 + c\*x^4)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{(x^2(b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2),x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*2\*(b + c\*x\*\*2))\*\* (3/2), x)

$$3.158 \quad \int \frac{A+Bx^2}{x^2(bx^2+cx^4)^{3/2}} dx$$

**Optimal.** Leaf size=137

$$\frac{3c(4bB - 5Ac) \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}} - \frac{3\sqrt{bx^2+cx^4}(4bB - 5Ac)}{8b^3x^3} + \frac{4bB - 5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

**Rubi [A]** time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2038, 2006, 2025, 2008, 206}

$$-\frac{3\sqrt{bx^2+cx^4}(4bB - 5Ac)}{8b^3x^3} + \frac{4bB - 5Ac}{4b^2x\sqrt{bx^2+cx^4}} + \frac{3c(4bB - 5Ac) \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}} - \frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] -A/(4\*b\*x^3\*Sqrt[b\*x^2 + c\*x^4]) + (4\*b\*B - 5\*A\*c)/(4\*b^2\*x\*Sqrt[b\*x^2 + c\*x^4]) - (3\*(4\*b\*B - 5\*A\*c)\*Sqrt[b\*x^2 + c\*x^4])/(8\*b^3\*x^3) + (3\*c\*(4\*b\*B - 5\*A\*c)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]])/(8\*b^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2006

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := -Simp[(a\*x^j + b\*x^n)^(p+1)/(a\*(n-j)\*(p+1)\*x^(j-1)), x] + Dist[(n\*p + n - j + 1)/(a\*(n-j)\*(p+1)), Int[(a\*x^j + b\*x^n)^(p+1)/x^j, x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]

#### Rule 2008

Int[1/Sqrt[(a\_.)\*(x\_)^2 + (b\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a\*x^2), x], x, x/Sqrt[a\*x^2 + b\*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

#### Rule 2025

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(j-1)\*(c\*x)^(m-j+1)\*(a\*x^j + b\*x^n)^(p+1))/(a\*(m+j\*p+1)), x] - Dist[(b\*(m+n\*p+n-j+1))/(a\*c^(n-j)\*(m+j\*p+1)), Int[(c\*x)^(m+n-j)\*(a\*x^j + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j\*p + 1, 0]

#### Rule 2038

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(jn\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*e^(j-1)\*(e\*x)^(m-j+1)\*(a\*x^j + b\*x^(j+n))^(p+1))/(a\*(m+j\*p+1)), x] + Dist[(a\*d\*(m+j\*p+1) - b\*c\*(m+n+p\*(j+n)+1))/(a\*e^n\*(m+j\*p+1)), Int[(e\*x)^(m+n)\*(a\*x^j + b\*x^(j+n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j+n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && (LtQ[m + j\*p, -1])

|| (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n\*p) - 1])) && (GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j\*p + 1, 0] && NeQ[m - n + j\*p + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^2 (bx^2 + cx^4)^{3/2}} dx &= -\frac{A}{4bx^3 \sqrt{bx^2 + cx^4}} - \frac{(-4bB + 5Ac) \int \frac{1}{(bx^2 + cx^4)^{3/2}} dx}{4b} \\ &= -\frac{A}{4bx^3 \sqrt{bx^2 + cx^4}} + \frac{4bB - 5Ac}{4b^2 x \sqrt{bx^2 + cx^4}} + \frac{(3(4bB - 5Ac)) \int \frac{1}{x^2 \sqrt{bx^2 + cx^4}} dx}{4b^2} \\ &= -\frac{A}{4bx^3 \sqrt{bx^2 + cx^4}} + \frac{4bB - 5Ac}{4b^2 x \sqrt{bx^2 + cx^4}} - \frac{3(4bB - 5Ac) \sqrt{bx^2 + cx^4}}{8b^3 x^3} - \frac{(3c(4bB - 5Ac))}{8b^3 x^3} \\ &= -\frac{A}{4bx^3 \sqrt{bx^2 + cx^4}} + \frac{4bB - 5Ac}{4b^2 x \sqrt{bx^2 + cx^4}} - \frac{3(4bB - 5Ac) \sqrt{bx^2 + cx^4}}{8b^3 x^3} + \frac{(3c(4bB - 5Ac))}{8b^3 x^3} \\ &= -\frac{A}{4bx^3 \sqrt{bx^2 + cx^4}} + \frac{4bB - 5Ac}{4b^2 x \sqrt{bx^2 + cx^4}} - \frac{3(4bB - 5Ac) \sqrt{bx^2 + cx^4}}{8b^3 x^3} + \frac{3c(4bB - 5Ac)}{8b^3 x^3} \end{aligned}$$

Mathematica [C] time = 0.05, size = 64, normalized size = 0.47

$$\frac{cx^4(5Ac - 4bB) {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{cx^2}{b} + 1\right) - Ab^2}{4b^3 x^3 \sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (- (A\*b^2) + c\*(-4\*b\*B + 5\*A\*c)\*x^4\*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c\*x^2)/b]) / (4\*b^3\*x^3\*Sqrt[x^2\*(b + c\*x^2)])

IntegrateAlgebraic [A] time = 1.10, size = 120, normalized size = 0.88

$$\frac{3(4bBc - 5Ac^2) \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2 + cx^4}}\right)}{8b^{7/2}} + \frac{\sqrt{bx^2 + cx^4} (-2Ab^2 + 5Abcx^2 + 15Ac^2x^4 - 4b^2Bx^2 - 12bBcx^4)}{8b^3x^5(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^2\*(b\*x^2 + c\*x^4)^(3/2)), x]

[Out] (Sqrt[b\*x^2 + c\*x^4]\*(-2\*A\*b^2 - 4\*b^2\*B\*x^2 + 5\*A\*b\*c\*x^2 - 12\*b\*B\*c\*x^4 + 15\*A\*c^2\*x^4)) / (8\*b^3\*x^5\*(b + c\*x^2)) + (3\*(4\*b\*B\*c - 5\*A\*c^2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[b\*x^2 + c\*x^4]]) / (8\*b^(7/2))

fricas [A] time = 0.45, size = 315, normalized size = 2.30

$$\frac{3((4Bb^2 - 5Ac^2)x^2 + (4Bb^2c - 5Abc^2)x^2)\sqrt{b} \log\left(\frac{c^2x^2 + 2cx + \sqrt{c^2x^2 + 4b}}{2c}\right) + 2(3(4Bb^2c - 5Abc^2)x^4 + 2Ab^2 + (4Bb^3 - 5Ab^2c)x^2)\sqrt{cx^2 + b} + 3((4Bb^2c - 5Abc^2)x^2 + (4Bb^2c - 5Abc^2)x^2)\sqrt{b} \arctan\left(\frac{\sqrt{cx^2 + b}}{cx + b}\right) + (3(4Bb^2c - 5Abc^2)x^4 + 2Ab^2 + (4Bb^3 - 5Ab^2c)x^2)\sqrt{cx^2 + b}}{16(b^4cx^2 + b^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^2/(c\*x^4+b\*x^2)^(3/2), x, algorithm="fricas")

[Out]  $[-1/16*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*\sqrt{b} * \log(-(c*x^3 + 2*b*x - 2*\sqrt{c*x^4 + b*x^2})*\sqrt{b})/x^3) + 2*(3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*\sqrt{c*x^4 + b*x^2})/(b^4*c*x^7 + b^5*x^5), -1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*\sqrt{-b}*\arctan(\sqrt{c*x^4 + b*x^2}*\sqrt{-b}/(c*x^3 + b*x)) + (3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*\sqrt{c*x^4 + b*x^2})/(b^4*c*x^7 + b^5*x^5)]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

[Out] `integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)`

**maple** [A] time = 0.06, size = 157, normalized size = 1.15

$$\frac{(cx^2 + b) \left( 15\sqrt{cx^2 + b} Ab^2c^2x^4 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 12\sqrt{cx^2 + b} Bb^2cx^4 \ln\left(\frac{2b+2\sqrt{cx^2+b}\sqrt{b}}{x}\right) - 15Ab^{\frac{3}{2}}c^2x^4 + 12Bb^{\frac{5}{2}}cx^4 - 5Ab^{\frac{5}{2}}cx^2 + 4Bb^{\frac{7}{2}}x^2 + 2Ab^{\frac{7}{2}} \right)}{8(cx^4 + bx^2)^{\frac{3}{2}}b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x)`

[Out]  $-1/8/x*(c*x^2+b)*(15*A*(c*x^2+b)^{(1/2)}*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)}))/x)*x^4*b*c^2-15*A*b^{(3/2)}*x^4*c^2-12*B*(c*x^2+b)^{(1/2)}*\ln(2*(b+(c*x^2+b)^{(1/2)}*b^{(1/2)}))/x*x^4*b^2*c+12*B*b^{(5/2)}*x^4*c-5*A*b^{(5/2)}*x^2*c+4*B*b^{(7/2)}*x^2+2*A*b^{(7/2)})/(c*x^4+b*x^2)^(3/2)/b^{(9/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)`

**mupad** [B] time = 1.26, size = 89, normalized size = 0.65

$$\frac{A \left(\frac{b}{cx^2} + 1\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{7}{2}; \frac{9}{2}; -\frac{b}{cx^2}\right)}{7x \left(cx^4 + bx^2\right)^{3/2}} - \frac{Bx \left(\frac{b}{cx^2} + 1\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{5}{2}; \frac{7}{2}; -\frac{b}{cx^2}\right)}{5 \left(cx^4 + bx^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x)`

[Out]  $-(A*(b/(c*x^2) + 1)^{(3/2)}*\operatorname{hypergeom}([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*x^2 + c*x^4)^{(3/2)}) - (B*x*(b/(c*x^2) + 1)^{(3/2)}*\operatorname{hypergeom}([3/2, 5/2], 7/2, -b/(c*x^2)))/(5*(b*x^2 + c*x^4)^{(3/2)})$



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2 (x^2 (b + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*2/(c\*x\*\*4+b\*x\*\*2)\*\*(3/2), x)

[Out] Integral((A + B\*x\*\*2)/(x\*\*2\*(x\*\*2\*(b + c\*x\*\*2))\*\*(3/2)), x)

$$3.159 \quad \int x^{7/2} (A + Bx^2) (bx^2 + cx^4) dx$$

Optimal. Leaf size=39

$$\frac{2}{17}x^{17/2}(Ac + bB) + \frac{2}{13}Abx^{13/2} + \frac{2}{21}Bcx^{21/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{17}x^{17/2}(Ac + bB) + \frac{2}{13}Abx^{13/2} + \frac{2}{21}Bcx^{21/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*A\*b\*x^(13/2))/13 + (2\*(b\*B + A\*c)\*x^(17/2))/17 + (2\*B\*c\*x^(21/2))/21

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{7/2} (A + Bx^2) (bx^2 + cx^4) dx &= \int x^{11/2} (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^{11/2} + (bB + Ac)x^{15/2} + Bcx^{19/2}) dx \\ &= \frac{2}{13}Abx^{13/2} + \frac{2}{17}(bB + Ac)x^{17/2} + \frac{2}{21}Bcx^{21/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{13/2} (273x^2(Ac + bB) + 357Ab + 221Bcx^4)}{4641}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*x^(13/2)\*(357\*A\*b + 273\*(b\*B + A\*c)\*x^2 + 221\*B\*c\*x^4))/4641

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.05

$$\frac{2(357Abx^{13/2} + 273Acx^{17/2} + 273bBx^{17/2} + 221Bcx^{21/2})}{4641}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*(357\*A\*b\*x^(13/2) + 273\*b\*B\*x^(17/2) + 273\*A\*c\*x^(17/2) + 221\*B\*c\*x^(21/2)))/4641

**fricas** [A] time = 0.41, size = 32, normalized size = 0.82

$$\frac{2}{4641} (221 B c x^{10} + 273 (B b + A c) x^8 + 357 A b x^6) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 2/4641\*(221\*B\*c\*x^10 + 273\*(B\*b + A\*c)\*x^8 + 357\*A\*b\*x^6)\*sqrt(x)

**giac** [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{21} B c x^{\frac{21}{2}} + \frac{2}{17} B b x^{\frac{17}{2}} + \frac{2}{17} A c x^{\frac{17}{2}} + \frac{2}{13} A b x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 2/21\*B\*c\*x^(21/2) + 2/17\*B\*b\*x^(17/2) + 2/17\*A\*c\*x^(17/2) + 2/13\*A\*b\*x^(13/2)

**maple** [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2 (221 B c x^4 + 273 A c x^2 + 273 B b x^2 + 357 A b) x^{\frac{13}{2}}}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x)

[Out] 2/4641\*x^(13/2)\*(221\*B\*c\*x^4+273\*A\*c\*x^2+273\*B\*b\*x^2+357\*A\*b)

**maxima** [A] time = 1.34, size = 27, normalized size = 0.69

$$\frac{2}{21} B c x^{\frac{21}{2}} + \frac{2}{17} (B b + A c) x^{\frac{17}{2}} + \frac{2}{13} A b x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 2/21\*B\*c\*x^(21/2) + 2/17\*(B\*b + A\*c)\*x^(17/2) + 2/13\*A\*b\*x^(13/2)

**mupad** [B] time = 0.06, size = 31, normalized size = 0.79

$$\frac{2 x^{13/2} (357 A b + 273 A c x^2 + 273 B b x^2 + 221 B c x^4)}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x)

[Out] (2\*x^(13/2)\*(357\*A\*b + 273\*A\*c\*x^2 + 273\*B\*b\*x^2 + 221\*B\*c\*x^4))/4641

**sympy** [A] time = 20.14, size = 46, normalized size = 1.18

$$\frac{2 A b x^{\frac{13}{2}}}{13} + \frac{2 A c x^{\frac{17}{2}}}{17} + \frac{2 B b x^{\frac{17}{2}}}{17} + \frac{2 B c x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(13/2)/13 + 2*A*c*x**(17/2)/17 + 2*B*b*x**(17/2)/17 + 2*B*c*x**(21/2)/21
```

$$3.160 \quad \int x^{5/2} (A + Bx^2) (bx^2 + cx^4) dx$$

Optimal. Leaf size=39

$$\frac{2}{15}x^{15/2}(Ac + bB) + \frac{2}{11}Abx^{11/2} + \frac{2}{19}Bcx^{19/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{15}x^{15/2}(Ac + bB) + \frac{2}{11}Abx^{11/2} + \frac{2}{19}Bcx^{19/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*A\*b\*x^(11/2))/11 + (2\*(b\*B + A\*c)\*x^(15/2))/15 + (2\*B\*c\*x^(19/2))/19

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{5/2} (A + Bx^2) (bx^2 + cx^4) dx &= \int x^{9/2} (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^{9/2} + (bB + Ac)x^{13/2} + Bcx^{17/2}) dx \\ &= \frac{2}{11}Abx^{11/2} + \frac{2}{15}(bB + Ac)x^{15/2} + \frac{2}{19}Bcx^{19/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{11/2} (209x^2(Ac + bB) + 285Ab + 165Bcx^4)}{3135}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*x^(11/2)\*(285\*A\*b + 209\*(b\*B + A\*c)\*x^2 + 165\*B\*c\*x^4))/3135

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.05

$$\frac{2(285Abx^{11/2} + 209Acx^{15/2} + 209bBx^{15/2} + 165Bcx^{19/2})}{3135}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*(285\*A\*b\*x^(11/2) + 209\*b\*B\*x^(15/2) + 209\*A\*c\*x^(15/2) + 165\*B\*c\*x^(19/2)))/3135

**fricas** [A] time = 0.40, size = 32, normalized size = 0.82

$$\frac{2}{3135} (165 B c x^9 + 209 (B b + A c) x^7 + 285 A b x^5) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 2/3135\*(165\*B\*c\*x^9 + 209\*(B\*b + A\*c)\*x^7 + 285\*A\*b\*x^5)\*sqrt(x)

**giac** [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{19} B c x^{\frac{19}{2}} + \frac{2}{15} B b x^{\frac{15}{2}} + \frac{2}{15} A c x^{\frac{15}{2}} + \frac{2}{11} A b x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 2/19\*B\*c\*x^(19/2) + 2/15\*B\*b\*x^(15/2) + 2/15\*A\*c\*x^(15/2) + 2/11\*A\*b\*x^(11/2)

**maple** [A] time = 0.06, size = 32, normalized size = 0.82

$$\frac{2(165 B c x^4 + 209 A c x^2 + 209 B b x^2 + 285 A b) x^{\frac{11}{2}}}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x)

[Out] 2/3135\*x^(11/2)\*(165\*B\*c\*x^4+209\*A\*c\*x^2+209\*B\*b\*x^2+285\*A\*b)

**maxima** [A] time = 1.35, size = 27, normalized size = 0.69

$$\frac{2}{19} B c x^{\frac{19}{2}} + \frac{2}{15} (B b + A c) x^{\frac{15}{2}} + \frac{2}{11} A b x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 2/19\*B\*c\*x^(19/2) + 2/15\*(B\*b + A\*c)\*x^(15/2) + 2/11\*A\*b\*x^(11/2)

**mupad** [B] time = 0.11, size = 31, normalized size = 0.79

$$\frac{2 x^{11/2} (285 A b + 209 A c x^2 + 209 B b x^2 + 165 B c x^4)}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x)

[Out] (2\*x^(11/2)\*(285\*A\*b + 209\*A\*c\*x^2 + 209\*B\*b\*x^2 + 165\*B\*c\*x^4))/3135

**sympy** [A] time = 11.25, size = 46, normalized size = 1.18

$$\frac{2 A b x^{\frac{11}{2}}}{11} + \frac{2 A c x^{\frac{15}{2}}}{15} + \frac{2 B b x^{\frac{15}{2}}}{15} + \frac{2 B c x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2), x)
```

```
[Out] 2*A*b*x**(11/2)/11 + 2*A*c*x**(15/2)/15 + 2*B*b*x**(15/2)/15 + 2*B*c*x**(19/2)/19
```

### 3.161 $\int x^{3/2} (A + Bx^2) (bx^2 + cx^4) dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(Ac + bB) + \frac{2}{9}Abx^{9/2} + \frac{2}{17}Bcx^{17/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{13}x^{13/2}(Ac + bB) + \frac{2}{9}Abx^{9/2} + \frac{2}{17}Bcx^{17/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*A\*b\*x^(9/2))/9 + (2\*(b\*B + A\*c)\*x^(13/2))/13 + (2\*B\*c\*x^(17/2))/17

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{3/2} (A + Bx^2) (bx^2 + cx^4) dx &= \int x^{7/2} (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^{7/2} + (bB + Ac)x^{11/2} + Bcx^{15/2}) dx \\ &= \frac{2}{9}Abx^{9/2} + \frac{2}{13}(bB + Ac)x^{13/2} + \frac{2}{17}Bcx^{17/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{9/2} (153x^2(Ac + bB) + 221Ab + 117Bcx^4)}{1989}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*x^(9/2)\*(221\*A\*b + 153\*(b\*B + A\*c)\*x^2 + 117\*B\*c\*x^4))/1989

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.05

$$\frac{2(221Abx^{9/2} + 153Acx^{13/2} + 153bBx^{13/2} + 117Bcx^{17/2})}{1989}$$

Antiderivative was successfully verified.



[In] IntegrateAlgebraic[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*(221\*A\*b\*x^(9/2) + 153\*b\*B\*x^(13/2) + 153\*A\*c\*x^(13/2) + 117\*B\*c\*x^(17/2)))/1989

**fricas** [A] time = 0.40, size = 32, normalized size = 0.82

$$\frac{2}{1989} (117 B c x^8 + 153 (B b + A c) x^6 + 221 A b x^4) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 2/1989\*(117\*B\*c\*x^8 + 153\*(B\*b + A\*c)\*x^6 + 221\*A\*b\*x^4)\*sqrt(x)

**giac** [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{17} B c x^{\frac{17}{2}} + \frac{2}{13} B b x^{\frac{13}{2}} + \frac{2}{13} A c x^{\frac{13}{2}} + \frac{2}{9} A b x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 2/17\*B\*c\*x^(17/2) + 2/13\*B\*b\*x^(13/2) + 2/13\*A\*c\*x^(13/2) + 2/9\*A\*b\*x^(9/2)

**maple** [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2 (117 B c x^4 + 153 A c x^2 + 153 B b x^2 + 221 A b) x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x)

[Out] 2/1989\*x^(9/2)\*(117\*B\*c\*x^4+153\*A\*c\*x^2+153\*B\*b\*x^2+221\*A\*b)

**maxima** [A] time = 1.33, size = 27, normalized size = 0.69

$$\frac{2}{17} B c x^{\frac{17}{2}} + \frac{2}{13} (B b + A c) x^{\frac{13}{2}} + \frac{2}{9} A b x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 2/17\*B\*c\*x^(17/2) + 2/13\*(B\*b + A\*c)\*x^(13/2) + 2/9\*A\*b\*x^(9/2)

**mupad** [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2 x^{9/2} (221 A b + 153 A c x^2 + 153 B b x^2 + 117 B c x^4)}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x)

[Out] (2\*x^(9/2)\*(221\*A\*b + 153\*A\*c\*x^2 + 153\*B\*b\*x^2 + 117\*B\*c\*x^4))/1989

**sympy** [A] time = 5.59, size = 46, normalized size = 1.18

$$\frac{2 A b x^{\frac{9}{2}}}{9} + \frac{2 A c x^{\frac{13}{2}}}{13} + \frac{2 B b x^{\frac{13}{2}}}{13} + \frac{2 B c x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2), x)
```

```
[Out] 2*A*b*x**(9/2)/9 + 2*A*c*x**(13/2)/13 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(17/2)/17
```

$$3.162 \quad \int \sqrt{x} (A + Bx^2) (bx^2 + cx^4) dx$$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(Ac + bB) + \frac{2}{7}Abx^{7/2} + \frac{2}{15}Bcx^{15/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{11}x^{11/2}(Ac + bB) + \frac{2}{7}Abx^{7/2} + \frac{2}{15}Bcx^{15/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*A\*b\*x^(7/2))/7 + (2\*(b\*B + A\*c)\*x^(11/2))/11 + (2\*B\*c\*x^(15/2))/15

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \sqrt{x} (A + Bx^2) (bx^2 + cx^4) dx &= \int x^{5/2} (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^{5/2} + (bB + Ac)x^{9/2} + Bcx^{13/2}) dx \\ &= \frac{2}{7}Abx^{7/2} + \frac{2}{11}(bB + Ac)x^{11/2} + \frac{2}{15}Bcx^{15/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{7/2} (105x^2(Ac + bB) + 165Ab + 77Bcx^4)}{1155}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*x^(7/2)\*(165\*A\*b + 105\*(b\*B + A\*c)\*x^2 + 77\*B\*c\*x^4))/1155

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.05

$$\frac{2(165Abx^{7/2} + 105Acx^{11/2} + 105bBx^{11/2} + 77Bcx^{15/2})}{1155}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x]

[Out] (2\*(165\*A\*b\*x^(7/2) + 105\*b\*B\*x^(11/2) + 105\*A\*c\*x^(11/2) + 77\*B\*c\*x^(15/2)))/1155

**fricas** [A] time = 0.39, size = 32, normalized size = 0.82

$$\frac{2}{1155} (77 B c x^7 + 105 (B b + A c) x^5 + 165 A b x^3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)\*x^(1/2),x, algorithm="fricas")

[Out] 2/1155\*(77\*B\*c\*x^7 + 105\*(B\*b + A\*c)\*x^5 + 165\*A\*b\*x^3)\*sqrt(x)

**giac** [A] time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{15} B c x^{\frac{15}{2}} + \frac{2}{11} B b x^{\frac{11}{2}} + \frac{2}{11} A c x^{\frac{11}{2}} + \frac{2}{7} A b x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)\*x^(1/2),x, algorithm="giac")

[Out] 2/15\*B\*c\*x^(15/2) + 2/11\*B\*b\*x^(11/2) + 2/11\*A\*c\*x^(11/2) + 2/7\*A\*b\*x^(7/2)

**maple** [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2 (77 B c x^4 + 105 A c x^2 + 105 B b x^2 + 165 A b) x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)\*x^(1/2),x)

[Out] 2/1155\*x^(7/2)\*(77\*B\*c\*x^4+105\*A\*c\*x^2+105\*B\*b\*x^2+165\*A\*b)

**maxima** [A] time = 1.31, size = 27, normalized size = 0.69

$$\frac{2}{15} B c x^{\frac{15}{2}} + \frac{2}{11} (B b + A c) x^{\frac{11}{2}} + \frac{2}{7} A b x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)\*x^(1/2),x, algorithm="maxima")

[Out] 2/15\*B\*c\*x^(15/2) + 2/11\*(B\*b + A\*c)\*x^(11/2) + 2/7\*A\*b\*x^(7/2)

**mupad** [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2 x^{7/2} (165 A b + 105 A c x^2 + 105 B b x^2 + 77 B c x^4)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4),x)

[Out] (2\*x^(7/2)\*(165\*A\*b + 105\*A\*c\*x^2 + 105\*B\*b\*x^2 + 77\*B\*c\*x^4))/1155

**sympy** [A] time = 2.34, size = 37, normalized size = 0.95

$$\frac{2 A b x^{\frac{7}{2}}}{7} + \frac{2 B c x^{\frac{15}{2}}}{15} + \frac{2 x^{\frac{11}{2}} (A c + B b)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*x\*\*(1/2),x)

[Out] 2\*A\*b\*x\*\*(7/2)/7 + 2\*B\*c\*x\*\*(15/2)/15 + 2\*x\*\*(11/2)\*(A\*c + B\*b)/11

$$3.163 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} dx$$

**Optimal.** Leaf size=39

$$\frac{2}{9}x^{9/2}(Ac + bB) + \frac{2}{5}Abx^{5/2} + \frac{2}{13}Bcx^{13/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{9}x^{9/2}(Ac + bB) + \frac{2}{5}Abx^{5/2} + \frac{2}{13}Bcx^{13/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/Sqrt[x], x]

[Out] (2\*A\*b\*x^(5/2))/5 + (2\*(b\*B + A\*c)\*x^(9/2))/9 + (2\*B\*c\*x^(13/2))/13

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{\sqrt{x}} dx &= \int x^{3/2} (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^{3/2} + (bB + Ac)x^{7/2} + Bcx^{11/2}) dx \\ &= \frac{2}{5}Abx^{5/2} + \frac{2}{9}(bB + Ac)x^{9/2} + \frac{2}{13}Bcx^{13/2} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{585}x^{5/2} (65x^2(Ac + bB) + 117Ab + 45Bcx^4)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/Sqrt[x], x]

[Out] (2\*x^(5/2)\*(117\*A\*b + 65\*(b\*B + A\*c)\*x^2 + 45\*B\*c\*x^4))/585

**IntegrateAlgebraic [A]** time = 0.03, size = 41, normalized size = 1.05

$$\frac{2}{585} (117Abx^{5/2} + 65Acx^{9/2} + 65bBx^{9/2} + 45Bcx^{13/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/Sqrt[x], x]

[Out] (2\*(117\*A\*b\*x^(5/2) + 65\*b\*B\*x^(9/2) + 65\*A\*c\*x^(9/2) + 45\*B\*c\*x^(13/2)))/585

**fricas** [A] time = 0.39, size = 32, normalized size = 0.82

$$\frac{2}{585} (45 B c x^6 + 65 (B b + A c) x^4 + 117 A b x^2) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(1/2), x, algorithm="fricas")

[Out] 2/585\*(45\*B\*c\*x^6 + 65\*(B\*b + A\*c)\*x^4 + 117\*A\*b\*x^2)\*sqrt(x)

**giac** [A] time = 0.21, size = 29, normalized size = 0.74

$$\frac{2}{13} B c x^{\frac{13}{2}} + \frac{2}{9} B b x^{\frac{9}{2}} + \frac{2}{9} A c x^{\frac{9}{2}} + \frac{2}{5} A b x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(1/2), x, algorithm="giac")

[Out] 2/13\*B\*c\*x^(13/2) + 2/9\*B\*b\*x^(9/2) + 2/9\*A\*c\*x^(9/2) + 2/5\*A\*b\*x^(5/2)

**maple** [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2 (45 B c x^4 + 65 A c x^2 + 65 B b x^2 + 117 A b) x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(1/2), x)

[Out] 2/585\*x^(5/2)\*(45\*B\*c\*x^4+65\*A\*c\*x^2+65\*B\*b\*x^2+117\*A\*b)

**maxima** [A] time = 1.37, size = 27, normalized size = 0.69

$$\frac{2}{13} B c x^{\frac{13}{2}} + \frac{2}{9} (B b + A c) x^{\frac{9}{2}} + \frac{2}{5} A b x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(1/2), x, algorithm="maxima")

[Out] 2/13\*B\*c\*x^(13/2) + 2/9\*(B\*b + A\*c)\*x^(9/2) + 2/5\*A\*b\*x^(5/2)

**mupad** [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2 x^{5/2} (117 A b + 65 A c x^2 + 65 B b x^2 + 45 B c x^4)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(1/2), x)

[Out] (2\*x^(5/2)\*(117\*A\*b + 65\*A\*c\*x^2 + 65\*B\*b\*x^2 + 45\*B\*c\*x^4))/585

**sympy** [A] time = 2.06, size = 46, normalized size = 1.18

$$\frac{2 A b x^{\frac{5}{2}}}{5} + \frac{2 A c x^{\frac{9}{2}}}{9} + \frac{2 B b x^{\frac{9}{2}}}{9} + \frac{2 B c x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*A*b*x**(5/2)/5 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(13/2)/13
```

$$3.164 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} dx$$

Optimal. Leaf size=39

$$\frac{2}{7}x^{7/2}(Ac + bB) + \frac{2}{3}Abx^{3/2} + \frac{2}{11}Bcx^{11/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{7}x^{7/2}(Ac + bB) + \frac{2}{3}Abx^{3/2} + \frac{2}{11}Bcx^{11/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(3/2), x]

[Out] (2\*A\*b\*x^(3/2))/3 + (2\*(b\*B + A\*c)\*x^(7/2))/7 + (2\*B\*c\*x^(11/2))/11

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^{3/2}} dx &= \int \sqrt{x} (A + Bx^2)(b + cx^2) dx \\ &= \int (Ab\sqrt{x} + (bB + Ac)x^{5/2} + Bcx^{9/2}) dx \\ &= \frac{2}{3}Abx^{3/2} + \frac{2}{7}(bB + Ac)x^{7/2} + \frac{2}{11}Bcx^{11/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{231}x^{3/2}(33x^2(Ac + bB) + 77Ab + 21Bcx^4)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(3/2), x]

[Out] (2\*x^(3/2)\*(77\*A\*b + 33\*(b\*B + A\*c)\*x^2 + 21\*B\*c\*x^4))/231

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.05

$$\frac{2}{231}(77Abx^{3/2} + 33Acx^{7/2} + 33bBx^{7/2} + 21Bcx^{11/2})$$

Antiderivative was successfully verified.



[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(3/2), x]

[Out] (2\*(77\*A\*b\*x^(3/2) + 33\*b\*B\*x^(7/2) + 33\*A\*c\*x^(7/2) + 21\*B\*c\*x^(11/2)))/231

**fricas** [A] time = 0.38, size = 30, normalized size = 0.77

$$\frac{2}{231} (21 Bcx^5 + 33 (Bb + Ac)x^3 + 77 Abx) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(3/2), x, algorithm="fricas")

[Out] 2/231\*(21\*B\*c\*x^5 + 33\*(B\*b + A\*c)\*x^3 + 77\*A\*b\*x)\*sqrt(x)

**giac** [A] time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{11} Bcx^{\frac{11}{2}} + \frac{2}{7} Bbx^{\frac{7}{2}} + \frac{2}{7} Acx^{\frac{7}{2}} + \frac{2}{3} Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(3/2), x, algorithm="giac")

[Out] 2/11\*B\*c\*x^(11/2) + 2/7\*B\*b\*x^(7/2) + 2/7\*A\*c\*x^(7/2) + 2/3\*A\*b\*x^(3/2)

**maple** [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2 (21 Bc x^4 + 33 Ac x^2 + 33 Bb x^2 + 77 Ab) x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(3/2), x)

[Out] 2/231\*x^(3/2)\*(21\*B\*c\*x^4+33\*A\*c\*x^2+33\*B\*b\*x^2+77\*A\*b)

**maxima** [A] time = 1.38, size = 27, normalized size = 0.69

$$\frac{2}{11} Bcx^{\frac{11}{2}} + \frac{2}{7} (Bb + Ac)x^{\frac{7}{2}} + \frac{2}{3} Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(3/2), x, algorithm="maxima")

[Out] 2/11\*B\*c\*x^(11/2) + 2/7\*(B\*b + A\*c)\*x^(7/2) + 2/3\*A\*b\*x^(3/2)

**mupad** [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2 x^{3/2} (77 A b + 33 A c x^2 + 33 B b x^2 + 21 B c x^4)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(3/2), x)

[Out] (2\*x^(3/2)\*(77\*A\*b + 33\*A\*c\*x^2 + 33\*B\*b\*x^2 + 21\*B\*c\*x^4))/231

**sympy** [A] time = 2.26, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Acx^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{7}{2}}}{7} + \frac{2Bcx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)/x\*\*(3/2), x)

[Out] 2\*A\*b\*x\*\*(3/2)/3 + 2\*A\*c\*x\*\*(7/2)/7 + 2\*B\*b\*x\*\*(7/2)/7 + 2\*B\*c\*x\*\*(11/2)/11

$$3.165 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}} dx$$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(Ac + bB) + 2Ab\sqrt{x} + \frac{2}{9}Bcx^{9/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{5}x^{5/2}(Ac + bB) + 2Ab\sqrt{x} + \frac{2}{9}Bcx^{9/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(5/2), x]

[Out] 2\*A\*b\*Sqrt[x] + (2\*(b\*B + A\*c)\*x^(5/2))/5 + (2\*B\*c\*x^(9/2))/9

Rule 448

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^{5/2}} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{\sqrt{x}} dx \\ &= \int \left( \frac{Ab}{\sqrt{x}} + (bB + Ac)x^{3/2} + Bcx^{7/2} \right) dx \\ &= 2Ab\sqrt{x} + \frac{2}{5}(bB + Ac)x^{5/2} + \frac{2}{9}Bcx^{9/2} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 0.89

$$\frac{2}{45}\sqrt{x} (9x^2(Ac + bB) + 45Ab + 5Bcx^4)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(5/2), x]

[Out] (2\*Sqrt[x]\*(45\*A\*b + 9\*(b\*B + A\*c)\*x^2 + 5\*B\*c\*x^4))/45

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.11

$$\frac{2}{45} (45Ab\sqrt{x} + 9Acx^{5/2} + 9bBx^{5/2} + 5Bcx^{9/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(5/2),x]

[Out] (2\*(45\*A\*b\*Sqrt[x] + 9\*b\*B\*x^(5/2) + 9\*A\*c\*x^(5/2) + 5\*B\*c\*x^(9/2)))/45

**fricas** [A] time = 0.39, size = 29, normalized size = 0.78

$$\frac{2}{45} (5 B c x^4 + 9 (B b + A c) x^2 + 45 A b) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(5/2),x, algorithm="fricas")

[Out] 2/45\*(5\*B\*c\*x^4 + 9\*(B\*b + A\*c)\*x^2 + 45\*A\*b)\*sqrt(x)

**giac** [A] time = 0.18, size = 29, normalized size = 0.78

$$\frac{2}{9} B c x^{\frac{9}{2}} + \frac{2}{5} B b x^{\frac{5}{2}} + \frac{2}{5} A c x^{\frac{5}{2}} + 2 A b \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(5/2),x, algorithm="giac")

[Out] 2/9\*B\*c\*x^(9/2) + 2/5\*B\*b\*x^(5/2) + 2/5\*A\*c\*x^(5/2) + 2\*A\*b\*sqrt(x)

**maple** [A] time = 0.05, size = 32, normalized size = 0.86

$$\frac{2 (5 B c x^4 + 9 A c x^2 + 9 B b x^2 + 45 A b) \sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(5/2),x)

[Out] 2/45\*x^(1/2)\*(5\*B\*c\*x^4+9\*A\*c\*x^2+9\*B\*b\*x^2+45\*A\*b)

**maxima** [A] time = 1.34, size = 27, normalized size = 0.73

$$\frac{2}{9} B c x^{\frac{9}{2}} + \frac{2}{5} (B b + A c) x^{\frac{5}{2}} + 2 A b \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(5/2),x, algorithm="maxima")

[Out] 2/9\*B\*c\*x^(9/2) + 2/5\*(B\*b + A\*c)\*x^(5/2) + 2\*A\*b\*sqrt(x)

**mupad** [B] time = 0.04, size = 31, normalized size = 0.84

$$\frac{2 \sqrt{x} (45 A b + 9 A c x^2 + 9 B b x^2 + 5 B c x^4)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(5/2),x)

[Out] (2\*x^(1/2)\*(45\*A\*b + 9\*A\*c\*x^2 + 9\*B\*b\*x^2 + 5\*B\*c\*x^4))/45

**sympy** [A] time = 2.62, size = 44, normalized size = 1.19

$$2 A b \sqrt{x} + \frac{2 A c x^{\frac{5}{2}}}{5} + \frac{2 B b x^{\frac{5}{2}}}{5} + \frac{2 B c x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*A*b*sqrt(x) + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9
```

$$3.166 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx$$

Optimal. Leaf size=37

$$\frac{2}{3}x^{3/2}(Ac + bB) - \frac{2Ab}{\sqrt{x}} + \frac{2}{7}Bcx^{7/2}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{2}{3}x^{3/2}(Ac + bB) - \frac{2Ab}{\sqrt{x}} + \frac{2}{7}Bcx^{7/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(7/2), x]

[Out] (-2\*A\*b)/Sqrt[x] + (2\*(b\*B + A\*c)\*x^(3/2))/3 + (2\*B\*c\*x^(7/2))/7

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)}{x^{7/2}} dx &= \int \frac{(A + Bx^2)(b + cx^2)}{x^{3/2}} dx \\ &= \int \left( \frac{Ab}{x^{3/2}} + (bB + Ac)\sqrt{x} + Bcx^{5/2} \right) dx \\ &= -\frac{2Ab}{\sqrt{x}} + \frac{2}{3}(bB + Ac)x^{3/2} + \frac{2}{7}Bcx^{7/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.95

$$\frac{2(-21Ab + 7Acx^2 + 7bBx^2 + 3Bcx^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(7/2), x]

[Out] (2\*(-21\*A\*b + 7\*b\*B\*x^2 + 7\*A\*c\*x^2 + 3\*B\*c\*x^4))/(21\*Sqrt[x])

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.95

$$\frac{2(-21Ab + 7Acx^2 + 7bBx^2 + 3Bcx^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(7/2), x]

[Out] (2\*(-21\*A\*b + 7\*b\*B\*x^2 + 7\*A\*c\*x^2 + 3\*B\*c\*x^4))/(21\*sqrt(x))

**fricas** [A] time = 0.38, size = 29, normalized size = 0.78

$$\frac{2(3Bcx^4 + 7(Bb + Ac)x^2 - 21Ab)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(7/2), x, algorithm="fricas")

[Out] 2/21\*(3\*B\*c\*x^4 + 7\*(B\*b + A\*c)\*x^2 - 21\*A\*b)/sqrt(x)

**giac** [A] time = 0.19, size = 29, normalized size = 0.78

$$\frac{2}{7}Bcx^{\frac{7}{2}} + \frac{2}{3}Bbx^{\frac{3}{2}} + \frac{2}{3}Acx^{\frac{3}{2}} - \frac{2Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(7/2), x, algorithm="giac")

[Out] 2/7\*B\*c\*x^(7/2) + 2/3\*B\*b\*x^(3/2) + 2/3\*A\*c\*x^(3/2) - 2\*A\*b/sqrt(x)

**maple** [A] time = 0.04, size = 32, normalized size = 0.86

$$\frac{2(-3Bcx^4 - 7Acx^2 - 7Bbx^2 + 21Ab)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(7/2), x)

[Out] -2/21/x^(1/2)\*(-3\*B\*c\*x^4-7\*A\*c\*x^2-7\*B\*b\*x^2+21\*A\*b)

**maxima** [A] time = 1.33, size = 27, normalized size = 0.73

$$\frac{2}{7}Bcx^{\frac{7}{2}} + \frac{2}{3}(Bb + Ac)x^{\frac{3}{2}} - \frac{2Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)/x^(7/2), x, algorithm="maxima")

[Out] 2/7\*B\*c\*x^(7/2) + 2/3\*(B\*b + A\*c)\*x^(3/2) - 2\*A\*b/sqrt(x)

**mupad** [B] time = 0.10, size = 31, normalized size = 0.84

$$\frac{14Acx^2 - 42Ab + 14Bbx^2 + 6Bcx^4}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4))/x^(7/2), x)

[Out] (14\*A\*c\*x^2 - 42\*A\*b + 14\*B\*b\*x^2 + 6\*B\*c\*x^4)/(21\*x^(1/2))

**sympy** [A] time = 3.92, size = 44, normalized size = 1.19

$$-\frac{2Ab}{\sqrt{x}} + \frac{2Acx^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{3}{2}}}{3} + \frac{2Bcx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(7/2),x)
```

```
[Out] -2*A*b/sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(7/2)/7
```

$$3.167 \quad \int x^{7/2} (A + Bx^2) (bx^2 + cx^4)^2 dx$$

**Optimal.** Leaf size=63

$$\frac{2}{17}Ab^2x^{17/2} + \frac{2}{25}cx^{25/2}(Ac + 2bB) + \frac{2}{21}bx^{21/2}(2Ac + bB) + \frac{2}{29}Bc^2x^{29/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{17}Ab^2x^{17/2} + \frac{2}{25}cx^{25/2}(Ac + 2bB) + \frac{2}{21}bx^{21/2}(2Ac + bB) + \frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*A\*b^2\*x^(17/2))/17 + (2\*b\*(b\*B + 2\*A\*c)\*x^(21/2))/21 + (2\*c\*(2\*b\*B + A\*c)\*x^(25/2))/25 + (2\*B\*c^2\*x^(29/2))/29

**Rule 448**

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int x^{7/2} (A + Bx^2) (bx^2 + cx^4)^2 dx &= \int x^{15/2} (A + Bx^2) (b + cx^2)^2 dx \\ &= \int (Ab^2x^{15/2} + b(bB + 2Ac)x^{19/2} + c(2bB + Ac)x^{23/2} + Bc^2x^{27/2}) dx \\ &= \frac{2}{17}Ab^2x^{17/2} + \frac{2}{21}b(bB + 2Ac)x^{21/2} + \frac{2}{25}c(2bB + Ac)x^{25/2} + \frac{2}{29}Bc^2x^{29/2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 63, normalized size = 1.00

$$\frac{2}{17}Ab^2x^{17/2} + \frac{2}{25}cx^{25/2}(Ac + 2bB) + \frac{2}{21}bx^{21/2}(2Ac + bB) + \frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*A\*b^2\*x^(17/2))/17 + (2\*b\*(b\*B + 2\*A\*c)\*x^(21/2))/21 + (2\*c\*(2\*b\*B + A\*c)\*x^(25/2))/25 + (2\*B\*c^2\*x^(29/2))/29

**IntegrateAlgebraic [A]** time = 0.05, size = 69, normalized size = 1.10

$$\frac{2(15225Ab^2x^{17/2} + 24650Abcx^{21/2} + 10353Ac^2x^{25/2} + 12325b^2Bx^{21/2} + 20706bBcx^{25/2} + 8925Bc^2x^{29/2})}{258825}$$



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*(15225\*A\*b^2\*x^(17/2) + 12325\*b^2\*B\*x^(21/2) + 24650\*A\*b\*c\*x^(21/2) + 20706\*b\*B\*c\*x^(25/2) + 10353\*A\*c^2\*x^(25/2) + 8925\*B\*c^2\*x^(29/2)))/258825

**fricas** [A] time = 0.40, size = 56, normalized size = 0.89

$$\frac{2}{258825} (8925 Bc^2x^{14} + 10353 (2 Bbc + Ac^2)x^{12} + 15225 Ab^2x^8 + 12325 (Bb^2 + 2 Abc)x^{10})\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 2/258825\*(8925\*B\*c^2\*x^14 + 10353\*(2\*B\*b\*c + A\*c^2)\*x^12 + 15225\*A\*b^2\*x^8 + 12325\*(B\*b^2 + 2\*A\*b\*c)\*x^10)\*sqrt(x)

**giac** [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{29} Bc^2x^{\frac{29}{2}} + \frac{4}{25} Bbcx^{\frac{25}{2}} + \frac{2}{25} Ac^2x^{\frac{25}{2}} + \frac{2}{21} Bb^2x^{\frac{21}{2}} + \frac{4}{21} Abcx^{\frac{21}{2}} + \frac{2}{17} Ab^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 2/29\*B\*c^2\*x^(29/2) + 4/25\*B\*b\*c\*x^(25/2) + 2/25\*A\*c^2\*x^(25/2) + 2/21\*B\*b^2\*x^(21/2) + 4/21\*A\*b\*c\*x^(21/2) + 2/17\*A\*b^2\*x^(17/2)

**maple** [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2 (8925B c^2x^6 + 10353A c^2x^4 + 20706Bbc x^4 + 24650Abc x^2 + 12325B b^2x^2 + 15225b^2A) x^{\frac{17}{2}}}{258825}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x)

[Out] 2/258825\*x^(17/2)\*(8925\*B\*c^2\*x^6+10353\*A\*c^2\*x^4+20706\*B\*b\*c\*x^4+24650\*A\*b\*c\*x^2+12325\*B\*b^2\*x^2+15225\*A\*b^2)

**maxima** [A] time = 1.30, size = 51, normalized size = 0.81

$$\frac{2}{29} Bc^2x^{\frac{29}{2}} + \frac{2}{25} (2 Bbc + Ac^2)x^{\frac{25}{2}} + \frac{2}{17} Ab^2x^{\frac{17}{2}} + \frac{2}{21} (Bb^2 + 2 Abc)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 2/29\*B\*c^2\*x^(29/2) + 2/25\*(2\*B\*b\*c + A\*c^2)\*x^(25/2) + 2/17\*A\*b^2\*x^(17/2) + 2/21\*(B\*b^2 + 2\*A\*b\*c)\*x^(21/2)

**mupad** [B] time = 0.13, size = 51, normalized size = 0.81

$$x^{21/2} \left( \frac{2 B b^2}{21} + \frac{4 A c b}{21} \right) + x^{25/2} \left( \frac{2 A c^2}{25} + \frac{4 B b c}{25} \right) + \frac{2 A b^2 x^{17/2}}{17} + \frac{2 B c^2 x^{29/2}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^{(21/2)} * ((2*B*b^2)/21 + (4*A*b*c)/21) + x^{(25/2)} * ((2*A*c^2)/25 + (4*B*b*c)/25) + (2*A*b^2*x^{(17/2)})/17 + (2*B*c^2*x^{(29/2)})/29$

sympy [A] time = 57.87, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{17}{2}}}{17} + \frac{4Abcx^{\frac{21}{2}}}{21} + \frac{2Ac^2x^{\frac{25}{2}}}{25} + \frac{2Bb^2x^{\frac{21}{2}}}{21} + \frac{4Bbcx^{\frac{25}{2}}}{25} + \frac{2Bc^2x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)`

[Out]  $2*A*b**2*x**(17/2)/17 + 4*A*b*c*x**(21/2)/21 + 2*A*c**2*x**(25/2)/25 + 2*B*b**2*x**(21/2)/21 + 4*B*b*c*x**(25/2)/25 + 2*B*c**2*x**(29/2)/29$

$$3.168 \quad \int x^{5/2} (A + Bx^2) (bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=63

$$\frac{2}{15}Ab^2x^{15/2} + \frac{2}{23}cx^{23/2}(Ac + 2bB) + \frac{2}{19}bx^{19/2}(2Ac + bB) + \frac{2}{27}Bc^2x^{27/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{15}Ab^2x^{15/2} + \frac{2}{23}cx^{23/2}(Ac + 2bB) + \frac{2}{19}bx^{19/2}(2Ac + bB) + \frac{2}{27}Bc^2x^{27/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*A\*b^2\*x^(15/2))/15 + (2\*b\*(b\*B + 2\*A\*c)\*x^(19/2))/19 + (2\*c\*(2\*b\*B + A\*c)\*x^(23/2))/23 + (2\*B\*c^2\*x^(27/2))/27

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{5/2} (A + Bx^2) (bx^2 + cx^4)^2 dx &= \int x^{13/2} (A + Bx^2) (b + cx^2)^2 dx \\ &= \int (Ab^2x^{13/2} + b(bB + 2Ac)x^{17/2} + c(2bB + Ac)x^{21/2} + Bc^2x^{25/2}) dx \\ &= \frac{2}{15}Ab^2x^{15/2} + \frac{2}{19}b(bB + 2Ac)x^{19/2} + \frac{2}{23}c(2bB + Ac)x^{23/2} + \frac{2}{27}Bc^2x^{27/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{15/2} (3933Ab^2 + 2565cx^4(Ac + 2bB) + 3105bx^2(2Ac + bB) + 2185Bc^2x^6)}{58995}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*x^(15/2)\*(3933\*A\*b^2 + 3105\*b\*(b\*B + 2\*A\*c)\*x^2 + 2565\*c\*(2\*b\*B + A\*c)\*x^4 + 2185\*B\*c^2\*x^6))/58995

**IntegrateAlgebraic [A]** time = 0.05, size = 69, normalized size = 1.10

$$\frac{2(3933Ab^2x^{15/2} + 6210Abcx^{19/2} + 2565Ac^2x^{23/2} + 3105b^2Bx^{19/2} + 5130bBcx^{23/2} + 2185Bc^2x^{27/2})}{58995}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*(3933\*A\*b^2\*x^(15/2) + 3105\*b^2\*B\*x^(19/2) + 6210\*A\*b\*c\*x^(19/2) + 5130\*b\*B\*c\*x^(23/2) + 2565\*A\*c^2\*x^(23/2) + 2185\*B\*c^2\*x^(27/2)))/58995

**fricas** [A] time = 0.40, size = 56, normalized size = 0.89

$$\frac{2}{58995} (2185 Bc^2x^{13} + 2565 (2 Bbc + Ac^2)x^{11} + 3933 Ab^2x^7 + 3105 (Bb^2 + 2 Abc)x^9)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 2/58995\*(2185\*B\*c^2\*x^13 + 2565\*(2\*B\*b\*c + A\*c^2)\*x^11 + 3933\*A\*b^2\*x^7 + 3105\*(B\*b^2 + 2\*A\*b\*c)\*x^9)\*sqrt(x)

**giac** [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{27} Bc^2x^{\frac{27}{2}} + \frac{4}{23} Bbcx^{\frac{23}{2}} + \frac{2}{23} Ac^2x^{\frac{23}{2}} + \frac{2}{19} Bb^2x^{\frac{19}{2}} + \frac{4}{19} Abcx^{\frac{19}{2}} + \frac{2}{15} Ab^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 2/27\*B\*c^2\*x^(27/2) + 4/23\*B\*b\*c\*x^(23/2) + 2/23\*A\*c^2\*x^(23/2) + 2/19\*B\*b^2\*x^(19/2) + 4/19\*A\*b\*c\*x^(19/2) + 2/15\*A\*b^2\*x^(15/2)

**maple** [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2 (2185B c^2x^6 + 2565A c^2x^4 + 5130Bbc x^4 + 6210Abc x^2 + 3105B b^2x^2 + 3933b^2A) x^{\frac{15}{2}}}{58995}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x)

[Out] 2/58995\*x^(15/2)\*(2185\*B\*c^2\*x^6+2565\*A\*c^2\*x^4+5130\*B\*b\*c\*x^4+6210\*A\*b\*c\*x^2+3105\*B\*b^2\*x^2+3933\*A\*b^2)

**maxima** [A] time = 1.34, size = 51, normalized size = 0.81

$$\frac{2}{27} Bc^2x^{\frac{27}{2}} + \frac{2}{23} (2 Bbc + Ac^2)x^{\frac{23}{2}} + \frac{2}{15} Ab^2x^{\frac{15}{2}} + \frac{2}{19} (Bb^2 + 2 Abc)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 2/27\*B\*c^2\*x^(27/2) + 2/23\*(2\*B\*b\*c + A\*c^2)\*x^(23/2) + 2/15\*A\*b^2\*x^(15/2) + 2/19\*(B\*b^2 + 2\*A\*b\*c)\*x^(19/2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{19/2} \left( \frac{2 B b^2}{19} + \frac{4 A c b}{19} \right) + x^{23/2} \left( \frac{2 A c^2}{23} + \frac{4 B b c}{23} \right) + \frac{2 A b^2 x^{15/2}}{15} + \frac{2 B c^2 x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^{(19/2)} * ((2*B*b^2)/19 + (4*A*b*c)/19) + x^{(23/2)} * ((2*A*c^2)/23 + (4*B*b*c)/23) + (2*A*b^2*x^{(15/2)})/15 + (2*B*c^2*x^{(27/2)})/27$

sympy [A] time = 39.82, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{4Abcx^{\frac{19}{2}}}{19} + \frac{2Ac^2x^{\frac{23}{2}}}{23} + \frac{2Bb^2x^{\frac{19}{2}}}{19} + \frac{4Bbcx^{\frac{23}{2}}}{23} + \frac{2Bc^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out]  $2*A*b**2*x**(15/2)/15 + 4*A*b*c*x**(19/2)/19 + 2*A*c**2*x**(23/2)/23 + 2*B*b**2*x**(19/2)/19 + 4*B*b*c*x**(23/2)/23 + 2*B*c**2*x**(27/2)/27$

$$3.169 \quad \int x^{3/2} (A + Bx^2) (bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=63

$$\frac{2}{13}Ab^2x^{13/2} + \frac{2}{21}cx^{21/2}(Ac + 2bB) + \frac{2}{17}bx^{17/2}(2Ac + bB) + \frac{2}{25}Bc^2x^{25/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{13}Ab^2x^{13/2} + \frac{2}{21}cx^{21/2}(Ac + 2bB) + \frac{2}{17}bx^{17/2}(2Ac + bB) + \frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*A\*b^2\*x^(13/2))/13 + (2\*b\*(b\*B + 2\*A\*c)\*x^(17/2))/17 + (2\*c\*(2\*b\*B + A\*c)\*x^(21/2))/21 + (2\*B\*c^2\*x^(25/2))/25

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{3/2} (A + Bx^2) (bx^2 + cx^4)^2 dx &= \int x^{11/2} (A + Bx^2) (b + cx^2)^2 dx \\ &= \int (Ab^2x^{11/2} + b(bB + 2Ac)x^{15/2} + c(2bB + Ac)x^{19/2} + Bc^2x^{23/2}) dx \\ &= \frac{2}{13}Ab^2x^{13/2} + \frac{2}{17}b(bB + 2Ac)x^{17/2} + \frac{2}{21}c(2bB + Ac)x^{21/2} + \frac{2}{25}Bc^2x^{25/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 63, normalized size = 1.00

$$\frac{2}{13}Ab^2x^{13/2} + \frac{2}{21}cx^{21/2}(Ac + 2bB) + \frac{2}{17}bx^{17/2}(2Ac + bB) + \frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*A\*b^2\*x^(13/2))/13 + (2\*b\*(b\*B + 2\*A\*c)\*x^(17/2))/17 + (2\*c\*(2\*b\*B + A\*c)\*x^(21/2))/21 + (2\*B\*c^2\*x^(25/2))/25

**IntegrateAlgebraic [A]** time = 0.04, size = 69, normalized size = 1.10

$$2 \left( 8925Ab^2x^{13/2} + 13650Abcx^{17/2} + 5525Ac^2x^{21/2} + 6825b^2Bx^{17/2} + 11050bBcx^{21/2} + 4641Bc^2x^{25/2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*(8925\*A\*b^2\*x^(13/2) + 6825\*b^2\*B\*x^(17/2) + 13650\*A\*b\*c\*x^(17/2) + 11050\*b\*B\*c\*x^(21/2) + 5525\*A\*c^2\*x^(21/2) + 4641\*B\*c^2\*x^(25/2)))/116025

**fricas** [A] time = 0.40, size = 56, normalized size = 0.89

$$\frac{2}{116025} (4641 Bc^2x^{12} + 5525 (2 Bbc + Ac^2)x^{10} + 8925 Ab^2x^6 + 6825 (Bb^2 + 2 Abc)x^8)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 2/116025\*(4641\*B\*c^2\*x^12 + 5525\*(2\*B\*b\*c + A\*c^2)\*x^10 + 8925\*A\*b^2\*x^6 + 6825\*(B\*b^2 + 2\*A\*b\*c)\*x^8)\*sqrt(x)

**giac** [A] time = 0.17, size = 53, normalized size = 0.84

$$\frac{2}{25} Bc^2x^{\frac{25}{2}} + \frac{4}{21} Bbcx^{\frac{21}{2}} + \frac{2}{21} Ac^2x^{\frac{21}{2}} + \frac{2}{17} Bb^2x^{\frac{17}{2}} + \frac{4}{17} Abcx^{\frac{17}{2}} + \frac{2}{13} Ab^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 2/25\*B\*c^2\*x^(25/2) + 4/21\*B\*b\*c\*x^(21/2) + 2/21\*A\*c^2\*x^(21/2) + 2/17\*B\*b^2\*x^(17/2) + 4/17\*A\*b\*c\*x^(17/2) + 2/13\*A\*b^2\*x^(13/2)

**maple** [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2(4641Bc^2x^6 + 5525Ac^2x^4 + 11050Bbcx^4 + 13650Abcx^2 + 6825Bb^2x^2 + 8925b^2A)x^{\frac{13}{2}}}{116025}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x)

[Out] 2/116025\*x^(13/2)\*(4641\*B\*c^2\*x^6+5525\*A\*c^2\*x^4+11050\*B\*b\*c\*x^4+13650\*A\*b\*c\*x^2+6825\*B\*b^2\*x^2+8925\*A\*b^2)

**maxima** [A] time = 1.30, size = 51, normalized size = 0.81

$$\frac{2}{25} Bc^2x^{\frac{25}{2}} + \frac{2}{21} (2 Bbc + Ac^2)x^{\frac{21}{2}} + \frac{2}{13} Ab^2x^{\frac{13}{2}} + \frac{2}{17} (Bb^2 + 2 Abc)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 2/25\*B\*c^2\*x^(25/2) + 2/21\*(2\*B\*b\*c + A\*c^2)\*x^(21/2) + 2/13\*A\*b^2\*x^(13/2) + 2/17\*(B\*b^2 + 2\*A\*b\*c)\*x^(17/2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{17/2} \left( \frac{2 B b^2}{17} + \frac{4 A c b}{17} \right) + x^{21/2} \left( \frac{2 A c^2}{21} + \frac{4 B b c}{21} \right) + \frac{2 A b^2 x^{13/2}}{13} + \frac{2 B c^2 x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^{(17/2)} * ((2*B*b^2)/17 + (4*A*b*c)/17) + x^{(21/2)} * ((2*A*c^2)/21 + (4*B*b*c)/21) + (2*A*b^2*x^{(13/2)})/13 + (2*B*c^2*x^{(25/2)})/25$

sympy [A] time = 20.95, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{4Abcx^{\frac{17}{2}}}{17} + \frac{2Ac^2x^{\frac{21}{2}}}{21} + \frac{2Bb^2x^{\frac{17}{2}}}{17} + \frac{4Bbcx^{\frac{21}{2}}}{21} + \frac{2Bc^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)`

[Out]  $2*A*b**2*x**(13/2)/13 + 4*A*b*c*x**(17/2)/17 + 2*A*c**2*x**(21/2)/21 + 2*B*b**2*x**(17/2)/17 + 4*B*b*c*x**(21/2)/21 + 2*B*c**2*x**(25/2)/25$



$$3.170 \quad \int \sqrt{x} (A + Bx^2) (bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=63

$$\frac{2}{11}Ab^2x^{11/2} + \frac{2}{19}cx^{19/2}(Ac + 2bB) + \frac{2}{15}bx^{15/2}(2Ac + bB) + \frac{2}{23}Bc^2x^{23/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{11}Ab^2x^{11/2} + \frac{2}{19}cx^{19/2}(Ac + 2bB) + \frac{2}{15}bx^{15/2}(2Ac + bB) + \frac{2}{23}Bc^2x^{23/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*A\*b^2\*x^(11/2))/11 + (2\*b\*(b\*B + 2\*A\*c)\*x^(15/2))/15 + (2\*c\*(2\*b\*B + A\*c)\*x^(19/2))/19 + (2\*B\*c^2\*x^(23/2))/23

Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \sqrt{x} (A + Bx^2) (bx^2 + cx^4)^2 dx &= \int x^{9/2} (A + Bx^2) (b + cx^2)^2 dx \\ &= \int (Ab^2x^{9/2} + b(bB + 2Ac)x^{13/2} + c(2bB + Ac)x^{17/2} + Bc^2x^{21/2}) dx \\ &= \frac{2}{11}Ab^2x^{11/2} + \frac{2}{15}b(bB + 2Ac)x^{15/2} + \frac{2}{19}c(2bB + Ac)x^{19/2} + \frac{2}{23}Bc^2x^{23/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{11/2} (6555Ab^2 + 3795cx^4(Ac + 2bB) + 4807bx^2(2Ac + bB) + 3135Bc^2x^6)}{72105}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*x^(11/2)\*(6555\*A\*b^2 + 4807\*b\*(b\*B + 2\*A\*c)\*x^2 + 3795\*c\*(2\*b\*B + A\*c)\*x^4 + 3135\*B\*c^2\*x^6))/72105

**IntegrateAlgebraic [A]** time = 0.05, size = 69, normalized size = 1.10

$$\frac{2(6555Ab^2x^{11/2} + 9614Abcx^{15/2} + 3795Ac^2x^{19/2} + 4807b^2Bx^{15/2} + 7590bBcx^{19/2} + 3135Bc^2x^{23/2})}{72105}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*(6555\*A\*b^2\*x^(11/2) + 4807\*b^2\*B\*x^(15/2) + 9614\*A\*b\*c\*x^(15/2) + 7590\*b\*B\*c\*x^(19/2) + 3795\*A\*c^2\*x^(19/2) + 3135\*B\*c^2\*x^(23/2)))/72105

**fricas** [A] time = 0.40, size = 56, normalized size = 0.89

$$\frac{2}{72105} (3135 Bc^2x^{11} + 3795 (2 Bbc + Ac^2)x^9 + 6555 Ab^2x^5 + 4807 (Bb^2 + 2 Abc)x^7)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2\*x^(1/2),x, algorithm="fricas")

[Out] 2/72105\*(3135\*B\*c^2\*x^11 + 3795\*(2\*B\*b\*c + A\*c^2)\*x^9 + 6555\*A\*b^2\*x^5 + 4807\*(B\*b^2 + 2\*A\*b\*c)\*x^7)\*sqrt(x)

**giac** [A] time = 0.18, size = 53, normalized size = 0.84

$$\frac{2}{23} Bc^2x^{\frac{23}{2}} + \frac{4}{19} Bbcx^{\frac{19}{2}} + \frac{2}{19} Ac^2x^{\frac{19}{2}} + \frac{2}{15} Bb^2x^{\frac{15}{2}} + \frac{4}{15} Abcx^{\frac{15}{2}} + \frac{2}{11} Ab^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2\*x^(1/2),x, algorithm="giac")

[Out] 2/23\*B\*c^2\*x^(23/2) + 4/19\*B\*b\*c\*x^(19/2) + 2/19\*A\*c^2\*x^(19/2) + 2/15\*B\*b^2\*x^(15/2) + 4/15\*A\*b\*c\*x^(15/2) + 2/11\*A\*b^2\*x^(11/2)

**maple** [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2 (3135B c^2x^6 + 3795A c^2x^4 + 7590Bbc x^4 + 9614Abc x^2 + 4807B b^2x^2 + 6555b^2A) x^{\frac{11}{2}}}{72105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2\*x^(1/2),x)

[Out] 2/72105\*x^(11/2)\*(3135\*B\*c^2\*x^6+3795\*A\*c^2\*x^4+7590\*B\*b\*c\*x^4+9614\*A\*b\*c\*x^2+4807\*B\*b^2\*x^2+6555\*A\*b^2)

**maxima** [A] time = 1.38, size = 51, normalized size = 0.81

$$\frac{2}{23} Bc^2x^{\frac{23}{2}} + \frac{2}{19} (2 Bbc + Ac^2)x^{\frac{19}{2}} + \frac{2}{11} Ab^2x^{\frac{11}{2}} + \frac{2}{15} (Bb^2 + 2 Abc)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2\*x^(1/2),x, algorithm="maxima")

[Out] 2/23\*B\*c^2\*x^(23/2) + 2/19\*(2\*B\*b\*c + A\*c^2)\*x^(19/2) + 2/11\*A\*b^2\*x^(11/2) + 2/15\*(B\*b^2 + 2\*A\*b\*c)\*x^(15/2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{15/2} \left( \frac{2 B b^2}{15} + \frac{4 A c b}{15} \right) + x^{19/2} \left( \frac{2 A c^2}{19} + \frac{4 B b c}{19} \right) + \frac{2 A b^2 x^{11/2}}{11} + \frac{2 B c^2 x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^{(15/2)} * ((2*B*b^2)/15 + (4*A*b*c)/15) + x^{(19/2)} * ((2*A*c^2)/19 + (4*B*b*c)/19) + (2*A*b^2*x^{(11/2)})/11 + (2*B*c^2*x^{(23/2)})/23$

sympy [A] time = 3.73, size = 66, normalized size = 1.05

$$\frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{2Bc^2x^{\frac{23}{2}}}{23} + \frac{2x^{\frac{19}{2}}(Ac^2 + 2Bbc)}{19} + \frac{2x^{\frac{15}{2}}(2Abc + Bb^2)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2\*x\*\*(1/2), x)

[Out]  $2*A*b**2*x**(11/2)/11 + 2*B*c**2*x**(23/2)/23 + 2*x**(19/2)*(A*c**2 + 2*B*b*c)/19 + 2*x**(15/2)*(2*A*b*c + B*b**2)/15$

$$3.171 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} dx$$

**Optimal.** Leaf size=63

$$\frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac + 2bB) + \frac{2}{13}bx^{13/2}(2Ac + bB) + \frac{2}{21}Bc^2x^{21/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac + 2bB) + \frac{2}{13}bx^{13/2}(2Ac + bB) + \frac{2}{21}Bc^2x^{21/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/Sqrt[x], x]

[Out] (2\*A\*b^2\*x^(9/2))/9 + (2\*b\*(b\*B + 2\*A\*c)\*x^(13/2))/13 + (2\*c\*(2\*b\*B + A\*c)\*x^(17/2))/17 + (2\*B\*c^2\*x^(21/2))/21

**Rule 448**

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{\sqrt{x}} dx &= \int x^{7/2} (A + Bx^2)(b + cx^2)^2 dx \\ &= \int (Ab^2x^{7/2} + b(bB + 2Ac)x^{11/2} + c(2bB + Ac)x^{15/2} + Bc^2x^{19/2}) dx \\ &= \frac{2}{9}Ab^2x^{9/2} + \frac{2}{13}b(bB + 2Ac)x^{13/2} + \frac{2}{17}c(2bB + Ac)x^{17/2} + \frac{2}{21}Bc^2x^{21/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}(1547Ab^2 + 819cx^4(Ac + 2bB) + 1071bx^2(2Ac + bB) + 663Bc^2x^6)}{13923}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/Sqrt[x], x]

[Out] (2\*x^(9/2)\*(1547\*A\*b^2 + 1071\*b\*(b\*B + 2\*A\*c)\*x^2 + 819\*c\*(2\*b\*B + A\*c)\*x^4 + 663\*B\*c^2\*x^6))/13923

**IntegrateAlgebraic [A]** time = 0.04, size = 69, normalized size = 1.10

$$\frac{2(1547Ab^2x^{9/2} + 2142Abcx^{13/2} + 819Ac^2x^{17/2} + 1071b^2Bx^{13/2} + 1638bBcx^{17/2} + 663Bc^2x^{21/2})}{13923}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/Sqrt[x], x]

[Out] (2\*(1547\*A\*b^2\*x^(9/2) + 1071\*b^2\*B\*x^(13/2) + 2142\*A\*b\*c\*x^(13/2) + 1638\*b\*B\*c\*x^(17/2) + 819\*A\*c^2\*x^(17/2) + 663\*B\*c^2\*x^(21/2)))/13923

**fricas [A]** time = 0.39, size = 56, normalized size = 0.89

$$\frac{2}{13923} (663 Bc^2x^{10} + 819 (2 Bbc + Ac^2)x^8 + 1547 Ab^2x^4 + 1071 (Bb^2 + 2 Abc)x^6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(1/2), x, algorithm="fricas")

[Out] 2/13923\*(663\*B\*c^2\*x^10 + 819\*(2\*B\*b\*c + A\*c^2)\*x^8 + 1547\*A\*b^2\*x^4 + 1071\*(B\*b^2 + 2\*A\*b\*c)\*x^6)\*sqrt(x)

**giac [A]** time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{21} Bc^2x^{\frac{21}{2}} + \frac{4}{17} Bbcx^{\frac{17}{2}} + \frac{2}{17} Ac^2x^{\frac{17}{2}} + \frac{2}{13} Bb^2x^{\frac{13}{2}} + \frac{4}{13} Abcx^{\frac{13}{2}} + \frac{2}{9} Ab^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(1/2), x, algorithm="giac")

[Out] 2/21\*B\*c^2\*x^(21/2) + 4/17\*B\*b\*c\*x^(17/2) + 2/17\*A\*c^2\*x^(17/2) + 2/13\*B\*b^2\*x^(13/2) + 4/13\*A\*b\*c\*x^(13/2) + 2/9\*A\*b^2\*x^(9/2)

**maple [A]** time = 0.05, size = 56, normalized size = 0.89

$$\frac{2(663Bc^2x^6 + 819Ac^2x^4 + 1638Bbcx^4 + 2142Abcx^2 + 1071Bb^2x^2 + 1547b^2A)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(1/2), x)

[Out] 2/13923\*x^(9/2)\*(663\*B\*c^2\*x^6+819\*A\*c^2\*x^4+1638\*B\*b\*c\*x^4+2142\*A\*b\*c\*x^2+1071\*B\*b^2\*x^2+1547\*A\*b^2)

**maxima [A]** time = 1.35, size = 51, normalized size = 0.81

$$\frac{2}{21} Bc^2x^{\frac{21}{2}} + \frac{2}{17} (2 Bbc + Ac^2)x^{\frac{17}{2}} + \frac{2}{9} Ab^2x^{\frac{9}{2}} + \frac{2}{13} (Bb^2 + 2 Abc)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(1/2), x, algorithm="maxima")

[Out] 2/21\*B\*c^2\*x^(21/2) + 2/17\*(2\*B\*b\*c + A\*c^2)\*x^(17/2) + 2/9\*A\*b^2\*x^(9/2) + 2/13\*(B\*b^2 + 2\*A\*b\*c)\*x^(13/2)

**mupad [B]** time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2} \left( \frac{2 B b^2}{13} + \frac{4 A c b}{13} \right) + x^{17/2} \left( \frac{2 A c^2}{17} + \frac{4 B b c}{17} \right) + \frac{2 A b^2 x^{9/2}}{9} + \frac{2 B c^2 x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(1/2), x)`

[Out]  $x^{13/2} * ((2*B*b^2)/13 + (4*A*b*c)/13) + x^{17/2} * ((2*A*c^2)/17 + (4*B*b*c)/17) + (2*A*b^2*x^{9/2})/9 + (2*B*c^2*x^{21/2})/21$

sympy [A] time = 9.94, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{4Abcx^{\frac{13}{2}}}{13} + \frac{2Ac^2x^{\frac{17}{2}}}{17} + \frac{2Bb^2x^{\frac{13}{2}}}{13} + \frac{4Bbcx^{\frac{17}{2}}}{17} + \frac{2Bc^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(1/2), x)`

[Out]  $2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(17/2)/17 + 2*B*b**2*x**(13/2)/13 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(21/2)/21$

$$3.172 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2}{7}Ab^2x^{7/2} + \frac{2}{15}cx^{15/2}(Ac + 2bB) + \frac{2}{11}bx^{11/2}(2Ac + bB) + \frac{2}{19}Bc^2x^{19/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{7}Ab^2x^{7/2} + \frac{2}{15}cx^{15/2}(Ac + 2bB) + \frac{2}{11}bx^{11/2}(2Ac + bB) + \frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(3/2), x]

[Out] (2\*A\*b^2\*x^(7/2))/7 + (2\*b\*(b\*B + 2\*A\*c)\*x^(11/2))/11 + (2\*c\*(2\*b\*B + A\*c)\*x^(15/2))/15 + (2\*B\*c^2\*x^(19/2))/19

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^{3/2}} dx &= \int x^{5/2} (A + Bx^2) (b + cx^2)^2 dx \\ &= \int (Ab^2x^{5/2} + b(bB + 2Ac)x^{9/2} + c(2bB + Ac)x^{13/2} + Bc^2x^{17/2}) dx \\ &= \frac{2}{7}Ab^2x^{7/2} + \frac{2}{11}b(bB + 2Ac)x^{11/2} + \frac{2}{15}c(2bB + Ac)x^{15/2} + \frac{2}{19}Bc^2x^{19/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 63, normalized size = 1.00

$$\frac{2}{7}Ab^2x^{7/2} + \frac{2}{15}cx^{15/2}(Ac + 2bB) + \frac{2}{11}bx^{11/2}(2Ac + bB) + \frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(3/2), x]

[Out] (2\*A\*b^2\*x^(7/2))/7 + (2\*b\*(b\*B + 2\*A\*c)\*x^(11/2))/11 + (2\*c\*(2\*b\*B + A\*c)\*x^(15/2))/15 + (2\*B\*c^2\*x^(19/2))/19

**IntegrateAlgebraic [A]** time = 0.04, size = 69, normalized size = 1.10

$$\frac{2(3135Ab^2x^{7/2} + 3990Abcx^{11/2} + 1463Ac^2x^{15/2} + 1995b^2Bx^{11/2} + 2926bBcx^{15/2} + 1155Bc^2x^{19/2})}{21945}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(3/2), x]

[Out] (2\*(3135\*A\*b^2\*x^(7/2) + 1995\*b^2\*B\*x^(11/2) + 3990\*A\*b\*c\*x^(11/2) + 2926\*b\*B\*c\*x^(15/2) + 1463\*A\*c^2\*x^(15/2) + 1155\*B\*c^2\*x^(19/2)))/21945

**fricas** [A] time = 0.39, size = 56, normalized size = 0.89

$$\frac{2}{21945} (1155 Bc^2x^9 + 1463 (2 Bbc + Ac^2)x^7 + 3135 Ab^2x^3 + 1995 (Bb^2 + 2 Abc)x^5)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(3/2), x, algorithm="fricas")

[Out] 2/21945\*(1155\*B\*c^2\*x^9 + 1463\*(2\*B\*b\*c + A\*c^2)\*x^7 + 3135\*A\*b^2\*x^3 + 1995\*(B\*b^2 + 2\*A\*b\*c)\*x^5)\*sqrt(x)

**giac** [A] time = 0.21, size = 53, normalized size = 0.84

$$\frac{2}{19} Bc^2x^{\frac{19}{2}} + \frac{4}{15} Bbcx^{\frac{15}{2}} + \frac{2}{15} Ac^2x^{\frac{15}{2}} + \frac{2}{11} Bb^2x^{\frac{11}{2}} + \frac{4}{11} Abcx^{\frac{11}{2}} + \frac{2}{7} Ab^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(3/2), x, algorithm="giac")

[Out] 2/19\*B\*c^2\*x^(19/2) + 4/15\*B\*b\*c\*x^(15/2) + 2/15\*A\*c^2\*x^(15/2) + 2/11\*B\*b^2\*x^(11/2) + 4/11\*A\*b\*c\*x^(11/2) + 2/7\*A\*b^2\*x^(7/2)

**maple** [A] time = 0.06, size = 56, normalized size = 0.89

$$\frac{2(1155Bc^2x^6 + 1463Ac^2x^4 + 2926Bbcx^4 + 3990Abcx^2 + 1995Bb^2x^2 + 3135b^2A)x^{\frac{7}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(3/2), x)

[Out] 2/21945\*x^(7/2)\*(1155\*B\*c^2\*x^6+1463\*A\*c^2\*x^4+2926\*B\*b\*c\*x^4+3990\*A\*b\*c\*x^2+1995\*B\*b^2\*x^2+3135\*A\*b^2)

**maxima** [A] time = 1.36, size = 51, normalized size = 0.81

$$\frac{2}{19} Bc^2x^{\frac{19}{2}} + \frac{2}{15} (2 Bbc + Ac^2)x^{\frac{15}{2}} + \frac{2}{7} Ab^2x^{\frac{7}{2}} + \frac{2}{11} (Bb^2 + 2 Abc)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(3/2), x, algorithm="maxima")

[Out] 2/19\*B\*c^2\*x^(19/2) + 2/15\*(2\*B\*b\*c + A\*c^2)\*x^(15/2) + 2/7\*A\*b^2\*x^(7/2) + 2/11\*(B\*b^2 + 2\*A\*b\*c)\*x^(11/2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2} \left( \frac{2 B b^2}{11} + \frac{4 A c b}{11} \right) + x^{15/2} \left( \frac{2 A c^2}{15} + \frac{4 B b c}{15} \right) + \frac{2 A b^2 x^{7/2}}{7} + \frac{2 B c^2 x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(3/2), x)



[Out]  $x^{(11/2)} * ((2*B*b^2)/11 + (4*A*b*c)/11) + x^{(15/2)} * ((2*A*c^2)/15 + (4*B*b*c)/15) + (2*A*b^2*x^{(7/2)})/7 + (2*B*c^2*x^{(19/2)})/19$

sympy [A] time = 10.48, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{4Abcx^{\frac{11}{2}}}{11} + \frac{2Ac^2x^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{11}{2}}}{11} + \frac{4Bbcx^{\frac{15}{2}}}{15} + \frac{2Bc^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*(3/2),x)

[Out]  $2*A*b**2*x**(7/2)/7 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(15/2)/15 + 2*B*b**2*x**(11/2)/11 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(19/2)/19$

$$3.173 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac + 2bB) + \frac{2}{9}bx^{9/2}(2Ac + bB) + \frac{2}{17}Bc^2x^{17/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac + 2bB) + \frac{2}{9}bx^{9/2}(2Ac + bB) + \frac{2}{17}Bc^2x^{17/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(5/2), x]

[Out] (2\*A\*b^2\*x^(5/2))/5 + (2\*b\*(b\*B + 2\*A\*c)\*x^(9/2))/9 + (2\*c\*(2\*b\*B + A\*c)\*x^(13/2))/13 + (2\*B\*c^2\*x^(17/2))/17

**Rule 448**

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} dx &= \int x^{3/2} (A+Bx^2)(b+cx^2)^2 dx \\ &= \int (Ab^2x^{3/2} + b(bB+2Ac)x^{7/2} + c(2bB+Ac)x^{11/2} + Bc^2x^{15/2}) dx \\ &= \frac{2}{5}Ab^2x^{5/2} + \frac{2}{9}b(bB+2Ac)x^{9/2} + \frac{2}{13}c(2bB+Ac)x^{13/2} + \frac{2}{17}Bc^2x^{17/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{5/2} (1989Ab^2 + 765cx^4(Ac + 2bB) + 1105bx^2(2Ac + bB) + 585Bc^2x^6)}{9945}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(5/2), x]

[Out] (2\*x^(5/2)\*(1989\*A\*b^2 + 1105\*b\*(b\*B + 2\*A\*c)\*x^2 + 765\*c\*(2\*b\*B + A\*c)\*x^4 + 585\*B\*c^2\*x^6))/9945

**IntegrateAlgebraic [A]** time = 0.05, size = 69, normalized size = 1.10

$$\frac{2(1989Ab^2x^{5/2} + 2210Abcx^{9/2} + 765Ac^2x^{13/2} + 1105b^2Bx^{9/2} + 1530bBcx^{13/2} + 585Bc^2x^{17/2})}{9945}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(5/2),x]

[Out] (2\*(1989\*A\*b^2\*x^(5/2) + 1105\*b^2\*B\*x^(9/2) + 2210\*A\*b\*c\*x^(9/2) + 1530\*b\*B\*c\*x^(13/2) + 765\*A\*c^2\*x^(13/2) + 585\*B\*c^2\*x^(17/2)))/9945

**fricas** [A] time = 0.39, size = 56, normalized size = 0.89

$$\frac{2}{9945} (585 Bc^2x^8 + 765 (2 Bbc + Ac^2)x^6 + 1989 Ab^2x^2 + 1105 (Bb^2 + 2 Abc)x^4)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(5/2),x, algorithm="fricas")

[Out] 2/9945\*(585\*B\*c^2\*x^8 + 765\*(2\*B\*b\*c + A\*c^2)\*x^6 + 1989\*A\*b^2\*x^2 + 1105\*(B\*b^2 + 2\*A\*b\*c)\*x^4)\*sqrt(x)

**giac** [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{17} Bc^2x^{\frac{17}{2}} + \frac{4}{13} Bbcx^{\frac{13}{2}} + \frac{2}{13} Ac^2x^{\frac{13}{2}} + \frac{2}{9} Bb^2x^{\frac{9}{2}} + \frac{4}{9} Abcx^{\frac{9}{2}} + \frac{2}{5} Ab^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(5/2),x, algorithm="giac")

[Out] 2/17\*B\*c^2\*x^(17/2) + 4/13\*B\*b\*c\*x^(13/2) + 2/13\*A\*c^2\*x^(13/2) + 2/9\*B\*b^2\*x^(9/2) + 4/9\*A\*b\*c\*x^(9/2) + 2/5\*A\*b^2\*x^(5/2)

**maple** [A] time = 0.06, size = 56, normalized size = 0.89

$$\frac{2 (585B c^2x^6 + 765A c^2x^4 + 1530Bbc x^4 + 2210Abc x^2 + 1105B b^2x^2 + 1989b^2A) x^{\frac{5}{2}}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(5/2),x)

[Out] 2/9945\*x^(5/2)\*(585\*B\*c^2\*x^6+765\*A\*c^2\*x^4+1530\*B\*b\*c\*x^4+2210\*A\*b\*c\*x^2+1105\*B\*b^2\*x^2+1989\*A\*b^2)

**maxima** [A] time = 1.37, size = 51, normalized size = 0.81

$$\frac{2}{17} Bc^2x^{\frac{17}{2}} + \frac{2}{13} (2 Bbc + Ac^2)x^{\frac{13}{2}} + \frac{2}{5} Ab^2x^{\frac{5}{2}} + \frac{2}{9} (Bb^2 + 2 Abc)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(5/2),x, algorithm="maxima")

[Out] 2/17\*B\*c^2\*x^(17/2) + 2/13\*(2\*B\*b\*c + A\*c^2)\*x^(13/2) + 2/5\*A\*b^2\*x^(5/2) + 2/9\*(B\*b^2 + 2\*A\*b\*c)\*x^(9/2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2} \left( \frac{2 B b^2}{9} + \frac{4 A c b}{9} \right) + x^{13/2} \left( \frac{2 A c^2}{13} + \frac{4 B b c}{13} \right) + \frac{2 A b^2 x^{5/2}}{5} + \frac{2 B c^2 x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(5/2),x)

[Out]  $x^{(9/2)} * ((2*B*b^2)/9 + (4*A*b*c)/9) + x^{(13/2)} * ((2*A*c^2)/13 + (4*B*b*c)/13) + (2*A*b^2*x^{(5/2)})/5 + (2*B*c^2*x^{(17/2)})/17$

sympy [A] time = 12.06, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{5}{2}}}{5} + \frac{4Abcx^{\frac{9}{2}}}{9} + \frac{2Ac^2x^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{9}{2}}}{9} + \frac{4Bbcx^{\frac{13}{2}}}{13} + \frac{2Bc^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*(5/2),x)

[Out]  $2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(9/2)/9 + 2*A*c**2*x**(13/2)/13 + 2*B*b**2*x**(9/2)/9 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(17/2)/17$

$$3.174 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2}{3}Ab^2x^{3/2} + \frac{2}{11}cx^{11/2}(Ac + 2bB) + \frac{2}{7}bx^{7/2}(2Ac + bB) + \frac{2}{15}Bc^2x^{15/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{3}Ab^2x^{3/2} + \frac{2}{11}cx^{11/2}(Ac + 2bB) + \frac{2}{7}bx^{7/2}(2Ac + bB) + \frac{2}{15}Bc^2x^{15/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(7/2), x]

[Out] (2\*A\*b^2\*x^(3/2))/3 + (2\*b\*(b\*B + 2\*A\*c)\*x^(7/2))/7 + (2\*c\*(2\*b\*B + A\*c)\*x^(11/2))/11 + (2\*B\*c^2\*x^(15/2))/15

**Rule 448**

Int[((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^(m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_.)^(m\_.)\*((a\_.)\*(x\_.)^(p\_.) + (b\_.)\*(x\_.)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}} dx &= \int \sqrt{x} (A+Bx^2)(b+cx^2)^2 dx \\ &= \int (Ab^2\sqrt{x} + b(bB+2Ac)x^{5/2} + c(2bB+Ac)x^{9/2} + Bc^2x^{13/2}) dx \\ &= \frac{2}{3}Ab^2x^{3/2} + \frac{2}{7}b(bB+2Ac)x^{7/2} + \frac{2}{11}c(2bB+Ac)x^{11/2} + \frac{2}{15}Bc^2x^{15/2} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}(385Ab^2 + 105cx^4(Ac + 2bB) + 165bx^2(2Ac + bB) + 77Bc^2x^6)}{1155}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(7/2), x]

[Out] (2\*x^(3/2)\*(385\*A\*b^2 + 165\*b\*(b\*B + 2\*A\*c)\*x^2 + 105\*c\*(2\*b\*B + A\*c)\*x^4 + 77\*B\*c^2\*x^6))/1155

**IntegrateAlgebraic [A]** time = 0.04, size = 69, normalized size = 1.10

$$\frac{2(385Ab^2x^{3/2} + 330Abcx^{7/2} + 105Ac^2x^{11/2} + 165b^2Bx^{7/2} + 210bBcx^{11/2} + 77Bc^2x^{15/2})}{1155}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(7/2), x]

[Out] (2\*(385\*A\*b^2\*x^(3/2) + 165\*b^2\*B\*x^(7/2) + 330\*A\*b\*c\*x^(7/2) + 210\*b\*B\*c\*x^(11/2) + 105\*A\*c^2\*x^(11/2) + 77\*B\*c^2\*x^(15/2)))/1155

**fricas** [A] time = 0.40, size = 54, normalized size = 0.86

$$\frac{2}{1155} (77 Bc^2x^7 + 105 (2 Bbc + Ac^2)x^5 + 385 Ab^2x + 165 (Bb^2 + 2 Abc)x^3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(7/2), x, algorithm="fricas")

[Out] 2/1155\*(77\*B\*c^2\*x^7 + 105\*(2\*B\*b\*c + A\*c^2)\*x^5 + 385\*A\*b^2\*x + 165\*(B\*b^2 + 2\*A\*b\*c)\*x^3)\*sqrt(x)

**giac** [A] time = 0.17, size = 53, normalized size = 0.84

$$\frac{2}{15} Bc^2x^{\frac{15}{2}} + \frac{4}{11} Bbcx^{\frac{11}{2}} + \frac{2}{11} Ac^2x^{\frac{11}{2}} + \frac{2}{7} Bb^2x^{\frac{7}{2}} + \frac{4}{7} Abcx^{\frac{7}{2}} + \frac{2}{3} Ab^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(7/2), x, algorithm="giac")

[Out] 2/15\*B\*c^2\*x^(15/2) + 4/11\*B\*b\*c\*x^(11/2) + 2/11\*A\*c^2\*x^(11/2) + 2/7\*B\*b^2\*x^(7/2) + 4/7\*A\*b\*c\*x^(7/2) + 2/3\*A\*b^2\*x^(3/2)

**maple** [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2(77Bc^2x^6 + 105Ac^2x^4 + 210Bbcx^4 + 330Abcx^2 + 165Bb^2x^2 + 385b^2A)x^{\frac{3}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(7/2), x)

[Out] 2/1155\*x^(3/2)\*(77\*B\*c^2\*x^6+105\*A\*c^2\*x^4+210\*B\*b\*c\*x^4+330\*A\*b\*c\*x^2+165\*B\*b^2\*x^2+385\*A\*b^2)

**maxima** [A] time = 1.33, size = 51, normalized size = 0.81

$$\frac{2}{15} Bc^2x^{\frac{15}{2}} + \frac{2}{11} (2 Bbc + Ac^2)x^{\frac{11}{2}} + \frac{2}{3} Ab^2x^{\frac{3}{2}} + \frac{2}{7} (Bb^2 + 2 Abc)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^2/x^(7/2), x, algorithm="maxima")

[Out] 2/15\*B\*c^2\*x^(15/2) + 2/11\*(2\*B\*b\*c + A\*c^2)\*x^(11/2) + 2/3\*A\*b^2\*x^(3/2) + 2/7\*(B\*b^2 + 2\*A\*b\*c)\*x^(7/2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{7/2} \left( \frac{2 B b^2}{7} + \frac{4 A c b}{7} \right) + x^{11/2} \left( \frac{2 A c^2}{11} + \frac{4 B b c}{11} \right) + \frac{2 A b^2 x^{3/2}}{3} + \frac{2 B c^2 x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^2)/x^(7/2), x)

[Out]  $x^{(7/2)} * ((2*B*b^2)/7 + (4*A*b*c)/7) + x^{(11/2)} * ((2*A*c^2)/11 + (4*B*b*c)/11) + (2*A*b^2*x^{(3/2)})/3 + (2*B*c^2*x^{(15/2)})/15$

sympy [A] time = 15.46, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{3}{2}}}{3} + \frac{4Abcx^{\frac{7}{2}}}{7} + \frac{2Ac^2x^{\frac{11}{2}}}{11} + \frac{2Bb^2x^{\frac{7}{2}}}{7} + \frac{4Bbcx^{\frac{11}{2}}}{11} + \frac{2Bc^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2/x\*\*(7/2),x)

[Out]  $2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(11/2)/11 + 2*B*b**2*x**(7/2)/7 + 4*B*b*c*x**(11/2)/11 + 2*B*c**2*x**(15/2)/15$

$$3.175 \quad \int x^{7/2} (A + Bx^2) (bx^2 + cx^4)^3 dx$$

**Optimal.** Leaf size=85

$$\frac{2}{21} Ab^3 x^{21/2} + \frac{2}{25} b^2 x^{25/2} (3Ac + bB) + \frac{2}{33} c^2 x^{33/2} (Ac + 3bB) + \frac{6}{29} bcx^{29/2} (Ac + bB) + \frac{2}{37} Bc^3 x^{37/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{25} b^2 x^{25/2} (3Ac + bB) + \frac{2}{21} Ab^3 x^{21/2} + \frac{2}{33} c^2 x^{33/2} (Ac + 3bB) + \frac{6}{29} bcx^{29/2} (Ac + bB) + \frac{2}{37} Bc^3 x^{37/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(21/2))/21 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(25/2))/25 + (6\*b\*c\*(b\*B + A\*c)\*x^(29/2))/29 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(33/2))/33 + (2\*B\*c^3\*x^(37/2))/37

#### Rule 448

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int x^{7/2} (A + Bx^2) (bx^2 + cx^4)^3 dx &= \int x^{19/2} (A + Bx^2) (b + cx^2)^3 dx \\ &= \int (Ab^3 x^{19/2} + b^2(bB + 3Ac)x^{23/2} + 3bc(bB + Ac)x^{27/2} + c^2(3bB + Ac)x^{31/2} + Bc^3 x^{35/2}) dx \\ &= \frac{2}{21} Ab^3 x^{21/2} + \frac{2}{25} b^2(bB + 3Ac)x^{25/2} + \frac{6}{29} bc(bB + Ac)x^{29/2} + \frac{2}{33} c^2(3bB + Ac)x^{33/2} + \frac{2}{37} Bc^3 x^{37/2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{21} Ab^3 x^{21/2} + \frac{2}{25} b^2 x^{25/2} (3Ac + bB) + \frac{2}{33} c^2 x^{33/2} (Ac + 3bB) + \frac{6}{29} bcx^{29/2} (Ac + bB) + \frac{2}{37} Bc^3 x^{37/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(21/2))/21 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(25/2))/25 + (6\*b\*c\*(b\*B + A\*c)\*x^(29/2))/29 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(33/2))/33 + (2\*B\*c^3\*x^(37/2))/37

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(295075Ab^3x^{21/2} + 743589Ab^2cx^{25/2} + 641025Abc^2x^{29/2} + 187775Ac^3x^{33/2} + 247863b^3Bx^{25/2} + 641025b^2Bcx^{29/2} + 563325bBc^2x^{33/2} + 167475Bc^3x^{37/2})}{6196575}$$



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(7/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*(295075\*A\*b^3\*x^(21/2) + 247863\*b^3\*B\*x^(25/2) + 743589\*A\*b^2\*c\*x^(25/2) + 641025\*b^2\*B\*c\*x^(29/2) + 641025\*A\*b\*c^2\*x^(29/2) + 563325\*b\*B\*c^2\*x^(33/2) + 187775\*A\*c^3\*x^(33/2) + 167475\*B\*c^3\*x^(37/2)))/6196575

**fricas** [A] time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{6196575} (167475 B c^3 x^{18} + 187775 (3 B b c^2 + A c^3) x^{16} + 641025 (B b^2 c + A b c^2) x^{14} + 295075 A b^3 x^{10} + 247863 (B b^3 + 3 A b^2 c) x^{12}) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 2/6196575\*(167475\*B\*c^3\*x^18 + 187775\*(3\*B\*b\*c^2 + A\*c^3)\*x^16 + 641025\*(B\*b^2\*c + A\*b\*c^2)\*x^14 + 295075\*A\*b^3\*x^10 + 247863\*(B\*b^3 + 3\*A\*b^2\*c)\*x^12)\*sqrt(x)

**giac** [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{37} B c^3 x^{\frac{37}{2}} + \frac{2}{11} B b c^2 x^{\frac{33}{2}} + \frac{2}{33} A c^3 x^{\frac{33}{2}} + \frac{6}{29} B b^2 c x^{\frac{29}{2}} + \frac{6}{29} A b c^2 x^{\frac{29}{2}} + \frac{2}{25} B b^3 x^{\frac{25}{2}} + \frac{6}{25} A b^2 c x^{\frac{25}{2}} + \frac{2}{21} A b^3 x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 2/37\*B\*c^3\*x^(37/2) + 2/11\*B\*b\*c^2\*x^(33/2) + 2/33\*A\*c^3\*x^(33/2) + 6/29\*B\*b^2\*c\*x^(29/2) + 6/29\*A\*b\*c^2\*x^(29/2) + 2/25\*B\*b^3\*x^(25/2) + 6/25\*A\*b^2\*c\*x^(25/2) + 2/21\*A\*b^3\*x^(21/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(167475 B c^3 x^8 + 187775 A c^3 x^6 + 563325 B b c^2 x^6 + 641025 A b c^2 x^4 + 641025 B b^2 c x^4 + 743589 A b^2 c x^2 + 247863 B b^3 x^2 + 295075 A b^3) x^{\frac{21}{2}}}{6196575}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x)

[Out] 2/6196575\*x^(21/2)\*(167475\*B\*c^3\*x^8+187775\*A\*c^3\*x^6+563325\*B\*b\*c^2\*x^6+641025\*A\*b\*c^2\*x^4+641025\*B\*b^2\*c\*x^4+743589\*A\*b^2\*c\*x^2+247863\*B\*b^3\*x^2+295075\*A\*b^3)

**maxima** [A] time = 1.32, size = 73, normalized size = 0.86

$$\frac{2}{37} B c^3 x^{\frac{37}{2}} + \frac{2}{33} (3 B b c^2 + A c^3) x^{\frac{33}{2}} + \frac{6}{29} (B b^2 c + A b c^2) x^{\frac{29}{2}} + \frac{2}{21} A b^3 x^{\frac{21}{2}} + \frac{2}{25} (B b^3 + 3 A b^2 c) x^{\frac{25}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 2/37\*B\*c^3\*x^(37/2) + 2/33\*(3\*B\*b\*c^2 + A\*c^3)\*x^(33/2) + 6/29\*(B\*b^2\*c + A\*b\*c^2)\*x^(29/2) + 2/21\*A\*b^3\*x^(21/2) + 2/25\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(25/2)

**mupad** [B] time = 0.10, size = 69, normalized size = 0.81

$$x^{25/2} \left( \frac{2 B b^3}{25} + \frac{6 A c b^2}{25} \right) + x^{33/2} \left( \frac{2 A c^3}{33} + \frac{2 B b c^2}{11} \right) + \frac{2 A b^3 x^{21/2}}{21} + \frac{2 B c^3 x^{37/2}}{37} + \frac{6 b c x^{29/2} (A c + B b)}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)`

[Out]  $x^{(25/2)} * ((2*B*b^3)/25 + (6*A*b^2*c)/25) + x^{(33/2)} * ((2*A*c^3)/33 + (2*B*b*c^2)/11) + (2*A*b^3*x^{(21/2)})/21 + (2*B*c^3*x^{(37/2)})/37 + (6*b*c*x^{(29/2)} * (A*c + B*b))/29$

sympy [A] time = 134.77, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{21}{2}}}{21} + \frac{6Ab^2cx^{\frac{25}{2}}}{25} + \frac{6Abc^2x^{\frac{29}{2}}}{29} + \frac{2Ac^3x^{\frac{33}{2}}}{33} + \frac{2Bb^3x^{\frac{25}{2}}}{25} + \frac{6Bb^2cx^{\frac{29}{2}}}{29} + \frac{2Bbc^2x^{\frac{33}{2}}}{11} + \frac{2Bc^3x^{\frac{37}{2}}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)`

[Out]  $2*A*b**3*x**(21/2)/21 + 6*A*b**2*c*x**(25/2)/25 + 6*A*b*c**2*x**(29/2)/29 + 2*A*c**3*x**(33/2)/33 + 2*B*b**3*x**(25/2)/25 + 6*B*b**2*c*x**(29/2)/29 + 2*B*b*c**2*x**(33/2)/11 + 2*B*c**3*x**(37/2)/37$

$$3.176 \quad \int x^{5/2} (A + Bx^2) (bx^2 + cx^4)^3 dx$$

**Optimal.** Leaf size=85

$$\frac{2}{19}Ab^3x^{19/2} + \frac{2}{23}b^2x^{23/2}(3Ac + bB) + \frac{2}{31}c^2x^{31/2}(Ac + 3bB) + \frac{2}{9}bcx^{27/2}(Ac + bB) + \frac{2}{35}Bc^3x^{35/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{23}b^2x^{23/2}(3Ac + bB) + \frac{2}{19}Ab^3x^{19/2} + \frac{2}{31}c^2x^{31/2}(Ac + 3bB) + \frac{2}{9}bcx^{27/2}(Ac + bB) + \frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(19/2))/19 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(23/2))/23 + (2\*b\*c\*(b\*B + A\*c)\*x^(27/2))/9 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(31/2))/31 + (2\*B\*c^3\*x^(35/2))/35

#### Rule 448

Int[((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1584

Int[(u\_.)\*(x\_.)^(m\_.)\*((a\_.)\*(x\_.)^(p\_.) + (b\_.)\*(x\_.)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int x^{5/2} (A + Bx^2) (bx^2 + cx^4)^3 dx &= \int x^{17/2} (A + Bx^2) (b + cx^2)^3 dx \\ &= \int (Ab^3x^{17/2} + b^2(bB + 3Ac)x^{21/2} + 3bc(bB + Ac)x^{25/2} + c^2(3bB + Ac)x^{29/2}) dx \\ &= \frac{2}{19}Ab^3x^{19/2} + \frac{2}{23}b^2(bB + 3Ac)x^{23/2} + \frac{2}{9}bc(bB + Ac)x^{27/2} + \frac{2}{31}c^2(3bB + Ac)x^{31/2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{19}Ab^3x^{19/2} + \frac{2}{23}b^2x^{23/2}(3Ac + bB) + \frac{2}{31}c^2x^{31/2}(Ac + 3bB) + \frac{2}{9}bcx^{27/2}(Ac + bB) + \frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(19/2))/19 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(23/2))/23 + (2\*b\*c\*(b\*B + A\*c)\*x^(27/2))/9 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(31/2))/31 + (2\*B\*c^3\*x^(35/2))/35

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(224595Ab^3x^{19/2} + 556605Ab^2cx^{23/2} + 474145Abc^2x^{27/2} + 137655Ac^3x^{31/2} + 185535b^3Bx^{23/2} + 474145b^2Bcx^{27/2} + 412965bBc^2x^{31/2} + 121923Bc^3x^{35/2})}{4267305}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*(224595\*A\*b^3\*x^(19/2) + 185535\*b^3\*B\*x^(23/2) + 556605\*A\*b^2\*c\*x^(23/2) + 474145\*b^2\*B\*c\*x^(27/2) + 474145\*A\*b\*c^2\*x^(27/2) + 412965\*b\*B\*c^2\*x^(31/2) + 137655\*A\*c^3\*x^(31/2) + 121923\*B\*c^3\*x^(35/2)))/4267305

**fricas** [A] time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{4267305} (121923 Bc^3x^{17} + 137655 (3Bbc^2 + Ac^3)x^{15} + 474145 (Bb^2c + Abc^2)x^{13} + 224595 Ab^3x^9 + 185535 (Bb^3 + 3Ab^2c)x^{11})\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 2/4267305\*(121923\*B\*c^3\*x^17 + 137655\*(3\*B\*b\*c^2 + A\*c^3)\*x^15 + 474145\*(B\*b^2\*c + A\*b\*c^2)\*x^13 + 224595\*A\*b^3\*x^9 + 185535\*(B\*b^3 + 3\*A\*b^2\*c)\*x^11)\*sqrt(x)

**giac** [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{35} Bc^3x^{\frac{35}{2}} + \frac{6}{31} Bbc^2x^{\frac{31}{2}} + \frac{2}{31} Ac^3x^{\frac{31}{2}} + \frac{2}{9} Bb^2cx^{\frac{27}{2}} + \frac{2}{9} Abc^2x^{\frac{27}{2}} + \frac{2}{23} Bb^3x^{\frac{23}{2}} + \frac{6}{23} Ab^2cx^{\frac{23}{2}} + \frac{2}{19} Ab^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 2/35\*B\*c^3\*x^(35/2) + 6/31\*B\*b\*c^2\*x^(31/2) + 2/31\*A\*c^3\*x^(31/2) + 2/9\*B\*b^2\*c\*x^(27/2) + 2/9\*A\*b\*c^2\*x^(27/2) + 2/23\*B\*b^3\*x^(23/2) + 6/23\*A\*b^2\*c\*x^(23/2) + 2/19\*A\*b^3\*x^(19/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(121923Bc^3x^8 + 137655Ac^3x^6 + 412965Bbc^2x^6 + 474145Abc^2x^4 + 474145Bb^2cx^4 + 556605Ab^2cx^2 + 185535Bb^3x^2 + 224595Ab^3)x^{\frac{19}{2}}}{4267305}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x)

[Out] 2/4267305\*x^(19/2)\*(121923\*B\*c^3\*x^8+137655\*A\*c^3\*x^6+412965\*B\*b\*c^2\*x^6+474145\*A\*b\*c^2\*x^4+474145\*B\*b^2\*c\*x^4+556605\*A\*b^2\*c\*x^2+185535\*B\*b^3\*x^2+224595\*A\*b^3)

**maxima** [A] time = 1.36, size = 73, normalized size = 0.86

$$\frac{2}{35} Bc^3x^{\frac{35}{2}} + \frac{2}{31} (3Bbc^2 + Ac^3)x^{\frac{31}{2}} + \frac{2}{9} (Bb^2c + Abc^2)x^{\frac{27}{2}} + \frac{2}{19} Ab^3x^{\frac{19}{2}} + \frac{2}{23} (Bb^3 + 3Ab^2c)x^{\frac{23}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 2/35\*B\*c^3\*x^(35/2) + 2/31\*(3\*B\*b\*c^2 + A\*c^3)\*x^(31/2) + 2/9\*(B\*b^2\*c + A\*b\*c^2)\*x^(27/2) + 2/19\*A\*b^3\*x^(19/2) + 2/23\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(23/2)

**mupad** [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{23/2} \left( \frac{2Bb^3}{23} + \frac{6Ac^3}{23} \right) + x^{31/2} \left( \frac{2Ac^3}{31} + \frac{6Bbc^2}{31} \right) + \frac{2Ab^3x^{19/2}}{19} + \frac{2Bc^3x^{35/2}}{35} + \frac{2bcx^{27/2}(Ac+Bb)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)`

[Out]  $x^{23/2} * ((2*B*b^3)/23 + (6*A*b^2*c)/23) + x^{31/2} * ((2*A*c^3)/31 + (6*B*b*c^2)/31) + (2*A*b^3*x^{19/2})/19 + (2*B*c^3*x^{35/2})/35 + (2*b*c*x^{27/2}) * (A*c + B*b)/9$

**sympy [A]** time = 92.32, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{6Ab^2cx^{\frac{23}{2}}}{23} + \frac{2Abc^2x^{\frac{27}{2}}}{9} + \frac{2Ac^3x^{\frac{31}{2}}}{31} + \frac{2Bb^3x^{\frac{23}{2}}}{23} + \frac{2Bb^2cx^{\frac{27}{2}}}{9} + \frac{6Bbc^2x^{\frac{31}{2}}}{31} + \frac{2Bc^3x^{\frac{35}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)`

[Out]  $2*A*b**3*x**(19/2)/19 + 6*A*b**2*c*x**(23/2)/23 + 2*A*b*c**2*x**(27/2)/9 + 2*A*c**3*x**(31/2)/31 + 2*B*b**3*x**(23/2)/23 + 2*B*b**2*c*x**(27/2)/9 + 6*B*b*c**2*x**(31/2)/31 + 2*B*c**3*x**(35/2)/35$

$$3.177 \quad \int x^{3/2} (A + Bx^2) (bx^2 + cx^4)^3 dx$$

**Optimal.** Leaf size=85

$$\frac{2}{17}Ab^3x^{17/2} + \frac{2}{21}b^2x^{21/2}(3Ac + bB) + \frac{2}{29}c^2x^{29/2}(Ac + 3bB) + \frac{6}{25}bcx^{25/2}(Ac + bB) + \frac{2}{33}Bc^3x^{33/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{21}b^2x^{21/2}(3Ac + bB) + \frac{2}{17}Ab^3x^{17/2} + \frac{2}{29}c^2x^{29/2}(Ac + 3bB) + \frac{6}{25}bcx^{25/2}(Ac + bB) + \frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(17/2))/17 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(21/2))/21 + (6\*b\*c\*(b\*B + A\*c)\*x^(25/2))/25 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(29/2))/29 + (2\*B\*c^3\*x^(33/2))/33

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int x^{3/2} (A + Bx^2) (bx^2 + cx^4)^3 dx &= \int x^{15/2} (A + Bx^2) (b + cx^2)^3 dx \\ &= \int (Ab^3x^{15/2} + b^2(bB + 3Ac)x^{19/2} + 3bc(bB + Ac)x^{23/2} + c^2(3bB + Ac)x^{27/2} + Ac^3x^{31/2}) dx \\ &= \frac{2}{17}Ab^3x^{17/2} + \frac{2}{21}b^2(bB + 3Ac)x^{21/2} + \frac{6}{25}bc(bB + Ac)x^{25/2} + \frac{2}{29}c^2(3bB + Ac)x^{29/2} + \frac{2}{33}Ac^3x^{33/2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{17}Ab^3x^{17/2} + \frac{2}{21}b^2x^{21/2}(3Ac + bB) + \frac{2}{29}c^2x^{29/2}(Ac + 3bB) + \frac{6}{25}bcx^{25/2}(Ac + bB) + \frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(17/2))/17 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(21/2))/21 + (6\*b\*c\*(b\*B + A\*c)\*x^(25/2))/25 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(29/2))/29 + (2\*B\*c^3\*x^(33/2))/33

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(167475Ab^3x^{17/2} + 406725Ab^2cx^{21/2} + 341649Abc^2x^{25/2} + 98175Ac^3x^{29/2} + 135575b^3Bx^{21/2} + 341649b^2Bcx^{25/2} + 294525bBc^2x^{29/2} + 86275Bc^3x^{33/2})}{2847075}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*(167475\*A\*b^3\*x^(17/2) + 135575\*b^3\*B\*x^(21/2) + 406725\*A\*b^2\*c\*x^(21/2) + 341649\*b^2\*B\*c\*x^(25/2) + 341649\*A\*b\*c^2\*x^(25/2) + 294525\*b\*B\*c^2\*x^(29/2) + 98175\*A\*c^3\*x^(29/2) + 86275\*B\*c^3\*x^(33/2)))/2847075

**fricas** [A] time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{2847075} (86275 Bc^3x^{16} + 98175 (3Bbc^2 + Ac^3)x^{14} + 341649 (Bb^2c + Abc^2)x^{12} + 167475 Ab^3x^8 + 135575 (Bb^3 + 3Ab^2c)x^{10})\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 2/2847075\*(86275\*B\*c^3\*x^16 + 98175\*(3\*B\*b\*c^2 + A\*c^3)\*x^14 + 341649\*(B\*b^2\*c + A\*b\*c^2)\*x^12 + 167475\*A\*b^3\*x^8 + 135575\*(B\*b^3 + 3\*A\*b^2\*c)\*x^10)\*sqrt(x)

**giac** [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{33} Bc^3x^{\frac{33}{2}} + \frac{6}{29} Bbc^2x^{\frac{29}{2}} + \frac{2}{29} Ac^3x^{\frac{29}{2}} + \frac{6}{25} Bb^2cx^{\frac{25}{2}} + \frac{6}{25} Abc^2x^{\frac{25}{2}} + \frac{2}{21} Bb^3x^{\frac{21}{2}} + \frac{2}{7} Ab^2cx^{\frac{21}{2}} + \frac{2}{17} Ab^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 2/33\*B\*c^3\*x^(33/2) + 6/29\*B\*b\*c^2\*x^(29/2) + 2/29\*A\*c^3\*x^(29/2) + 6/25\*B\*b^2\*c\*x^(25/2) + 6/25\*A\*b\*c^2\*x^(25/2) + 2/21\*B\*b^3\*x^(21/2) + 2/7\*A\*b^2\*c\*x^(21/2) + 2/17\*A\*b^3\*x^(17/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(86275Bc^3x^8 + 98175Ac^3x^6 + 294525Bbc^2x^6 + 341649Abc^2x^4 + 341649Bb^2cx^4 + 406725Ab^2cx^2 + 135575Bb^3x^2 + 167475Ab^3)x^{\frac{17}{2}}}{2847075}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x)

[Out] 2/2847075\*x^(17/2)\*(86275\*B\*c^3\*x^8+98175\*A\*c^3\*x^6+294525\*B\*b\*c^2\*x^6+341649\*A\*b\*c^2\*x^4+341649\*B\*b^2\*c\*x^4+406725\*A\*b^2\*c\*x^2+135575\*B\*b^3\*x^2+167475\*A\*b^3)

**maxima** [A] time = 1.38, size = 73, normalized size = 0.86

$$\frac{2}{33} Bc^3x^{\frac{33}{2}} + \frac{2}{29} (3Bbc^2 + Ac^3)x^{\frac{29}{2}} + \frac{6}{25} (Bb^2c + Abc^2)x^{\frac{25}{2}} + \frac{2}{17} Ab^3x^{\frac{17}{2}} + \frac{2}{21} (Bb^3 + 3Ab^2c)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 2/33\*B\*c^3\*x^(33/2) + 2/29\*(3\*B\*b\*c^2 + A\*c^3)\*x^(29/2) + 6/25\*(B\*b^2\*c + A\*b\*c^2)\*x^(25/2) + 2/17\*A\*b^3\*x^(17/2) + 2/21\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(21/2)

**mupad** [B] time = 0.04, size = 69, normalized size = 0.81

$$x^{21/2} \left( \frac{2Bb^3}{21} + \frac{2Ac^2}{7} \right) + x^{29/2} \left( \frac{2Ac^3}{29} + \frac{6Bbc^2}{29} \right) + \frac{2Ab^3x^{17/2}}{17} + \frac{2Bc^3x^{33/2}}{33} + \frac{6bcx^{25/2}(Ac+Bb)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)`

[Out]  $x^{(21/2)} * ((2*B*b^3)/21 + (2*A*b^2*c)/7) + x^{(29/2)} * ((2*A*c^3)/29 + (6*B*b*c^2)/29) + (2*A*b^3*x^{(17/2)})/17 + (2*B*c^3*x^{(33/2)})/33 + (6*b*c*x^{(25/2)} * (A*c + B*b))/25$

sympy [A] time = 54.01, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{17}{2}}}{17} + \frac{2Ab^2cx^{\frac{21}{2}}}{7} + \frac{6Abc^2x^{\frac{25}{2}}}{25} + \frac{2Ac^3x^{\frac{29}{2}}}{29} + \frac{2Bb^3x^{\frac{21}{2}}}{21} + \frac{6Bb^2cx^{\frac{25}{2}}}{25} + \frac{6Bbc^2x^{\frac{29}{2}}}{29} + \frac{2Bc^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)`

[Out]  $2*A*b**3*x**(17/2)/17 + 2*A*b**2*c*x**(21/2)/7 + 6*A*b*c**2*x**(25/2)/25 + 2*A*c**3*x**(29/2)/29 + 2*B*b**3*x**(21/2)/21 + 6*B*b**2*c*x**(25/2)/25 + 6*B*b*c**2*x**(29/2)/29 + 2*B*c**3*x**(33/2)/33$



$$3.178 \quad \int \sqrt{x} (A + Bx^2) (bx^2 + cx^4)^3 dx$$

**Optimal.** Leaf size=85

$$\frac{2}{15}Ab^3x^{15/2} + \frac{2}{19}b^2x^{19/2}(3Ac + bB) + \frac{2}{27}c^2x^{27/2}(Ac + 3bB) + \frac{6}{23}bcx^{23/2}(Ac + bB) + \frac{2}{31}Bc^3x^{31/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{19}b^2x^{19/2}(3Ac + bB) + \frac{2}{15}Ab^3x^{15/2} + \frac{2}{27}c^2x^{27/2}(Ac + 3bB) + \frac{6}{23}bcx^{23/2}(Ac + bB) + \frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(15/2))/15 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(19/2))/19 + (6\*b\*c\*(b\*B + A\*c)\*x^(23/2))/23 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(27/2))/27 + (2\*B\*c^3\*x^(31/2))/31

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[q, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \sqrt{x} (A + Bx^2) (bx^2 + cx^4)^3 dx &= \int x^{13/2} (A + Bx^2) (b + cx^2)^3 dx \\ &= \int (Ab^3x^{13/2} + b^2(bB + 3Ac)x^{17/2} + 3bc(bB + Ac)x^{21/2} + c^2(3bB + Ac)x^{25/2}) dx \\ &= \frac{2}{15}Ab^3x^{15/2} + \frac{2}{19}b^2(bB + 3Ac)x^{19/2} + \frac{6}{23}bc(bB + Ac)x^{23/2} + \frac{2}{27}c^2(3bB + Ac)x^{27/2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{15}Ab^3x^{15/2} + \frac{2}{19}b^2x^{19/2}(3Ac + bB) + \frac{2}{27}c^2x^{27/2}(Ac + 3bB) + \frac{6}{23}bcx^{23/2}(Ac + bB) + \frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*A\*b^3\*x^(15/2))/15 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(19/2))/19 + (6\*b\*c\*(b\*B + A\*c)\*x^(23/2))/23 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(27/2))/27 + (2\*B\*c^3\*x^(31/2))/31

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(121923Ab^3x^{15/2} + 288765Ab^2cx^{19/2} + 238545Abc^2x^{23/2} + 67735Ac^3x^{27/2} + 96255b^3Bx^{19/2} + 238545b^2Bcx^{23/2} + 203205bBc^2x^{27/2} + 58995Bc^3x^{31/2})}{1828845}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*(121923\*A\*b^3\*x^(15/2) + 96255\*b^3\*B\*x^(19/2) + 288765\*A\*b^2\*c\*x^(19/2) + 238545\*b^2\*B\*c\*x^(23/2) + 238545\*A\*b\*c^2\*x^(23/2) + 203205\*b\*B\*c^2\*x^(27/2) + 67735\*A\*c^3\*x^(27/2) + 58995\*B\*c^3\*x^(31/2)))/1828845

**fricas** [A] time = 0.39, size = 78, normalized size = 0.92

$$\frac{2}{1828845} (58995 B c^3 x^{15} + 67735 (3 B b c^2 + A c^3) x^{13} + 238545 (B b^2 c + A b c^2) x^{11} + 121923 A b^3 x^7 + 96255 (B b^3 + 3 A b^2 c) x^9) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3\*x^(1/2),x, algorithm="fricas")

[Out] 2/1828845\*(58995\*B\*c^3\*x^15 + 67735\*(3\*B\*b\*c^2 + A\*c^3)\*x^13 + 238545\*(B\*b^2\*c + A\*b\*c^2)\*x^11 + 121923\*A\*b^3\*x^7 + 96255\*(B\*b^3 + 3\*A\*b^2\*c)\*x^9)\*sqrt(x)

**giac** [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{31} B c^3 x^{\frac{31}{2}} + \frac{2}{9} B b c^2 x^{\frac{27}{2}} + \frac{2}{27} A c^3 x^{\frac{27}{2}} + \frac{6}{23} B b^2 c x^{\frac{23}{2}} + \frac{6}{23} A b c^2 x^{\frac{23}{2}} + \frac{2}{19} B b^3 x^{\frac{19}{2}} + \frac{6}{19} A b^2 c x^{\frac{19}{2}} + \frac{2}{15} A b^3 x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3\*x^(1/2),x, algorithm="giac")

[Out] 2/31\*B\*c^3\*x^(31/2) + 2/9\*B\*b\*c^2\*x^(27/2) + 2/27\*A\*c^3\*x^(27/2) + 6/23\*B\*b^2\*c\*x^(23/2) + 6/23\*A\*b\*c^2\*x^(23/2) + 2/19\*B\*b^3\*x^(19/2) + 6/19\*A\*b^2\*c\*x^(19/2) + 2/15\*A\*b^3\*x^(15/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(58995 B c^3 x^8 + 67735 A c^3 x^6 + 203205 B b c^2 x^6 + 238545 A b c^2 x^4 + 238545 B b^2 c x^4 + 288765 A b^2 c x^2 + 96255 B b^3 x^2 + 121923 A b^3) x^{\frac{15}{2}}}{1828845}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3\*x^(1/2),x)

[Out] 2/1828845\*x^(15/2)\*(58995\*B\*c^3\*x^8+67735\*A\*c^3\*x^6+203205\*B\*b\*c^2\*x^6+238545\*A\*b\*c^2\*x^4+238545\*B\*b^2\*c\*x^4+288765\*A\*b^2\*c\*x^2+96255\*B\*b^3\*x^2+121923\*A\*b^3)

**maxima** [A] time = 1.33, size = 73, normalized size = 0.86

$$\frac{2}{31} B c^3 x^{\frac{31}{2}} + \frac{2}{27} (3 B b c^2 + A c^3) x^{\frac{27}{2}} + \frac{6}{23} (B b^2 c + A b c^2) x^{\frac{23}{2}} + \frac{2}{15} A b^3 x^{\frac{15}{2}} + \frac{2}{19} (B b^3 + 3 A b^2 c) x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3\*x^(1/2),x, algorithm="maxima")

[Out] 2/31\*B\*c^3\*x^(31/2) + 2/27\*(3\*B\*b\*c^2 + A\*c^3)\*x^(27/2) + 6/23\*(B\*b^2\*c + A\*b\*c^2)\*x^(23/2) + 2/15\*A\*b^3\*x^(15/2) + 2/19\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(19/2)

**mupad** [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{19/2} \left( \frac{2 B b^3}{19} + \frac{6 A c b^2}{19} \right) + x^{27/2} \left( \frac{2 A c^3}{27} + \frac{2 B b c^2}{9} \right) + \frac{2 A b^3 x^{15/2}}{15} + \frac{2 B c^3 x^{31/2}}{31} + \frac{6 b c x^{23/2} (A c + B b)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)`

[Out]  $x^{19/2} * ((2*B*b^3)/19 + (6*A*b^2*c)/19) + x^{27/2} * ((2*A*c^3)/27 + (2*B*b*c^2)/9) + (2*A*b^3*x^{15/2})/15 + (2*B*c^3*x^{31/2})/31 + (6*b*c*x^{23/2} * (A*c + B*b))/23$

**sympy [A]** time = 5.51, size = 95, normalized size = 1.12

$$\frac{2Ab^3x^{\frac{15}{2}}}{15} + \frac{2Bc^3x^{\frac{31}{2}}}{31} + \frac{2x^{\frac{27}{2}}(Ac^3 + 3Bbc^2)}{27} + \frac{2x^{\frac{23}{2}}(3Abc^2 + 3Bb^2c)}{23} + \frac{2x^{\frac{19}{2}}(3Ab^2c + Bb^3)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3*x**(1/2),x)`

[Out]  $2*A*b**3*x**(15/2)/15 + 2*B*c**3*x**(31/2)/31 + 2*x**(27/2)*(A*c**3 + 3*B*b*c**2)/27 + 2*x**(23/2)*(3*A*b*c**2 + 3*B*b**2*c)/23 + 2*x**(19/2)*(3*A*b**2*c + B*b**3)/19$

$$3.179 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}} dx$$

**Optimal.** Leaf size=85

$$\frac{2}{13}Ab^3x^{13/2} + \frac{2}{17}b^2x^{17/2}(3Ac + bB) + \frac{2}{25}c^2x^{25/2}(Ac + 3bB) + \frac{2}{7}bcx^{21/2}(Ac + bB) + \frac{2}{29}Bc^3x^{29/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{17}b^2x^{17/2}(3Ac + bB) + \frac{2}{13}Ab^3x^{13/2} + \frac{2}{25}c^2x^{25/2}(Ac + 3bB) + \frac{2}{7}bcx^{21/2}(Ac + bB) + \frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/Sqrt[x], x]

[Out] (2\*A\*b^3\*x^(13/2))/13 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(17/2))/17 + (2\*b\*c\*(b\*B + A\*c)\*x^(21/2))/7 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(25/2))/25 + (2\*B\*c^3\*x^(29/2))/29

#### Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{\sqrt{x}} dx &= \int x^{11/2} (A + Bx^2) (b + cx^2)^3 dx \\ &= \int (Ab^3x^{11/2} + b^2(bB + 3Ac)x^{15/2} + 3bc(bB + Ac)x^{19/2} + c^2(3bB + Ac)x^{23/2} + Bc^3x^{27/2}) dx \\ &= \frac{2}{13}Ab^3x^{13/2} + \frac{2}{17}b^2(bB + 3Ac)x^{17/2} + \frac{2}{7}bc(bB + Ac)x^{21/2} + \frac{2}{25}c^2(3bB + Ac)x^{25/2} + \frac{2}{29}Bc^3x^{29/2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{13}Ab^3x^{13/2} + \frac{2}{17}b^2x^{17/2}(3Ac + bB) + \frac{2}{25}c^2x^{25/2}(Ac + 3bB) + \frac{2}{7}bcx^{21/2}(Ac + bB) + \frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/Sqrt[x], x]

[Out] (2\*A\*b^3\*x^(13/2))/13 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(17/2))/17 + (2\*b\*c\*(b\*B + A\*c)\*x^(21/2))/7 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(25/2))/25 + (2\*B\*c^3\*x^(29/2))/29

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(86275Ab^3x^{13/2} + 197925Ab^2cx^{17/2} + 160225Abc^2x^{21/2} + 44863Ac^3x^{25/2} + 65975b^3Bx^{17/2} + 160225b^2Bcx^{21/2} + 134589bBc^2x^{25/2} + 38675Bc^3x^{29/2})}{1121575}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/sqrt[x], x]

[Out] (2\*(86275\*A\*b^3\*x^(13/2) + 65975\*b^3\*B\*x^(17/2) + 197925\*A\*b^2\*c\*x^(17/2) + 160225\*b^2\*B\*c\*x^(21/2) + 160225\*A\*b\*c^2\*x^(21/2) + 134589\*b\*B\*c^2\*x^(25/2) + 44863\*A\*c^3\*x^(25/2) + 38675\*B\*c^3\*x^(29/2)))/1121575

**fricas** [A] time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{1121575} (38675 B c^3 x^{14} + 44863 (3 B b c^2 + A c^3) x^{12} + 160225 (B b^2 c + A b c^2) x^{10} + 86275 A b^3 x^6 + 65975 (B b^3 + 3 A b^2 c) x^8) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(1/2), x, algorithm="fricas")

[Out] 2/1121575\*(38675\*B\*c^3\*x^14 + 44863\*(3\*B\*b\*c^2 + A\*c^3)\*x^12 + 160225\*(B\*b^2\*c + A\*b\*c^2)\*x^10 + 86275\*A\*b^3\*x^6 + 65975\*(B\*b^3 + 3\*A\*b^2\*c)\*x^8)\*sqrt(x)

**giac** [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{29} B c^3 x^{\frac{29}{2}} + \frac{6}{25} B b c^2 x^{\frac{25}{2}} + \frac{2}{25} A c^3 x^{\frac{25}{2}} + \frac{2}{7} B b^2 c x^{\frac{21}{2}} + \frac{2}{7} A b c^2 x^{\frac{21}{2}} + \frac{2}{17} B b^3 x^{\frac{17}{2}} + \frac{6}{17} A b^2 c x^{\frac{17}{2}} + \frac{2}{13} A b^3 x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(1/2), x, algorithm="giac")

[Out] 2/29\*B\*c^3\*x^(29/2) + 6/25\*B\*b\*c^2\*x^(25/2) + 2/25\*A\*c^3\*x^(25/2) + 2/7\*B\*b^2\*c\*x^(21/2) + 2/7\*A\*b\*c^2\*x^(21/2) + 2/17\*B\*b^3\*x^(17/2) + 6/17\*A\*b^2\*c\*x^(17/2) + 2/13\*A\*b^3\*x^(13/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(38675 B c^3 x^8 + 44863 A c^3 x^6 + 134589 B b c^2 x^6 + 160225 A b c^2 x^4 + 160225 B b^2 c x^4 + 197925 A b^2 c x^2 + 65975 B b^3 x^2 + 86275 A b^3) x^{\frac{13}{2}}}{1121575}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(1/2), x)

[Out] 2/1121575\*x^(13/2)\*(38675\*B\*c^3\*x^8+44863\*A\*c^3\*x^6+134589\*B\*b\*c^2\*x^6+160225\*A\*b\*c^2\*x^4+160225\*B\*b^2\*c\*x^4+197925\*A\*b^2\*c\*x^2+65975\*B\*b^3\*x^2+86275\*A\*b^3)

**maxima** [A] time = 1.36, size = 73, normalized size = 0.86

$$\frac{2}{29} B c^3 x^{\frac{29}{2}} + \frac{2}{25} (3 B b c^2 + A c^3) x^{\frac{25}{2}} + \frac{2}{7} (B b^2 c + A b c^2) x^{\frac{21}{2}} + \frac{2}{13} A b^3 x^{\frac{13}{2}} + \frac{2}{17} (B b^3 + 3 A b^2 c) x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(1/2), x, algorithm="maxima")

[Out] 2/29\*B\*c^3\*x^(29/2) + 2/25\*(3\*B\*b\*c^2 + A\*c^3)\*x^(25/2) + 2/7\*(B\*b^2\*c + A\*b\*c^2)\*x^(21/2) + 2/13\*A\*b^3\*x^(13/2) + 2/17\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(17/2)

**mupad** [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{17/2} \left( \frac{2 B b^3}{17} + \frac{6 A c b^2}{17} \right) + x^{25/2} \left( \frac{2 A c^3}{25} + \frac{6 B b c^2}{25} \right) + \frac{2 A b^3 x^{13/2}}{13} + \frac{2 B c^3 x^{29/2}}{29} + \frac{2 b c x^{21/2} (A c + B b)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(1/2), x)`

[Out]  $x^{(17/2)} * ((2*B*b^3)/17 + (6*A*b^2*c)/17) + x^{(25/2)} * ((2*A*c^3)/25 + (6*B*b*c^2)/25) + (2*A*b^3*x^{(13/2)})/13 + (2*B*c^3*x^{(29/2)})/29 + (2*b*c*x^{(21/2)} * (A*c + B*b))/7$

sympy [A] time = 29.88, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{13}{2}}}{13} + \frac{6Ab^2cx^{\frac{17}{2}}}{17} + \frac{2Abc^2x^{\frac{21}{2}}}{7} + \frac{2Ac^3x^{\frac{25}{2}}}{25} + \frac{2Bb^3x^{\frac{17}{2}}}{17} + \frac{2Bb^2cx^{\frac{21}{2}}}{7} + \frac{6Bbc^2x^{\frac{25}{2}}}{25} + \frac{2Bc^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(1/2), x)`

[Out]  $2*A*b**3*x**(13/2)/13 + 6*A*b**2*c*x**(17/2)/17 + 2*A*b*c**2*x**(21/2)/7 + 2*A*c**3*x**(25/2)/25 + 2*B*b**3*x**(17/2)/17 + 2*B*b**2*c*x**(21/2)/7 + 6*B*b*c**2*x**(25/2)/25 + 2*B*c**3*x**(29/2)/29$

$$3.180 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}} dx$$

**Optimal.** Leaf size=85

$$\frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2x^{15/2}(3Ac + bB) + \frac{2}{23}c^2x^{23/2}(Ac + 3bB) + \frac{6}{19}bcx^{19/2}(Ac + bB) + \frac{2}{27}Bc^3x^{27/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{15}b^2x^{15/2}(3Ac + bB) + \frac{2}{11}Ab^3x^{11/2} + \frac{2}{23}c^2x^{23/2}(Ac + 3bB) + \frac{6}{19}bcx^{19/2}(Ac + bB) + \frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(3/2), x]

[Out] (2\*A\*b^3\*x^(11/2))/11 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(15/2))/15 + (6\*b\*c\*(b\*B + A\*c)\*x^(19/2))/19 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(23/2))/23 + (2\*B\*c^3\*x^(27/2))/27

**Rule 448**

Int[((e\_)\*(x\_))^(m\_)\*((a\_)+(b\_)\*(x\_)^(n\_))^(p\_)\*((c\_)+(d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{3/2}} dx &= \int x^{9/2} (A + Bx^2)(b + cx^2)^3 dx \\ &= \int (Ab^3x^{9/2} + b^2(bB + 3Ac)x^{13/2} + 3bc(bB + Ac)x^{17/2} + c^2(3bB + Ac)x^{21/2} + Bc^3x^{25/2}) dx \\ &= \frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2(bB + 3Ac)x^{15/2} + \frac{6}{19}bc(bB + Ac)x^{19/2} + \frac{2}{23}c^2(3bB + Ac)x^{23/2} + \frac{2}{27}Bc^3x^{27/2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2x^{15/2}(3Ac + bB) + \frac{2}{23}c^2x^{23/2}(Ac + 3bB) + \frac{6}{19}bcx^{19/2}(Ac + bB) + \frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(3/2), x]

[Out] (2\*A\*b^3\*x^(11/2))/11 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(15/2))/15 + (6\*b\*c\*(b\*B + A\*c)\*x^(19/2))/19 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(23/2))/23 + (2\*B\*c^3\*x^(27/2))/27

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(58995Ab^3x^{11/2} + 129789Ab^2cx^{15/2} + 102465Abc^2x^{19/2} + 28215Ac^3x^{23/2} + 43263b^3Bx^{15/2} + 102465b^2Bcx^{19/2} + 84645bBc^2x^{23/2} + 24035Bc^3x^{27/2})}{648945}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(3/2), x]

[Out] (2\*(58995\*A\*b^3\*x^(11/2) + 43263\*b^3\*B\*x^(15/2) + 129789\*A\*b^2\*c\*x^(15/2) + 102465\*b^2\*B\*c\*x^(19/2) + 102465\*A\*b\*c^2\*x^(19/2) + 84645\*b\*B\*c^2\*x^(23/2) + 28215\*A\*c^3\*x^(23/2) + 24035\*B\*c^3\*x^(27/2)))/648945

**fricas [A]** time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{648945} (24035 Bc^3x^{13} + 28215 (3Bbc^2 + Ac^3)x^{11} + 102465 (Bb^2c + Abc^2)x^9 + 58995 Ab^3x^5 + 43263 (Bb^3 + 3Ab^2c)x^7)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(3/2), x, algorithm="fricas")

[Out] 2/648945\*(24035\*B\*c^3\*x^13 + 28215\*(3\*B\*b\*c^2 + A\*c^3)\*x^11 + 102465\*(B\*b^2\*c + A\*b\*c^2)\*x^9 + 58995\*A\*b^3\*x^5 + 43263\*(B\*b^3 + 3\*A\*b^2\*c)\*x^7)\*sqrt(x)

**giac [A]** time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{27} Bc^3x^{\frac{27}{2}} + \frac{6}{23} Bbc^2x^{\frac{23}{2}} + \frac{2}{23} Ac^3x^{\frac{23}{2}} + \frac{6}{19} Bb^2cx^{\frac{19}{2}} + \frac{6}{19} Abc^2x^{\frac{19}{2}} + \frac{2}{15} Bb^3x^{\frac{15}{2}} + \frac{2}{5} Ab^2cx^{\frac{15}{2}} + \frac{2}{11} Ab^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(3/2), x, algorithm="giac")

[Out] 2/27\*B\*c^3\*x^(27/2) + 6/23\*B\*b\*c^2\*x^(23/2) + 2/23\*A\*c^3\*x^(23/2) + 6/19\*B\*b^2\*c\*x^(19/2) + 6/19\*A\*b\*c^2\*x^(19/2) + 2/15\*B\*b^3\*x^(15/2) + 2/5\*A\*b^2\*c\*x^(15/2) + 2/11\*A\*b^3\*x^(11/2)

**maple [A]** time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(24035Bc^3x^8 + 28215Ac^3x^6 + 84645Bbc^2x^6 + 102465Abc^2x^4 + 102465Bb^2cx^4 + 129789Ab^2cx^2 + 43263Bb^3x^2 + 58995Ab^3)x^{\frac{11}{2}}}{648945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(3/2), x)

[Out] 2/648945\*x^(11/2)\*(24035\*B\*c^3\*x^8+28215\*A\*c^3\*x^6+84645\*B\*b\*c^2\*x^6+102465\*A\*b\*c^2\*x^4+102465\*B\*b^2\*c\*x^4+129789\*A\*b^2\*c\*x^2+43263\*B\*b^3\*x^2+58995\*A\*b^3)

**maxima [A]** time = 1.32, size = 73, normalized size = 0.86

$$\frac{2}{27} Bc^3x^{\frac{27}{2}} + \frac{2}{23} (3Bbc^2 + Ac^3)x^{\frac{23}{2}} + \frac{6}{19} (Bb^2c + Abc^2)x^{\frac{19}{2}} + \frac{2}{11} Ab^3x^{\frac{11}{2}} + \frac{2}{15} (Bb^3 + 3Ab^2c)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(3/2), x, algorithm="maxima")

[Out] 2/27\*B\*c^3\*x^(27/2) + 2/23\*(3\*B\*b\*c^2 + A\*c^3)\*x^(23/2) + 6/19\*(B\*b^2\*c + A\*b\*c^2)\*x^(19/2) + 2/11\*A\*b^3\*x^(11/2) + 2/15\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(15/2)

**mupad [B]** time = 0.03, size = 69, normalized size = 0.81

$$x^{15/2} \left( \frac{2Bb^3}{15} + \frac{2Ac^2b^2}{5} \right) + x^{23/2} \left( \frac{2Ac^3}{23} + \frac{6Bbc^2}{23} \right) + \frac{2Ab^3x^{11/2}}{11} + \frac{2Bc^3x^{27/2}}{27} + \frac{6bcx^{19/2}(Ac+Bb)}{19}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2), x)`

[Out]  $x^{(15/2)} * ((2*B*b^3)/15 + (2*A*b^2*c)/5) + x^{(23/2)} * ((2*A*c^3)/23 + (6*B*b*c^2)/23) + (2*A*b^3*x^{(11/2)})/11 + (2*B*c^3*x^{(27/2)})/27 + (6*b*c*x^{(19/2)} * (A*c + B*b))/19$

**sympy [A]** time = 31.77, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ab^2cx^{\frac{15}{2}}}{5} + \frac{6Abc^2x^{\frac{19}{2}}}{19} + \frac{2Ac^3x^{\frac{23}{2}}}{23} + \frac{2Bb^3x^{\frac{15}{2}}}{15} + \frac{6Bb^2cx^{\frac{19}{2}}}{19} + \frac{6Bbc^2x^{\frac{23}{2}}}{23} + \frac{2Bc^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(3/2), x)`

[Out]  $2*A*b**3*x**(11/2)/11 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(19/2)/19 + 2*A*c**3*x**(23/2)/23 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(19/2)/19 + 6*B*b*c**2*x**(23/2)/23 + 2*B*c**3*x**(27/2)/27$

$$3.181 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} dx$$

**Optimal.** Leaf size=85

$$\frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2x^{13/2}(3Ac + bB) + \frac{2}{21}c^2x^{21/2}(Ac + 3bB) + \frac{6}{17}bcx^{17/2}(Ac + bB) + \frac{2}{25}Bc^3x^{25/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{13}b^2x^{13/2}(3Ac + bB) + \frac{2}{9}Ab^3x^{9/2} + \frac{2}{21}c^2x^{21/2}(Ac + 3bB) + \frac{6}{17}bcx^{17/2}(Ac + bB) + \frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(5/2), x]

[Out] (2\*A\*b^3\*x^(9/2))/9 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(13/2))/13 + (6\*b\*c\*(b\*B + A\*c)\*x^(17/2))/17 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(21/2))/21 + (2\*B\*c^3\*x^(25/2))/25

#### Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} dx &= \int x^{7/2} (A+Bx^2)(b+cx^2)^3 dx \\ &= \int (Ab^3x^{7/2} + b^2(bB+3Ac)x^{11/2} + 3bc(bB+Ac)x^{15/2} + c^2(3bB+Ac)x^{19/2} + Bc^3x^{23/2}) dx \\ &= \frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2(bB+3Ac)x^{13/2} + \frac{6}{17}bc(bB+Ac)x^{17/2} + \frac{2}{21}c^2(3bB+Ac)x^{21/2} + \frac{2}{25}Bc^3x^{25/2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2x^{13/2}(3Ac + bB) + \frac{2}{21}c^2x^{21/2}(Ac + 3bB) + \frac{6}{17}bcx^{17/2}(Ac + bB) + \frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(5/2), x]

[Out] (2\*A\*b^3\*x^(9/2))/9 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(13/2))/13 + (6\*b\*c\*(b\*B + A\*c)\*x^(17/2))/17 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(21/2))/21 + (2\*B\*c^3\*x^(25/2))/25

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(38675Ab^3x^{9/2} + 80325Ab^2cx^{13/2} + 61425Abc^2x^{17/2} + 16575Ac^3x^{21/2} + 26775b^3Bx^{13/2} + 61425b^2Bcx^{17/2} + 49725bBc^2x^{21/2} + 13923Bc^3x^{25/2})}{348075}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(5/2), x]

[Out] (2\*(38675\*A\*b^3\*x^(9/2) + 26775\*b^3\*B\*x^(13/2) + 80325\*A\*b^2\*c\*x^(13/2) + 61425\*b^2\*B\*c\*x^(17/2) + 61425\*A\*b\*c^2\*x^(17/2) + 49725\*b\*B\*c^2\*x^(21/2) + 16575\*A\*c^3\*x^(21/2) + 13923\*B\*c^3\*x^(25/2)))/348075

**fricas** [A] time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{348075} (13923 B c^3 x^{12} + 16575 (3 B b c^2 + A c^3) x^{10} + 61425 (B b^2 c + A b c^2) x^8 + 38675 A b^3 x^4 + 26775 (B b^3 + 3 A b^2 c) x^6) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(5/2), x, algorithm="fricas")

[Out] 2/348075\*(13923\*B\*c^3\*x^12 + 16575\*(3\*B\*b\*c^2 + A\*c^3)\*x^10 + 61425\*(B\*b^2\*c + A\*b\*c^2)\*x^8 + 38675\*A\*b^3\*x^4 + 26775\*(B\*b^3 + 3\*A\*b^2\*c)\*x^6)\*sqrt(x)

**giac** [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{25} B c^3 x^{\frac{25}{2}} + \frac{2}{7} B b c^2 x^{\frac{21}{2}} + \frac{2}{21} A c^3 x^{\frac{21}{2}} + \frac{6}{17} B b^2 c x^{\frac{17}{2}} + \frac{6}{17} A b c^2 x^{\frac{17}{2}} + \frac{2}{13} B b^3 x^{\frac{13}{2}} + \frac{6}{13} A b^2 c x^{\frac{13}{2}} + \frac{2}{9} A b^3 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(5/2), x, algorithm="giac")

[Out] 2/25\*B\*c^3\*x^(25/2) + 2/7\*B\*b\*c^2\*x^(21/2) + 2/21\*A\*c^3\*x^(21/2) + 6/17\*B\*b^2\*c\*x^(17/2) + 6/17\*A\*b\*c^2\*x^(17/2) + 2/13\*B\*b^3\*x^(13/2) + 6/13\*A\*b^2\*c\*x^(13/2) + 2/9\*A\*b^3\*x^(9/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(13923 B c^3 x^8 + 16575 A c^3 x^6 + 49725 B b c^2 x^6 + 61425 A b c^2 x^4 + 61425 B b^2 c x^4 + 80325 A b^2 c x^2 + 26775 B b^3 x^2 + 38675 A b^3) x^{\frac{9}{2}}}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(5/2), x)

[Out] 2/348075\*x^(9/2)\*(13923\*B\*c^3\*x^8+16575\*A\*c^3\*x^6+49725\*B\*b\*c^2\*x^6+61425\*A\*b\*c^2\*x^4+61425\*B\*b^2\*c\*x^4+80325\*A\*b^2\*c\*x^2+26775\*B\*b^3\*x^2+38675\*A\*b^3)

**maxima** [A] time = 1.40, size = 73, normalized size = 0.86

$$\frac{2}{25} B c^3 x^{\frac{25}{2}} + \frac{2}{21} (3 B b c^2 + A c^3) x^{\frac{21}{2}} + \frac{6}{17} (B b^2 c + A b c^2) x^{\frac{17}{2}} + \frac{2}{9} A b^3 x^{\frac{9}{2}} + \frac{2}{13} (B b^3 + 3 A b^2 c) x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(5/2), x, algorithm="maxima")

[Out] 2/25\*B\*c^3\*x^(25/2) + 2/21\*(3\*B\*b\*c^2 + A\*c^3)\*x^(21/2) + 6/17\*(B\*b^2\*c + A\*b\*c^2)\*x^(17/2) + 2/9\*A\*b^3\*x^(9/2) + 2/13\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(13/2)

**mupad** [B] time = 0.04, size = 69, normalized size = 0.81

$$x^{13/2} \left( \frac{2 B b^3}{13} + \frac{6 A c b^2}{13} \right) + x^{21/2} \left( \frac{2 A c^3}{21} + \frac{2 B b c^2}{7} \right) + \frac{2 A b^3 x^{9/2}}{9} + \frac{2 B c^3 x^{25/2}}{25} + \frac{6 b c x^{17/2} (A c + B b)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(5/2), x)

[Out]  $x^{(13/2)} * ((2*B*b^3)/13 + (6*A*b^2*c)/13) + x^{(21/2)} * ((2*A*c^3)/21 + (2*B*b*c^2)/7) + (2*A*b^3*x^{(9/2)})/9 + (2*B*c^3*x^{(25/2)})/25 + (6*b*c*x^{(17/2)}*(A*c + B*b))/17$

**sympy** [A] time = 35.91, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{9}{2}}}{9} + \frac{6Ab^2cx^{\frac{13}{2}}}{13} + \frac{6Abc^2x^{\frac{17}{2}}}{17} + \frac{2Ac^3x^{\frac{21}{2}}}{21} + \frac{2Bb^3x^{\frac{13}{2}}}{13} + \frac{6Bb^2cx^{\frac{17}{2}}}{17} + \frac{2Bbc^2x^{\frac{21}{2}}}{7} + \frac{2Bc^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*(5/2),x)

[Out]  $2*A*b**3*x**(9/2)/9 + 6*A*b**2*c*x**(13/2)/13 + 6*A*b*c**2*x**(17/2)/17 + 2*A*c**3*x**(21/2)/21 + 2*B*b**3*x**(13/2)/13 + 6*B*b**2*c*x**(17/2)/17 + 2*B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(25/2)/25$

$$3.182 \quad \int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}} dx$$

**Optimal.** Leaf size=85

$$\frac{2}{7}Ab^3x^{7/2} + \frac{2}{11}b^2x^{11/2}(3Ac + bB) + \frac{2}{19}c^2x^{19/2}(Ac + 3bB) + \frac{2}{5}bcx^{15/2}(Ac + bB) + \frac{2}{23}Bc^3x^{23/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1584, 448}

$$\frac{2}{11}b^2x^{11/2}(3Ac + bB) + \frac{2}{7}Ab^3x^{7/2} + \frac{2}{19}c^2x^{19/2}(Ac + 3bB) + \frac{2}{5}bcx^{15/2}(Ac + bB) + \frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(7/2), x]

[Out] (2\*A\*b^3\*x^(7/2))/7 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(11/2))/11 + (2\*b\*c\*(b\*B + A\*c)\*x^(15/2))/5 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(19/2))/19 + (2\*B\*c^3\*x^(23/2))/23

#### Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^{7/2}} dx &= \int x^{5/2} (A + Bx^2)(b + cx^2)^3 dx \\ &= \int (Ab^3x^{5/2} + b^2(bB + 3Ac)x^{9/2} + 3bc(bB + Ac)x^{13/2} + c^2(3bB + Ac)x^{17/2} + Bc^3x^{21/2}) dx \\ &= \frac{2}{7}Ab^3x^{7/2} + \frac{2}{11}b^2(bB + 3Ac)x^{11/2} + \frac{2}{5}bc(bB + Ac)x^{15/2} + \frac{2}{19}c^2(3bB + Ac)x^{19/2} + \frac{2}{23}Bc^3x^{23/2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{7}Ab^3x^{7/2} + \frac{2}{11}b^2x^{11/2}(3Ac + bB) + \frac{2}{19}c^2x^{19/2}(Ac + 3bB) + \frac{2}{5}bcx^{15/2}(Ac + bB) + \frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(7/2), x]

[Out] (2\*A\*b^3\*x^(7/2))/7 + (2\*b^2\*(b\*B + 3\*A\*c)\*x^(11/2))/11 + (2\*b\*c\*(b\*B + A\*c)\*x^(15/2))/5 + (2\*c^2\*(3\*b\*B + A\*c)\*x^(19/2))/19 + (2\*B\*c^3\*x^(23/2))/23

**IntegrateAlgebraic [A]** time = 0.06, size = 97, normalized size = 1.14

$$\frac{2(24035Ab^3x^{7/2} + 45885Ab^2cx^{11/2} + 33649Abc^2x^{15/2} + 8855Ac^3x^{19/2} + 15295b^3Bx^{11/2} + 33649b^2Bcx^{15/2} + 26565bBc^2x^{19/2} + 7315Bc^3x^{23/2})}{168245}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(7/2), x]

[Out] (2\*(24035\*A\*b^3\*x^(7/2) + 15295\*b^3\*B\*x^(11/2) + 45885\*A\*b^2\*c\*x^(11/2) + 33649\*b^2\*B\*c\*x^(15/2) + 33649\*A\*b\*c^2\*x^(15/2) + 26565\*b\*B\*c^2\*x^(19/2) + 8855\*A\*c^3\*x^(19/2) + 7315\*B\*c^3\*x^(23/2)))/168245

**fricas** [A] time = 0.40, size = 78, normalized size = 0.92

$$\frac{2}{168245} (7315 B c^3 x^{11} + 8855 (3 B b c^2 + A c^3) x^9 + 33649 (B b^2 c + A b c^2) x^7 + 24035 A b^3 x^3 + 15295 (B b^3 + 3 A b^2 c) x^5) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(7/2), x, algorithm="fricas")

[Out] 2/168245\*(7315\*B\*c^3\*x^11 + 8855\*(3\*B\*b\*c^2 + A\*c^3)\*x^9 + 33649\*(B\*b^2\*c + A\*b\*c^2)\*x^7 + 24035\*A\*b^3\*x^3 + 15295\*(B\*b^3 + 3\*A\*b^2\*c)\*x^5)\*sqrt(x)

**giac** [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{23} B c^3 x^{\frac{23}{2}} + \frac{6}{19} B b c^2 x^{\frac{19}{2}} + \frac{2}{19} A c^3 x^{\frac{19}{2}} + \frac{2}{5} B b^2 c x^{\frac{15}{2}} + \frac{2}{5} A b c^2 x^{\frac{15}{2}} + \frac{2}{11} B b^3 x^{\frac{11}{2}} + \frac{6}{11} A b^2 c x^{\frac{11}{2}} + \frac{2}{7} A b^3 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(7/2), x, algorithm="giac")

[Out] 2/23\*B\*c^3\*x^(23/2) + 6/19\*B\*b\*c^2\*x^(19/2) + 2/19\*A\*c^3\*x^(19/2) + 2/5\*B\*b^2\*c\*x^(15/2) + 2/5\*A\*b\*c^2\*x^(15/2) + 2/11\*B\*b^3\*x^(11/2) + 6/11\*A\*b^2\*c\*x^(11/2) + 2/7\*A\*b^3\*x^(7/2)

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2(7315 B c^3 x^8 + 8855 A c^3 x^6 + 26565 B b c^2 x^6 + 33649 A b c^2 x^4 + 33649 B b^2 c x^4 + 45885 A b^2 c x^2 + 15295 B b^3 x^2 + 24035 A b^3) x^{\frac{7}{2}}}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(7/2), x)

[Out] 2/168245\*x^(7/2)\*(7315\*B\*c^3\*x^8+8855\*A\*c^3\*x^6+26565\*B\*b\*c^2\*x^6+33649\*A\*b\*c^2\*x^4+33649\*B\*b^2\*c\*x^4+45885\*A\*b^2\*c\*x^2+15295\*B\*b^3\*x^2+24035\*A\*b^3)

**maxima** [A] time = 1.34, size = 73, normalized size = 0.86

$$\frac{2}{23} B c^3 x^{\frac{23}{2}} + \frac{2}{19} (3 B b c^2 + A c^3) x^{\frac{19}{2}} + \frac{2}{5} (B b^2 c + A b c^2) x^{\frac{15}{2}} + \frac{2}{7} A b^3 x^{\frac{7}{2}} + \frac{2}{11} (B b^3 + 3 A b^2 c) x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*(c\*x^4+b\*x^2)^3/x^(7/2), x, algorithm="maxima")

[Out] 2/23\*B\*c^3\*x^(23/2) + 2/19\*(3\*B\*b\*c^2 + A\*c^3)\*x^(19/2) + 2/5\*(B\*b^2\*c + A\*b\*c^2)\*x^(15/2) + 2/7\*A\*b^3\*x^(7/2) + 2/11\*(B\*b^3 + 3\*A\*b^2\*c)\*x^(11/2)

**mupad** [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2} \left( \frac{2 B b^3}{11} + \frac{6 A c b^2}{11} \right) + x^{19/2} \left( \frac{2 A c^3}{19} + \frac{6 B b c^2}{19} \right) + \frac{2 A b^3 x^{7/2}}{7} + \frac{2 B c^3 x^{23/2}}{23} + \frac{2 b c x^{15/2} (A c + B b)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*x^2)\*(b\*x^2 + c\*x^4)^3)/x^(7/2), x)

[Out]  $x^{(11/2)} * ((2*B*b^3)/11 + (6*A*b^2*c)/11) + x^{(19/2)} * ((2*A*c^3)/19 + (6*B*b*c^2)/19) + (2*A*b^3*x^{(7/2)})/7 + (2*B*c^3*x^{(23/2)})/23 + (2*b*c*x^{(15/2)}*(A*c + B*b))/5$

**sympy [A]** time = 53.84, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{7}{2}}}{7} + \frac{6Ab^2cx^{\frac{11}{2}}}{11} + \frac{2Abc^2x^{\frac{15}{2}}}{5} + \frac{2Ac^3x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{11}{2}}}{11} + \frac{2Bb^2cx^{\frac{15}{2}}}{5} + \frac{6Bbc^2x^{\frac{19}{2}}}{19} + \frac{2Bc^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3/x\*\*(7/2), x)

[Out]  $2*A*b**3*x**(7/2)/7 + 6*A*b**2*c*x**(11/2)/11 + 2*A*b*c**2*x**(15/2)/5 + 2*A*c**3*x**(19/2)/19 + 2*B*b**3*x**(11/2)/11 + 2*B*b**2*c*x**(15/2)/5 + 6*B*b*c**2*x**(19/2)/19 + 2*B*c**3*x**(23/2)/23$

$$3.183 \quad \int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=278

$$\frac{b^{7/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB - Ac)}{2\sqrt{2}c^{15/4}}$$

**Rubi [A]** time = 0.27, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1584, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{7/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2}c^{15/4}} - \frac{2x^{7/2}(bB - Ac)}{7c^2} + \frac{2bx^{3/2}(bB - Ac)}{3c^3} + \frac{2Bx^{11/2}}{11c}$$

Antiderivative was successfully verified.

[In] Int[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*b\*(b\*B - A\*c)\*x^(3/2))/(3\*c^3) - (2\*(b\*B - A\*c)\*x^(7/2))/(7\*c^2) + (2\*B\*x^(11/2))/(11\*c) + (b^(7/4)\*(b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*c^(15/4)) - (b^(7/4)\*(b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*c^(15/4)) - (b^(7/4)\*(b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*c^(15/4)) + (b^(7/4)\*(b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*c^(15/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^(n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(b\*e\*(m+n\*(p+1)+1)), x] - Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(b\*(m+n\*(p+1)+1)), x]



+ 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] :=> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :=> Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :=> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4} dx &= \int \frac{x^{9/2}(A+Bx^2)}{b+cx^2} dx \\
&= \frac{2Bx^{11/2}}{11c} - \frac{\left(2\left(\frac{11bB}{2} - \frac{11Ac}{2}\right)\right) \int \frac{x^{9/2}}{b+cx^2} dx}{11c} \\
&= -\frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b(bB-Ac)) \int \frac{x^{5/2}}{b+cx^2} dx}{c^2} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(b^2(bB-Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{c^3} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(2b^2(bB-Ac)) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^3} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b^2(bB-Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^{7/2}} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(b^2(bB-Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx\right)}{2c^4} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{b^{7/4}(bB-Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} - \sqrt{2}\sqrt{c}^{15/4}\right)}{2\sqrt{2}c^{15/4}} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{b^{7/4}(bB-Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}}{c^{15/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 133, normalized size = 0.48

$$\frac{2x^{3/2}(-11bc(7A+3Bx^2)+3c^2x^2(11A+7Bx^2)+77b^2B)}{231c^3} + \frac{b(-b)^{3/4}(bB-Ac) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}} + \frac{(-b)^{7/4}(bB-Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(13/2)\*(A+B\*x^2))/(b\*x^2+c\*x^4),x]

[Out] (2\*x^(3/2)\*(77\*b^2\*B-11\*b\*c\*(7\*A+3\*B\*x^2)+3\*c^2\*x^2\*(11\*A+7\*B\*x^2)))/(231\*c^3)+((-b)^(3/4)\*b\*(b\*B-A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/c^(15/4)+((-b)^(7/4)\*(b\*B-A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/c^(15/4)

**IntegrateAlgebraic [A]** time = 0.24, size = 180, normalized size = 0.65

$$\frac{(b^{11/4}B-Ab^{7/4}c) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{15/4}} + \frac{(b^{11/4}B-Ab^{7/4}c) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{15/4}} + \frac{2x^{3/2}(-77Abc+33Ac^2x^2+77b^2B-33bBcx^2+21Bc^2x^4)}{231c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(13/2)\*(A+B\*x^2))/(b\*x^2+c\*x^4),x]

[Out] (2\*x^(3/2)\*(77\*b^2\*B-77\*A\*b\*c-33\*b\*B\*c\*x^2+33\*A\*c^2\*x^2+21\*B\*c^2\*x^4))/(231\*c^3)+((b^(11/4)\*B-A\*b^(7/4)\*c)\*ArcTan[(Sqrt[b]-Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*c^(15/4))+((b^(11/4)\*B-A\*b^(7/4)\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b]+Sqrt[c]\*x)]/(Sqrt[2]\*c^(15/4)))

**fricas [B]** time = 0.45, size = 920, normalized size = 3.31

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 
$$-1/462*(924*c^3*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{1/4}*\arctan((\sqrt{c}*(B^6*b^{16} - 6*A*B^5*b^{15}*c + 15*A^2*B^4*b^{14}*c^2 - 20*A^3*B^3*b^{13}*c^3 + 15*A^4*B^2*b^{12}*c^4 - 6*A^5*B*b^{11}*c^5 + A^6*b^{10}*c^6)*x - (B^4*b^{11}*c^7 - 4*A*B^3*b^{10}*c^8 + 6*A^2*B^2*b^9*c^9 - 4*A^3*B*b^8*c^{10} + A^4*b^7*c^{11})*\sqrt{c}*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15}))*c^4*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{1/4} + (B^3*b^8*c^4 - 3*A*B^2*b^7*c^5 + 3*A^2*B*b^6*c^6 - A^3*b^5*c^7)*\sqrt{c}*(x)*(- (B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{1/4}))/ (B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4) - 231*c^3*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{1/4}*\log(c^{11}*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{3/4} - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*\sqrt{c}*(x)) + 231*c^3*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{1/4}*\log(-c^{11}*(-(B^4*b^{11} - 4*A*B^3*b^{10}*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^{15})^{3/4} - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*\sqrt{c}*(x)) - 4*(21*B*c^2*x^5 - 33*(B*b*c - A*c^2)*x^3 + 77*(B*b^2 - A*b*c)*x)*\sqrt{c}*(x))/c^3$$

**giac [A]** time = 0.24, size = 298, normalized size = 1.07

$$\frac{\sqrt{2} (bc)^{\frac{3}{2}} Bb^2 - (bc)^{\frac{3}{2}} Abc \arctan\left(\frac{\sqrt{2}(\frac{b}{c})^{\frac{1}{2}} + \sqrt{c}}{(\frac{b}{c})^{\frac{1}{2}}}\right)}{2c^6} - \frac{\sqrt{2} (bc)^{\frac{3}{2}} Bb^2 - (bc)^{\frac{3}{2}} Abc \arctan\left(\frac{\sqrt{2}(\frac{b}{c})^{\frac{1}{2}} - \sqrt{c}}{(\frac{b}{c})^{\frac{1}{2}}}\right)}{2c^6} + \frac{\sqrt{2} (bc)^{\frac{3}{2}} Bb^2 - (bc)^{\frac{3}{2}} Abc \log\left(\sqrt{2}\sqrt{c}\left(\frac{b}{c}\right)^{\frac{1}{2}} + x + \sqrt{c}\right)}{4c^6} - \frac{\sqrt{2} (bc)^{\frac{3}{2}} Bb^2 - (bc)^{\frac{3}{2}} Abc \log\left(-\sqrt{2}\sqrt{c}\left(\frac{b}{c}\right)^{\frac{1}{2}} + x + \sqrt{c}\right)}{4c^6} + \frac{2(21Bc^{10}x^{\frac{11}{2}} - 33Bb^2c^8x^{\frac{7}{2}} + 33A^2c^{10}x^{\frac{11}{2}} - 77Ab^2c^8x^{\frac{7}{2}})}{231c^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 
$$-1/2*\sqrt{2}*((bc^3)^{(3/4)}*B*b^2 - (bc^3)^{(3/4)}*A*b*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*\sqrt{c}*(x))/ (b/c)^{(1/4)})/c^6 - 1/2*\sqrt{2}*((bc^3)^{(3/4)}*B*b^2 - (bc^3)^{(3/4)}*A*b*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*\sqrt{c}*(x))/ (b/c)^{(1/4)})/c^6 + 1/4*\sqrt{2}*((bc^3)^{(3/4)}*B*b^2 - (bc^3)^{(3/4)}*A*b*c)*\log(\sqrt{2}*\sqrt{c}*(b/c)^{(1/4)} + x + \sqrt{c}*(b/c)^{(1/4)})/c^6 - 1/4*\sqrt{2}*((bc^3)^{(3/4)}*B*b^2 - (bc^3)^{(3/4)}*A*b*c)*\log(-\sqrt{2}*\sqrt{c}*(b/c)^{(1/4)} + x + \sqrt{c}*(b/c)^{(1/4)})/c^6 + 2/231*(21*B*c^{10}*x^{(11/2)} - 33*B*b^2*c^8*x^{(7/2)} + 33*A*c^{10}*x^{(11/2)} + 77*B*b^2*c^8*x^{(3/2)} - 77*A*b*c^9*x^{(3/2)})/c^{11}$$

**maple [A]** time = 0.08, size = 336, normalized size = 1.21

$$\frac{2Bx^{\frac{11}{2}}}{11c} + \frac{2Ax^{\frac{7}{2}}}{7c} - \frac{2Bbx^{\frac{7}{2}}}{7c^2} - \frac{2Abx^{\frac{3}{2}}}{3c^2} + \frac{2Bb^2x^{\frac{3}{2}}}{3c^3} + \frac{\sqrt{2}Ab^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{(\frac{b}{c})^{\frac{1}{2}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{2}}c^3} + \frac{\sqrt{2}Ab^2\arctan\left(\frac{\sqrt{2}\sqrt{c}}{(\frac{b}{c})^{\frac{1}{2}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{2}}c^3} + \frac{\sqrt{2}Ab^2\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{2}\sqrt{c}+\sqrt{c}}{x+\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{2}\sqrt{c}-\sqrt{c}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{2}}c^3} - \frac{\sqrt{2}Bb^3\arctan\left(\frac{\sqrt{2}\sqrt{c}}{(\frac{b}{c})^{\frac{1}{2}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{2}}c^4} - \frac{\sqrt{2}Bb^3\arctan\left(\frac{\sqrt{2}\sqrt{c}}{(\frac{b}{c})^{\frac{1}{2}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{2}}c^4} - \frac{\sqrt{2}Bb^3\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{2}\sqrt{c}+\sqrt{c}}{x+\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{2}\sqrt{c}-\sqrt{c}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out] 
$$2/11*B*x^{(11/2)}/c+2/7/c*x^{(7/2)}*A-2/7/c^2*x^{(7/2)}*bB-2/3/c^2*x^{(3/2)}*A*b+2/3/c^3*x^{(3/2)}*B*b^2+1/4*b^2/c^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x-(b/c)^{(1/4)})*x^{(1/2)}*2^{(1/2)}+(b/c)^{(1/2)})/(x+(b/c)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(b/c)^{(1/2)}))+1/2*b^2/c^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+1/2*b^2/c^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)-1/4*b^3/c^4/(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x-(b/c)^{(1/4)})*x^{(1/2)}*2^{(1/2)}+(b/c)^{(1/2)})/(x+$$

$(b/c)^{1/4} * x^{1/2} * 2^{1/2} + (b/c)^{1/2} - 1/2 * b^3/c^4 / (b/c)^{1/4} * 2^{1/2} * B * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} + 1) - 1/2 * b^3/c^4 / (b/c)^{1/4} * 2^{1/2} * B * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} - 1)$

**maxima [A]** time = 3.07, size = 237, normalized size = 0.85

$$\frac{(Bb^3 - Ab^2c) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\frac{1}{4} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\frac{1}{4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2} \frac{1}{4} \sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{\frac{1}{b^4 c^4}} + \frac{\sqrt{2} \log\left(-\sqrt{2} \frac{1}{4} \sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{\frac{1}{b^4 c^4}} \right)}{4c^3} + \frac{2(21Bc^2x^{\frac{11}{2}} - 33(Bbc - Ac^2)x^{\frac{7}{2}} + 77(Bb^2 - Abc)x^{\frac{3}{2}})}{231c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $-1/4 * (B * b^3 - A * b^2 * c) * (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * b^{1/4} * c^{1/4} + 2 * \sqrt{2} * \sqrt{c} * \sqrt{x}) / \sqrt{(\sqrt{2} * b^{1/4} * c^{1/4}) * \sqrt{c}})) / (\sqrt{2} * \sqrt{b} * \sqrt{c}) * \sqrt{c} + 2 * \sqrt{2} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * b^{1/4} * c^{1/4} - 2 * \sqrt{2} * \sqrt{c} * \sqrt{x}) / \sqrt{(\sqrt{2} * b^{1/4} * c^{1/4}) * \sqrt{c}})) / (\sqrt{2} * \sqrt{b} * \sqrt{c}) * \sqrt{c} - \sqrt{2} * \log(\sqrt{2} * b^{1/4} * c^{1/4} * \sqrt{x} + \sqrt{c} * x + \sqrt{b}) / (b^{1/4} * c^{3/4}) + \sqrt{2} * \log(-\sqrt{2} * b^{1/4} * c^{1/4} * \sqrt{x} + \sqrt{c} * x + \sqrt{b}) / (b^{1/4} * c^{3/4}) / c^3 + 2/231 * (21 * B * c^2 * x^{11/2} - 33 * (B * b * c - A * c^2) * x^{7/2} + 77 * (B * b^2 - A * b * c) * x^{3/2}) / c^3$

**mupad [B]** time = 0.23, size = 115, normalized size = 0.41

$$x^{7/2} \left( \frac{2A}{7c} - \frac{2Bb}{7c^2} \right) + \frac{2Bx^{11/2}}{11c} + \frac{(-b)^{7/4} \operatorname{atan}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (Ac - Bb)}{c^{15/4}} - \frac{bx^{3/2} \left(\frac{2A}{c} - \frac{2Bb}{c^2}\right)}{3c} + \frac{(-b)^{7/4} \operatorname{atan}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (Ac - Bb) \operatorname{li}}{c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out]  $x^{7/2} * ((2*A)/(7*c) - (2*B*b)/(7*c^2)) + (2*B*x^{11/2})/(11*c) + ((-b)^{7/4} * \operatorname{atan}(c^{1/4} * x^{1/2} / (-b)^{1/4}) * (A*c - B*b)) / c^{15/4} + ((-b)^{7/4} * \operatorname{atan}(c^{1/4} * x^{1/2} * 1i / (-b)^{1/4}) * (A*c - B*b) * 1i) / c^{15/4} - (b*x^{3/2} * ((2*A)/c - (2*B*b)/c^2)) / (3*c)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(13/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] Timed out

$$3.184 \quad \int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=276

$$\frac{b^{5/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{13/4}} - \frac{b^{5/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{13/4}} + \frac{b^{5/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{\sqrt{2} c^{13/4}} - \frac{b^{5/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{\sqrt{2} c^{13/4}} - \frac{2x^{5/2}(bB - Ac)}{5c^2} + \frac{2b\sqrt{x}(bB - Ac)}{c^3} + \frac{2Bx^{9/2}}{9c}$$

**Rubi [A]** time = 0.25, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1584, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{13/4}} - \frac{b^{5/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{13/4}} + \frac{b^{5/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{\sqrt{2} c^{13/4}} - \frac{b^{5/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{\sqrt{2} c^{13/4}} - \frac{2x^{5/2}(bB - Ac)}{5c^2} + \frac{2b\sqrt{x}(bB - Ac)}{c^3} + \frac{2Bx^{9/2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*b\*(b\*B - A\*c)\*Sqrt[x])/c^3 - (2\*(b\*B - A\*c)\*x^(5/2))/(5\*c^2) + (2\*B\*x^(9/2))/(9\*c) + (b^(5/4)\*(b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*c^(13/4)) - (b^(5/4)\*(b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*c^(13/4)) + (b^(5/4)\*(b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*c^(13/4)) - (b^(5/4)\*(b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*c^(13/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), x]

+ 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := SImp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^n, x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11/2} (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{x^{7/2} (A + Bx^2)}{b + cx^2} dx \\
&= \frac{2Bx^{9/2}}{9c} - \frac{\left(2\left(\frac{9bB}{2} - \frac{9Ac}{2}\right)\right) \int \frac{x^{7/2}}{b+cx^2} dx}{9c} \\
&= -\frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{(b(bB - Ac)) \int \frac{x^{3/2}}{b+cx^2} dx}{c^2} \\
&= \frac{2b(bB - Ac)\sqrt{x}}{c^3} - \frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^2(bB - Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{c^3} \\
&= \frac{2b(bB - Ac)\sqrt{x}}{c^3} - \frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(2b^2(bB - Ac)) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{c^3} \\
&= \frac{2b(bB - Ac)\sqrt{x}}{c^3} - \frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^{3/2}(bB - Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, \sqrt{x}\right)}{c^3} \\
&= \frac{2b(bB - Ac)\sqrt{x}}{c^3} - \frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^{3/2}(bB - Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, \sqrt{x}\right)}{2c^{7/2}} \\
&= \frac{2b(bB - Ac)\sqrt{x}}{c^3} - \frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right)}{2\sqrt{2}c^{13/4}} \\
&= \frac{2b(bB - Ac)\sqrt{x}}{c^3} - \frac{2(bB - Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 227, normalized size = 0.82

$$\frac{45\sqrt{2}b^{5/4}(bB - Ac) \left( \log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) - \log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) \right) + \frac{90\sqrt{2}b^{5/4}(bB - Ac) \left( \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) - \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}} + 1\right) \right)}{\sqrt[4]{c}}}{180c^3} + \frac{72cx^{5/2}(Ac - bB) + 360b\sqrt{x}(bB - Ac) + 40Bc^2x^{9/2}}{180c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (360\*b\*(b\*B - A\*c)\*Sqrt[x] + 72\*c\*(-(b\*B) + A\*c)\*x^(5/2) + 40\*B\*c^2\*x^(9/2) + (90\*Sqrt[2]\*b^(5/4)\*(b\*B - A\*c)\*(ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]))/c^(1/4) + (45\*Sqrt[2]\*b^(5/4)\*(b\*B - A\*c)\*(Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]))/c^(1/4))/(180\*c^3)

**IntegrateAlgebraic [A]** time = 0.30, size = 181, normalized size = 0.66

$$\frac{(b^{9/4}B - Ab^{5/4}c) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - (b^{9/4}B - Ab^{5/4}c) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{13/4}} + \frac{2\sqrt{x}(-45Abc + 9Ac^2x^2 + 45b^2B - 9bBcx^2 + 5Bc^2x^4)}{45c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*Sqrt[x]\*(45\*b^2\*B - 45\*A\*b\*c - 9\*b\*B\*c\*x^2 + 9\*A\*c^2\*x^2 + 5\*B\*c^2\*x^4))/(45\*c^3) + ((b^(9/4)\*B - A\*b^(5/4)\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*c^(13/4)) - ((b^(9/4)\*B - A\*b^(5/4)\*c

)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*c^(13/4))

**fricas [B]** time = 0.43, size = 714, normalized size = 2.59

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 1/90\*(180\*c^3\*(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(1/4)\*arctan((sqrt(c^6\*sqrt(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13) + (B^2\*b^4 - 2\*A\*B\*b^3\*c + A^2\*b^2\*c^2)\*x)\*c^10\*(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(3/4) + (B\*b^2\*c^10 - A\*b\*c^11)\*sqrt(x)\*(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(3/4))/(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4) + 45\*c^3\*(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(1/4)\*log(c^3\*(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(1/4) - (B\*b^2 - A\*b\*c)\*sqrt(x)) - 45\*c^3\*(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(1/4)\*log(-(B^4\*b^9 - 4\*A\*B^3\*b^8\*c + 6\*A^2\*B^2\*b^7\*c^2 - 4\*A^3\*B\*b^6\*c^3 + A^4\*b^5\*c^4)/c^13)^(1/4) - (B\*b^2 - A\*b\*c)\*sqrt(x)) + 4\*(5\*B\*c^2\*x^4 + 45\*B\*b^2 - 45\*A\*b\*c - 9\*(B\*b\*c - A\*c^2)\*x^2)\*sqrt(x))/c^3

**giac [A]** time = 0.20, size = 298, normalized size = 1.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] -1/2\*sqrt(2)\*((b\*c^3)^(1/4)\*B\*b^2 - (b\*c^3)^(1/4)\*A\*b\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2\*sqrt(2)\*((b\*c^3)^(1/4)\*B\*b^2 - (b\*c^3)^(1/4)\*A\*b\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/c^4 - 1/4\*sqrt(2)\*((b\*c^3)^(1/4)\*B\*b^2 - (b\*c^3)^(1/4)\*A\*b\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 1/4\*sqrt(2)\*((b\*c^3)^(1/4)\*B\*b^2 - (b\*c^3)^(1/4)\*A\*b\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2/45\*(5\*B\*c^8\*x^(9/2) - 9\*B\*b\*c^7\*x^(5/2) + 9\*A\*c^8\*x^(5/2) + 45\*B\*b^2\*c^6\*sqrt(x) - 45\*A\*b\*c^7\*sqrt(x))/c^9

**maple [A]** time = 0.05, size = 330, normalized size = 1.20

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out] 2/9\*B\*x^(9/2)/c+2/5/c\*A\*x^(5/2)-2/5/c^2\*B\*x^(5/2)\*b-2/c^2\*A\*b\*x^(1/2)+2/c^3\*B\*b^2\*x^(1/2)+1/2\*b/c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)+1/4\*b/c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))+1/2\*b/c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-1/2\*b^2/c^3\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-1/4\*b^2/c^3\*(b/c)^(1/4)\*2^(1/2)\*B\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-1/2\*b^2/c^3\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)



**maxima** [A] time = 3.01, size = 259, normalized size = 0.94

$$\frac{\left( \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\frac{1}{\sqrt{2b^2c^2}+2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\frac{1}{\sqrt{2b^2c^2}-2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2b^2c^2}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{2}}c^{\frac{1}{4}}} - \frac{\sqrt{2}(Bb-Ac)\log\left(-\sqrt{2b^2c^2}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{2}}c^{\frac{1}{4}}} \right)^2}{4c^3} + \frac{2\left(5Bc^2x^2-9(Bbc-Ac^2)x^2+45(Bb^2-Abc)\sqrt{x}\right)}{45c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $-1/4*(2*\sqrt{2}*(B*b - A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} + 2*\sqrt{c}*\sqrt{x}))/\sqrt{c}*\sqrt{x})/\sqrt{c} + 2*\sqrt{2}*(B*b - A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} - 2*\sqrt{c}*\sqrt{x}))/\sqrt{c} + \sqrt{2}*(B*b - A*c)*\log(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{(3/4)}*c^{(1/4)}) - \sqrt{2}*(B*b - A*c)*\log(-\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{(3/4)}*c^{(1/4)}) + b^2/c^3 + 2/45*(5*B*c^2*x^{(9/2)} - 9*(B*b*c - A*c^2)*x^{(5/2)} + 45*(B*b^2 - A*b*c)*\sqrt{x})/c^3$

**mupad** [B] time = 0.25, size = 788, normalized size = 2.86

$$\frac{(-B)\operatorname{atan}\left(\frac{\sqrt{2}\left(\frac{1}{\sqrt{2b^2c^2}+2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{2\sqrt{2}b^{\frac{1}{2}}c^{\frac{1}{4}}} + \frac{2B\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(\frac{1}{\sqrt{2b^2c^2}-2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{2\sqrt{2}b^{\frac{1}{2}}c^{\frac{1}{4}}} - \frac{2\sqrt{2}(Bb-Ac)\log\left(\sqrt{2b^2c^2}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{2b^{\frac{1}{2}}c^{\frac{1}{4}}} + \frac{2\sqrt{2}(Bb-Ac)\log\left(-\sqrt{2b^2c^2}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{2b^{\frac{1}{2}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out]  $x^{(5/2)}*((2*A)/(5*c) - (2*B*b)/(5*c^2)) + (2*B*x^{(9/2)})/(9*c) - ((-b)^{(5/4)}* \operatorname{atan}(((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b)*1i)/(2*c^{(13/4)} + ((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b)*1i)/(2*c^{(13/4)})))/(((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b))/(2*c^{(13/4)} - ((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b))/(2*c^{(13/4)})))*(A*c - B*b)*1i)/c^{(13/4)} - ((b)^{(5/4)}*\operatorname{atan}(((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b)*1i)/(2*c^{(13/4)})))*(A*c - B*b))/(2*c^{(13/4)} + ((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b)*1i)/(2*c^{(13/4)})))/(((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b))/(2*c^{(13/4)} - ((b)^{(5/4)}*((16*x^{(1/2)}*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((b)^{(5/4)}*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^{(13/4)})))*(A*c - B*b))/(2*c^{(13/4)})))*(A*c - B*b)*1i)/c^{(13/4)} - (b*x^{(1/2)}*((2*A)/c - (2*B*b)/c^2))/c$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(11/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] Timed out

$$3.185 \quad \int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=257

$$\frac{b^{3/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{11/4}} - \frac{b^{3/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{11/4}} - \frac{b^{3/4}(bB - Ac)}{3c^2} + \frac{2Bx^{7/2}}{7c}$$

**Rubi [A]** time = 0.22, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{3/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{11/4}} - \frac{b^{3/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} c^{11/4}} - \frac{b^{3/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2} c^{11/4}} + \frac{b^{3/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2} c^{11/4}} - \frac{2x^{3/2}(bB - Ac)}{3c^2} + \frac{2Bx^{7/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4), x]
```

```
[Out] (-2*(b*B - A*c)*x^(3/2))/(3*c^2) + (2*B*x^(7/2))/(7*c) - (b^(3/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*c^(11/4)) + (b^(3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*c^(11/4)) + (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

#### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^(n*(m-n+1)))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m+1)-1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 459

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(b*e*(m+n*(p+1)+1)), x] - Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(b*(m+n*(p+1)+1)), x]
```

+ 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] :=> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :=> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :=> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2} (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{x^{5/2} (A + Bx^2)}{b + cx^2} dx \\
&= \frac{2Bx^{7/2}}{7c} - \frac{\left(2\left(\frac{7bB}{2} - \frac{7Ac}{2}\right)\right) \int \frac{x^{5/2}}{b+cx^2} dx}{7c} \\
&= -\frac{2(bB - Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB - Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{c^2} \\
&= -\frac{2(bB - Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(2b(bB - Ac)) \operatorname{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{2(bB - Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{(b(bB - Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(b(bB - Ac)) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2c^3} \\
&= -\frac{2(bB - Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{b^{3/4}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{11/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 110, normalized size = 0.43

$$\frac{2c^{3/4}x^{3/2}(7Ac - 7bB + 3Bcx^2) - 21(-b)^{3/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right) + 21(-b)^{3/4}(bB - Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{21c^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*c^(3/4)\*x^(3/2)\*(-7\*b\*B + 7\*A\*c + 3\*B\*c\*x^2) - 21\*(-b)^(3/4)\*(b\*B - A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + 21\*(-b)^(3/4)\*(b\*B - A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/(21\*c^(11/4))

**IntegrateAlgebraic [A]** time = 0.21, size = 160, normalized size = 0.62

$$-\frac{(b^{7/4}B - Ab^{3/4}c) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{11/4}} - \frac{(b^{7/4}B - Ab^{3/4}c) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{11/4}} + \frac{2x^{3/2}(7Ac - 7bB + 3Bcx^2)}{21c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*x^(3/2)\*(-7\*b\*B + 7\*A\*c + 3\*B\*c\*x^2))/(21\*c^2) - ((b^(7/4)\*B - A\*b^(3/4)\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*c^(11/4)) - ((b^(7/4)\*B - A\*b^(3/4)\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*c^(11/4))

**fricas [B]** time = 0.45, size = 899, normalized size = 3.50

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out]  $\frac{1}{42} \cdot (84 \cdot c^2 \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{1/4} \cdot \arctan\left(\frac{\sqrt{c} \cdot (B^6 \cdot b^{10} - 6 \cdot A \cdot B^5 \cdot b^9 \cdot c + 15 \cdot A^2 \cdot B^4 \cdot b^8 \cdot c^2 - 20 \cdot A^3 \cdot B^3 \cdot b^7 \cdot c^3 + 15 \cdot A^4 \cdot B^2 \cdot b^6 \cdot c^4 - 6 \cdot A^5 \cdot B \cdot b^5 \cdot c^5 + A^6 \cdot b^4 \cdot c^6) \cdot x - (B^4 \cdot b^7 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^8 + A^4 \cdot b^3 \cdot c^9) \cdot \sqrt{c}}{\sqrt{c} \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11}}\right) \cdot c^3 \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{1/4} + (B^3 \cdot b^5 \cdot c^3 - 3 \cdot A \cdot B^2 \cdot b^4 \cdot c^4 + 3 \cdot A^2 \cdot B \cdot b^3 \cdot c^5 - A^3 \cdot b^2 \cdot c^6) \cdot \sqrt{x} \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{1/4} + (B^3 \cdot b^5 \cdot c^3 - 3 \cdot A \cdot B^2 \cdot b^4 \cdot c^4 + 3 \cdot A^2 \cdot B \cdot b^3 \cdot c^5 - A^3 \cdot b^2 \cdot c^6) \cdot \sqrt{x} \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{1/4} - 21 \cdot c^2 \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{1/4} \cdot \log(c^8 \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{3/4} - (B^3 \cdot b^5 \cdot c^3 - 3 \cdot A \cdot B^2 \cdot b^4 \cdot c^4 + 3 \cdot A^2 \cdot B \cdot b^3 \cdot c^5 - A^3 \cdot b^2 \cdot c^6) \cdot \sqrt{x} + 21 \cdot c^2 \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{1/4} \cdot \log(-c^8 \cdot (-B^4 \cdot b^7 - 4 \cdot A \cdot B^3 \cdot b^6 \cdot c + 6 \cdot A^2 \cdot B^2 \cdot b^5 \cdot c^2 - 4 \cdot A^3 \cdot B \cdot b^4 \cdot c^3 + A^4 \cdot b^3 \cdot c^4) / c^{11})^{3/4} - (B^3 \cdot b^5 \cdot c^3 - 3 \cdot A \cdot B^2 \cdot b^4 \cdot c^4 + 3 \cdot A^2 \cdot B \cdot b^3 \cdot c^5 - A^3 \cdot b^2 \cdot c^6) \cdot \sqrt{x} + 4 \cdot (3 \cdot B \cdot c \cdot x^3 - 7 \cdot (B \cdot b - A \cdot c) \cdot x) \cdot \sqrt{x} / c^2$

**giac** [A] time = 0.21, size = 264, normalized size = 1.03

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{3}{4}} Ac \right) \arctan\left(\frac{\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + \sqrt{x}}{z\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{3}{4}} Ac \right) \arctan\left(\frac{-\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}}{z\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{3}{4}} Ac \right) \log\left(\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4c^5} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{3}{4}} Ac \right) \log\left(-\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4c^5} + \frac{2 \left( 3 B b c^{\frac{1}{2}} - 7 B b c^{\frac{3}{2}} x^2 + 7 A c^{\frac{1}{2}} x^3 \right)}{21 c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \arctan\left(\frac{1}{2} \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (b/c)^{\frac{1}{4}} + 2 \cdot \sqrt{x}) / (b/c)^{\frac{1}{4}}\right) / c^5 + \frac{1}{2} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \arctan\left(-\frac{1}{2} \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (b/c)^{\frac{1}{4}} - 2 \cdot \sqrt{x}) / (b/c)^{\frac{1}{4}}\right) / c^5 - \frac{1}{4} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \log(\sqrt{2} \cdot \sqrt{x} \cdot (b/c)^{\frac{1}{4}} + x + \sqrt{b/c}) / c^5 + \frac{1}{4} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \log(-\sqrt{2} \cdot \sqrt{x} \cdot (b/c)^{\frac{1}{4}} + x + \sqrt{b/c}) / c^5 + \frac{2}{21} \cdot (3 \cdot B \cdot c^6 \cdot x^{\frac{7}{2}} - 7 \cdot B \cdot b \cdot c^5 \cdot x^{\frac{3}{2}} + 7 \cdot A \cdot c^6 \cdot x^{\frac{3}{2}}) / c^7$

**maple** [A] time = 0.05, size = 308, normalized size = 1.20

$$\frac{\frac{2Bx^{\frac{7}{2}}}{7c} + \frac{2Ax^{\frac{3}{2}}}{3c} - \frac{2Bbx^{\frac{3}{2}}}{3c^2} - \frac{\sqrt{2} Ab \arctan\left(\frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} - \frac{\sqrt{2} Ab \arctan\left(\frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} - \frac{\sqrt{2} Ab \ln\left(\frac{x \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}\right)}{4 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} + \frac{\sqrt{2} B b^2 \arctan\left(\frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^3} + \frac{\sqrt{2} B b^2 \arctan\left(\frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^3} + \frac{\sqrt{2} B b^2 \ln\left(\frac{x \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}\right)}{4 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out]  $\frac{2}{7} \cdot B \cdot x^{\frac{7}{2}} / c + \frac{2}{3} \cdot c \cdot x^{\frac{3}{2}} \cdot A - \frac{2}{3} \cdot c^2 \cdot x^{\frac{3}{2}} \cdot b \cdot B - \frac{1}{2} \cdot b \cdot c^2 / (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot A \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} - 1\right) - \frac{1}{4} \cdot b \cdot c^2 / (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot A \cdot \ln\left(\frac{(x - (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}) \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}}{(x + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}) \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}}\right) - \frac{1}{2} \cdot b \cdot c^2 / (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot A \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} + 1\right) + \frac{1}{2} \cdot b^2 \cdot c^3 / (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot B \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} - 1\right) + \frac{1}{4} \cdot b^2 \cdot c^3 / (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot B \cdot \ln\left(\frac{(x - (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}) \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}}{(x + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}) \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}}}\right) + \frac{1}{2} \cdot b^2 \cdot c^3 / (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot B \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} + 1\right)$

**maxima** [A] time = 3.13, size = 214, normalized size = 0.83

$$(Bb^2 - Abc) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \right) + \frac{2\left(3Bcx^{\frac{7}{2}} - 7(Bb - Ac)x^{\frac{3}{2}}\right)}{21c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 1/4\*(B\*b^2 - A\*b\*c)\*(2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(sqrt(b)\*sqrt(c))\*sqrt(c) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(sqrt(b)\*sqrt(c))\*sqrt(c) - sqrt(2)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(1/4)\*c^(3/4)) + sqrt(2)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(1/4)\*c^(3/4))/c^2 + 2/21\*(3\*B\*c\*x^(7/2) - 7\*(B\*b - A\*c)\*x^(3/2))/c^2

**mupad** [B] time = 0.24, size = 92, normalized size = 0.36

$$x^{3/2} \left( \frac{2A}{3c} - \frac{2Bb}{3c^2} \right) + \frac{2Bx^{7/2}}{7c} + \frac{(-b)^{3/4} \operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right) (Ac - Bb)}{c^{11/4}} + \frac{(-b)^{3/4} \operatorname{atan}\left(\frac{c^{1/4}\sqrt{x} 1i}{(-b)^{1/4}}\right) (Ac - Bb) 1i}{c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out] x^(3/2)\*((2\*A)/(3\*c) - (2\*B\*b)/(3\*c^2)) + (2\*B\*x^(7/2))/(7\*c) + ((-b)^(3/4)\*atan((c^(1/4)\*x^(1/2))/(-b)^(1/4))\*(A\*c - B\*b))/c^(11/4) + ((-b)^(3/4)\*atan((c^(1/4)\*x^(1/2)\*1i)/(-b)^(1/4))\*(A\*c - B\*b)\*1i)/c^(11/4)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(9/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] Timed out

$$3.186 \quad \int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=255

$$\frac{\sqrt[4]{b}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} - \frac{\sqrt[4]{b}(bB)}{c^2}$$

**Rubi [A]** time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2\sqrt{x}(bB - Ac)}{c^2} - \frac{\sqrt[4]{b}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} - \frac{\sqrt[4]{b}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2}c^{9/4}} + \frac{2Bx^{5/2}}{5c}$$

Antiderivative was successfully verified.

[In] Int[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (-2\*(b\*B - A\*c)\*Sqrt[x])/c^2 + (2\*B\*x^(5/2))/(5\*c) - (b^(1/4)\*(b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*c^(9/4)) + (b^(1/4)\*(b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*c^(9/4)) - (b^(1/4)\*(b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*c^(9/4)) + (b^(1/4)\*(b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*c^(9/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(b\*e\*(m+n\*(p+1)+1)), x] - Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(b\*(m+n\*(p+1)+1)), x]

+ 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^n, x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rubi steps



$$\begin{aligned}
\int \frac{x^{7/2} (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{x^{3/2} (A + Bx^2)}{b + cx^2} dx \\
&= \frac{2Bx^{5/2}}{5c} - \frac{\left(2\left(\frac{5bB}{2} - \frac{5Ac}{2}\right)\right) \int \frac{x^{3/2}}{b+cx^2} dx}{5c} \\
&= -\frac{2(bB - Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(b(bB - Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{c^2} \\
&= -\frac{2(bB - Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(2b(bB - Ac)) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{2(bB - Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(\sqrt{b}(bB - Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^2} + \frac{(\sqrt{b}(bB - Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2c^{5/2}} \\
&= -\frac{2(bB - Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB - Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 208, normalized size = 0.82

$$\frac{-40\sqrt{x}(bB - Ac) + \frac{5\sqrt{2}\sqrt[4]{b}(Ac - bB)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) - \log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right)\right)}{\sqrt[4]{c}} - \frac{10\sqrt{2}\sqrt[4]{b}(bB - Ac)\left(\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) - \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}} + 1\right)\right)}{\sqrt[4]{c}} + 8Bcx^{5/2}}{20c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (-40\*(b\*B - A\*c)\*Sqrt[x] + 8\*B\*c\*x^(5/2) - (10\*Sqrt[2]\*b^(1/4)\*(b\*B - A\*c)\*(ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]))/c^(1/4) + (5\*Sqrt[2]\*b^(1/4)\*(-b\*B) + A\*c)\*(Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/c^(1/4))/(20\*c^2)

**IntegrateAlgebraic [A]** time = 0.22, size = 165, normalized size = 0.65

$$\frac{(b^{5/4}B - A\sqrt[4]{b}c) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{9/4}} + \frac{(b^{5/4}B - A\sqrt[4]{b}c) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{9/4}} + \frac{2(5Ac\sqrt{x} - 5bB\sqrt{x} + Bcx^{5/2})}{5c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*(-5\*b\*B\*Sqrt[x] + 5\*A\*c\*Sqrt[x] + B\*c\*x^(5/2)))/(5\*c^2) - ((b^(5/4)\*B - A\*b^(1/4)\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*c^(9/4)) + ((b^(5/4)\*B - A\*b^(1/4)\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*c^(9/4))

**fricas [B]** time = 0.43, size = 660, normalized size = 2.59

$$\frac{2\sqrt{2}\sqrt[4]{b}(bB - Ac)\left(\log\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)\right) + 8Bcx^{5/2}}{20c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 
$$-1/10*(20*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(1/4)}*\arctan\left(\frac{\sqrt{c^4*\sqrt{-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9}}}{z^{(1/4)}}\right) + (B^2*b^2 - 2*A*B*b*c + A^2*c^2)*x)*c^7*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(3/4)} + (B*b*c^7 - A*c^8)*\sqrt{x}*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(3/4)} + (B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9 + 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(1/4)}*\log(c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(1/4)} - (B*b - A*c)*\sqrt{x}) - 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(1/4)}*\log(-c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^{(1/4)} - (B*b - A*c)*\sqrt{x}) - 4*(B*c*x^2 - 5*B*b + 5*A*c)*\sqrt{x})/c^2$$

**giac** [A] time = 0.20, size = 263, normalized size = 1.03

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} + 2\sqrt{x} \right)}{z^{(1/4)}} \right)}{2c^3} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} - 2\sqrt{x} \right)}{z^{(1/4)}} \right)}{2c^3} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{z}{c} \right)^{\frac{1}{4}} + x + \sqrt{x} \right)}{4c^3} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{z}{c} \right)^{\frac{1}{4}} + x + \sqrt{x} \right)}{4c^3} + \frac{2 \left( Bc^4 x^{\frac{5}{2}} - 5 Bb c^3 \sqrt{x} + 5 A c^4 \sqrt{x} \right)}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 
$$1/2*\sqrt{2}*((bc^3)^{(1/4)}*B*b - (bc^3)^{(1/4)}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*\sqrt{x}))/((b/c)^{(1/4)})/c^3 + 1/2*\sqrt{2}*((bc^3)^{(1/4)}*B*b - (bc^3)^{(1/4)}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*\sqrt{x}))/((b/c)^{(1/4)})/c^3 + 1/4*\sqrt{2}*((bc^3)^{(1/4)}*B*b - (bc^3)^{(1/4)}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c})/c^3 - 1/4*\sqrt{2}*((bc^3)^{(1/4)}*B*b - (bc^3)^{(1/4)}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c})/c^3 + 2/5*(B*c^4*x^{(5/2)} - 5*B*b*c^3*\sqrt{x} + 5*A*c^4*\sqrt{x})/c^5$$

**maple** [A] time = 0.06, size = 299, normalized size = 1.17

$$\frac{2Bx^{\frac{5}{2}}}{5c} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} - 1 \right)}{2c} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} + 1 \right)}{2c} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A \ln \left( \frac{\sqrt{2} \sqrt{x} \sqrt{b/c} + \sqrt{x}}{\sqrt{2} \sqrt{x} \sqrt{b/c} - \sqrt{x}} \right)}{4c} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} Bb \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} - 1 \right)}{2c^2} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} Bb \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} + 1 \right)}{2c^2} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} Bb \ln \left( \frac{\sqrt{2} \sqrt{x} \sqrt{b/c} + \sqrt{x}}{\sqrt{2} \sqrt{x} \sqrt{b/c} - \sqrt{x}} \right)}{4c^2} + \frac{2A\sqrt{x}}{c} - \frac{2Bb\sqrt{x}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out] 
$$2/5*B*x^{(5/2)}/c + 2/c*A*x^{(1/2)} - 2/c^2*b*B*x^{(1/2)} - 1/2/c*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1) - 1/2/c*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1) - 1/4/c*(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})) + 1/2/c^2*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)*b + 1/2/c^2*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)*b + 1/4/c^2*(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))*b$$

**maxima** [A] time = 2.99, size = 235, normalized size = 0.92

$$\left( \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}c^2+2\sqrt{x}}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}c^2-2\sqrt{x}}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}\sqrt{\frac{1}{4}c^2\sqrt{x}+\sqrt{cx}+\sqrt{b}}\right)}{\frac{3}{b^{\frac{1}{4}}c^{\frac{1}{4}}}} - \frac{\sqrt{2}(Bb-Ac)\log\left(-\sqrt{2}\sqrt{\frac{1}{4}c^2\sqrt{x}+\sqrt{cx}+\sqrt{b}}\right)}{\frac{3}{b^{\frac{1}{4}}c^{\frac{1}{4}}}} \right) b + \frac{2\left(Bcx^{\frac{5}{2}} - 5(Bb - Ac)\sqrt{x}\right)}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.



```
1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c**2 - 2*B*b*  
sqrt(x)/c**2 + 2*B*x**(5/2)/(5*c), True))
```

$$3.187 \quad \int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=237

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} \sqrt[4]{b} c^{7/4}} + \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} \sqrt[4]{b} c^{7/4}} + \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{\sqrt{2} \sqrt[4]{b} c^{7/4}}$$

**Rubi [A]** time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {1584, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} \sqrt[4]{b} c^{7/4}} + \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} \sqrt[4]{b} c^{7/4}} + \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2} \sqrt[4]{b} c^{7/4}} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2} \sqrt[4]{b} c^{7/4}} + \frac{2Bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x]

[Out] (2\*B\*x^(3/2))/(3\*c) + ((b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(1/4)\*c^(7/4)) - ((b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(1/4)\*c^(7/4)) - ((b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(1/4)\*c^(7/4)) + ((b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(1/4)\*c^(7/4)))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \ :> \ \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \text{EqQ}[c^2d - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

### Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

### Rule 1584

$\text{Int}[(u_.)x^{(m_.)}((a_.)x^{(p_.)} + (b_.)x^{(q_.)})^{(n_.)}, x\_Symbol] \ :> \ \text{Int}[u^m x^{(m+n)p} (a + bx^{(q-p)})^n, x] \ /; \ \text{FreeQ}[\{a, b, m, p, q\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p]$

### Rubi steps

$$\begin{aligned} \int \frac{x^{5/2} (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{\sqrt{x} (A + Bx^2)}{b + cx^2} dx \\ &= \frac{2Bx^{3/2}}{3c} - \frac{\left(2\left(\frac{3bB}{2} - \frac{3Ac}{2}\right)\right) \int \frac{\sqrt{x}}{b+cx^2} dx}{3c} \\ &= \frac{2Bx^{3/2}}{3c} - \frac{\left(4\left(\frac{3bB}{2} - \frac{3Ac}{2}\right)\right) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{3c} \\ &= \frac{2Bx^{3/2}}{3c} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b} - \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b} + \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{c^{3/2}} \\ &= \frac{2Bx^{3/2}}{3c} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{2c^2} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{2c^2} \\ &= \frac{2Bx^{3/2}}{3c} - \frac{(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB - Ac) \log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} \\ &= \frac{2Bx^{3/2}}{3c} + \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB - Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 95, normalized size = 0.40

$$\frac{(3Ac - 3bB) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right) + 3(bB - Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right) + 2\sqrt[4]{-b} Bc^{3/4}x^{3/2}}{3\sqrt[4]{-b} c^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*(-b)^(1/4)\*B\*c^(3/4)\*x^(3/2) + (-3\*b\*B + 3\*A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + 3\*(b\*B - A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)]/(3\*(-b)^(1/4)\*c^(7/4))

**IntegrateAlgebraic [A]** time = 0.21, size = 135, normalized size = 0.57

$$\frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB - Ac) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{2Bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*B\*x^(3/2))/(3\*c) + ((b\*B - A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(1/4)\*c^(7/4)) + ((b\*B - A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*b^(1/4)\*c^(7/4))

**fricas [B]** time = 0.46, size = 834, normalized size = 3.52

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/6\*(4\*B\*x^(3/2) - 12\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(1/4)\*arctan((sqrt((B^6\*b^6 - 6\*A\*B^5\*b^5\*c + 15\*A^2\*B^4\*b^4\*c^2 - 20\*A^3\*B^3\*b^3\*c^3 + 15\*A^4\*B^2\*b^2\*c^4 - 6\*A^5\*B\*b\*c^5 + A^6\*c^6)\*x - (B^4\*b^5\*c^3 - 4\*A\*B^3\*b^4\*c^4 + 6\*A^2\*B^2\*b^3\*c^5 - 4\*A^3\*B\*b^2\*c^6 + A^4\*b\*c^7)\*sqrt(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))))\*c^2\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(1/4) + (B^3\*b^3\*c^2 - 3\*A\*B^2\*b^2\*c^3 + 3\*A^2\*B\*b\*c^4 - A^3\*c^5)\*sqrt(x)\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(1/4))/(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4) + 3\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(1/4)\*log(b\*c^5\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(3/4) - (B^3\*b^3 - 3\*A\*B^2\*b^2\*c + 3\*A^2\*B\*b\*c^2 - A^3\*c^3)\*sqrt(x)) - 3\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(1/4)\*log(-b\*c^5\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b\*c^7))^(3/4) - (B^3\*b^3 - 3\*A\*B^2\*b^2\*c + 3\*A^2\*B\*b\*c^2 - A^3\*c^3)\*sqrt(x)))/c

**giac [A]** time = 0.23, size = 251, normalized size = 1.06

$$\frac{\sqrt{2}\left((bc^3)^{\frac{3}{2}}Bb - (bc^3)^{\frac{3}{2}}Ac\right) \arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + 2\sqrt{c}}}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2bc^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{2}}Bb - (bc^3)^{\frac{3}{2}}Ac\right) \arctan\left(-\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} - 2\sqrt{c}}}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2bc^4} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{2}}Bb - (bc^3)^{\frac{3}{2}}Ac\right) \log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4bc^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{2}}Bb - (bc^3)^{\frac{3}{2}}Ac\right) \log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out]  $2/3*B*x^{3/2}/c - 1/2*\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} + 2*\sqrt{x})/(b/c)^{1/4})/(b*c^4) - 1/2*\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} - 2*\sqrt{x})/(b/c)^{1/4})/(b*c^4) + 1/4*\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b*c^4) - 1/4*\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b*c^4)$

**maple [A]** time = 0.05, size = 280, normalized size = 1.18

$$\frac{2Bx^{\frac{3}{2}}}{3c} + \frac{\sqrt{2} A \arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c} + \frac{\sqrt{2} A \arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c} + \frac{\sqrt{2} A \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c} - \frac{\sqrt{2} B b \arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2} - \frac{\sqrt{2} B b \arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2} - \frac{\sqrt{2} B b \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x)

[Out]  $2/3*B*x^{3/2}/c+1/2/c/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1)+1/2/c/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)+1/4/c/(b/c)^{1/4}*2^{1/2}*A*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})/(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))-1/2/c^2/(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1)*b-1/2/c^2/(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)*b-1/4/c^2/(b/c)^{1/4}*2^{1/2}*B*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})/(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))*b$

**maxima [A]** time = 3.02, size = 194, normalized size = 0.82

$$\frac{2Bx^{\frac{3}{2}}}{3c} - \frac{(Bb - Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}\sqrt{c}}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}\sqrt{c}}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $2/3*B*x^{3/2}/c - 1/4*(B*b - A*c)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x})/\sqrt{(\sqrt{b}*\sqrt{c})}))/(\sqrt{(\sqrt{b}*\sqrt{c})}*\sqrt{c}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x})/\sqrt{(\sqrt{b}*\sqrt{c})}))/(\sqrt{(\sqrt{b}*\sqrt{c})}*\sqrt{c}) - \sqrt{2}*\log(\sqrt{2}*(b/c)^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{1/4}*c^{3/4}) + \sqrt{2}*\log(-\sqrt{2}*(b/c)^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{1/4}*c^{3/4}))/c$

**mupad [B]** time = 0.15, size = 71, normalized size = 0.30

$$\frac{2Bx^{3/2}}{3c} + \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac - Bb)}{(-b)^{1/4}c^{7/4}} - \frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac - Bb)}{(-b)^{1/4}c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out]  $(2*B*x^{3/2})/(3*c) + (\operatorname{atan}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(A*c - B*b))/((-b)^{1/4}*c^{7/4}) - (\operatorname{atanh}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(A*c - B*b))/((-b)^{1/4}*c^{7/4})$



**sympy [A]** time = 65.60, size = 459, normalized size = 1.94

$$\begin{cases} \sqrt{c} \left( \frac{2A}{\sqrt{c}} + \frac{2Bx^{\frac{3}{2}}}{3} \right) & \text{for } b = 0 \wedge c = 0 \\ \frac{2Ax^{\frac{3}{2}} + 2Bx^{\frac{7}{2}}}{3} & \text{for } c = 0 \\ \frac{2A}{c} + \frac{2Bx^{\frac{3}{2}}}{c} & \text{for } b = 0 \\ \frac{(-1)^{\frac{3}{4}} A \left( \frac{1}{c} \right)^{\frac{3}{4}} \log\left(-\sqrt{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{c}\right)}{2\sqrt[4]{c}} + \frac{(-1)^{\frac{3}{4}} A \left( \frac{1}{c} \right)^{\frac{3}{4}} \log\left(\sqrt{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{c}\right)}{2\sqrt[4]{c}} - \frac{(-1)^{\frac{3}{4}} A \left( \frac{1}{c} \right)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{c}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{\sqrt[4]{c}} + \frac{2(-1)^{\frac{3}{4}} A \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{c}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{\sqrt[4]{c} \sqrt[4]{x}} + \frac{(-1)^{\frac{3}{4}} B b \left( \frac{1}{c} \right)^{\frac{3}{4}} \log\left(-\sqrt{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{c}\right)}{2} - \frac{(-1)^{\frac{3}{4}} B b \left( \frac{1}{c} \right)^{\frac{3}{4}} \log\left(\sqrt{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{c}\right)}{2} + \frac{(-1)^{\frac{3}{4}} B b \left( \frac{1}{c} \right)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{c}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{c} - \frac{2(-1)^{\frac{3}{4}} B b \left( \frac{1}{c} \right)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{c}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{c^{\frac{3}{4}} \sqrt[4]{x}} + \frac{2Bx^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2), x)

[Out] Piecewise((zoo\*(-2\*A/sqrt(x) + 2\*B\*x\*\*(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((2\*A\*x\*\*(3/2)/3 + 2\*B\*x\*\*(7/2)/7)/b, Eq(c, 0)), ((-2\*A/sqrt(x) + 2\*B\*x\*\*(3/2)/3)/c, Eq(b, 0)), ((-1)\*\*(3/4)\*A\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(1/4)) + (-1)\*\*(3/4)\*A\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(1/4)) - (-1)\*\*(3/4)\*A\*(1/c)\*\*(3/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(1/4) + 2\*(-1)\*\*(3/4)\*A\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/(b\*\*(1/4)\*c\*(1/c)\*\*(1/4)) + (-1)\*\*(3/4)\*B\*b\*\*(3/4)\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*c) - (-1)\*\*(3/4)\*B\*b\*\*(3/4)\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*c) + (-1)\*\*(3/4)\*B\*b\*\*(3/4)\*(1/c)\*\*(3/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/c - 2\*(-1)\*\*(3/4)\*B\*b\*\*(3/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/(c\*\*2\*(1/c)\*\*(1/4)) + 2\*B\*x\*\*(3/2)/(3\*c), True))

$$3.188 \quad \int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=235

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{3/4} c^{5/4}} - \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{3/4} c^{5/4}} + \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{\sqrt{2} b^{3/4} c^{5/4}}$$

**Rubi [A]** time = 0.18, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {1584, 459, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{3/4} c^{5/4}} - \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{3/4} c^{5/4}} + \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{\sqrt{2} b^{3/4} c^{5/4}} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x} + 1\right)}{\sqrt{2} b^{3/4} c^{5/4}} + \frac{2B\sqrt{x}}{c}$$

Antiderivative was successfully verified.

[In] Int[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*B\*Sqrt[x])/c + ((b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/ (Sqrt[2]\*b^(3/4)\*c^(5/4)) - ((b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/ (Sqrt[2]\*b^(3/4)\*c^(5/4)) + ((b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/ (2\*Sqrt[2]\*b^(3/4)\*c^(5/4)) - ((b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/ (2\*Sqrt[2]\*b^(3/4)\*c^(5/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 628

$\text{Int}[\{(d_) + (e_)(x_)\}/\{(a_) + (b_)(x_) + (c_)(x_)^2\}, x\_Symbol] \ :> \ \text{Simp}[\{d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]\}/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - b^2e, 0]$

### Rule 1162

$\text{Int}[\{(d_) + (e_)(x_)^2\}/\{(a_) + (c_)(x_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

### Rule 1165

$\text{Int}[\{(d_) + (e_)(x_)^2\}/\{(a_) + (c_)(x_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

### Rule 1584

$\text{Int}[(u_)(x_)^{(m_)} \cdot \{(a_)(x_)^{(p_)} + (b_)(x_)^{(q_)}\}^{(n_)}, x\_Symbol] \ :> \ \text{Int}[u \cdot x^{(m + n \cdot p)} \cdot (a + b \cdot x^{(q - p)})^n, x] \ /; \ \text{FreeQ}[\{a, b, m, p, q\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p]$

### Rubi steps

$$\begin{aligned} \int \frac{x^{3/2} (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{A + Bx^2}{\sqrt{x} (b + cx^2)} dx \\ &= \frac{2B\sqrt{x}}{c} - \frac{\left(2\left(\frac{bB}{2} - \frac{Ac}{2}\right)\right) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{c} \\ &= \frac{2B\sqrt{x}}{c} - \frac{\left(4\left(\frac{bB}{2} - \frac{Ac}{2}\right)\right) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{c} \\ &= \frac{2B\sqrt{x}}{c} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b} - \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{\sqrt{b}c} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b} + \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{\sqrt{b}c} \\ &= \frac{2B\sqrt{x}}{c} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{2\sqrt{b}c^{3/2}} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}}} dx, x, \sqrt{x}\right)}{2\sqrt{b}c^{3/2}} \\ &= \frac{2B\sqrt{x}}{c} + \frac{(bB - Ac) \log(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x)}{2\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB - Ac) \log(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x)}{2\sqrt{2}b^{3/4}c^{5/4}} \\ &= \frac{2B\sqrt{x}}{c} + \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB - Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} + \frac{(bB - Ac) \log(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x)}{2\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB - Ac) \log(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x)}{2\sqrt{2}b^{3/4}c^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 166, normalized size = 0.71

$$\frac{(bB - Ac) \left( \log \left( -\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x \right) - \log \left( \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x \right) + 2 \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} \right) - 2 \tan^{-1} \left( \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1 \right) \right)}{2\sqrt{2} b^{3/4} c^{5/4}} + \frac{2B\sqrt{x}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*B\*Sqrt[x])/c + ((b\*B - A\*c)\*(2\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 2\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]))/(2\*Sqrt[2]\*b^(3/4)\*c^(5/4))

**IntegrateAlgebraic [A]** time = 0.20, size = 134, normalized size = 0.57

$$\frac{(bB - Ac) \tan^{-1} \left( \frac{\sqrt{b} - \sqrt{c} x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}} \right)}{\sqrt{2} b^{3/4} c^{5/4}} - \frac{(bB - Ac) \tanh^{-1} \left( \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x} \right)}{\sqrt{2} b^{3/4} c^{5/4}} + \frac{2B\sqrt{x}}{c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (2\*B\*Sqrt[x])/c + ((b\*B - A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(3/4)\*c^(5/4)) - ((b\*B - A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]/(Sqrt[b] + Sqrt[c]\*x)))/(Sqrt[2]\*b^(3/4)\*c^(5/4))

**fricas [B]** time = 0.44, size = 645, normalized size = 2.74

$$\frac{c \left( \frac{2 \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} \arctan \left( \frac{\sqrt{b} - \sqrt{c} x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}} \right) - (bB - Ac) \tanh^{-1} \left( \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x} \right)}{\sqrt{2} b^{3/4} c^{5/4}} \right) + 2B\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/2\*(4\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(1/4)\*arctan((sqrt(b^2\*c^2\*sqrt(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5)) + (B^2\*b^2 - 2\*A\*B\*b\*c + A^2\*c^2)\*x)\*b^2\*c^4\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(3/4) + (B\*b^3\*c^4 - A\*b^2\*c^5)\*sqrt(x)\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(3/4))/(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)) + c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(1/4)\*log(b\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(1/4) - (B\*b - A\*c)\*sqrt(x)) - c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(1/4)\*log(-b\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^5))^(1/4) - (B\*b - A\*c)\*sqrt(x)) + 4\*B\*sqrt(x))/c

**giac [A]** time = 0.18, size = 251, normalized size = 1.07

$$\frac{2B\sqrt{x}}{c} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} + 2\sqrt{c} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2bc^2} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} - 2\sqrt{c} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2bc^2} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \log \left( \sqrt{2} \sqrt{c} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{b}} \right)}{4bc^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \log \left( -\sqrt{2} \sqrt{c} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{b}} \right)}{4bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2), x, algorithm="giac")

[Out]  $2*B*\sqrt{x}/c - 1/2*\sqrt{2}*((b*c^3)^{1/4}*B*b - (b*c^3)^{1/4}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} + 2*\sqrt{x})/(b/c)^{1/4})/(b*c^2) - 1/2*\sqrt{2}*((b*c^3)^{1/4}*B*b - (b*c^3)^{1/4}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} - 2*\sqrt{x})/(b/c)^{1/4})/(b*c^2) - 1/4*\sqrt{2}*((b*c^3)^{1/4}*B*b - (b*c^3)^{1/4}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b*c^2) + 1/4*\sqrt{2}*((b*c^3)^{1/4}*B*b - (b*c^3)^{1/4}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b*c^2)$

**maple [A]** time = 0.06, size = 277, normalized size = 1.18

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}\right)}{4b} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}\right)}{4c} + \frac{2B\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2), x)`

[Out]  $2*B*x^{1/2}/c + 1/2*(b/c)^{1/4}/b*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1) + 1/2*(b/c)^{1/4}/b*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1) + 1/4*(b/c)^{1/4}/b*2^{1/2}*A*\ln((x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))/((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})) - 1/2/c*(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1) - 1/2/c*(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1) - 1/4/c*(b/c)^{1/4}*2^{1/2}*B*\ln((x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))/((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))$

**maxima [A]** time = 3.16, size = 218, normalized size = 0.93

$$\frac{2B\sqrt{x}}{c} - \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}}c^{\frac{1}{4}}}} - \frac{\sqrt{2}(Bb-Ac)\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2), x, algorithm="maxima")`

[Out]  $2*B*\sqrt{x}/c - 1/4*(2*\sqrt{2}*(B*b - A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x})/\sqrt{b}*\sqrt{c}))/(\sqrt{b}*\sqrt{c}) + 2*\sqrt{2}*(B*b - A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x})/\sqrt{b}*\sqrt{c}))/(\sqrt{b}*\sqrt{c}) + \sqrt{2}*(B*b - A*c)*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}x + \sqrt{b})/(b^{3/4}*c^{1/4}) - \sqrt{2}*(B*b - A*c)*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}x + \sqrt{b})/(b^{3/4}*c^{1/4})/c$

**mupad [B]** time = 0.29, size = 739, normalized size = 3.14

$$\frac{2B\sqrt{x}}{c} - \frac{\operatorname{atan}\left(\frac{(A-B)\sqrt{(16A^2-32AB+16B^2)}}{2(-b)^{3/4}c^{1/4}}\right)}{(-b)^{3/4}c^{1/4}} + \frac{\operatorname{atan}\left(\frac{(A+B)\sqrt{(16A^2-32AB+16B^2)}}{2(-b)^{3/4}c^{1/4}}\right)}{(-b)^{3/4}c^{1/4}}}{(A-B)} + \frac{\operatorname{atan}\left(\frac{(A-B)\sqrt{(16A^2-32AB+16B^2)}}{2(-b)^{3/4}c^{1/4}}\right)}{(-b)^{3/4}c^{1/4}} - \frac{\operatorname{atan}\left(\frac{(A+B)\sqrt{(16A^2-32AB+16B^2)}}{2(-b)^{3/4}c^{1/4}}\right)}{(-b)^{3/4}c^{1/4}}}{(A-B)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4), x)`

[Out]  $(2*B*x^{1/2})/c - (\operatorname{atan}(\frac{(A*c - B*b)*(x^{1/2}*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))}{2*(-b)^{3/4}*c^{5/4}})) * i) / (2*(-b)^{3/4}*c^{5/4}) + ((A*c - B*b)*(x^{1/2}*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)) / (2*(-b)^{3/4}*c^{5/4})) * i) / (2*(-b)^{3/4}*c^{5/4}) / (((A*c - B*b)*(x^{1/2}*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)) / (2*(-b)^{3/4}*c^{5/4}))) / (2*(-b)^{3/4}*c^{5/4}) - ((A*c - B*b)*(x^{1/2}*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)) / (2*(-b)^{3/4}*c^{5/4}))) / (2*(-b)^{3/4}*c^{5/4}))) * (A*c - B$



$$3.189 \quad \int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4} dx$$

**Optimal.** Leaf size=235

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{5/4} c^{3/4}} - \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{5/4} c^{3/4}} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{\sqrt{2} b^{5/4} c^{3/4}}$$

**Rubi [A]** time = 0.19, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {1584, 453, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{5/4} c^{3/4}} - \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{5/4} c^{3/4}} - \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{\sqrt{2} b^{5/4} c^{3/4}} + \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{\sqrt{2} b^{5/4} c^{3/4}} - \frac{2A}{b\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (-2\*A)/(b\*Sqrt[x]) - ((b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(5/4)\*c^(3/4)) + ((b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(5/4)\*c^(3/4)) + ((b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(5/4)\*c^(3/4)) - ((b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(5/4)\*c^(3/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 453

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x} (A + Bx^2)}{bx^2 + cx^4} dx &= \int \frac{A + Bx^2}{x^{3/2} (b + cx^2)} dx \\
 &= -\frac{2A}{b\sqrt{x}} - \frac{\left(2\left(-\frac{bB}{2} + \frac{Ac}{2}\right)\right) \int \frac{\sqrt{x}}{b+cx^2} dx}{b} \\
 &= -\frac{2A}{b\sqrt{x}} - \frac{\left(4\left(-\frac{bB}{2} + \frac{Ac}{2}\right)\right) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b} \\
 &= -\frac{2A}{b\sqrt{x}} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b\sqrt{c}} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b\sqrt{c}} \\
 &= -\frac{2A}{b\sqrt{x}} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2bc} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2bc} \\
 &= -\frac{2A}{b\sqrt{x}} + \frac{(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB - Ac) \log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} \\
 &= -\frac{2A}{b\sqrt{x}} - \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB - Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB - Ac) \log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}
 \end{aligned}$$



**Mathematica [A]** time = 0.10, size = 74, normalized size = 0.31

$$\frac{(bB - Ac) \left( \tan^{-1} \left( \frac{\sqrt[4]{c} \sqrt{x}}{\sqrt[4]{-b}} \right) + \tanh^{-1} \left( \frac{b \sqrt[4]{c} \sqrt{x}}{(-b)^{5/4}} \right) \right)}{\sqrt[4]{-b} c^{3/4}} - \frac{2A}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] ((-2\*A)/Sqrt[x] + ((b\*B - A\*c)\*(ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b\*c^(1/4)\*Sqrt[x])/(-b)^(5/4)]))/((-b)^(1/4)\*c^(3/4)))/b

**IntegrateAlgebraic [A]** time = 0.20, size = 135, normalized size = 0.57

$$-\frac{(bB - Ac) \tan^{-1} \left( \frac{\sqrt{b} - \sqrt{c} x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}} \right)}{\sqrt{2} b^{5/4} c^{3/4}} - \frac{(bB - Ac) \tanh^{-1} \left( \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x} \right)}{\sqrt{2} b^{5/4} c^{3/4}} - \frac{2A}{b\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4), x]

[Out] (-2\*A)/(b\*Sqrt[x]) - ((b\*B - A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(5/4)\*c^(3/4)) - ((b\*B - A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*b^(5/4)\*c^(3/4))

**fricas [B]** time = 0.45, size = 843, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*x^(1/2)/(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] 1/2\*(4\*b\*x\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(1/4)\*arctan((sqrt((B^6\*b^6 - 6\*A\*B^5\*b^5\*c + 15\*A^2\*B^4\*b^4\*c^2 - 20\*A^3\*B^3\*b^3\*c^3 + 15\*A^4\*B^2\*b^2\*c^4 - 6\*A^5\*B\*b\*c^5 + A^6\*c^6)\*x - (B^4\*b^7\*c - 4\*A\*B^3\*b^6\*c^2 + 6\*A^2\*B^2\*b^5\*c^3 - 4\*A^3\*B\*b^4\*c^4 + A^4\*b^3\*c^5)\*sqrt(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))))\*b\*c\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(1/4) + (B^3\*b^4\*c - 3\*A\*B^2\*b^3\*c^2 + 3\*A^2\*B\*b^2\*c^3 - A^3\*b\*c^4)\*sqrt(x)\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(1/4))/(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4) - b\*x\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(1/4)\*log(b^4\*c^2\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(3/4) - (B^3\*b^3 - 3\*A\*B^2\*b^2\*c + 3\*A^2\*B\*b\*c^2 - A^3\*c^3)\*sqrt(x)) + b\*x\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(1/4)\*log(-b^4\*c^2\*(-(B^4\*b^4 - 4\*A\*B^3\*b^3\*c + 6\*A^2\*B^2\*b^2\*c^2 - 4\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^5\*c^3))^(3/4) - (B^3\*b^3 - 3\*A\*B^2\*b^2\*c + 3\*A^2\*B\*b\*c^2 - A^3\*c^3)\*sqrt(x)) - 4\*A\*sqrt(x))/(b\*x)

**giac [A]** time = 0.19, size = 251, normalized size = 1.07

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{2}} + 2\sqrt{c} \right)}{z \left( \frac{b}{c} \right)^{\frac{1}{2}}} \right)}{2b^2c^3} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{2}} - 2\sqrt{c} \right)}{z \left( \frac{b}{c} \right)^{\frac{1}{2}}} \right)}{2b^2c^3} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{2}} + x + \sqrt{\frac{b}{c}} \right)}{4b^2c^3} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{2}} + x + \sqrt{\frac{b}{c}} \right)}{4b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*x^(1/2)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 
$$-2A/(b\sqrt{x}) + 1/2\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\arctan(1/2\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} + 2*\sqrt{x})/(b/c)^{1/4})/(b^2*c^3) + 1/2\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\arctan(-1/2\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} - 2*\sqrt{x})/(b/c)^{1/4})/(b^2*c^3) - 1/4\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^2*c^3) + 1/4\sqrt{2}*((b*c^3)^{3/4}*B*b - (b*c^3)^{3/4}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^2*c^3)$$

**maple [A]** time = 0.06, size = 277, normalized size = 1.18

$$\frac{\sqrt{2} A \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b} - \frac{\sqrt{2} A \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b} - \frac{\sqrt{2} A \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}} b} + \frac{\sqrt{2} B \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} c} + \frac{\sqrt{2} B \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} c} + \frac{\sqrt{2} B \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}} c} - \frac{2A}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)\*x^(1/2)/(c\*x^4+b\*x^2),x)

[Out] 
$$-1/2/b/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)-1/4/b/(b/c)^{1/4}*2^{1/2}*A*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})/(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))-1/2/b/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1)+1/2/c/(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)+1/4/c/(b/c)^{1/4}*2^{1/2}*B*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})/(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))+1/2/c/(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1)-2*A/b/x^{1/2}$$

**maxima [A]** time = 3.01, size = 194, normalized size = 0.83

$$(Bb - Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \right) - \frac{2A}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*x^(1/2)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out] 
$$1/4*(B*b - A*c)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x})/\sqrt{\sqrt{b}*\sqrt{c}}))/(\sqrt{\sqrt{b}*\sqrt{c}}*\sqrt{c}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x})/\sqrt{\sqrt{b}*\sqrt{c}}))/(\sqrt{\sqrt{b}*\sqrt{c}}*\sqrt{c}) - \sqrt{2}*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{1/4}*c^{3/4}) + \sqrt{2}*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{1/4}*c^{3/4}))/b - 2*A/(b*\sqrt{x})$$

**mupad [B]** time = 0.22, size = 71, normalized size = 0.30

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac - Bb)}{(-b)^{5/4}c^{3/4}} - \frac{2A}{b\sqrt{x}} - \frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac - Bb)}{(-b)^{5/4}c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4),x)

[Out] 
$$\left(\operatorname{atan}\left(\frac{c^{1/4}*x^{1/2}}{(-b)^{1/4}}\right)/(b^{5/4}*c^{3/4})\right)*(Ac - Bb) - (2*A)/(b*x^{1/2}) - \left(\operatorname{atanh}\left(\frac{c^{1/4}*x^{1/2}}{(-b)^{1/4}}\right)/(b^{5/4}*c^{3/4})\right)*(Ac - Bb)$$

sympy [A] time = 18.45, size = 456, normalized size = 1.94

$$\begin{cases}
 \frac{2A}{5x} \left( -\frac{2A}{5x} \frac{2B}{\sqrt{x}} \right) & \text{for } b=0 \wedge c=0 \\
 \frac{2A}{5x} \frac{2B}{\sqrt{x}} & \text{for } b=0 \\
 \frac{2A}{5x} \frac{2Bx}{b} & \text{for } c=0 \\
 \frac{2A}{b\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} A c \left(\frac{1}{c}\right)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{x}\right)}{2b^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}} A c \left(\frac{1}{c}\right)^{\frac{3}{4}} \log\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{x}\right)}{2b^{\frac{5}{4}}} + \frac{(-1)^{\frac{3}{4}} A c \left(\frac{1}{c}\right)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{b^{\frac{5}{4}}} - \frac{2(-1)^{\frac{3}{4}} A c \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{b^{\frac{5}{4}} \sqrt[4]{x}} - \frac{(-1)^{\frac{3}{4}} B c \left(\frac{1}{c}\right)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{x}\right)}{2\sqrt[4]{c}} + \frac{(-1)^{\frac{3}{4}} B c \left(\frac{1}{c}\right)^{\frac{3}{4}} \log\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{x} + \sqrt{x}\right)}{2\sqrt[4]{c}} - \frac{(-1)^{\frac{3}{4}} B c \left(\frac{1}{c}\right)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{\sqrt[4]{c}} + \frac{2(-1)^{\frac{3}{4}} B c \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{x}}\right)}{\sqrt[4]{c} \sqrt[4]{x}} & \text{otherwise}
 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*x\*\*(1/2)/(c\*x\*\*4+b\*x\*\*2), x)

[Out] Piecewise((zoo\*(-2\*A/(5\*x\*\*(5/2)) - 2\*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-2\*A/(5\*x\*\*(5/2)) - 2\*B/sqrt(x))/c, Eq(b, 0)), ((-2\*A/sqrt(x) + 2\*B\*x\*\*(3/2)/3)/b, Eq(c, 0)), (-2\*A/(b\*sqrt(x)) + (-1)\*\*(3/4)\*A\*c\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(5/4)) - (-1)\*\*(3/4)\*A\*c\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(5/4)) + (-1)\*\*(3/4)\*A\*c\*(1/c)\*\*(3/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(5/4) - 2\*(-1)\*\*(3/4)\*A\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/(b\*\*(5/4)\*(1/c)\*\*(1/4)) - (-1)\*\*(3/4)\*B\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(1/4)) + (-1)\*\*(3/4)\*B\*(1/c)\*\*(3/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(1/4)) - (-1)\*\*(3/4)\*B\*(1/c)\*\*(3/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(1/4) + 2\*(-1)\*\*(3/4)\*B\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/(b\*\*(1/4)\*c\*(1/c)\*\*(1/4)), True))

$$3.190 \quad \int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=237

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{7/4} \sqrt[4]{c}} + \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{7/4} \sqrt[4]{c}} - \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x}{\sqrt{2} b^{7/4} \sqrt[4]{c}}\right)}{\sqrt{2} b^{7/4} \sqrt[4]{c}}$$

**Rubi [A]** time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {1584, 453, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{7/4} \sqrt[4]{c}} + \frac{(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{7/4} \sqrt[4]{c}} - \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2} b^{7/4} \sqrt[4]{c}} + \frac{(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2} b^{7/4} \sqrt[4]{c}} - \frac{2A}{3bx^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)),x]

[Out] (-2\*A)/(3\*b\*x^(3/2)) - ((b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(7/4)\*c^(1/4)) + ((b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(7/4)\*c^(1/4)) - ((b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(7/4)\*c^(1/4)) + ((b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(7/4)\*c^(1/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 453

Int[((e\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^2}{\sqrt{x}(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^{5/2}(b + cx^2)} dx \\
 &= -\frac{2A}{3bx^{3/2}} - \frac{\left(2\left(-\frac{3bB}{2} + \frac{3Ac}{2}\right)\right) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{3b} \\
 &= -\frac{2A}{3bx^{3/2}} - \frac{\left(4\left(-\frac{3bB}{2} + \frac{3Ac}{2}\right)\right) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{3b} \\
 &= -\frac{2A}{3bx^{3/2}} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b} - \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^{3/2}} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{\sqrt{b} + \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^{3/2}} \\
 &= -\frac{2A}{3bx^{3/2}} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{2b^{3/2}\sqrt{c}} + \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{2b^{3/2}\sqrt{c}} \\
 &= -\frac{2A}{3bx^{3/2}} - \frac{(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB - Ac) \log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} \\
 &= -\frac{2A}{3bx^{3/2}} - \frac{(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB - Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{(bB - Ac)}{b^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 168, normalized size = 0.71

$$\frac{(bB - Ac) \left( \log \left( -\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x \right) - \log \left( \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x \right) + 2 \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} \right) - 2 \tan^{-1} \left( \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1 \right) \right)}{2\sqrt{2} b^{7/4} \sqrt[4]{c}} - \frac{2A}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)), x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.22, size = 136, normalized size = 0.57

$$-\frac{(bB - Ac) \tan^{-1} \left( \frac{\sqrt{b} - \sqrt{c} x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}} \right)}{\sqrt{2} b^{7/4} \sqrt[4]{c}} + \frac{(bB - Ac) \tanh^{-1} \left( \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x} \right)}{\sqrt{2} b^{7/4} \sqrt[4]{c}} - \frac{2A}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)), x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(7/4)*c^(1/4))
```

**fricas [B]** time = 0.44, size = 653, normalized size = 2.76

$$\frac{12 b^2 \left( \frac{2 b^2 A^2 \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 b^2 c^2} \arctan \left( \frac{\sqrt{2} \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 \sqrt{b^2 c^2 + 4 A^2 c^2}} \right) + \frac{2 b^2 A^2 \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 b^2 c^2} \arctan \left( \frac{\sqrt{2} \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 \sqrt{b^2 c^2 + 4 A^2 c^2}} \right) + \frac{2 b^2 A^2 \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 b^2 c^2} \arctan \left( \frac{\sqrt{2} \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 \sqrt{b^2 c^2 + 4 A^2 c^2}} \right) + \frac{2 b^2 A^2 \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 b^2 c^2} \arctan \left( \frac{\sqrt{2} \sqrt{b^2 c^2 + 4 A^2 c^2}}{2 \sqrt{b^2 c^2 + 4 A^2 c^2}} \right) \right)}{6 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2), x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c)) + (B^2*b^2 - 2*A*B*b*c + A^2*c^2)*x)*b^5*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(3/4) + (B*b^6*c - A*b^5*c^2)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4) + 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4)*log(b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) - 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4)*log(-b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) + 4*A*sqrt(x))/(b*x^2)
```

**giac [A]** time = 0.23, size = 251, normalized size = 1.06

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + 2\sqrt{c}}}{2 \left(\frac{b}{c}\right)^{\frac{1}{2}}} \right)}{2 b^2 c} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} - 2\sqrt{c}}}{2 \left(\frac{b}{c}\right)^{\frac{1}{2}}} \right)}{2 b^2 c} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + x} + \sqrt{\frac{b}{c}} \right)}{4 b^2 c} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + x} + \sqrt{\frac{b}{c}} \right)}{4 b^2 c} - \frac{2A}{3 b x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2), x, algorithm="giac")
```

[Out]  $\frac{1}{2}\sqrt{2}((b^2c^3)^{1/4}Bb - (b^2c^3)^{1/4}Ac)\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}}{b/c} + \sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}((b^2c^3)^{1/4}Bb - (b^2c^3)^{1/4}Ac)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}}{b/c} - \sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}((b^2c^3)^{1/4}Bb - (b^2c^3)^{1/4}Ac)\log\left(\frac{\sqrt{2}\sqrt{x}(b/c) + x + \sqrt{b/c}}{b^2c}\right) - \frac{1}{4}\sqrt{2}((b^2c^3)^{1/4}Bb - (b^2c^3)^{1/4}Ac)\log\left(\frac{-\sqrt{2}\sqrt{x}(b/c) + x + \sqrt{b/c}}{b^2c}\right) - \frac{2}{3}A/(b^2x^{3/2})$

**maple [A]** time = 0.06, size = 280, normalized size = 1.18

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}Ac\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right) - \left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}Ac\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right) - \left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}Ac\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}\right) + \left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right) + \left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right) + \left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{b}{c}}}\right) - \frac{2A}{3bx^{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(c\*x^4+b\*x^2)/x^(1/2), x)

[Out]  $-2/3A/b/x^{3/2} - 1/4/b^2*(b/c)^{1/4}*2^{1/2}*A*\ln\left(\frac{(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})}{(x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})}\right)*c - 1/2/b^2*(b/c)^{1/4}*2^{1/2}*A*\arctan\left(\frac{2^{1/2}}{(b/c)^{1/4}*x^{1/2}+1}\right)*c - 1/2/b^2*(b/c)^{1/4}*2^{1/2}*A*\arctan\left(\frac{2^{1/2}}{(b/c)^{1/4}*x^{1/2}-1}\right)*c + 1/4/b*(b/c)^{1/4}*2^{1/2}*B*\ln\left(\frac{(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})}{(x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})}\right) + 1/2/b*(b/c)^{1/4}*2^{1/2}*B*\arctan\left(\frac{2^{1/2}}{(b/c)^{1/4}*x^{1/2}+1}\right) + 1/2/b*(b/c)^{1/4}*2^{1/2}*B*\arctan\left(\frac{2^{1/2}}{(b/c)^{1/4}*x^{1/2}-1}\right)$

**maxima [A]** time = 3.07, size = 218, normalized size = 0.92

$$\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right) + 2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right) + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b} - \frac{\sqrt{2}(Bb-Ac)\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b} - \frac{2A}{3bx^{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)/x^(1/2), x, algorithm="maxima")

[Out]  $\frac{1}{4}*(2*\sqrt{2}*(B*b - A*c)*\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}}{b^{1/4}}*c^{1/4} + \sqrt{c}\sqrt{x}\right)\right) + 2*\sqrt{2}*(B*b - A*c)*\arctan\left(-\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}}{b^{1/4}}*c^{1/4} - \sqrt{c}\sqrt{x}\right)\right) + \sqrt{2}*(B*b - A*c)*\log\left(\frac{\sqrt{2}\sqrt{x}(b/c)^{1/4} + \sqrt{c}x + \sqrt{b}}{b^{3/4}c^{1/4}}\right) - \sqrt{2}*(B*b - A*c)*\log\left(\frac{-\sqrt{2}\sqrt{x}(b/c)^{1/4} + \sqrt{c}x + \sqrt{b}}{b^{3/4}c^{1/4}}\right) + \sqrt{2}*(B*b - A*c)*\log\left(\frac{-\sqrt{2}\sqrt{x}(b/c)^{1/4} + \sqrt{c}x + \sqrt{b}}{b^{3/4}c^{1/4}}\right))/b - \frac{2}{3}A/(b^2x^{3/2})$

**mupad [B]** time = 0.30, size = 811, normalized size = 3.42

$$\frac{2A}{3bx^{\frac{3}{2}}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x}}{2\sqrt{b}\sqrt{c}}\right) + \frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right) + 2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right) + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b} - \frac{\sqrt{2}(Bb-Ac)\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b} - \frac{2A}{3bx^{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^(1/2)\*(b\*x^2 + c\*x^4)), x)

[Out]  $-\frac{(2A)/(3b^2x^{3/2}) - \operatorname{atan}\left(\frac{(Ac - Bb)(x^{1/2}(16A^2b^3c^5 + 16B^2b^5c^3 - 32ABb^4c^4) - ((Ac - Bb)(32A^2b^5c^4 - 32B^2b^6c^3))}{(2(-b)^{7/4}c^{1/4})}\right)}{(2(-b)^{7/4}c^{1/4})} + \frac{(Ac - Bb)(x^{1/2}(16A^2b^3c^5 + 16B^2b^5c^3 - 32ABb^4c^4) + ((Ac - Bb)(32A^2b^5c^4 - 32B^2b^6c^3))}{(2(-b)^{7/4}c^{1/4})}}{(2(-b)^{7/4}c^{1/4})} + \frac{((Ac - Bb)(x^{1/2}(16A^2b^3c^5 + 16B^2b^5c^3 - 32ABb^4c^4) - ((Ac - Bb)(32A^2b^5c^4 - 32B^2b^6c^3)))/(2(-b)^{7/4}c^{1/4}))}{(2(-b)^{7/4}c^{1/4})} - \frac{((Ac - Bb)(x^{1/2}(16A^2b^3c^5 + 16B^2b^5c^3 - 32ABb^4c^4) - ((Ac - Bb)(32A^2b^5c^4 - 32B^2b^6c^3)))/(2(-b)^{7/4}c^{1/4}))}{(2(-b)^{7/4}c^{1/4})} - \frac{2A}{3bx^{\frac{3}{2}}}$

$$\begin{aligned}
& 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4)) \\
& - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))))/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))))/(2*(-b)^(7/4)*c^(1/4)))/((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)) \\
& - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b))/((-b)^(7/4)*c^(1/4))
\end{aligned}$$

**sympy [A]** time = 27.10, size = 364, normalized size = 1.54

$$\begin{cases}
\frac{2A}{7x^{\frac{7}{2}} - \frac{2B}{3x^{\frac{3}{2}}}} & \text{for } b = 0 \wedge c = 0 \\
-\frac{2A}{7x^{\frac{7}{2}} - \frac{2B}{3x^{\frac{3}{2}}}} & \text{for } b = 0 \\
-\frac{2A}{3x^{\frac{3}{2}} + 2B\sqrt{x}} & \text{for } c = 0 \\
-\frac{2A}{3bx^{\frac{3}{2}}} + \frac{\sqrt{-1}Ac\sqrt[4]{c}\log(-\sqrt{-1}\sqrt[4]{b}\sqrt[4]{c} + \sqrt{x})}{2b^{\frac{7}{4}}} - \frac{\sqrt{-1}Ac\sqrt[4]{c}\log(\sqrt{-1}\sqrt[4]{b}\sqrt[4]{c} + \sqrt{x})}{2b^{\frac{7}{4}}} + \frac{\sqrt{-1}Ac\sqrt[4]{c}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{c}}\right)}{b^{\frac{7}{4}}} - \frac{\sqrt{-1}B\sqrt[4]{c}\log(-\sqrt{-1}\sqrt[4]{b}\sqrt[4]{c} + \sqrt{x})}{2b^{\frac{3}{4}}} + \frac{\sqrt{-1}B\sqrt[4]{c}\log(\sqrt{-1}\sqrt[4]{b}\sqrt[4]{c} + \sqrt{x})}{2b^{\frac{3}{4}}} - \frac{\sqrt{-1}B\sqrt[4]{c}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{c}}\right)}{b^{\frac{3}{4}}} & \text{otherwise}
\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)/x\*\*(1/2), x)

[Out] Piecewise((zoo\*(-2\*A/(7\*x\*\*(7/2)) - 2\*B/(3\*x\*\*(3/2))), Eq(b, 0) & Eq(c, 0)), ((-2\*A/(7\*x\*\*(7/2)) - 2\*B/(3\*x\*\*(3/2)))/c, Eq(b, 0)), ((-2\*A/(3\*x\*\*(3/2)) + 2\*B\*sqrt(x))/b, Eq(c, 0)), (-2\*A/(3\*b\*x\*\*(3/2)) + (-1)\*\*(1/4)\*A\*c\*(1/c)\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(7/4)) - (-1)\*\*(1/4)\*A\*c\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(7/4)) + (-1)\*\*(1/4)\*A\*c\*(1/c)\*\*(1/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(7/4) - (-1)\*\*(1/4)\*B\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(3/4)) + (-1)\*\*(1/4)\*B\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(3/4)) - (-1)\*\*(1/4)\*B\*(1/c)\*\*(1/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(3/4), True))



$$3.191 \quad \int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=255

$$\frac{\sqrt[4]{c}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{9/4}} + \frac{\sqrt[4]{c}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{9/4}} + \frac{\sqrt[4]{c}(bB - Ac)}{5bx^{5/2}}$$

**Rubi [A]** time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2(bB - Ac)}{b^2 \sqrt{x}} - \frac{\sqrt[4]{c}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{9/4}} + \frac{\sqrt[4]{c}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{9/4}} + \frac{\sqrt[4]{c}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2} b^{9/4}} - \frac{\sqrt[4]{c}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2} b^{9/4}} - \frac{2A}{5bx^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)),x]

[Out]  $(-2*A)/(5*b*x^{5/2}) - (2*(b*B - A*c))/(b^2*\text{Sqrt}[x]) + (c^{1/4}*(b*B - A*c)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*\text{Sqrt}[x])/b^{1/4}])/(2*\text{Sqrt}[2]*b^{9/4}) - (c^{1/4}*(b*B - A*c)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*\text{Sqrt}[x])/b^{1/4}])/(2*\text{Sqrt}[2]*b^{9/4}) - (c^{1/4}*(b*B - A*c)*\text{Log}[\text{Sqrt}[b] - \text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[c]*x])/(2*\text{Sqrt}[2]*b^{9/4}) + (c^{1/4}*(b*B - A*c)*\text{Log}[\text{Sqrt}[b] + \text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[c]*x])/(2*\text{Sqrt}[2]*b^{9/4})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^{3/2}(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^{7/2}(b + cx^2)} dx \\
&= \frac{2A}{5bx^{5/2}} - \frac{\left(2\left(-\frac{5bB}{2} + \frac{5Ac}{2}\right)\right) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{5b} \\
&= \frac{2A}{5bx^{5/2}} - \frac{2(bB - Ac)}{b^2\sqrt{x}} - \frac{(c(bB - Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{b^2} \\
&= \frac{2A}{5bx^{5/2}} - \frac{2(bB - Ac)}{b^2\sqrt{x}} - \frac{(2c(bB - Ac)) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2A}{5bx^{5/2}} - \frac{2(bB - Ac)}{b^2\sqrt{x}} + \frac{(\sqrt{c}(bB - Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^2} - \frac{(\sqrt{c}(bB - Ac)) \text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{c}x^2} dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2A}{5bx^{5/2}} - \frac{2(bB - Ac)}{b^2\sqrt{x}} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{2b^2} - \frac{(bB - Ac) \text{Subst}\left(\int \frac{1}{\sqrt{b} - \sqrt{c}x^2} dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2A}{5bx^{5/2}} - \frac{2(bB - Ac)}{b^2\sqrt{x}} - \frac{\sqrt[4]{c}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{c}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{c}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{2}b^{9/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 46, normalized size = 0.18

$$\frac{2\left(5x^2(bB - Ac) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\frac{cx^2}{b}\right) + Ab\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*(A\*b + 5\*(b\*B - A\*c)\*x^2\*Hypergeometric2F1[-1/4, 1, 3/4, -((c\*x^2)/b)])/(5\*b^2\*x^(5/2))

**IntegrateAlgebraic [A]** time = 0.23, size = 160, normalized size = 0.63

$$\frac{(bB\sqrt[4]{c} - Ac^{5/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{9/4}} + \frac{(bB\sqrt[4]{c} - Ac^{5/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{9/4}} - \frac{2(Ab - 5Acx^2 + 5bBx^2)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*(A\*b + 5\*b\*B\*x^2 - 5\*A\*c\*x^2))/(5\*b^2\*x^(5/2)) + ((b\*B\*c^(1/4) - A\*c^(5/4))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(9/4)) + ((b\*B\*c^(1/4) - A\*c^(5/4))\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]/(Sqrt[b] + Sqrt[c]\*x)))/(Sqrt[2]\*b^(9/4))

**fricas [B]** time = 0.44, size = 883, normalized size = 3.46

-----

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*arctan((sqrt((B^6*b^6*c^2 - 6*A*B^5*b^5*c^3 + 15*A^2*B^4*b^4*c^4 - 20*A^3*B^3*b^3*c^5 + 15*A^4*B^2*b^2*c^6 - 6*A^5*B*b*c^7 + A^6*c^8)*x - (B^4*b^9*c - 4*A*B^3*b^8*c^2 + 6*A^2*B^2*b^7*c^3 - 4*A^3*B*b^6*c^4 + A^4*b^5*c^5)*sqrt(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)))*b^2*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4) + (B^3*b^5*c - 3*A*B^2*b^4*c^2 + 3*A^2*B*b^3*c^3 - A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4))/(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)) - 5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) + 5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(-b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) + 4*(5*(B*b - A*c)*x^2 + A*b)*sqrt(x))/(b^2*x^3)
```

**giac** [A] time = 0.24, size = 268, normalized size = 1.05

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \left( \frac{b}{c} \right)^{\frac{1}{4}} + \sqrt{x} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2 b^{\frac{3}{2}} c^2} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \left( \frac{b}{c} \right)^{\frac{1}{4}} - \sqrt{x} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2 b^{\frac{3}{2}} c^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{4 b^{\frac{3}{2}} c^2} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{4 b^{\frac{3}{2}} c^2} - \frac{2(5 B b^2 - 5 A c x^2 + A b)}{5 b^2 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) - 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 2/5*(5*B*b*x^2 - 5*A*c*x^2 + A*b)/(b^2*x^(5/2))
```

**maple** [A] time = 0.06, size = 299, normalized size = 1.17

$$\frac{\sqrt{2} Ac \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}} b^2} + \frac{\sqrt{2} Ac \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}} b^2} + \frac{\sqrt{2} Ac \ln \left( \frac{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{4 \left( \frac{b}{c} \right)^{\frac{1}{4}} b^2} - \frac{\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}} b} - \frac{\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}} b} - \frac{\sqrt{2} B \ln \left( \frac{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{4 \left( \frac{b}{c} \right)^{\frac{1}{4}} b} + \frac{2Ac}{b^2 \sqrt{x}} - \frac{2B}{b \sqrt{x}} - \frac{2A}{5 b x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/2/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c+1/4/b^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))*c+1/2/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*c-1/2/b/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/b/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-2/5*A/b/x^(5/2)+2/b^2/x^(1/2)*A*c-2/b/x^(1/2)*B
```

**maxima [A]** time = 3.08, size = 213, normalized size = 0.84

$$\frac{(Bbc - Ac^2) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \right)}{4b^2} - \frac{2(5(Bb - Ac)x^2 + Ab)}{5b^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(3/2)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $-1/4*(B*b*c - A*c^2)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x}))/\sqrt{c})/\sqrt{b} + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x}))/\sqrt{c} - \sqrt{2}*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*\sqrt{b})/(b^{1/4}*c^{3/4}) + \sqrt{2}*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*\sqrt{b})/(b^{1/4}*c^{3/4}))/b^2 - 2/5*(5*(B*b - A*c)*x^2 + A*b)/(b^2*x^{5/2})$

**mupad [B]** time = 0.23, size = 90, normalized size = 0.35

$$\frac{(-c)^{1/4} \operatorname{atan}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (Ac - Bb)}{b^{9/4}} - \frac{2A}{5b} - \frac{2x^2(Ac - Bb)}{b^2} - \frac{(-c)^{1/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (Ac - Bb)}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)),x)

[Out]  $((-c)^{1/4}*\operatorname{atan}(((c)^{1/4}*x^{1/2}))/b^{1/4})*(A*c - B*b))/b^{9/4} - ((2*A)/(5*b) - (2*x^2*(A*c - B*b))/b^2)/x^{5/2} - ((-c)^{1/4}*\operatorname{atanh}(((c)^{1/4}*x^{1/2}))/b^{1/4})*(A*c - B*b))/b^{9/4}$

**sympy [A]** time = 108.60, size = 366, normalized size = 1.44

$$A \begin{cases} \frac{2}{x^{\frac{9}{2}}} & \text{for } b=0 \wedge c=0 \\ -\frac{2}{9c^{\frac{5}{2}}} & \text{for } b=0 \\ -\frac{2}{5b^{\frac{5}{2}}} & \text{for } c=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{2c}{b^2\sqrt{x}} - \frac{(-1)^{\frac{3}{4}}c \log\left(-\sqrt{-1}\sqrt{b}\sqrt{\frac{x}{c}} + \sqrt{x}\right)}{2b^{\frac{3}{4}}\sqrt{c}} + \frac{(-1)^{\frac{3}{4}}c \log\left(\sqrt{-1}\sqrt{b}\sqrt{\frac{x}{c}} + \sqrt{x}\right)}{2b^{\frac{3}{4}}\sqrt{c}} + \frac{(-1)^{\frac{3}{4}}c \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt{b}\sqrt{c}}\right)}{b^{\frac{3}{4}}\sqrt{c}} & \text{otherwise} \end{cases} + B \begin{cases} \frac{2}{x^{\frac{5}{2}}} & \text{for } b=0 \wedge c=0 \\ -\frac{2}{5c^{\frac{5}{2}}} & \text{for } b=0 \\ \frac{2}{b\sqrt{c}} & \text{for } c=0 \\ \frac{2}{b\sqrt{c}} + \frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt{-1}\sqrt{b}\sqrt{\frac{x}{c}} + \sqrt{x}\right)}{2b^{\frac{3}{4}}\sqrt{c}} - \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt{-1}\sqrt{b}\sqrt{\frac{x}{c}} + \sqrt{x}\right)}{2b^{\frac{3}{4}}\sqrt{c}} - \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt{b}\sqrt{c}}\right)}{b^{\frac{3}{4}}\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*(3/2)/(c\*x\*\*4+b\*x\*\*2),x)

[Out]  $A*\operatorname{Piecewise}((zoo/x^{9/2}), \operatorname{Eq}(b, 0) \& \operatorname{Eq}(c, 0)), (-2/(9*c*x^{9/2})), \operatorname{Eq}(b, 0)), (-2/(5*b*x^{5/2})), \operatorname{Eq}(c, 0)), (-2/(5*b*x^{5/2}) + 2*c/(b**2*\sqrt{x}) - (-1)**(3/4)*c*\log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + \sqrt{x}))/ (2*b**(9/4)*(1/c)**(1/4)) + (-1)**(3/4)*c*\log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + \sqrt{x}))/ (2*b**(9/4)*(1/c)**(1/4)) + (-1)**(3/4)*c*\operatorname{atan}((-1)**(3/4)*\sqrt{x}/(b**(1/4)*(1/c)**(1/4)))/ (b**(9/4)*(1/c)**(1/4)), \operatorname{True})) + B*\operatorname{Piecewise}((zoo/x^{5/2}), \operatorname{Eq}(b, 0) \& \operatorname{Eq}(c, 0)), (-2/(5*c*x^{5/2})), \operatorname{Eq}(b, 0)), (-2/(b*\sqrt{x})), \operatorname{Eq}(c, 0)), (-2/(b*\sqrt{x}) + (-1)**(3/4)*\log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + \sqrt{x}))/ (2*b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)*\log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + \sqrt{x}))/ (2*b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)*\operatorname{atan}((-1)**(3/4)*\sqrt{x}/(b**(1/4)*(1/c)**(1/4)))/ (b**(5/4)*(1/c)**(1/4)), \operatorname{True}))$

$$3.192 \quad \int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=257

$$\frac{c^{3/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{11/4}} - \frac{c^{3/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{11/4}} + \frac{c^{3/4}(bB - Ac)}{2\sqrt{2} b^{11/4}}$$

**Rubi [A]** time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 453, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{11/4}} - \frac{c^{3/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{2\sqrt{2} b^{11/4}} + \frac{c^{3/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2} b^{11/4}} - \frac{c^{3/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2} b^{11/4}} - \frac{2(bB - Ac)}{3b^2 x^{3/2}} - \frac{2A}{7bx^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^(5/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*A)/(7\*b\*x^(7/2)) - (2\*(b\*B - A\*c))/(3\*b^2\*x^(3/2)) + (c^(3/4)\*(b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(11/4)) - (c^(3/4)\*(b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(11/4))) + (c^(3/4)\*(b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(11/4)) - (c^(3/4)\*(b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(2\*Sqrt[2]\*b^(11/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^(p), x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x]

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^{5/2}(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^{9/2}(b + cx^2)} dx \\
&= -\frac{2A}{7bx^{7/2}} - \frac{\left(2\left(-\frac{7bB}{2} + \frac{7Ac}{2}\right)\right) \int \frac{1}{x^{5/2}(b+cx^2)} dx}{7b} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB - Ac)}{3b^2x^{3/2}} - \frac{(c(bB - Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{b^2} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB - Ac)}{3b^2x^{3/2}} - \frac{(2c(bB - Ac)) \operatorname{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{b^2} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB - Ac)}{3b^2x^{3/2}} - \frac{(c(bB - Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(c(bB - Ac)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{c}x^2} dx, x, \sqrt{x}\right)}{b^{5/2}} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB - Ac)}{3b^2x^{3/2}} - \frac{(\sqrt{c}(bB - Ac)) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2b^{5/2}} - \frac{(\sqrt{c}(bB - Ac)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x} dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{11/4}} \\
&= -\frac{2A}{7bx^{7/2}} - \frac{2(bB - Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB - Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 47, normalized size = 0.18

$$\frac{14x^2(Ac - bB) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\frac{cx^2}{b}\right) - 6Ab}{21b^2x^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^(5/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-6\*A\*b + 14\*(-(b\*B) + A\*c)\*x^2\*Hypergeometric2F1[-3/4, 1, 1/4, -(c\*x^2)/b])/ (21\*b^2\*x^(7/2))

**IntegrateAlgebraic [A]** time = 0.23, size = 162, normalized size = 0.63

$$\frac{(bBc^{3/4} - Ac^{7/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{11/4}} - \frac{(bBc^{3/4} - Ac^{7/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{11/4}} - \frac{2(3Ab - 7Acx^2 + 7bBx^2)}{21b^2x^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^(5/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*(3\*A\*b + 7\*b\*B\*x^2 - 7\*A\*c\*x^2))/(21\*b^2\*x^(7/2)) + ((b\*B\*c^(3/4) - A\*c^(7/4))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(11/4)) - ((b\*B\*c^(3/4) - A\*c^(7/4))\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*b^(11/4))

**fricas [B]** time = 0.43, size = 707, normalized size = 2.75

44724 (21b^2x^{7/2} + 14x^2(Ac - bB) {}\_2F\_1(-3/4, 1; 1/4; -cx^2/b) - 6Ab) / (21b^2x^{7/2})



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(5/2)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out]  $\frac{1}{42} \cdot (84 \cdot b^2 \cdot x^4 \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{1/4} \cdot \arctan\left(\frac{\sqrt{b^6 \cdot \sqrt{c} \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11}}}{b^2 \cdot c^2 - 2 \cdot A \cdot B \cdot b \cdot c^3 + A^2 \cdot c^4} \cdot x\right) \cdot b^8 \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{3/4} + (B \cdot b^9 \cdot c - A \cdot b^8 \cdot c^2) \cdot \sqrt{x} \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{3/4} / (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) + 21 \cdot b^2 \cdot x^4 \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{1/4} \cdot \log(b^3 \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{1/4} - (B \cdot b \cdot c - A \cdot c^2) \cdot \sqrt{x}) - 21 \cdot b^2 \cdot x^4 \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{1/4} \cdot \log(-b^3 \cdot (- (B^4 \cdot b^4 \cdot c^3 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^4 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^5 - 4 \cdot A^3 \cdot B \cdot b \cdot c^6 + A^4 \cdot c^7) / b^{11})^{1/4} - (B \cdot b \cdot c - A \cdot c^2) \cdot \sqrt{x}) - 4 \cdot (7 \cdot (B \cdot b - A \cdot c) \cdot x^2 + 3 \cdot A \cdot b) \cdot \sqrt{x}) / (b^2 \cdot x^4)$

**giac** [A] time = 0.19, size = 257, normalized size = 1.00

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \arctan\left(\frac{\sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} + 2\sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}}\right)}{2b^3} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \arctan\left(\frac{\sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} - 2\sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}}\right)}{2b^3} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \log\left(\sqrt{2} \sqrt{c} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{x}\right)}{4b^3} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb - (bc^3)^{\frac{1}{4}} Ac \right) \log\left(-\sqrt{2} \sqrt{c} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{x}\right)}{4b^3} - \frac{2(7Bb^2 - 7Ac^2 + 3Ab)}{21b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(5/2)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out]  $-\frac{1}{2} \cdot \sqrt{2} \cdot ((bc^3)^{\frac{1}{4}} \cdot B \cdot b - (bc^3)^{\frac{1}{4}} \cdot A \cdot c) \cdot \arctan\left(\frac{1}{2} \cdot \sqrt{2} \cdot \left(\frac{b}{c}\right)^{\frac{1}{4}} + 2 \cdot \sqrt{x}\right) / (b/c)^{\frac{1}{4}} / b^3 - \frac{1}{2} \cdot \sqrt{2} \cdot ((bc^3)^{\frac{1}{4}} \cdot B \cdot b - (bc^3)^{\frac{1}{4}} \cdot A \cdot c) \cdot \arctan\left(-\frac{1}{2} \cdot \sqrt{2} \cdot \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2 \cdot \sqrt{x}\right) / (b/c)^{\frac{1}{4}} / b^3 - \frac{1}{4} \cdot \sqrt{2} \cdot ((bc^3)^{\frac{1}{4}} \cdot B \cdot b - (bc^3)^{\frac{1}{4}} \cdot A \cdot c) \cdot \log\left(\sqrt{2} \cdot \sqrt{c} \cdot \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{x}\right) / b^3 + \frac{1}{4} \cdot \sqrt{2} \cdot ((bc^3)^{\frac{1}{4}} \cdot B \cdot b - (bc^3)^{\frac{1}{4}} \cdot A \cdot c) \cdot \log\left(-\sqrt{2} \cdot \sqrt{c} \cdot \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{x}\right) / b^3 - \frac{2}{21} \cdot (7 \cdot B \cdot b \cdot x^2 - 7 \cdot A \cdot c \cdot x^2 + 3 \cdot A \cdot b) / (b^2 \cdot x^{\frac{7}{2}})$

**maple** [A] time = 0.06, size = 308, normalized size = 1.20

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2} \sqrt{c} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2} \sqrt{c} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \ln\left(\frac{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{x}}{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{x}}\right)}{4b^3} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c \arctan\left(\frac{\sqrt{2} \sqrt{c} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c \arctan\left(\frac{\sqrt{2} \sqrt{c} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2} - \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c \ln\left(\frac{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{x}}{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{x}}\right)}{4b^2} + \frac{2Ac}{3b^2x^{\frac{7}{2}}} - \frac{2B}{3bx^{\frac{7}{2}}} - \frac{2A}{7bx^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^(5/2)/(c\*x^4+b\*x^2),x)

[Out]  $\frac{1}{2} \cdot c^2 / b^3 \cdot (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot A \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} - 1\right) + \frac{1}{4} \cdot c^2 / b^3 \cdot (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot A \cdot \ln\left(\frac{x + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{2}}}{x - (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{2}}}\right) + \frac{1}{2} \cdot c^2 / b^3 \cdot (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot A \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} + 1\right) - \frac{1}{2} \cdot c / b^2 \cdot (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot B \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} - 1\right) - \frac{1}{4} \cdot c / b^2 \cdot (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot B \cdot \ln\left(\frac{x + (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{2}}}{x - (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + (b/c)^{\frac{1}{2}}}\right) - \frac{1}{2} \cdot c / b^2 \cdot (b/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot B \cdot \arctan\left(2^{\frac{1}{2}} / (b/c)^{\frac{1}{4}} \cdot x^{\frac{1}{2}} + 1\right) - \frac{2}{7} \cdot A / b \cdot x^{\frac{7}{2}} + \frac{2}{3} \cdot b^2 / x^{\frac{3}{2}} \cdot A \cdot c - \frac{2}{3} \cdot b / x^{\frac{3}{2}} \cdot B$

**maxima** [A] time = 2.93, size = 247, normalized size = 0.96

$$\frac{2 \sqrt{2} (Bbc - Ac^2) \arctan\left(\frac{\sqrt{2} \left( \sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} + 2 \sqrt{c} \sqrt{x} \right)}{2 \sqrt{b^{\frac{1}{4}} c^{\frac{1}{4}}}}\right)}{\sqrt{b} \sqrt{b^{\frac{1}{4}} c^{\frac{1}{4}}}} + \frac{2 \sqrt{2} (Bbc - Ac^2) \arctan\left(\frac{\sqrt{2} \left( \sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} - 2 \sqrt{c} \sqrt{x} \right)}{2 \sqrt{b^{\frac{1}{4}} c^{\frac{1}{4}}}}\right)}{\sqrt{b} \sqrt{b^{\frac{1}{4}} c^{\frac{1}{4}}}} + \frac{\sqrt{2} (Bbc - Ac^2) \log\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c} x + \sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}} c^{\frac{1}{4}}}} - \frac{\sqrt{2} (Bbc - Ac^2) \log\left(-\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c} x + \sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}} c^{\frac{1}{4}}}} - \frac{2(7(Bb - Ac)x^2 + 3Ab)}{21b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(5/2)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $-1/4*(2*\sqrt{2}*(B*b*c - A*c^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x})/\sqrt{\sqrt{b}*\sqrt{c}}))/(\sqrt{b}*\sqrt{\sqrt{b}*\sqrt{c}}) + 2*\sqrt{2}*(B*b*c - A*c^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x})/\sqrt{\sqrt{b}*\sqrt{c}}))/(\sqrt{b}*\sqrt{\sqrt{b}*\sqrt{c}}) + \sqrt{2}*(B*b*c - A*c^2)*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{3/4}*c^{1/4}) - \sqrt{2}*(B*b*c - A*c^2)*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{3/4}*c^{1/4})/b^2 - 2/21*(7*(B*b - A*c)*x^2 + 3*A*b)/(b^2*x^{7/2})$

mupad [B] time = 0.32, size = 555, normalized size = 2.16

$$\frac{2A}{7b} \frac{2^2(Ac-Bb)}{3b^2} + \frac{(-c)^{3/4} \operatorname{atan}\left(\frac{(-c)^{3/4}(Ac-Bb)\sqrt{(16c^2b^2-32ABb^2+16b^2b^2)c}}{2314}, \frac{(-c)^{3/4}(Ac-Bb)\sqrt{(16c^2b^2-32ABb^2+16b^2b^2)c}}{2314}\right)}{2314} + \frac{(-c)^{3/4}(Ac-Bb)\sqrt{(16c^2b^2-32ABb^2+16b^2b^2)c}}{2314} \operatorname{atan}\left(\frac{(-c)^{3/4}(Ac-Bb)\sqrt{(16c^2b^2-32ABb^2+16b^2b^2)c}}{2314}, \frac{(-c)^{3/4}(Ac-Bb)\sqrt{(16c^2b^2-32ABb^2+16b^2b^2)c}}{2314}\right) \frac{(Ac-Bb)}{b^{11/4}} - \frac{(-c)^{3/4} \operatorname{atan}\left(\frac{A^3c^3\sqrt{1-2b^2c^2}\sqrt{1-A^2Bb^2}\sqrt{1+A^2Bb^2}\sqrt{1-2b^2c^2}\sqrt{1-2b^2c^2}}{b^{14}(c-3A^2Bb+3AB^2b^2-b^2b^2)}\right)}{b^{11/4}} (Ac-Bb) 11$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^(5/2)\*(b\*x^2 + c\*x^4)),x)

[Out]  $((-c)^{3/4}*\operatorname{atan}((((c)^{3/4}*(A*c - B*b)*(x^{1/2}*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) - ((c)^{3/4}*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^{10}*c^4)*1i)/(2*b^{11/4}))))/(2*b^{11/4}) + ((c)^{3/4}*(A*c - B*b)*(x^{1/2}*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((c)^{3/4}*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^{10}*c^4)*1i)/(2*b^{11/4}))))/(2*b^{11/4}))/((((c)^{3/4}*(A*c - B*b)*(x^{1/2}*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) - ((c)^{3/4}*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^{10}*c^4)*1i)/(2*b^{11/4}))) * 1i)/(2*b^{11/4}) - ((c)^{3/4}*(A*c - B*b)*(x^{1/2}*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((c)^{3/4}*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^{10}*c^4)*1i)/(2*b^{11/4}))) * 1i)/(2*b^{11/4}))) * (A*c - B*b))/b^{11/4} - ((2*A)/(7*b) - (2*x^2*(A*c - B*b))/(3*b^2))/x^{7/2} - ((c)^{3/4}*\operatorname{atan}((A^3*c^8*x^{1/2}*1i - B^3*b^3*c^5*x^{1/2}*1i - A^2*B*b*c^7*x^{1/2}*3i + A*B^2*b^2*c^6*x^{1/2}*3i)/(b^{1/4}*(c)^{19/4}*(c*(c*(A^3*c - 3*A^2*B*b) + 3*A*B^2*b^2) - B^3*b^3))))*(A*c - B*b)*1i)/b^{11/4}$

sympy [A] time = 109.30, size = 405, normalized size = 1.58

$$\begin{cases} \operatorname{atan}\left(\frac{2A}{11x^2} - \frac{2B}{7x^2}\right) & \text{for } b = 0 \wedge c = 0 \\ \frac{2A}{11x^2} - \frac{2B}{7x^2} & \text{for } b = 0 \\ \frac{2A}{7x^2} - \frac{2B}{3x^2} & \text{for } c = 0 \\ \frac{2A}{7bx^2} + \frac{2Ac}{3bx^2} - \frac{\sqrt{-1}Ac^2\sqrt{\frac{1}{c}}\log\left(\frac{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}\right)}{2b^{\frac{11}{4}}} + \frac{\sqrt{-1}Ac^2\sqrt{\frac{1}{c}}\log\left(\frac{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}\right)}{2b^{\frac{11}{4}}} - \frac{\sqrt{-1}Ac^2\sqrt{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)\sqrt{\frac{1}{c}}}{\sqrt{c}\sqrt{\frac{1}{c}}}\right)}{b^{\frac{11}{4}}} - \frac{2B}{3bx^{\frac{7}{2}}} + \frac{\sqrt{-1}Bc\sqrt{\frac{1}{c}}\log\left(\frac{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}\right)}{2b^{\frac{7}{4}}} - \frac{\sqrt{-1}Bc\sqrt{\frac{1}{c}}\log\left(\frac{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}{\sqrt{-1}\sqrt{c}\sqrt{\frac{1}{c}} + \sqrt{c}}\right)}{2b^{\frac{7}{4}}} + \frac{\sqrt{-1}Bc\sqrt{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)\sqrt{\frac{1}{c}}}{\sqrt{c}\sqrt{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*(5/2)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] Piecewise((zoo\*(-2\*A/(11\*x\*\*(11/2)) - 2\*B/(7\*x\*\*(7/2))), Eq(b, 0) & Eq(c, 0)), ((-2\*A/(11\*x\*\*(11/2)) - 2\*B/(7\*x\*\*(7/2)))/c, Eq(b, 0)), ((-2\*A/(7\*x\*\*(7/2)) - 2\*B/(3\*x\*\*(3/2)))/b, Eq(c, 0)), (-2\*A/(7\*b\*x\*\*(7/2)) + 2\*A\*c/(3\*b\*\*2\*x\*\*(3/2)) - (-1)\*\*(1/4)\*A\*c\*\*2\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(11/4)) + (-1)\*\*(1/4)\*A\*c\*\*2\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(11/4)) - (-1)\*\*(1/4)\*A\*c\*\*2\*(1/c)\*\*(1/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(11/4) - 2\*B/(3\*b\*x\*\*(3/2)) + (-1)\*\*(1/4)\*B\*c\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(7/4)) - (-1)\*\*(1/4)\*B\*c\*(1/c)\*\*(1/4)\*log((-1)\*\*(1/4)\*b\*\*(1/4)\*(1/c)\*\*(1/4) + sqrt(x))/(2\*b\*\*(7/4)) + (-1)\*\*(1/4)\*B\*c\*(1/c)\*\*(1/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(b\*\*(1/4)\*(1/c)\*\*(1/4)))/b\*\*(7/4), True))

$$3.193 \quad \int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=276

$$\frac{c^{5/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{13/4}} - \frac{c^{5/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{13/4}} - \frac{c^{5/4}(bB - Ac)}{2\sqrt{2} b^{13/4}}$$

**Rubi [A]** time = 0.24, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{13/4}} - \frac{c^{5/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{13/4}} - \frac{c^{5/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{\sqrt{2} b^{13/4}} + \frac{c^{5/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{\sqrt{2} b^{13/4}} - \frac{2(bB - Ac)}{5b^2 x^{9/2}} + \frac{2c(bB - Ac)}{b^3 \sqrt{x}} - \frac{2A}{9bx^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^(7/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*A)/(9\*b\*x^(9/2)) - (2\*(b\*B - A\*c))/(5\*b^2\*x^(5/2)) + (2\*c\*(b\*B - A\*c))/(b^3\*Sqrt[x]) - (c^(5/4)\*(b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(13/4)) + (c^(5/4)\*(b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(Sqrt[2]\*b^(13/4)) + (c^(5/4)\*(b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*b^(13/4)) - (c^(5/4)\*(b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*b^(13/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n))/c^n)^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)),

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^{7/2}(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^{11/2}(b + cx^2)} dx \\
&= \frac{2A}{9bx^{9/2}} - \frac{\left(2\left(-\frac{9bB}{2} + \frac{9Ac}{2}\right)\right) \int \frac{1}{x^{7/2}(b+cx^2)} dx}{9b} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} - \frac{(c(bB - Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{b^2} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} + \frac{2c(bB - Ac)}{b^3\sqrt{x}} + \frac{(c^2(bB - Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{b^3} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} + \frac{2c(bB - Ac)}{b^3\sqrt{x}} + \frac{(2c^2(bB - Ac)) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} + \frac{2c(bB - Ac)}{b^3\sqrt{x}} - \frac{(c^{3/2}(bB - Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} + \frac{2c(bB - Ac)}{b^3\sqrt{x}} + \frac{(c(bB - Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2b^3} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} + \frac{2c(bB - Ac)}{b^3\sqrt{x}} + \frac{c^{5/4}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\right)}{2\sqrt{2}b^{13/4}} \\
&= \frac{2A}{9bx^{9/2}} - \frac{2(bB - Ac)}{5b^2x^{5/2}} + \frac{2c(bB - Ac)}{b^3\sqrt{x}} - \frac{c^{5/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} + \frac{c^{5/4}(bB - Ac)}{b^3}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 47, normalized size = 0.17

$$\frac{2\left(9x^2(Ac - bB) {}_2F_1\left(-\frac{5}{4}, 1; -\frac{1}{4}; -\frac{cx^2}{b}\right) - 5Ab\right)}{45b^2x^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^(7/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (2\*(-5\*A\*b + 9\*(-(b\*B) + A\*c))\*x^2\*Hypergeometric2F1[-5/4, 1, -1/4, -((c\*x^2)/b)])/(45\*b^2\*x^(9/2))

**IntegrateAlgebraic [A]** time = 0.25, size = 185, normalized size = 0.67

$$\frac{(bBc^{5/4} - Ac^{9/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - (bBc^{5/4} - Ac^{9/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{13/4}} - \frac{2(5Ab^2 - 9Abcx^2 + 45Ac^2x^4 + 9b^2Bx^2 - 45bBcx^4)}{45b^3x^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^(7/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*(5\*A\*b^2 + 9\*b^2\*B\*x^2 - 9\*A\*b\*c\*x^2 - 45\*b\*B\*c\*x^4 + 45\*A\*c^2\*x^4))/(45\*b^3\*x^(9/2)) - ((b\*B\*c^(5/4) - A\*c^(9/4))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(13/4)) - ((b\*B\*c^(5/4) - A\*c^(9/4))\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[b] + Sqrt[c]\*x)]/(Sqrt[2]\*b^(13/4))

**fricas** [B] time = 0.45, size = 931, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(7/2)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 
$$\frac{1}{90} \cdot (180 \cdot b^3 \cdot x^5 \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{1/4} \cdot \arctan\left(\frac{\sqrt{(B^6 \cdot b^6 \cdot c^8 - 6 \cdot A \cdot B^5 \cdot b^5 \cdot c^9 + 15 \cdot A^2 \cdot B^4 \cdot b^4 \cdot c^{10} - 20 \cdot A^3 \cdot B^3 \cdot b^3 \cdot c^{11} + 15 \cdot A^4 \cdot B^2 \cdot b^2 \cdot c^{12} - 6 \cdot A^5 \cdot B \cdot b \cdot c^{13} + A^6 \cdot c^{14}) \cdot x - (B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) \cdot \sqrt{-(B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13}}}{(B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13}}\right) \cdot b^3 \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{1/4} + (B^3 \cdot b^6 \cdot c^4 - 3 \cdot A \cdot B^2 \cdot b^5 \cdot c^5 + 3 \cdot A^2 \cdot B \cdot b^4 \cdot c^6 - A^3 \cdot b^3 \cdot c^7) \cdot \sqrt{x} \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{1/4} / (B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) - 45 \cdot b^3 \cdot x^5 \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{1/4} \cdot \log(b^{10} \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{3/4} - (B^3 \cdot b^6 \cdot c^4 - 3 \cdot A \cdot B^2 \cdot b^5 \cdot c^5 + 3 \cdot A^2 \cdot B \cdot b^4 \cdot c^6 - A^3 \cdot b^3 \cdot c^7) \cdot \sqrt{x} + 45 \cdot b^3 \cdot x^5 \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{1/4} \cdot \log(-b^{10} \cdot (-B^4 \cdot b^4 \cdot c^5 - 4 \cdot A \cdot B^3 \cdot b^3 \cdot c^6 + 6 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^7 - 4 \cdot A^3 \cdot B \cdot b \cdot c^8 + A^4 \cdot c^9) / b^{13})^{3/4} - (B^3 \cdot b^6 \cdot c^4 - 3 \cdot A \cdot B^2 \cdot b^5 \cdot c^5 + 3 \cdot A^2 \cdot B \cdot b^4 \cdot c^6 - A^3 \cdot b^3 \cdot c^7) \cdot \sqrt{x} + 4 \cdot (45 \cdot (B \cdot b \cdot c - A \cdot c^2) \cdot x^4 - 5 \cdot A \cdot b^2 - 9 \cdot (B \cdot b^2 - A \cdot b \cdot c) \cdot x^2) \cdot \sqrt{x} / (b^3 \cdot x^5)$$

**giac** [A] time = 0.20, size = 291, normalized size = 1.05

$$\frac{\sqrt{(bc)^2 Bb - (bc)^2 Ac} \arctan\left(\frac{\sqrt{z} \sqrt{\frac{z}{b^2} + 2\sqrt{z}}}{z \sqrt{\frac{z}{b^2}}}\right) + \sqrt{(bc)^2 Bb - (bc)^2 Ac} \arctan\left(-\frac{\sqrt{z} \sqrt{\frac{z}{b^2} + 2\sqrt{z}}}{z \sqrt{\frac{z}{b^2}}}\right) - \sqrt{(bc)^2 Bb - (bc)^2 Ac} \log\left(\sqrt{z} \sqrt{\frac{z}{b^2} + x + \sqrt{z}}\right) + \sqrt{(bc)^2 Bb - (bc)^2 Ac} \log\left(-\sqrt{z} \sqrt{\frac{z}{b^2} + x + \sqrt{z}}\right) + 2(45 Bbcx^4 - 45 Ac^2 x^4 - 9 Bb^2 x^2 + 9 Abcx^2 - 5 Ab^2)}{4b^4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(7/2)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 
$$\frac{1}{2} \sqrt{2} \cdot ((b \cdot c^3)^{3/4} \cdot B \cdot b - (b \cdot c^3)^{3/4} \cdot A \cdot c) \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (\sqrt{2} \cdot (b/c)^{1/4} + 2 \cdot \sqrt{x}) / (b/c)^{1/4}\right) / (b^4 \cdot c) + \frac{1}{2} \sqrt{2} \cdot ((b \cdot c^3)^{3/4} \cdot B \cdot b - (b \cdot c^3)^{3/4} \cdot A \cdot c) \cdot \arctan\left(-\frac{1}{2} \sqrt{2} \cdot (\sqrt{2} \cdot (b/c)^{1/4} - 2 \cdot \sqrt{x}) / (b/c)^{1/4}\right) / (b^4 \cdot c) - \frac{1}{4} \sqrt{2} \cdot ((b \cdot c^3)^{3/4} \cdot B \cdot b - (b \cdot c^3)^{3/4} \cdot A \cdot c) \cdot \log(\sqrt{2} \cdot \sqrt{x} \cdot (b/c)^{1/4} + x + \sqrt{b/c}) / (b^4 \cdot c) + \frac{1}{4} \sqrt{2} \cdot ((b \cdot c^3)^{3/4} \cdot B \cdot b - (b \cdot c^3)^{3/4} \cdot A \cdot c) \cdot \log(-\sqrt{2} \cdot \sqrt{x} \cdot (b/c)^{1/4} + x + \sqrt{b/c}) / (b^4 \cdot c) + \frac{2}{45} \cdot (45 \cdot B \cdot b \cdot c \cdot x^4 - 45 \cdot A \cdot c^2 \cdot x^4 - 9 \cdot B \cdot b^2 \cdot x^2 + 9 \cdot A \cdot b \cdot c \cdot x^2 - 5 \cdot A \cdot b^2) / (b^3 \cdot x^{9/2})$$

**maple** [A] time = 0.06, size = 330, normalized size = 1.20

$$\frac{\sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{x}{b}\right)^{1/4}}\right) - \sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{x}{b}\right)^{1/4}}\right) - \sqrt{2} A c^2 \ln\left(\frac{x - \left(\frac{x}{b}\right)^{1/4} \sqrt{2} \sqrt{x} + \sqrt{x}}{x + \left(\frac{x}{b}\right)^{1/4} \sqrt{2} \sqrt{x} + \sqrt{x}}\right) + \sqrt{2} B c \arctan\left(\frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{x}{b}\right)^{1/4}}\right) + \sqrt{2} B c \arctan\left(\frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{x}{b}\right)^{1/4}}\right) + \sqrt{2} B c \ln\left(\frac{x - \left(\frac{x}{b}\right)^{1/4} \sqrt{2} \sqrt{x} + \sqrt{x}}{x + \left(\frac{x}{b}\right)^{1/4} \sqrt{2} \sqrt{x} + \sqrt{x}}\right) - \frac{2 A c^2}{b^3 \sqrt{x}} + \frac{2 B c}{b^2 \sqrt{x}} + \frac{2 A c}{5 b^2 x^2} - \frac{2 B}{5 b x^2} - \frac{2 A}{9 b x^2}}{4 \left(\frac{x}{b}\right)^{1/4} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^(7/2)/(c\*x^4+b\*x^2),x)

[Out] 
$$-1/2 \cdot c^2 / b^3 \cdot (b/c)^{1/4} \cdot 2^{1/2} \cdot A \cdot \arctan(2^{1/2} / (b/c)^{1/4} \cdot x^{1/2} - 1) - 1/4 \cdot c^2 / b^3 \cdot (b/c)^{1/4} \cdot 2^{1/2} \cdot A \cdot \ln((x - (b/c)^{1/4} \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4} \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2})) - 1/2 \cdot c^2 / b^3 \cdot (b/c)^{1/4} \cdot 2^{1/2} \cdot A \cdot \arctan(2^{1/2} / (b/c)^{1/4} \cdot x^{1/2} + 1) + 1/2 \cdot c / b^2 \cdot (b/c)^{1/4} \cdot 2^{1/2} \cdot B \cdot \arctan(2^{1/2} / (b/c)^{1/4} \cdot x^{1/2} - 1) + 1/4 \cdot c / b^2 \cdot (b/c)^{1/4} \cdot 2^{1/2} \cdot B \cdot \ln((x - (b/c)^{1/4} \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4} \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}))$$

$+(b/c)^{(1/2)})+1/2*c/b^2/(b/c)^{(1/4)*2^{(1/2)*B*\arctan(2^{(1/2)/(b/c)^{(1/4)*x^{(1/2)+1}-2/9*A/b/x^{(9/2)+2/5/b^2/x^{(5/2)*A*c-2/5/b/x^{(5/2)*B-2/b^3*c^2/x^{(1/2)*A+2/b^2*c/x^{(1/2)*B}}$

**maxima [A]** time = 3.11, size = 237, normalized size = 0.86

$$\frac{(Bbc^2 - Ac^3) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} \right)}{4b^3} + \frac{2(45(Bbc - Ac^2)x^4 - 5Ab^2 - 9(Bb^2 - Abc)x^2)}{45b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(7/2)/(c\*x^4+b\*x^2),x, algorithm="maxima")

[Out]  $\frac{1}{4}(B*b*c^2 - A*c^3)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} + 2*\sqrt{c}*\sqrt{x})/\sqrt{(\sqrt{b}*\sqrt{c})}))/(\sqrt{(\sqrt{b}*\sqrt{c})}*\sqrt{c}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} - 2*\sqrt{c}*\sqrt{x})/\sqrt{(\sqrt{b}*\sqrt{c})}))/(\sqrt{(\sqrt{b}*\sqrt{c})}*\sqrt{c}) - \sqrt{2}*\log(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{(1/4)}*c^{(3/4)}) + \sqrt{2}*\log(-\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c}*x + \sqrt{b})/(b^{(1/4)}*c^{(3/4)})/b^3 + 2/45*(45*(B*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2 - A*b*c)*x^2)/(b^3*x^{(9/2)})$

**mupad [B]** time = 0.23, size = 107, normalized size = 0.39

$$\frac{(-c)^{5/4} \operatorname{atan}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (Ac - Bb)}{b^{13/4}} - \frac{\frac{2A}{9b} - \frac{2x^2(Ac - Bb)}{5b^2} + \frac{2cx^4(Ac - Bb)}{b^3}}{x^{9/2}} - \frac{(-c)^{5/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (Ac - Bb)}{b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^(7/2)\*(b\*x^2 + c\*x^4)),x)

[Out]  $((-c)^{(5/4)*\operatorname{atan}(((c)^{(1/4)*x^{(1/2)})/b^{(1/4)})*(A*c - B*b)})/b^{(13/4)} - ((2*A)/(9*b) - (2*x^2*(A*c - B*b))/(5*b^2) + (2*c*x^4*(A*c - B*b))/b^3)/x^{(9/2)} - ((c)^{(5/4)*\operatorname{atanh}(((c)^{(1/4)*x^{(1/2)})/b^{(1/4)})*(A*c - B*b)})/b^{(13/4)}$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*(7/2)/(c\*x\*\*4+b\*x\*\*2),x)

[Out] Timed out

$$3.194 \quad \int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)} dx$$

**Optimal.** Leaf size=278

$$\frac{c^{7/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{15/4}} + \frac{c^{7/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{15/4}} - \frac{c^{7/4}(bB - Ac)}{11bx^{11/2}}$$

**Rubi [A]** time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 453, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{7/4}(bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{15/4}} + \frac{c^{7/4}(bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{2\sqrt{2} b^{15/4}} - \frac{c^{7/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2} b^{15/4}} + \frac{c^{7/4}(bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{\sqrt{2} b^{15/4}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} - \frac{2A}{11bx^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^(9/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*A)/(11\*b\*x^(11/2)) - (2\*(b\*B - A\*c))/(7\*b^2\*x^(7/2)) + (2\*c\*(b\*B - A\*c))/(3\*b^3\*x^(3/2)) - (c^(7/4)\*(b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(Sqrt[2]\*b^(15/4)) + (c^(7/4)\*(b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(Sqrt[2]\*b^(15/4)) - (c^(7/4)\*(b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*b^(15/4)) + (c^(7/4)\*(b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(2\*Sqrt[2]\*b^(15/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^(p), x], (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)),



```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^{9/2}(bx^2 + cx^4)} dx &= \int \frac{A + Bx^2}{x^{13/2}(b + cx^2)} dx \\
&= -\frac{2A}{11bx^{11/2}} - \frac{\left(2\left(-\frac{11bB}{2} + \frac{11Ac}{2}\right)\right) \int \frac{1}{x^{9/2}(b+cx^2)} dx}{11b} \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} - \frac{(c(bB - Ac)) \int \frac{1}{x^{5/2}(b+cx^2)} dx}{b^2} \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} + \frac{(c^2(bB - Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{b^3} \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} + \frac{(2c^2(bB - Ac)) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{b^3} \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} + \frac{(c^2(bB - Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{b^{7/2}} + \dots \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} + \frac{(c^{3/2}(bB - Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{2b^{7/2}} \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} \\
&= -\frac{2A}{11bx^{11/2}} - \frac{2(bB - Ac)}{7b^2x^{7/2}} + \frac{2c(bB - Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB - Ac)}{231b^3x^{11/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 47, normalized size = 0.17

$$\frac{-22x^2(bB - Ac) {}_2F_1\left(-\frac{7}{4}, 1; -\frac{3}{4}; -\frac{cx^2}{b}\right) - 14Ab}{77b^2x^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^(9/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-14\*A\*b - 22\*(b\*B - A\*c)\*x^2\*Hypergeometric2F1[-7/4, 1, -3/4, -(c\*x^2)/b])/((77\*b^2\*x^(11/2))

**IntegrateAlgebraic [A]** time = 0.24, size = 184, normalized size = 0.66

$$-\frac{(bBc^{7/4} - Ac^{11/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{15/4}} + \frac{(bBc^{7/4} - Ac^{11/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{15/4}} - \frac{2(21Ab^2 - 33Abcx^2 + 77Ac^2x^4 + 33b^2Bx^2 - 77bBcx^4)}{231b^3x^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^(9/2)\*(b\*x^2 + c\*x^4)), x]

[Out] (-2\*(21\*A\*b^2 + 33\*b^2\*B\*x^2 - 33\*A\*b\*c\*x^2 - 77\*b\*B\*c\*x^4 + 77\*A\*c^2\*x^4))/(231\*b^3\*x^(11/2)) - ((b\*B\*c^(7/4) - A\*c^(11/4))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(Sqrt[2]\*b^(15/4)) + ((b\*B\*c^(7/4) - A\*c^(11/4))\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]/(Sqrt[b] + Sqrt[c]\*x))/(Sqrt[2]\*b^(15/4))

**fricas [B]** time = 0.44, size = 734, normalized size = 2.64

$$\frac{\sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \arctan \left( \frac{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x}}{z(x)^{\frac{1}{2}}} \right) + \sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \arctan \left( \frac{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x}}{z(x)^{\frac{1}{2}}} \right) + \sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{z(x)^{\frac{1}{2}}}{z(x)^{\frac{1}{2}}} + x + \sqrt{x} \right) \right) - \sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{z(x)^{\frac{1}{2}}}{z(x)^{\frac{1}{2}}} + x + \sqrt{x} \right) \right) + \frac{2(77Bbcx^4 - 77Ac^2x^4 - 33Bb^2x^2 + 33Abcx^2 - 21Ab^2)}{231b^2x^2}}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(9/2)/(c\*x^4+b\*x^2),x, algorithm="fricas")

[Out] 
$$-1/462*(924*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{1/4}*\arctan((\sqrt{x}(\sqrt{b^8*\sqrt{x}(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15}) + (B^2*b^2*c^4 - 2*A*B*b*c^5 + A^2*c^6)*x)*b^{11}*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{3/4} + (B*b^{12}*c^2 - A*b^{11}*c^3)*\sqrt{x}*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{3/4})/(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})) + 231*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{1/4}*\log(b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{1/4} - (B*b*c^2 - A*c^3)*\sqrt{x}) - 231*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{1/4}*\log(-b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^{10} + A^4*c^{11})/b^{15})^{1/4} - (B*b*c^2 - A*c^3)*\sqrt{x}) - 4*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)*\sqrt{x})/(b^3*x^6)$$

**giac [A]** time = 0.19, size = 291, normalized size = 1.05

$$\frac{\sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \arctan \left( \frac{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x}}{z(x)^{\frac{1}{2}}} \right) + \sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \arctan \left( \frac{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x}}{z(x)^{\frac{1}{2}}} \right) + \sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{z(x)^{\frac{1}{2}}}{z(x)^{\frac{1}{2}}} + x + \sqrt{x} \right) \right) - \sqrt{2} \left( (bc^2)^{\frac{1}{2}} Bbc - (bc^2)^{\frac{1}{2}} Ac^2 \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{z(x)^{\frac{1}{2}}}{z(x)^{\frac{1}{2}}} + x + \sqrt{x} \right) \right) + \frac{2(77Bbcx^4 - 77Ac^2x^4 - 33Bb^2x^2 + 33Abcx^2 - 21Ab^2)}{231b^2x^2}}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(9/2)/(c\*x^4+b\*x^2),x, algorithm="giac")

[Out] 
$$1/2*\sqrt{2}*(b*c^3)^{(1/4)}*B*b*c - (b*c^3)^{(1/4)}*A*c^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*\sqrt{x}))/b^4 + 1/2*\sqrt{2}*(b*c^3)^{(1/4)}*B*b*c - (b*c^3)^{(1/4)}*A*c^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*\sqrt{x}))/b^4 + 1/4*\sqrt{2}*(b*c^3)^{(1/4)}*B*b*c - (b*c^3)^{(1/4)}*A*c^2)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c}))/b^4 - 1/4*\sqrt{2}*(b*c^3)^{(1/4)}*B*b*c - (b*c^3)^{(1/4)}*A*c^2)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c}))/b^4 + 2/231*(77*B*b*c*x^4 - 77*A*c^2*x^4 - 33*B*b^2*x^2 + 33*A*b*c*x^2 - 21*A*b^2)/(b^3*x^{(11/2)})$$

**maple [A]** time = 0.06, size = 336, normalized size = 1.21

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right) - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right) + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \ln \left( \frac{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x} + \sqrt{2}}{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x} + \sqrt{2}} \right) + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c^2 \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right) + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c^2 \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right) + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c^2 \ln \left( \frac{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x} + \sqrt{2}}{\sqrt{2} \sqrt{x} \sqrt{2} \sqrt{x} + \sqrt{2}} \right) - \frac{2Ac^2}{3b^3x^2} + \frac{2Bc}{3b^2x^2} + \frac{2Ac}{7b^3x^2} - \frac{2B}{7b^2x^2} - \frac{2A}{11b^3x^2}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^(9/2)/(c\*x^4+b\*x^2),x)

[Out] 
$$-1/2*c^3/b^4*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)-1/4*c^3/b^4*(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))-1/2*c^3/b^4*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+1/2*c^2/b^3*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)+1/4*c^2/b^3*(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+1/2*c^2/b^3*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)-2/11*A/b/x^{(11/2)}+2/7/b^2/x^{(7/2)}*A*c-2/7/b/x^{(7/2)}*B-2/3/b^3*c^2/x^{(3/2)}*A+2/3/b^2*c/x^{(3/2)}*B$$

**maxima [A]** time = 3.12, size = 276, normalized size = 0.99

$$\frac{2\sqrt{2}(Bbc^2 - Ac^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{c}\right)}{2\sqrt{b}\sqrt{c}}\right) + 2\sqrt{2}(Bbc^2 - Ac^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{c}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(Bbc^2 - Ac^3) \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}\right) - \sqrt{2}(Bbc^2 - Ac^3) \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}}c^{\frac{1}{4}}}} + \frac{2(77(Bbc - Ac^2)x^4 - 21Ab^2 - 33(Bb^2 - Abc)x^2)}{231b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)
)) + 2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
- 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)
)) + sqrt(2)*(B*b*c^2 - A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c^2 - A*c^3)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4))/b^3 + 2/2
31*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)/(b^3*x^(11/
2))
```

**mupad [B]** time = 0.35, size = 563, normalized size = 2.03

$$\frac{(-c)^{7/4} \operatorname{atan}\left(\frac{2b^2\sqrt{c}\sqrt{c^2-3b^2}\sqrt{c}-3b^2Bb^2\sqrt{c}+3Aa^2\sqrt{c}\sqrt{c}}{184(-c)^{3/4}\sqrt{c}(c^2-3A^2Bb)+3Aa^2\sqrt{c}\sqrt{c}}\right)(Ac-Bb) - \frac{2A}{11b} - \frac{2c^2(Ac-Bb)}{2b^2} + \frac{2c^2(Ac-Bb)}{3b^2}}{b^{15/4}} - \frac{(-c)^{7/4} \operatorname{atan}\left(\frac{(-c)^{7/4}(Ac-Bb)\sqrt{c}(16a^2\sqrt{c}^2-32Aa^2\sqrt{c}^2+16a^2\sqrt{c}^2)}{23154} - \frac{(-c)^{7/4}(Ac-Bb)(22A^2\sqrt{c}^2-32Bb^2\sqrt{c}^2)}{23154}\right) + \frac{(-c)^{7/4}(Ac-Bb)\sqrt{c}(16a^2\sqrt{c}^2-32Aa^2\sqrt{c}^2+16a^2\sqrt{c}^2)}{23154}}{b^{15/4}}}{b^{15/4}} (Ac - Bb) + 11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(7/4)*atan((A^3*c^10*x^(1/2) - B^3*b^3*c^7*x^(1/2) - 3*A^2*B*b*c^9*x^(
1/2) + 3*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(c*(c*(A^3*c - 3*A^2*
B*b) + 3*A*B^2*b^2) - B^3*b^3)))*(Ac - B*b))/b^(15/4) - ((-c)^(7/4)*atan((
(-c)^(7/4)*(Ac - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B
*b^10*c^8) - ((-c)^(7/4)*(Ac - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(
15/4))))*1i)/(2*b^(15/4)) + ((-c)^(7/4)*(Ac - B*b)*(x^(1/2)*(16*A^2*b^9*c^
9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) + ((-c)^(7/4)*(Ac - B*b)*(32*A*b^13
*c^6 - 32*B*b^14*c^5))/(2*b^(15/4))))*1i)/(2*b^(15/4)))/(((c)^(7/4)*(Ac -
B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) - ((-c)^(
7/4)*(Ac - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(15/4))))/(2*b^(15/
4)) - ((-c)^(7/4)*(Ac - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 -
32*A*B*b^10*c^8) + ((-c)^(7/4)*(Ac - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))
/(2*b^(15/4))))/(2*b^(15/4)))*(Ac - B*b)*1i)/b^(15/4) - ((2*A)/(11*b) - (
2*x^2*(Ac - B*b))/(7*b^2) + (2*c*x^4*(Ac - B*b))/(3*b^3))/x^(11/2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

$$3.195 \quad \int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=332

$$\frac{b^{5/4}(13bB - 9Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{17/4}} - \frac{b^{5/4}(13bB - 9Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{17/4}} + \dots$$

**Rubi [A]** time = 0.28, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}(13bB - 9Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{17/4}} - \frac{b^{5/4}(13bB - 9Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{17/4}} + \frac{b^{5/4}(13bB - 9Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}}\right)}{4\sqrt{2} c^{17/4}} - \frac{b^{5/4}(13bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}} + 1\right)}{4\sqrt{2} c^{17/4}} + \frac{x^{9/2}(13bB - 9Ac)}{18bc^2} - \frac{x^{5/2}(13bB - 9Ac)}{10c^3} + \frac{b\sqrt{x}(13bB - 9Ac)}{2c^4} - \frac{x^{13/2}(bB - Ac)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(19/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*(13\*b\*B - 9\*A\*c)\*Sqrt[x])/(2\*c^4) - ((13\*b\*B - 9\*A\*c)\*x^(5/2))/(10\*c^3) + ((13\*b\*B - 9\*A\*c)\*x^(9/2))/(18\*b\*c^2) - ((b\*B - A\*c)\*x^(13/2))/(2\*b\*c\*(b + c\*x^2)) + (b^(5/4)\*(13\*b\*B - 9\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*c^(17/4)) - (b^(5/4)\*(13\*b\*B - 9\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*c^(17/4)) + (b^(5/4)\*(13\*b\*B - 9\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*c^(17/4)) - (b^(5/4)\*(13\*b\*B - 9\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*c^(17/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x^{19/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^{11/2} (A + Bx^2)}{(b + cx^2)^2} dx \\
 &= -\frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{\left(\frac{13bB}{2} - \frac{9Ac}{2}\right) \int \frac{x^{11/2}}{b+cx^2} dx}{2bc} \\
 &= \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} - \frac{(13bB - 9Ac) \int \frac{x^{7/2}}{b+cx^2} dx}{4c^2} \\
 &= -\frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{(b(13bB - 9Ac)) \int \frac{x^{3/2}}{b+cx^2} dx}{4c^3} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} - \frac{(b^2(13bB - 9Ac)) \int \frac{x^{-1/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} - \frac{(b^2(13bB - 9Ac)) \int \frac{x^{-3/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} - \frac{(b^3(13bB - 9Ac)) \int \frac{x^{-5/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} - \frac{(b^4(13bB - 9Ac)) \int \frac{x^{-7/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} - \frac{(b^5(13bB - 9Ac)) \int \frac{x^{-9/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{(b^6(13bB - 9Ac)) \int \frac{x^{-11/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{(b^7(13bB - 9Ac)) \int \frac{x^{-13/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{(b^8(13bB - 9Ac)) \int \frac{x^{-15/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{(b^9(13bB - 9Ac)) \int \frac{x^{-17/2}}{b+cx^2} dx}{4c^4} \\
 &= \frac{b(13bB - 9Ac)\sqrt{x}}{2c^4} - \frac{(13bB - 9Ac)x^{5/2}}{10c^3} + \frac{(13bB - 9Ac)x^{9/2}}{18bc^2} - \frac{(bB - Ac)x^{13/2}}{2bc(b + cx^2)} + \frac{(b^{10}(13bB - 9Ac)) \int \frac{x^{-19/2}}{b+cx^2} dx}{4c^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.67, size = 417, normalized size = 1.26

$$\frac{90\sqrt{2}^{10}(13bB - 9Ac)\tan^{-1}\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 90\sqrt{2}^{10}(13bB - 9Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 405\sqrt{2}^{10}B\log\left(-\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\right) + 405\sqrt{2}^{10}B\log\left(\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\right) - \frac{90\sqrt{2}^{10}c}{720c^{17/4}} - 2880A^{5/4}\sqrt{c} + 288A^{9/4}c^{3/2} + 585\sqrt{2}^{10}B\log\left(-\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\right) - 585\sqrt{2}^{10}B\log\left(\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\right) + \frac{2070\sqrt{2}^{10}}{720c^{17/4}} + 4320B^2\sqrt{c}\sqrt{x} - 576B^2c^{3/2} + 160B^2c^{5/2}}{720c^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(19/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (4320\*b^2\*B\*c^(1/4)\*Sqrt[x] - 2880\*A\*b\*c^(5/4)\*Sqrt[x] - 576\*b\*B\*c^(5/4)\*x^(5/2) + 288\*A\*c^(9/4)\*x^(5/2) + 160\*B\*c^(9/4)\*x^(9/2) + (360\*b^3\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2) - (360\*A\*b^2\*c^(5/4)\*Sqrt[x])/(b + c\*x^2) + 90\*Sqrt[2]\*b^(5/4)\*(13\*b\*B - 9\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 90\*Sqrt[2]\*b^(5/4)\*(13\*b\*B - 9\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 585\*Sqrt[2]\*b^(9/4)\*B\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 405\*Sqrt[2]\*A\*b^(5/4)\*c\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 585\*Sqrt[2]\*b^(9/4)\*B\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 405\*Sqrt[2]\*A\*b^(5/4)\*c\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(720\*c^(17/4))

**IntegrateAlgebraic [A]** time = 0.85, size = 443, normalized size = 1.33

$$\frac{9AB^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 9AB^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) + x^{5/2}(468B^2Bc - 324Ab^2c) + \sqrt{x}(585B^2B - 405Ab^2c) + x^{9/2}(36Ac^3 - 52Bb^2c) + 20Bc^3x^{13/2}}{4\sqrt{2}c^{13/4}} + \frac{9AB^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 9AB^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) + x^{5/2}(468B^2Bc - 324Ab^2c) + \sqrt{x}(585B^2B - 405Ab^2c) + x^{9/2}(36Ac^3 - 52Bb^2c) + 20Bc^3x^{13/2}}{4\sqrt{2}c^{13/4}} + \frac{2\sqrt{2}B^{9/4}B\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 2\sqrt{2}B^{9/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right)}{c^{17/4}} - \frac{3b^{9/4}B\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 3b^{9/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right)}{4\sqrt{2}c^{17/4}} + \frac{2\sqrt{2}B^{9/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 2\sqrt{2}B^{9/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right)}{c^{17/4}} + \frac{3b^{9/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) - 3b^{9/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right)}{4\sqrt{2}c^{17/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(19/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((585\*b^3\*B - 405\*A\*b^2\*c)\*Sqrt[x] + (468\*b^2\*B\*c - 324\*A\*b\*c^2)\*x^(5/2) + (-52\*b\*B\*c^2 + 36\*A\*c^3)\*x^(9/2) + 20\*B\*c^3\*x^(13/2))/(90\*c^4\*(b + c\*x^2)) - (3\*b^(9/4)\*B\*ArcTan[(b^(1/4)/(Sqrt[2]\*c^(1/4)) - (c^(1/4)\*x)/(Sqrt[2]\*b^(1/4))]/Sqrt[x]])/(4\*Sqrt[2]\*c^(17/4)) + (2\*Sqrt[2]\*b^(9/4)\*B\*ArcTan[(b^(1/4)/(Sqrt[2]\*c^(1/4)) - (c^(1/4)\*x)/(Sqrt[2]\*b^(1/4))]/Sqrt[x]])/c^(17/4) - (9\*A\*b^(5/4)\*ArcTan[(b^(1/4)/(Sqrt[2]\*c^(1/4)) - (c^(1/4)\*x)/(Sqrt[2]\*b^(1/4))]/Sqrt[x]])/(4\*Sqrt[2]\*c^(13/4)) + (3\*b^(9/4)\*B\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]/(Sqrt[b] + Sqrt[c]\*x)))/(4\*Sqrt[2]\*c^(17/4)) - (2\*Sqrt[2]\*b^(9/4)\*B\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]/(Sqrt[b] + Sqrt[c]\*x)))/c^(17/4) + (9\*A\*b^(5/4)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]/(Sqrt[b] + Sqrt[c]\*x)))/(4\*Sqrt[2]\*c^(13/4))

**fricas** [B] time = 0.46, size = 804, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 1/360\*(180\*(c^5\*x^2 + b\*c^4)\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(1/4)\*arctan((sqrt(c^8\*sqrt(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17) + (169\*B^2\*b^4 - 234\*A\*B\*b^3\*c + 81\*A^2\*b^2\*c^2)\*x)\*c^13\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(3/4) + (13\*B\*b^2\*c^13 - 9\*A\*b\*c^14)\*sqrt(x)\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(3/4))/(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4) + 45\*(c^5\*x^2 + b\*c^4)\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(1/4)\*log(c^4\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(1/4) - (13\*B\*b^2 - 9\*A\*b\*c)\*sqrt(x)) - 45\*(c^5\*x^2 + b\*c^4)\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(1/4)\*log(-c^4\*(-(28561\*B^4\*b^9 - 79092\*A\*B^3\*b^8\*c + 82134\*A^2\*B^2\*b^7\*c^2 - 37908\*A^3\*B\*b^6\*c^3 + 6561\*A^4\*b^5\*c^4)/c^17)^(1/4) - (13\*B\*b^2 - 9\*A\*b\*c)\*sqrt(x)) + 4\*(20\*B\*c^3\*x^6 - 4\*(13\*B\*b\*c^2 - 9\*A\*c^3)\*x^4 + 585\*B\*b^3 - 405\*A\*b^2\*c + 36\*(13\*B\*b^2\*c - 9\*A\*b\*c^2)\*x^2)\*sqrt(x))/(c^5\*x^2 + b\*c^4)

**giac** [A] time = 0.22, size = 335, normalized size = 1.01

$$\frac{\sqrt{2} \left( 13 (bc)^{\frac{1}{2}} Bc^2 - 9 (bc)^{\frac{1}{2}} Abc \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{c} \right)^{\frac{1}{2}} + \sqrt{c}}{z(c)^{\frac{1}{2}}} \right)}{8c^5} - \frac{\sqrt{2} \left( 13 (bc)^{\frac{1}{2}} Bc^2 - 9 (bc)^{\frac{1}{2}} Abc \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{c} \right)^{\frac{1}{2}} + \sqrt{c}}{z(c)^{\frac{1}{2}}} \right)}{8c^5} - \frac{\sqrt{2} \left( 13 (bc)^{\frac{1}{2}} Bc^2 - 9 (bc)^{\frac{1}{2}} Abc \right) \log \left( \sqrt{2} \sqrt{c} \left( \frac{z}{c} \right)^{\frac{1}{2}} + z + \sqrt{c} \right)}{16c^5} + \frac{\sqrt{2} \left( 13 (bc)^{\frac{1}{2}} Bc^2 - 9 (bc)^{\frac{1}{2}} Abc \right) \log \left( -\sqrt{2} \sqrt{c} \left( \frac{z}{c} \right)^{\frac{1}{2}} + z + \sqrt{c} \right)}{16c^5} + \frac{BB^2\sqrt{c} - AB^2c\sqrt{c}}{2(c^2+b)^2} + \frac{2(5Bb^3c^2 - 18Bbc^3z^2 + 9Ab^3c^2 + 135Bb^2c^4\sqrt{c} - 90Ab^3c\sqrt{c})}{45c^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] -1/8\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b^2 - 9\*(b\*c^3)^(1/4)\*A\*b\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/c^5 - 1/8\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b^2 - 9\*(b\*c^3)^(1/4)\*A\*b\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/c^5 - 1/16\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b^2 - 9\*(b\*c^3)^(1/4)\*A\*b\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/16\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b^2 - 9\*(b\*c^3)^(1/4)\*A\*b\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/2\*(B\*b^3\*sqrt(x) - A\*b^2\*c\*sqrt(x))/((c\*x^2 + b)\*c^4) + 2/45\*(5\*B\*c^16\*x^(9/2) - 18\*B\*b\*c^15\*x^



$$(5/2) + 9A*c^16*x^(5/2) + 135*B*b^2*c^14*sqrt(x) - 90*A*b*c^15*sqrt(x))/c^18$$

**maple [A]** time = 0.06, size = 372, normalized size = 1.12

$$\frac{2Bx^{\frac{5}{2}}}{9c^2} + \frac{2Ax^{\frac{5}{2}}}{5c^2} - \frac{4Bbx^{\frac{3}{2}}}{5c^3} - \frac{A^2\sqrt{x}}{2(c^2+b)c^3} + \frac{B^2\sqrt{x}}{2(c^2+b)c^4} + \frac{9\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Ab\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^3} + \frac{9\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Ab\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^3} + \frac{9\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Ab\ln\left(\frac{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}-\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}+\sqrt{x}}\right)}{16c^4} + \frac{13\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Bb^2\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^4} + \frac{13\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Bb^2\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^4} + \frac{13\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Bb^2\ln\left(\frac{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}-\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}+\sqrt{x}}\right)}{16c^4} - \frac{4Ab\sqrt{x}}{c^3} + \frac{6Bb^2\sqrt{x}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(19/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 2/9/c^2\*B\*x^(9/2)+2/5/c^2\*A\*x^(5/2)-4/5/c^3\*B\*x^(5/2)\*b-4/c^3\*A\*b\*x^(1/2)+6/c^4\*B\*b^2\*x^(1/2)-1/2\*b^2/c^3\*x^(1/2)/(c\*x^2+b)\*A+1/2\*b^3/c^4\*x^(1/2)/(c\*x^2+b)\*B+9/8\*b/c^3\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)+9/8\*b/c^3\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)+9/16\*b/c^3\*(b/c)^(1/4)\*2^(1/2)\*A\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-13/8\*b^2/c^4\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-13/8\*b^2/c^4\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-13/16\*b^2/c^4\*(b/c)^(1/4)\*2^(1/2)\*B\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))

**maxima [A]** time = 3.07, size = 298, normalized size = 0.90

$$\frac{(Bb^3 - Ab^2c)\sqrt{x}}{2(c^2x^2 + bc^4)} \left[ \frac{2\sqrt{2}(13Bb - 9Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\frac{1}{4} + z\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(13Bb - 9Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\frac{1}{4} - z\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(13Bb - 9Ac)\log\left(\frac{\sqrt{2}\frac{1}{4}\sqrt{x} + \sqrt{c}z + \sqrt{b}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}(13Bb - 9Ac)\log\left(-\sqrt{2}\frac{1}{4}\sqrt{x} + \sqrt{c}z + \sqrt{b}\right)}{b^{\frac{1}{4}}} \right] b^{\frac{3}{4}} + \frac{2(5Bc^2x^2 - 9(2Bbc - Ac^2)x^{\frac{5}{2}} + 45(3Bb^2 - 2Abc)\sqrt{x})}{45c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 1/2\*(B\*b^3 - A\*b^2\*c)\*sqrt(x)/(c^5\*x^2 + b\*c^4) - 1/16\*(2\*sqrt(2)\*(13\*B\*b - 9\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(b)\*sqrt(sqrt(b)\*sqrt(c)) + 2\*sqrt(2)\*(13\*B\*b - 9\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(b)\*sqrt(sqrt(b)\*sqrt(c)) + sqrt(2)\*(13\*B\*b - 9\*A\*c)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4)) - sqrt(2)\*(13\*B\*b - 9\*A\*c)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4)))\*b^2/c^4 + 2/45\*(5\*B\*c^2\*x^2 - 9\*(2\*B\*b\*c - A\*c^2)\*x^(5/2) + 45\*(3\*B\*b^2 - 2\*A\*b\*c)\*sqrt(x))/c^4

**mupad [B]** time = 0.22, size = 857, normalized size = 2.58

$$\frac{x^{\frac{5}{2}}\left(\frac{2A}{5c^2} + \frac{2B}{5c^2}\right) - \frac{4Bbx^{\frac{3}{2}}}{5c^3} - \frac{A^2\sqrt{x}}{2(c^2+b)c^3} + \frac{B^2\sqrt{x}}{2(c^2+b)c^4} + \frac{9\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Ab\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^3} + \frac{9\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Ab\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^3} + \frac{9\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Ab\ln\left(\frac{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}-\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}+\sqrt{x}}\right)}{16c^4} + \frac{13\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Bb^2\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^4} + \frac{13\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Bb^2\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{2}}}\right)}{8c^4} + \frac{13\left(\frac{b}{c}\right)^{\frac{3}{2}}\sqrt{2}Bb^2\ln\left(\frac{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}-\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{2}}\sqrt{x}+\sqrt{x}}\right)}{16c^4} - \frac{4Ab\sqrt{x}}{c^3} + \frac{6Bb^2\sqrt{x}}{c^4}}{2(c^2x^2 + bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(19/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out] x^(5/2)\*((2\*A)/(5\*c^2) - (4\*B\*b)/(5\*c^3)) - x^(1/2)\*((2\*b\*((2\*A)/c^2 - (4\*B\*b)/c^3))/c + (2\*B\*b^2)/c^4) + (2\*B\*x^(9/2))/(9\*c^2) + (x^(1/2)\*((B\*b^3)/2 - (A\*b^2\*c)/2))/(b\*c^4 + c^5\*x^2) + ((-b)^(5/4)\*atan((((-b)^(5/4))\*((x^(1/2))\*((169\*B^2\*b^6 + 81\*A^2\*b^4\*c^2 - 234\*A\*B\*b^5\*c))/c^5 + ((-b)^(5/4)\*(9\*A\*c - 13\*B\*b)\*(13\*B\*b^4 - 9\*A\*b^3\*c))/c^(21/4))\*(9\*A\*c - 13\*B\*b)\*1i)/(8\*c^(17/4))) + ((-b)^(5/4)\*((x^(1/2)\*(169\*B^2\*b^6 + 81\*A^2\*b^4\*c^2 - 234\*A\*B\*b^5\*c))/c^5 - ((-b)^(5/4)\*(9\*A\*c - 13\*B\*b)\*(13\*B\*b^4 - 9\*A\*b^3\*c))/c^(21/4))\*(9\*A\*c - 13\*B\*b)\*1i)/(8\*c^(17/4)))/((((-b)^(5/4))\*((x^(1/2)\*(169\*B^2\*b^6 + 81\*A^2\*b^4\*c^2 - 234\*A\*B\*b^5\*c))/c^5 + ((-b)^(5/4)\*(9\*A\*c - 13\*B\*b)\*(13\*B\*b^4 - 9\*A\*b^3\*c))/c^(21/4))\*(9\*A\*c - 13\*B\*b)\*1i)/(8\*c^(17/4))) - ((-b)^(5/4)\*((x^(1/2)\*(169\*B^2\*b^6 + 81\*A^2\*b^4\*c^2 - 234\*A\*B\*b^5\*c))/c^5 - ((-b)^(5/4)\*(9\*A\*c - 13\*B\*b)\*(13\*B\*b^4 - 9\*A\*b^3\*c))/c^(21/4))\*(9\*A\*c - 13\*B\*b)\*1i)/(8\*c^(17/4)))/((((-b)^(5/4))\*((x^(1/2)\*(169\*B^2\*b^6 + 81\*A^2\*b^4\*c^2 - 234\*A\*B\*b^5\*c))/c^5 - ((-b)^(5/4)\*(9\*A\*c - 13\*B\*b)\*(13\*B\*b^4 - 9\*A\*b^3\*c))/c^(21/4))\*(9\*A\*c - 13\*B\*b)\*1i)/(8\*c^(17/4)))

```

*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))))*(9
*A*c - 13*B*b)*1i)/(4*c^(17/4)) - ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)*
169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 1
3*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))
+ ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5
+ ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c
- 13*B*b))/(8*c^(17/4)))/((((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*
c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^
3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2
)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c
- 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(1
7/4))))*(9*A*c - 13*B*b))/(4*c^(17/4))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(19/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.196 \quad \int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=310

$$\frac{b^{3/4}(11bB - 7Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{15/4}} - \frac{b^{3/4}(11bB - 7Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{15/4}} - \frac{b^3}{2bc(b+cx^2)}$$

Rubi [A] time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{3/4}(11bB - 7Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{15/4}} - \frac{b^{3/4}(11bB - 7Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} c^{15/4}} - \frac{b^{3/4}(11bB - 7Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} c^{15/4}} + \frac{b^{3/4}(11bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{4\sqrt{2} c^{15/4}} + \frac{x^{7/2}(11bB - 7Ac)}{14bc^2} - \frac{x^{3/2}(11bB - 7Ac)}{6c^3} - \frac{x^{1/2}(bB - Ac)}{2bc(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((11\*b\*B - 7\*A\*c)\*x^(3/2))/(6\*c^3) + ((11\*b\*B - 7\*A\*c)\*x^(7/2))/(14\*b\*c^2) - ((b\*B - A\*c)\*x^(11/2))/(2\*b\*c\*(b + c\*x^2)) - (b^(3/4)\*(11\*b\*B - 7\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*c^(15/4)) + (b^(3/4)\*(11\*b\*B - 7\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*c^(15/4)) + (b^(3/4)\*(11\*b\*B - 7\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*c^(15/4)) - (b^(3/4)\*(11\*b\*B - 7\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*c^(15/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{17/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^{9/2} (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} + \frac{\left(\frac{11bB}{2} - \frac{7Ac}{2}\right) \int \frac{x^{9/2}}{b+cx^2} dx}{2bc} \\
&= \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} - \frac{(11bB - 7Ac) \int \frac{x^{5/2}}{b+cx^2} dx}{4c^2} \\
&= -\frac{(11bB - 7Ac)x^{3/2}}{6c^3} + \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} + \frac{(b(11bB - 7Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{4c^3} \\
&= -\frac{(11bB - 7Ac)x^{3/2}}{6c^3} + \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} + \frac{(b(11bB - 7Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{b+cx^2} dx\right)}{2c^3} \\
&= -\frac{(11bB - 7Ac)x^{3/2}}{6c^3} + \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} - \frac{(b(11bB - 7Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{b+cx^2} dx\right)}{4c^{7/2}} \\
&= -\frac{(11bB - 7Ac)x^{3/2}}{6c^3} + \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} + \frac{(b(11bB - 7Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{b+cx^2} dx\right)}{8c^3} \\
&= -\frac{(11bB - 7Ac)x^{3/2}}{6c^3} + \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} + \frac{b^{3/4}(11bB - 7Ac) \log\left(\sqrt{\frac{b+cx^2}{b}}\right)}{8\sqrt{2}c^3} \\
&= -\frac{(11bB - 7Ac)x^{3/2}}{6c^3} + \frac{(11bB - 7Ac)x^{7/2}}{14bc^2} - \frac{(bB - Ac)x^{11/2}}{2bc(b + cx^2)} - \frac{b^{3/4}(11bB - 7Ac) \tan^{-1}\left(\sqrt{\frac{b+cx^2}{b}}\right)}{4\sqrt{2}c^{15/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.30, size = 154, normalized size = 0.50

$$\frac{(-b)^{3/4}(3bB - 2Ac) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{-b}}\right)}{c^{15/4}} + \frac{(-b)^{3/4}(3bB - 2Ac) \operatorname{tanh}^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{-b}}\right)}{c^{15/4}} + \frac{2x^{3/2}(Ac - bB) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3c^3} + \frac{2x^{3/2}(Ac - 2bB)}{3c^3} + \frac{2Bx^{7/2}}{7c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*(-2\*b\*B + A\*c)\*x^(3/2))/(3\*c^3) + (2\*B\*x^(7/2))/(7\*c^2) - ((-b)^(3/4)\*(3\*b\*B - 2\*A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)]/c^(15/4) + ((-b)^(3/4)\*(3\*b\*B - 2\*A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)]/c^(15/4) + (2\*(-(b\*B) + A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -(c\*x^2)/b])/(3\*c^3)

**IntegrateAlgebraic [A]** time = 0.75, size = 362, normalized size = 1.17

$$\frac{\sqrt{2}Ab^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{c}}\right)}{c^{15/4}} - \frac{Ab^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{c}}\right)}{4\sqrt{2}c^{15/4}} + \frac{7Ab^{3/4} \operatorname{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b+\sqrt{c}x}}\right)}{4\sqrt{2}c^{15/4}} + \frac{x^{3/2}(49Abc - 77b^2B) + x^{7/2}(28Ac^2 - 44bBc) + 12Bc^2x^{11/2}}{42c^3(b + cx^2)} - \frac{11b^{7/4}B \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{c}}\right)}{4\sqrt{2}c^{15/4}} - \frac{11b^{7/4}B \operatorname{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b+\sqrt{c}x}}\right)}{4\sqrt{2}c^{15/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((-77\*b^2\*B + 49\*A\*b\*c)\*x^(3/2) + (-44\*b\*B\*c + 28\*A\*c^2)\*x^(7/2) + 12\*B\*c^2\*x^(11/2))/(42\*c^3\*(b + c\*x^2)) - (11\*b^(7/4)\*B\*ArcTan[(b^(1/4)/(Sqrt[2]\*c^

$$\begin{aligned} & \left( \frac{1}{4} \right) - \left( c^{(1/4)} * x \right) / \left( \text{Sqrt}[2] * b^{(1/4)} \right) / \text{Sqrt}[x] \right) / \left( 4 * \text{Sqrt}[2] * c^{(15/4)} \right) - \left( A * b^{(3/4)} * \text{ArcTan} \left[ \left( b^{(1/4)} / \left( \text{Sqrt}[2] * c^{(1/4)} \right) - \left( c^{(1/4)} * x \right) / \left( \text{Sqrt}[2] * b^{(1/4)} \right) \right] / \text{Sqrt}[x] \right) \right) / \left( 4 * \text{Sqrt}[2] * c^{(11/4)} \right) + \left( \text{Sqrt}[2] * A * b^{(3/4)} * \text{ArcTan} \left[ \left( b^{(1/4)} / \left( \text{Sqrt}[2] * c^{(1/4)} \right) - \left( c^{(1/4)} * x \right) / \left( \text{Sqrt}[2] * b^{(1/4)} \right) \right] / \text{Sqrt}[x] \right) \right) / c^{(11/4)} - \left( 11 * b^{(7/4)} * B * \text{ArcTanh} \left[ \left( \text{Sqrt}[2] * b^{(1/4)} * c^{(1/4)} * \text{Sqrt}[x] \right) / \left( \text{Sqrt}[b] + \text{Sqrt}[c] * x \right) \right] \right) / \left( 4 * \text{Sqrt}[2] * c^{(15/4)} \right) + \left( 7 * A * b^{(3/4)} * \text{ArcTanh} \left[ \left( \text{Sqrt}[2] * b^{(1/4)} * c^{(1/4)} * \text{Sqrt}[x] \right) / \left( \text{Sqrt}[b] + \text{Sqrt}[c] * x \right) \right] \right) / \left( 4 * \text{Sqrt}[2] * c^{(11/4)} \right) \end{aligned}$$

**fricas [B]** time = 0.45, size = 989, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{168} * (84 * (c^4 * x^2 + b * c^3) * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(1/4)} * \arctan \left( \frac{\sqrt{\left( (1771561 * B^6 * b^{10} - 6764142 * A * B^5 * b^9 * c + 10761135 * A^2 * B^4 * b^8 * c^2 - 9130660 * A^3 * B^3 * b^7 * c^3 + 4357815 * A^4 * B^2 * b^6 * c^4 - 1109262 * A^5 * B * b^5 * c^5 + 117649 * A^6 * b^4 * c^6) * x - (14641 * B^4 * b^7 * c^7 - 37268 * A * B^3 * b^6 * c^8 + 35574 * A^2 * B^2 * b^5 * c^9 - 15092 * A^3 * B * b^4 * c^{10} + 2401 * A^4 * b^3 * c^{11}) * \sqrt{- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15}}}{(14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15}} \right) * c^4 * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(1/4)} + (1331 * B^3 * b^5 * c^4 - 2541 * A * B^2 * b^4 * c^5 + 1617 * A^2 * B * b^3 * c^6 - 343 * A^3 * b^2 * c^7) * \sqrt{x} * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(1/4)} / (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) - 21 * (c^4 * x^2 + b * c^3) * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(1/4)} * \log \left( c^{11} * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(3/4)} - (1331 * B^3 * b^5 - 2541 * A * B^2 * b^4 * c + 1617 * A^2 * B * b^3 * c^2 - 343 * A^3 * b^2 * c^3) * \sqrt{x} \right) + 21 * (c^4 * x^2 + b * c^3) * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(1/4)} * \log \left( - c^{11} * (- (14641 * B^4 * b^7 - 37268 * A * B^3 * b^6 * c + 35574 * A^2 * B^2 * b^5 * c^2 - 15092 * A^3 * B * b^4 * c^3 + 2401 * A^4 * b^3 * c^4) / c^{15})^{(3/4)} - (1331 * B^3 * b^5 - 2541 * A * B^2 * b^4 * c + 1617 * A^2 * B * b^3 * c^2 - 343 * A^3 * b^2 * c^3) * \sqrt{x} \right) + 4 * (12 * B * c^2 * x^5 - 4 * (11 * B * b * c - 7 * A * c^2) * x^3 - 7 * (11 * B * b^2 - 7 * A * b * c) * x) * \sqrt{x} / (c^4 * x^2 + b * c^3)$$

**giac [A]** time = 0.27, size = 299, normalized size = 0.96

$$\frac{\sqrt{2} \left( (11 (bc)^{\frac{1}{2}} Bb - 7 (bc)^{\frac{3}{2}} A \right) \arctan \left( \frac{\sqrt{2} \sqrt{(b/c)^{\frac{1}{2}} + x}}{z^{(1/4)}} \right) + \sqrt{2} \left( (11 (bc)^{\frac{1}{2}} Bb - 7 (bc)^{\frac{3}{2}} A \right) \arctan \left( - \frac{\sqrt{2} \sqrt{(b/c)^{\frac{1}{2}} + x}}{z^{(1/4)}} \right) - \sqrt{2} \left( (11 (bc)^{\frac{1}{2}} Bb - 7 (bc)^{\frac{3}{2}} A \right) \log \left( \sqrt{2} \sqrt{(b/c)^{\frac{1}{2}} + x} + \sqrt{c} \right) + \sqrt{2} \left( (11 (bc)^{\frac{1}{2}} Bb - 7 (bc)^{\frac{3}{2}} A \right) \log \left( - \sqrt{2} \sqrt{(b/c)^{\frac{1}{2}} + x} + \sqrt{c} \right) + \frac{2 \left( 3 B b^{13} x^3 - 14 B b^{11} x^2 + 7 A c^{12} x^2 \right)}{21 c^{14}}}{8 c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2 * (B * b^2 * x^{(3/2)} - A * b * c * x^{(3/2)}) / ((c * x^2 + b) * c^3) + 1/8 * \sqrt{2} * (11 * (b * c^3)^{(3/4)} * B * b - 7 * (b * c^3)^{(3/4)} * A * c) * \arctan \left( \frac{1/2 * \sqrt{2} * (\sqrt{2} * (b/c)^{(1/4)} + 2 * \sqrt{x})}{(b/c)^{(1/4)}} \right) / c^6 + 1/8 * \sqrt{2} * (11 * (b * c^3)^{(3/4)} * B * b - 7 * (b * c^3)^{(3/4)} * A * c) * \arctan \left( \frac{-1/2 * \sqrt{2} * (\sqrt{2} * (b/c)^{(1/4)} - 2 * \sqrt{x})}{(b/c)^{(1/4)}} \right) / c^6 - 1/16 * \sqrt{2} * (11 * (b * c^3)^{(3/4)} * B * b - 7 * (b * c^3)^{(3/4)} * A * c) * \log \left( \frac{\sqrt{2} * \sqrt{x} * (b/c)^{(1/4)} + x + \sqrt{b/c}}{c^6} \right) + 1/16 * \sqrt{2} * (11 * (b * c^3)^{(3/4)} * B * b - 7 * (b * c^3)^{(3/4)} * A * c) * \log \left( \frac{-\sqrt{2} * \sqrt{x} * (b/c)^{(1/4)} + x + \sqrt{b/c}}{c^6} \right) + 2/21 * (3 * B * c^{12} * x^{(7/2)} - 14 * B * b * c^{11} * x^{(3/2)} + 7 * A * c^{12} * x^{(3/2)}) / c^{14} \end{aligned}$$

**maple [A]** time = 0.06, size = 348, normalized size = 1.12

$$\frac{2Bx^{\frac{3}{2}}}{7c^2} + \frac{ABx^{\frac{3}{2}}}{2(c^2x^2+b)c^2} - \frac{Bb^2x^{\frac{3}{2}}}{2(c^2x^2+b)c^3} + \frac{2Aa^{\frac{3}{2}}}{3c^2} - \frac{4Bba^{\frac{3}{2}}}{3c^3} - \frac{7\sqrt{2}A\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{c}}{c}\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}}c^3} - \frac{7\sqrt{2}A\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{c}}{c}+1\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}}c^3} - \frac{7\sqrt{2}Ab\ln\left(\frac{c+\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}{c+\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}-\sqrt{c}}\right)}{16\left(\frac{c}{b}\right)^{\frac{1}{4}}c^3} + \frac{11\sqrt{2}Bb^2\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{c}}{c}-1\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}}c^4} + \frac{11\sqrt{2}Bb^2\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{c}}{c}+1\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}}c^4} + \frac{11\sqrt{2}Bb^2\ln\left(\frac{c+\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}{c+\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}-\sqrt{c}}\right)}{16\left(\frac{c}{b}\right)^{\frac{1}{4}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $2/7/c^2*B*x^{(7/2)}+2/3/c^2*x^{(3/2)}*A-4/3/c^3*x^{(3/2)}*b*B+1/2*b/c^2*x^{(3/2)}/(c*x^2+b)*A-1/2*b^2/c^3*x^{(3/2)}/(c*x^2+b)*B-7/16*b/c^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))-7/8*b/c^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)})*x^{(1/2)}+1)-7/8*b/c^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)})*x^{(1/2)}-1)+11/16*b^2/c^4/(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+11/8*b^2/c^4/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)})*x^{(1/2)}+1)+11/8*b^2/c^4/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)})*x^{(1/2)}-1)$

**maxima [A]** time = 3.11, size = 247, normalized size = 0.80

$$\frac{(11Bb^2 - 7Abc) \left( \frac{2\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}\left(\sqrt{2b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}\left(\sqrt{2b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16c^3} + \frac{2(3Bcx^{\frac{7}{2}} - 7(2Bb - Ac)x^{\frac{3}{2}})}{21c^3} - \frac{(Bb^2 - Abc)x^{\frac{3}{2}}}{2(c^4x^2 + bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-1/2*(B*b^2 - A*b*c)*x^{(3/2)}/(c^4*x^2 + b*c^3) + 1/16*(11*B*b^2 - 7*A*b*c)*(2*\sqrt{2}*\operatorname{arctan}(1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} + 2*\sqrt{c})*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})*\sqrt{c}})/(\sqrt{(\sqrt{b}*\sqrt{c})*\sqrt{c}})*\sqrt{c} + 2*\sqrt{2}*\operatorname{arctan}(-1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} - 2*\sqrt{c})*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})*\sqrt{c}})/(\sqrt{(\sqrt{b}*\sqrt{c})*\sqrt{c}})*\sqrt{c} - \sqrt{2}*\log(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*x^{(1/4)}*\sqrt{x} + \sqrt{c}x + \sqrt{b}))/(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*x^{(1/4)}*\sqrt{x} + \sqrt{c}x + \sqrt{b}))/(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*x^{(1/4)}*\sqrt{x} + \sqrt{c}x + \sqrt{b}))/c^3 + 2/21*(3*B*c*x^{(7/2)} - 7*(2*B*b - A*c)*x^{(3/2)})/c^3$

**mupad [B]** time = 0.17, size = 127, normalized size = 0.41

$$x^{3/2} \left( \frac{2A}{3c^2} - \frac{4Bb}{3c^3} \right) + \frac{2Bx^{7/2}}{7c^2} - \frac{x^{3/2} \left( \frac{Bb^2}{2} - \frac{Abc}{2} \right)}{c^4x^2 + bc^3} + \frac{(-b)^{3/4} \operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right) (7Ac - 11Bb)}{4c^{15/4}} + \frac{(-b)^{3/4} \operatorname{atan}\left(\frac{c^{1/4}\sqrt{x} \operatorname{li}}{(-b)^{1/4}}\right) (7Ac - 11Bb) \operatorname{li}}{4c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^{(3/2)}*((2*A)/(3*c^2) - (4*B*b)/(3*c^3)) + (2*B*x^{(7/2)})/(7*c^2) - (x^{(3/2)})*((B*b^2)/2 - (A*b*c)/2)/(b*c^3 + c^4*x^2) + ((-b)^{(3/4)}*\operatorname{atan}((c^{(1/4)}*x^{(1/2)})/(-b)^{(1/4)})*(7*A*c - 11*B*b))/(4*c^{(15/4)}) + ((-b)^{(3/4)}*\operatorname{atan}((c^{(1/4)}*x^{(1/2)}*1i)/(-b)^{(1/4)})*(7*A*c - 11*B*b)*1i)/(4*c^{(15/4)})$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(17/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.197 \quad \int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=310

$$-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}$$

**Rubi [A]** time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{5/2}(9bB-5Ac)}{10bc^2}-\frac{\sqrt{x}(9bB-5Ac)}{2c^3}-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{13/4}}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((9\*b\*B - 5\*A\*c)\*Sqrt[x])/(2\*c^3) + ((9\*b\*B - 5\*A\*c)\*x^(5/2))/(10\*b\*c^2) - ((b\*B - A\*c)\*x^(9/2))/(2\*b\*c\*(b + c\*x^2)) - (b^(1/4)\*(9\*b\*B - 5\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*c^(13/4)) + (b^(1/4)\*(9\*b\*B - 5\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*c^(13/4)) - (b^(1/4)\*(9\*b\*B - 5\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*c^(13/4)) + (b^(1/4)\*(9\*b\*B - 5\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*c^(13/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[(b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*



```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{15/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^{7/2} (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} + \frac{\left(\frac{9bB}{2} - \frac{5Ac}{2}\right) \int \frac{x^{7/2}}{b+cx^2} dx}{2bc} \\
&= \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} - \frac{(9bB - 5Ac) \int \frac{x^{3/2}}{b+cx^2} dx}{4c^2} \\
&= -\frac{(9bB - 5Ac)\sqrt{x}}{2c^3} + \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} + \frac{(b(9bB - 5Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{4c^3} \\
&= -\frac{(9bB - 5Ac)\sqrt{x}}{2c^3} + \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} + \frac{(b(9bB - 5Ac)) \text{Subst}\left(\int \frac{1}{b+cx^4} dx\right)}{2c^3} \\
&= -\frac{(9bB - 5Ac)\sqrt{x}}{2c^3} + \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} + \frac{(\sqrt{b}(9bB - 5Ac)) \text{Subst}\left(\int \frac{\sqrt{b}}{b+cx^4} dx\right)}{4c^3} \\
&= -\frac{(9bB - 5Ac)\sqrt{x}}{2c^3} + \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} + \frac{(\sqrt{b}(9bB - 5Ac)) \text{Subst}\left(\int \frac{\sqrt{b}}{\sqrt{c}} dx\right)}{8c^{7/2}} \\
&= -\frac{(9bB - 5Ac)\sqrt{x}}{2c^3} + \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} - \frac{\sqrt[4]{b}(9bB - 5Ac) \log(\sqrt{b} - \sqrt{2})}{8\sqrt{2}c^{13/4}} \\
&= -\frac{(9bB - 5Ac)\sqrt{x}}{2c^3} + \frac{(9bB - 5Ac)x^{5/2}}{10bc^2} - \frac{(bB - Ac)x^{9/2}}{2bc(b + cx^2)} - \frac{\sqrt[4]{b}(9bB - 5Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}}{4}\sqrt{\frac{b}{c}}\right)}{4\sqrt{2}c^{13/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.55, size = 385, normalized size = 1.24

$$\frac{-10\sqrt{2}\sqrt{b}(9bB - 5Ac)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{b}\sqrt{c}}{4}\right) + 10\sqrt{2}\sqrt{b}(9bB - 5Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}}{4}\right) + \frac{40b^{5/4}c}{3c^{13/4}} + 25\sqrt{2}A\sqrt{b}\sqrt{c}\log(-\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) - 25\sqrt{2}A\sqrt{b}\sqrt{c}\log(\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) + 160Ac^{5/4}\sqrt{c} - 45\sqrt{2}B^{3/4}\log(-\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) + 45\sqrt{2}B^{3/4}\log(\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) - \frac{40b^{5/4}c}{3c^{13/4}} - 320B^{3/4}\sqrt{c} + 320B^{3/4}c^{1/2}}{8c^{13/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (-320\*b\*B\*c^(1/4)\*Sqrt[x] + 160\*A\*c^(5/4)\*Sqrt[x] + 32\*B\*c^(5/4)\*x^(5/2) - (40\*b^2\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2) + (40\*A\*b\*c^(5/4)\*Sqrt[x])/(b + c\*x^2) - 10\*Sqrt[2]\*b^(1/4)\*(9\*b\*B - 5\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 10\*Sqrt[2]\*b^(1/4)\*(9\*b\*B - 5\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 45\*Sqrt[2]\*b^(5/4)\*B\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 25\*Sqrt[2]\*A\*b^(1/4)\*c\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 45\*Sqrt[2]\*b^(5/4)\*B\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 25\*Sqrt[2]\*A\*b^(1/4)\*c\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(80\*c^(13/4))

**IntegrateAlgebraic [A]** time = 0.68, size = 208, normalized size = 0.67

$$-\frac{(9b^{5/4}B - 5A\sqrt[4]{bc}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{cx}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}c^{13/4}} + \frac{(9b^{5/4}B - 5A\sqrt[4]{bc}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{cx}}\right)}{4\sqrt{2}c^{13/4}} + \frac{25Abc\sqrt{x} + 20Ac^2x^{5/2} - 45b^2B\sqrt{x} - 36bBcx^{5/2} + 4Bc^2x^{9/2}}{10c^3(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (-45\*b^2\*B\*Sqrt[x] + 25\*A\*b\*c\*Sqrt[x] - 36\*b\*B\*c\*x^(5/2) + 20\*A\*c^2\*x^(5/2) + 4\*B\*c^2\*x^(9/2))/(10\*c^3\*(b + c\*x^2)) - ((9\*b^(5/4)\*B - 5\*A\*b^(1/4)\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(4\*Sqrt[2]\*c^(13/4)) + ((9\*b^(5/4)\*B - 5\*A\*b^(1/4)\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(4\*Sqrt[2]\*c^(13/4))

**fricas [B]** time = 0.45, size = 748, normalized size = 2.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] -1/40\*(20\*(c^4\*x^2 + b\*c^3)\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(1/4)\*arctan((sqrt(c^6\*sqrt(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13) + (81\*B^2\*b^2 - 90\*A\*B\*b\*c + 25\*A^2\*c^2)\*x)\*c^10\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(3/4) + (9\*B\*b\*c^10 - 5\*A\*c^11)\*sqrt(x)\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(3/4))/(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)) + 5\*(c^4\*x^2 + b\*c^3)\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(1/4)\*log(c^3\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(1/4) - (9\*B\*b - 5\*A\*c)\*sqrt(x)) - 5\*(c^4\*x^2 + b\*c^3)\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(1/4)\*log(-c^3\*(-(6561\*B^4\*b^5 - 14580\*A\*B^3\*b^4\*c + 12150\*A^2\*B^2\*b^3\*c^2 - 4500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^13)^(1/4) - (9\*B\*b - 5\*A\*c)\*sqrt(x)) - 4\*(4\*B\*c^2\*x^4 - 45\*B\*b^2 + 25\*A\*b\*c - 4\*(9\*B\*b\*c - 5\*A\*c^2)\*x^2)\*sqrt(x))/(c^4\*x^2 + b\*c^3)

**giac [A]** time = 0.19, size = 298, normalized size = 0.96

$$\frac{\sqrt{c} \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Arctan} \left( \frac{\sqrt{c} \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{c}}{2 \left( \frac{5}{2} \right)^{\frac{1}{2}}} \right)}{8c^4} + \frac{\sqrt{c} \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Arctan} \left( \frac{\sqrt{c} \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{c}}{2 \left( \frac{5}{2} \right)^{\frac{1}{2}}} \right)}{8c^4} + \frac{\sqrt{2} \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Bb} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{c} \left( \frac{5}{2} \right)^{\frac{1}{2}} + x + \sqrt{c}}{2 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{c}} \right)}{16c^4} - \frac{\sqrt{2} \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Bb} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{c} \left( \frac{5}{2} \right)^{\frac{1}{2}} - x + \sqrt{c}}{2 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{c}} \right)}{16c^4} - \frac{2 \operatorname{Bb}^2 \sqrt{c} - 10 \operatorname{Bb} c^2 \sqrt{c} + 5 A c^4 \sqrt{c}}{2 (c x^2 + b) c^3} + \frac{2 \operatorname{Bb}^2 \sqrt{c} - 10 \operatorname{Bb} c^2 \sqrt{c} + 5 A c^4 \sqrt{c}}{5 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 1/8\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/c^4 + 1/8\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/c^4 + 1/16\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/16\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2\*(B\*b^2\*sqrt(x) - A\*b\*c\*sqrt(x))/((c\*x^2 + b)\*c^3) + 2/5\*(B\*c^8\*x^(5/2) - 10\*B\*b\*c^7\*sqrt(x) + 5\*A\*c^8\*sqrt(x))/c^10

**maple [A]** time = 0.07, size = 339, normalized size = 1.09

$$\frac{2 \operatorname{Bb}^2 \sqrt{c} + 2 (c x^2 + b) c^2}{5 c^2} - \frac{2 \operatorname{Bb}^2 \sqrt{c}}{2 (c x^2 + b) c^2} - \frac{5 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Arctan} \left( \frac{\sqrt{2} \sqrt{c} - 1}{\left( \frac{5}{2} \right)^{\frac{1}{2}}} \right)}{8 c^2} - \frac{5 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Arctan} \left( \frac{\sqrt{2} \sqrt{c} + 1}{\left( \frac{5}{2} \right)^{\frac{1}{2}}} \right)}{8 c^2} - \frac{5 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} A \ln \left( \frac{x \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \sqrt{c} + \sqrt{c}}{x \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \sqrt{c} + \sqrt{c}} \right)}{16 c^2} + \frac{9 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Bb} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{c} - 1}{\left( \frac{5}{2} \right)^{\frac{1}{2}}} \right)}{8 c^3} + \frac{9 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Bb} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{c} + 1}{\left( \frac{5}{2} \right)^{\frac{1}{2}}} \right)}{8 c^3} + \frac{9 \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \operatorname{Bb} \ln \left( \frac{x \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \sqrt{c} + \sqrt{c}}{x \left( \frac{5}{2} \right)^{\frac{1}{2}} \sqrt{2} \sqrt{c} + \sqrt{c}} \right)}{16 c^3} + \frac{2 A \sqrt{c} - 4 \operatorname{Bb} \sqrt{c}}{c^2} - \frac{2 A \sqrt{c} - 4 \operatorname{Bb} \sqrt{c}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $2/5/c^2*B*x^{(5/2)}+2/c^2*A*x^{(1/2)}-4/c^3*b*B*x^{(1/2)}+1/2*b/c^2*x^{(1/2)}/(c*x^2+b)*A-1/2*b^2/c^3*x^{(1/2)}/(c*x^2+b)*B-5/8/c^2*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)-5/16/c^2*(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))-5/8/c^2*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+9/8*b/c^3*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)+9/16*b/c^3*(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+9/8*b/c^3*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)$

**maxima** [A] time = 3.03, size = 271, normalized size = 0.87

$$\frac{(Bb^2 - Abc)\sqrt{x}}{2(c^4x^2 + bc^3)} + \frac{\left( \frac{2\sqrt{2}(9Bb-5Ac)\arctan\left(\frac{\sqrt{2}\sqrt{b^4c^4+2\sqrt{c}x}}{2\sqrt{b^4c^4}}\right)}{\sqrt{b}\sqrt{b^4c^4}} + \frac{2\sqrt{2}(9Bb-5Ac)\arctan\left(\frac{\sqrt{2}\sqrt{b^4c^4-2\sqrt{c}x}}{2\sqrt{b^4c^4}}\right)}{\sqrt{b}\sqrt{b^4c^4}} + \frac{\sqrt{2}(9Bb-5Ac)\log\left(\frac{\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right) - \sqrt{2}(9Bb-5Ac)\log\left(\frac{\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}} \right)}{16c^3} + \frac{2(Bcx^{\frac{5}{2}} - 5(2Bb - Ac)\sqrt{x})}{5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-1/2*(B*b^2 - A*b*c)*\text{sqrt}(x)/(c^4*x^2 + b*c^3) + 1/16*(2*\text{sqrt}(2)*(9*B*b - 5*A*c)*\arctan(1/2*\text{sqrt}(2)*(sqrt(2)*b^{(1/4)}*c^{(1/4)} + 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(b)*\text{sqrt}(sqrt(b)*\text{sqrt}(c))) + 2*\text{sqrt}(2)*(9*B*b - 5*A*c)*\arctan(-1/2*\text{sqrt}(2)*(sqrt(2)*b^{(1/4)}*c^{(1/4)} - 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(b)*\text{sqrt}(sqrt(b)*\text{sqrt}(c))) + \text{sqrt}(2)*(9*B*b - 5*A*c)*\log(\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/(b^{(3/4)}*c^{(1/4)}) - \text{sqrt}(2)*(9*B*b - 5*A*c)*\log(-\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/(b^{(3/4)}*c^{(1/4)}) + b/c^3 + 2/5*(B*c*x^{(5/2)} - 5*(2*B*b - A*c)*\text{sqrt}(x))/c^3$

**mapad** [B] time = 0.26, size = 823, normalized size = 2.65

$$\sqrt{\frac{(2A - 4Bb)}{c^2}} + \frac{2Bb^2}{c^2} + \frac{\sqrt{\frac{(2A - 4Bb)}{c^2}}}{c^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $x^{(1/2)}*((2*A)/c^2 - (4*B*b)/c^3) + (2*B*x^{(5/2)})/(5*c^2) - (x^{(1/2)}*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^{(1/4)}*\text{atan}((((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 - ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^{(13/4)}))*(5*A*c - 9*B*b)*1i)/(8*c^{(13/4)}) + ((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 + ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^{(13/4)}))*(5*A*c - 9*B*b)*1i)/(8*c^{(13/4)})))/((((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 - ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^{(13/4)}))*(5*A*c - 9*B*b))/(8*c^{(13/4)}) - ((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 + ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^{(13/4)}))*(5*A*c - 9*B*b))/(8*c^{(13/4)})))*((5*A*c - 9*B*b)*1i)/(4*c^{(13/4)}) + ((-b)^{(1/4)}*\text{atan}((((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 - ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^{(13/4)}))*(5*A*c - 9*B*b))/(8*c^{(13/4)}) + ((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 + ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^{(13/4)}))*(5*A*c - 9*B*b))/(8*c^{(13/4)})))/((((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 - ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^{(13/4)}))*(5*A*c - 9*B*b)*1i)/(8*c^{(13/4)}) - ((-b)^{(1/4)}*((x^{(1/2)}*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)))/c^3 + ((-b)^{(1/4)}*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^{(13/4)}))*(5*A*c - 9*B*b)*1i)/(8*c^{(13/4)})))*((5*A*c - 9*B*b))/(4*c^{(13/4)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(15/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.198 \quad \int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=289

$$-\frac{(7bB-3Ac)\log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

**Rubi [A]** time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{3/2}(7bB-3Ac)}{6bc^2}-\frac{(7bB-3Ac)\log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((7\*b\*B - 3\*A\*c)\*x^(3/2))/(6\*b\*c^2) - ((b\*B - A\*c)\*x^(7/2))/(2\*b\*c\*(b + c\*x^2)) + ((7\*b\*B - 3\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*b^(1/4)\*c^(11/4)) - ((7\*b\*B - 3\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*b^(1/4)\*c^(11/4)) - ((7\*b\*B - 3\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(1/4)\*c^(11/4)) + ((7\*b\*B - 3\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(1/4)\*c^(11/4))

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^{5/2} (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{(bB - Ac)x^{7/2}}{2bc(b + cx^2)} + \frac{\left(\frac{7bB}{2} - \frac{3Ac}{2}\right) \int \frac{x^{5/2}}{b+cx^2} dx}{2bc} \\
&= \frac{(7bB - 3Ac)x^{3/2}}{6bc^2} - \frac{(bB - Ac)x^{7/2}}{2bc(b + cx^2)} - \frac{(7bB - 3Ac) \int \frac{\sqrt{x}}{b+cx^2} dx}{4c^2} \\
&= \frac{(7bB - 3Ac)x^{3/2}}{6bc^2} - \frac{(bB - Ac)x^{7/2}}{2bc(b + cx^2)} - \frac{(7bB - 3Ac) \operatorname{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{2c^2} \\
&= \frac{(7bB - 3Ac)x^{3/2}}{6bc^2} - \frac{(bB - Ac)x^{7/2}}{2bc(b + cx^2)} + \frac{(7bB - 3Ac) \operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{4c^{5/2}} - \frac{(7bB - 3Ac) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{8c^3} \\
&= \frac{(7bB - 3Ac)x^{3/2}}{6bc^2} - \frac{(bB - Ac)x^{7/2}}{2bc(b + cx^2)} - \frac{(7bB - 3Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{(7bB - 3Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}} - \frac{(7bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.26, size = 136, normalized size = 0.47

$$\frac{(3Ac - 6bB) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) + (6bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) + 2\sqrt[4]{-b} Bc^{3/4} x^{3/2}}{3\sqrt[4]{-b} c^{11/4}} + \frac{2x^{3/2}(bB - Ac) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3bc^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (2\*(-b)^(1/4)\*B\*c^(3/4)\*x^(3/2) + (-6\*b\*B + 3\*A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + (6\*b\*B - 3\*A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/(3\*(-b)^(1/4)\*c^(11/4)) + (2\*(b\*B - A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -(c\*x^2)/b])/(3\*b\*c^2)

**IntegrateAlgebraic [A]** time = 0.72, size = 167, normalized size = 0.58

$$\frac{(7bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{(7bB - 3Ac) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{x^{3/2}(-3Ac + 7bB + 4Bcx^2)}{6c^2(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (x^(3/2)\*(7\*b\*B - 3\*A\*c + 4\*B\*c\*x^2))/(6\*c^2\*(b + c\*x^2)) + ((7\*b\*B - 3\*A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(4\*Sqrt[



$2] * b^{(1/4)} * c^{(11/4)} + ((7 * b * B - 3 * A * c) * \text{ArcTanh}[(\text{Sqrt}[2] * b^{(1/4)} * c^{(1/4)} * \text{Sqrt}[x]) / (\text{Sqrt}[b] + \text{Sqrt}[c] * x)]) / (4 * \text{Sqrt}[2] * b^{(1/4)} * c^{(11/4)})$

**fricas [B]** time = 0.45, size = 925, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 
$$-1/24 * (12 * (c^3 * x^2 + b * c^2) * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(1/4)} * \arctan(\text{sqrt}((117649 * B^6 * b^6 - 302526 * A * B^5 * b^5 * c + 324135 * A^2 * B^4 * b^4 * c^2 - 185220 * A^3 * B^3 * b^3 * c^3 + 59535 * A^4 * B^2 * b^2 * c^4 - 10206 * A^5 * B * b * c^5 + 729 * A^6 * c^6) * x - (2401 * B^4 * b^5 * c^5 - 4116 * A * B^3 * b^4 * c^6 + 2646 * A^2 * B^2 * b^3 * c^7 - 756 * A^3 * B * b^2 * c^8 + 81 * A^4 * b * c^9) * \text{sqrt}(- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))) * c^3 * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(1/4)} + (343 * B^3 * b^3 * c^3 - 441 * A * B^2 * b^2 * c^4 + 189 * A^2 * B * b * c^5 - 27 * A^3 * c^6) * \text{sqrt}(x) * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(1/4)} / (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) - 3 * (c^3 * x^2 + b * c^2) * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(1/4)} * \log(b * c^8 * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(3/4)} - (343 * B^3 * b^3 - 441 * A * B^2 * b^2 * c + 189 * A^2 * B * b * c^2 - 27 * A^3 * c^3) * \text{sqrt}(x)) + 3 * (c^3 * x^2 + b * c^2) * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(1/4)} * \log(-b * c^8 * (- (2401 * B^4 * b^4 - 4116 * A * B^3 * b^3 * c + 2646 * A^2 * B^2 * b^2 * c^2 - 756 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b * c^{11}))^{(3/4)} - (343 * B^3 * b^3 - 441 * A * B^2 * b^2 * c + 189 * A^2 * B * b * c^2 - 27 * A^3 * c^3) * \text{sqrt}(x)) - 4 * (4 * B * c * x^3 + (7 * B * b - 3 * A * c) * x) * \text{sqrt}(x) / (c^3 * x^2 + b * c^2)$$

**giac [A]** time = 0.21, size = 283, normalized size = 0.98

$$\frac{\frac{2 B c^2}{3 c^2} + \frac{B b x^3}{2 (c x^2 + b) c^2} - \frac{A c x^3}{2 (c x^2 + b) c^2}}{8 b c^3} - \frac{\sqrt{2} \left( (b c^3)^{\frac{3}{4}} B b - 3 (b c^3)^{\frac{3}{4}} A c \right) \arctan\left(\frac{\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} + x \sqrt{\frac{b}{c}}}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 b c^3} - \frac{\sqrt{2} \left( (b c^3)^{\frac{3}{4}} B b - 3 (b c^3)^{\frac{3}{4}} A c \right) \arctan\left(\frac{\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} - x \sqrt{\frac{b}{c}}}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 b c^3} + \frac{\sqrt{2} \left( (b c^3)^{\frac{3}{4}} B b - 3 (b c^3)^{\frac{3}{4}} A c \right) \log\left(\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} + x \sqrt{\frac{b}{c}}}\right)}{16 b c^3} - \frac{\sqrt{2} \left( (b c^3)^{\frac{3}{4}} B b - 3 (b c^3)^{\frac{3}{4}} A c \right) \log\left(-\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} + x \sqrt{\frac{b}{c}}}\right)}{16 b c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 
$$2/3 * B * x^{(3/2)} / c^2 + 1/2 * (B * b * x^{(3/2)} - A * c * x^{(3/2)}) / ((c * x^2 + b) * c^2) - 1/8 * \text{sqrt}(2) * (7 * (b * c^3)^{(3/4)} * B * b - 3 * (b * c^3)^{(3/4)} * A * c) * \arctan(1/2 * \text{sqrt}(2) * (\text{sqrt}(2) * (b/c)^{(1/4)} + 2 * \text{sqrt}(x)) / (b/c)^{(1/4)}) / (b * c^5) - 1/8 * \text{sqrt}(2) * (7 * (b * c^3)^{(3/4)} * B * b - 3 * (b * c^3)^{(3/4)} * A * c) * \arctan(-1/2 * \text{sqrt}(2) * (\text{sqrt}(2) * (b/c)^{(1/4)} - 2 * \text{sqrt}(x)) / (b/c)^{(1/4)}) / (b * c^5) + 1/16 * \text{sqrt}(2) * (7 * (b * c^3)^{(3/4)} * B * b - 3 * (b * c^3)^{(3/4)} * A * c) * \log(\text{sqrt}(2) * \text{sqrt}(x) * (b/c)^{(1/4)} + x + \text{sqrt}(b/c)) / (b * c^5) - 1/16 * \text{sqrt}(2) * (7 * (b * c^3)^{(3/4)} * B * b - 3 * (b * c^3)^{(3/4)} * A * c) * \log(-\text{sqrt}(2) * \text{sqrt}(x) * (b/c)^{(1/4)} + x + \text{sqrt}(b/c)) / (b * c^5)$$

**maple [A]** time = 0.06, size = 317, normalized size = 1.10

$$-\frac{A x^3}{2 (c x^2 + b) c} + \frac{B b x^3}{2 (c x^2 + b) c^2} + \frac{2 B x^3}{3 c^2} + \frac{3 \sqrt{2} A \arctan\left(\frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} + \frac{3 \sqrt{2} A \arctan\left(\frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} + \frac{3 \sqrt{2} A \ln\left(\frac{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}\right)}{16 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} - \frac{7 \sqrt{2} B b \arctan\left(\frac{\sqrt{2} \sqrt{x} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^3} - \frac{7 \sqrt{2} B b \arctan\left(\frac{\sqrt{2} \sqrt{x} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^3} - \frac{7 \sqrt{2} B b \ln\left(\frac{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}\right)}{16 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $\frac{2}{3}Bx^{3/2}/c^2 - 1/2/c*x^{3/2}/(c*x^2+b)*A + 1/2/c^2*x^{3/2}/(c*x^2+b)*b*B - 7/16/c^3/(b/c)^{1/4}*2^{1/2}*b*B*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))/((x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})) - 7/8/c^3/(b/c)^{1/4}*2^{1/2}*b*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1) - 7/8/c^3/(b/c)^{1/4}*2^{1/2}*b*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1) + 3/16/c^2/(b/c)^{1/4}*2^{1/2}*A*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))/((x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})) + 3/8/c^2/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1) + 3/8/c^2/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)$

**maxima** [A] time = 3.31, size = 223, normalized size = 0.77

$$\frac{(Bb - 3Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\frac{1}{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\frac{1}{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{2(c^3x^2 + bc^2) + \frac{2Bx^3}{3c^2}} - \frac{16c^2}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}(Bb - A*c)*x^{3/2}/(c^3*x^2 + b*c^2) + \frac{2}{3}B*x^{3/2}/c^2 - \frac{1}{16}(7*B*b - 3*A*c)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x})})/\sqrt{(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x})}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x})})/\sqrt{(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x})}) - \sqrt{2}*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}x + \sqrt{b})/(b^{1/4}*c^{3/4}) + \sqrt{2}*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}x + \sqrt{b})/(b^{1/4}*c^{3/4}))/c^2$

**mupad** [B] time = 0.25, size = 106, normalized size = 0.37

$$\frac{2Bx^{3/2}}{3c^2} - \frac{x^{3/2}\left(\frac{Ac}{2} - \frac{Bb}{2}\right)}{c^3x^2 + bc^2} + \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(3Ac - 7Bb)}{4(-b)^{1/4}c^{11/4}} + \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}1i}{(-b)^{1/4}}\right)(3Ac - 7Bb)1i}{4(-b)^{1/4}c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)`

[Out]  $\frac{2*B*x^{3/2}}{3*c^2} - \frac{x^{3/2}*((A*c)/2 - (B*b)/2)}{(b*c^2 + c^3*x^2)} + \left(\operatorname{atan}\left(\frac{c^{1/4}*x^{1/2}}{(-b)^{1/4}}\right)*(3*A*c - 7*B*b)\right)/(4*(-b)^{1/4}*c^{11/4}) + \left(\operatorname{atan}\left(\frac{c^{1/4}*x^{1/2}*1i}{(-b)^{1/4}}\right)*(3*A*c - 7*B*b)*1i\right)/(4*(-b)^{1/4}*c^{11/4})$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)`

[Out] Timed out

$$3.199 \quad \int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=289

$$\frac{(5bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{3/4} c^{9/4}} - \frac{(5bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{3/4} c^{9/4}} + \frac{(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}}\right)}{4\sqrt{2} b^{3/4} c^{9/4}}$$

**Rubi [A]** time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(5bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{3/4} c^{9/4}} - \frac{(5bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{3/4} c^{9/4}} + \frac{(5bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}}\right)}{4\sqrt{2} b^{3/4} c^{9/4}} - \frac{(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}} + 1\right)}{4\sqrt{2} b^{3/4} c^{9/4}} + \frac{\sqrt{x} (5bB - Ac)}{2bc^2} - \frac{x^{5/2} (bB - Ac)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((5\*b\*B - A\*c)\*Sqrt[x])/(2\*b\*c^2) - ((b\*B - A\*c)\*x^(5/2))/(2\*b\*c\*(b + c\*x^2)) + ((5\*b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(3/4)\*c^(9/4)) - ((5\*b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(3/4)\*c^(9/4)) + ((5\*b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(3/4)\*c^(9/4)) - ((5\*b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(3/4)\*c^(9/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{x^{3/2} (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{(bB - Ac)x^{5/2}}{2bc(b + cx^2)} + \frac{\left(\frac{5bB}{2} - \frac{Ac}{2}\right) \int \frac{x^{3/2}}{b+cx^2} dx}{2bc} \\
&= \frac{(5bB - Ac)\sqrt{x}}{2bc^2} - \frac{(bB - Ac)x^{5/2}}{2bc(b + cx^2)} - \frac{(5bB - Ac) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{4c^2} \\
&= \frac{(5bB - Ac)\sqrt{x}}{2bc^2} - \frac{(bB - Ac)x^{5/2}}{2bc(b + cx^2)} - \frac{(5bB - Ac) \operatorname{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{2c^2} \\
&= \frac{(5bB - Ac)\sqrt{x}}{2bc^2} - \frac{(bB - Ac)x^{5/2}}{2bc(b + cx^2)} - \frac{(5bB - Ac) \operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{4\sqrt{b}c^2} - \frac{(5bB - Ac) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{8\sqrt{b}c^{5/2}} \\
&= \frac{(5bB - Ac)\sqrt{x}}{2bc^2} - \frac{(bB - Ac)x^{5/2}}{2bc(b + cx^2)} + \frac{(5bB - Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.53, size = 354, normalized size = 1.22

$$\frac{2\sqrt{2}(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{16c^{9/4}} - \frac{2\sqrt{2}(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{16c^{9/4}} - \frac{\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{16c^{9/4}} + \frac{\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{16c^{9/4}} - \frac{8A\sqrt{2}\sqrt{x}}{8b+4c^2} + \frac{8bB\sqrt{2}\sqrt{x}}{8b+4c^2} + 5\sqrt{2}\sqrt[4]{b}B\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) - 5\sqrt{2}\sqrt[4]{b}B\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) + 32B\sqrt[4]{c}\sqrt{x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (32\*B\*c^(1/4)\*Sqrt[x] + (8\*b\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2) - (8\*A\*c^(5/4)\*Sqrt[x])/(b + c\*x^2) + (2\*Sqrt[2]\*(5\*b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(b^(3/4)) - (2\*Sqrt[2]\*(5\*b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(b^(3/4)) + 5\*Sqrt[2]\*b^(1/4)\*B\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - (Sqrt[2]\*A\*c\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/b^(3/4) - 5\*Sqrt[2]\*b^(1/4)\*B\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + (Sqrt[2]\*A\*c\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/b^(3/4))/(16\*c^(9/4))

**IntegrateAlgebraic [A]** time = 0.71, size = 167, normalized size = 0.58

$$\frac{(5bB - Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} + \frac{\sqrt{x}(-Ac + 5bB + 4Bcx^2)}{2c^2(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out]  $(\text{Sqrt}[x]*(5*b*B - A*c + 4*B*c*x^2))/(2*c^2*(b + c*x^2)) + ((5*b*B - A*c)*\text{ArcTan}[(\text{Sqrt}[b] - \text{Sqrt}[c]*x)/(\text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x])])/(4*\text{Sqrt}[2]*b^{3/4}*c^{9/4}) - ((5*b*B - A*c)*\text{ArcTanh}[(\text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x])]/(\text{Sqrt}[b] + \text{Sqrt}[c]*x))/(4*\text{Sqrt}[2]*b^{3/4}*c^{9/4})$

**fricas [B]** time = 0.45, size = 725, normalized size = 2.51

$$\frac{\sqrt{2} \sqrt{x} \left( 5 b B - A c + 4 B c x^2 \right)}{2 c^2 (b + c x^2)} + \frac{\left( 5 b B - A c \right) \text{ArcTan} \left( \frac{\sqrt{b} - \sqrt{c} x}{\sqrt{2} b^{1/4} c^{1/4} \sqrt{x}} \right)}{4 \sqrt{2} b^{3/4} c^{9/4}} - \frac{\left( 5 b B - A c \right) \text{ArcTanh} \left( \frac{\sqrt{2} b^{1/4} c^{1/4} \sqrt{x}}{\sqrt{b} + \sqrt{c} x} \right)}{4 \sqrt{2} b^{3/4} c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out]  $\frac{1}{8} * (4 * (c^3 * x^2 + b * c^2) * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{1/4} * \arctan(\frac{\sqrt{b^2 * c^4 * \sqrt{x} - (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9)}}{x}) * b^2 * c^7 * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{3/4} + (5 * B * b^3 * c^7 - A * b^2 * c^8) * \sqrt{x} * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{3/4}) / (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) + (c^3 * x^2 + b * c^2) * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{1/4} * \log(b * c^2 * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{1/4} - (5 * B * b - A * c) * \sqrt{x}) - (c^3 * x^2 + b * c^2) * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{1/4} * \log(-b * c^2 * (- (625 * B^4 * b^4 - 500 * A * B^3 * b^3 * c + 150 * A^2 * B^2 * b^2 * c^2 - 20 * A^3 * B * b * c^3 + A^4 * c^4) / (b^3 * c^9))^{1/4} - (5 * B * b - A * c) * \sqrt{x}) + 4 * (4 * B * c * x^2 + 5 * B * b - A * c) * \sqrt{x}) / (c^3 * x^2 + b * c^2)$

**giac [A]** time = 0.19, size = 283, normalized size = 0.98

$$\frac{\sqrt{2} \sqrt{x} \left( 5 (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}}^{\frac{1}{2}} + \sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8 bc^3} - \frac{\sqrt{2} \left( 5 (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}}^{\frac{1}{2}} - \sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8 bc^3} - \frac{\sqrt{2} \left( 5 (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16 bc^3} + \frac{\sqrt{2} \left( 5 (bc^3)^{\frac{1}{2}} Bb - (bc^3)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16 bc^3} + \frac{Bb \sqrt{x} - A c \sqrt{x}}{2 (c x^2 + b) c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $2 * B * \sqrt{x} / c^2 - 1/8 * \sqrt{2} * (5 * (b * c^3)^{1/4} * B * b - (b * c^3)^{1/4} * A * c) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * (b/c)^{1/4} + 2 * \sqrt{x}) / (b/c)^{1/4}) / (b * c^3) - 1/8 * \sqrt{2} * (5 * (b * c^3)^{1/4} * B * b - (b * c^3)^{1/4} * A * c) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * (b/c)^{1/4} - 2 * \sqrt{x}) / (b/c)^{1/4}) / (b * c^3) - 1/16 * \sqrt{2} * (5 * (b * c^3)^{1/4} * B * b - (b * c^3)^{1/4} * A * c) * \log(\sqrt{2} * \sqrt{x} * (b/c)^{1/4} + x + \sqrt{b/c}) / (b * c^3) + 1/16 * \sqrt{2} * (5 * (b * c^3)^{1/4} * B * b - (b * c^3)^{1/4} * A * c) * \log(-\sqrt{2} * \sqrt{x} * (b/c)^{1/4} + x + \sqrt{b/c}) / (b * c^3) + 1/2 * (B * b * \sqrt{x} - A * c * \sqrt{x}) / ((c * x^2 + b) * c^2)$

**maple [A]** time = 0.06, size = 323, normalized size = 1.12

$$\frac{A \sqrt{x}}{2 (c x^2 + b) c} + \frac{B b \sqrt{x}}{2 (c x^2 + b) c^2} + \frac{\left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} - 1 \right)}{8 b c} + \frac{\left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} + 1 \right)}{8 b c} + \frac{\left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} A \ln \left( \frac{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{16 b c} - \frac{5 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} - 1 \right)}{8 c^2} - \frac{5 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x}}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} + 1 \right)}{8 c^2} - \frac{5 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \ln \left( \frac{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{16 c^2} + \frac{2 B \sqrt{x}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $2 * B / c^2 * x^{1/2} - 1/2 / c * x^{1/2} / (c * x^2 + b) * A + 1/2 / c^2 * x^{1/2} / (c * x^2 + b) * b * B + 1/8 / c * (b/c)^{1/4} / b * 2^{1/2} * A * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} - 1) + 1/16 / c * (b/c)^{1/4} / b * 2^{1/2} * A * \ln((x + (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2}) / (x - (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2})) + 1/8 / c * (b/c)^{1/4} / b * 2^{1/2} * A * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} + 1) - 5/8 / c^2 * (b/c)^{1/4} * 2^{1/2} * B * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} - 1) - 5/8 / c^2 * (b/c)^{1/4} * 2^{1/2} * B * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} + 1) - 5/8 / c^2 * (b/c)^{1/4} * 2^{1/2} * B * \ln((x + (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2}) / (x - (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2})) + 2 * B * \sqrt{x} / c^2$

$$\frac{1}{(b/c)^{1/4}} x^{1/2-1} - \frac{5}{16} \frac{1}{c^2} (b/c)^{1/4} 2^{1/2} B \ln\left(\frac{(x+(b/c)^{1/4}) 2^{1/2} (1/2) x^{1/2} + (b/c)^{1/2}}{(x-(b/c)^{1/4}) 2^{1/2} x^{1/2} + (b/c)^{1/2}}\right) - \frac{5}{8} \frac{1}{c^2} (b/c)^{1/4} 2^{1/2} B \arctan\left(\frac{2^{1/2}}{(b/c)^{1/4} x^{1/2} + 1}\right)$$

**maxima [A]** time = 3.11, size = 250, normalized size = 0.87

$$\frac{(Bb - Ac)\sqrt{x}}{2(c^3x^2 + bc^2)} + \frac{2B\sqrt{x}}{c^2} - \frac{2\sqrt{2}(5Bb - Ac) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(5Bb - Ac) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(5Bb - Ac) \log\left(\sqrt{2}b^{1/4}c^{1/4}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{3/4}c^{1/4}} - \frac{\sqrt{2}(5Bb - Ac) \log\left(-\sqrt{2}b^{1/4}c^{1/4}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{3/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}(Bb - Ac)\sqrt{x}/(c^3x^2 + bc^2) + 2B\sqrt{x}/c^2 - \frac{1}{16}(2\sqrt{2}(5Bb - Ac)\arctan(1/2\sqrt{2}(\sqrt{2}b^{1/4}c^{1/4} + 2\sqrt{c}\sqrt{x})/\sqrt{b}\sqrt{c}))/(\sqrt{b}\sqrt{b}\sqrt{c}) + 2\sqrt{2}(5Bb - Ac)\arctan(-1/2\sqrt{2}(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x})/\sqrt{b}\sqrt{c})/(\sqrt{b}\sqrt{b}\sqrt{c}) + \sqrt{2}(5Bb - Ac)\log(\sqrt{2}b^{1/4}c^{1/4}\sqrt{x} + \sqrt{c}x + \sqrt{b})/(b^{3/4}c^{1/4}) - \sqrt{2}(5Bb - Ac)\log(-\sqrt{2}b^{1/4}c^{1/4}\sqrt{x} + \sqrt{c}x + \sqrt{b})/(b^{3/4}c^{1/4})/c^2$

**mupad [B]** time = 0.21, size = 744, normalized size = 2.57

$$\frac{2B\sqrt{x}}{c^2} - \frac{\sqrt{x}\left(\frac{Ac}{2} - \frac{Bb}{2}\right)}{c^3x^2 + bc^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}} + \frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}} + (Ac - 5Bb) \operatorname{li}\left(\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}}\right) + \frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}} + (Ac - 5Bb) \operatorname{li}\left(\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $\frac{2B\sqrt{x}}{c^2} - \frac{\sqrt{x}\left(\frac{Ac}{2} - \frac{Bb}{2}\right)}{c^3x^2 + bc^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}} + \frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}} + (Ac - 5Bb) \operatorname{li}\left(\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}}\right) + \frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}} + (Ac - 5Bb) \operatorname{li}\left(\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{2}b^{1/4}c^{1/4} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4(-b)^{3/4}c^{1/4}}\right)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(11/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.200 \quad \int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=261

$$\frac{(Ac + 3bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} b^{5/4} c^{7/4}} - \frac{(Ac + 3bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} b^{5/4} c^{7/4}} - \frac{(Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x}\right)}{4\sqrt{2} b^{5/4} c^{7/4}}$$

**Rubi [A]** time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {1584, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(Ac + 3bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} b^{5/4} c^{7/4}} - \frac{(Ac + 3bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{8\sqrt{2} b^{5/4} c^{7/4}} - \frac{(Ac + 3bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{2} b^{5/4} c^{7/4}} + \frac{(Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{4\sqrt{2} b^{5/4} c^{7/4}} - \frac{x^{3/2}(bB - Ac)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((b\*B - A\*c)\*x^(3/2))/(2\*b\*c\*(b + c\*x^2)) - ((3\*b\*B + A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(5/4)\*c^(7/4)) + ((3\*b\*B + A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(5/4)\*c^(7/4))) + ((3\*b\*B + A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(8\*Sqrt[2]\*b^(5/4)\*c^(7/4)) - ((3\*b\*B + A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(8\*Sqrt[2]\*b^(5/4)\*c^(7/4)))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*b\*e\*n\*(p + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(e\*x)^m\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n\*(p + 1))]))

#### Rule 617



```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{\sqrt{x} (A + Bx^2)}{(b + cx^2)^2} dx \\
&= -\frac{(bB - Ac)x^{3/2}}{2bc(b + cx^2)} + \frac{\left(\frac{3bB}{2} + \frac{Ac}{2}\right) \int \frac{\sqrt{x}}{b+cx^2} dx}{2bc} \\
&= -\frac{(bB - Ac)x^{3/2}}{2bc(b + cx^2)} + \frac{\left(\frac{3bB}{2} + \frac{Ac}{2}\right) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{bc} \\
&= -\frac{(bB - Ac)x^{3/2}}{2bc(b + cx^2)} - \frac{(3bB + Ac) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{4bc^{3/2}} + \frac{(3bB + Ac) \text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{4bc^{3/2}} \\
&= -\frac{(bB - Ac)x^{3/2}}{2bc(b + cx^2)} + \frac{(3bB + Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{8bc^2} + \frac{(3bB + Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{8bc^2} \\
&= -\frac{(bB - Ac)x^{3/2}}{2bc(b + cx^2)} + \frac{(3bB + Ac) \log\left(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{c} x\right)}{8\sqrt{2} b^{5/4} c^{7/4}} - \frac{(3bB + Ac) \log\left(\sqrt{b} + \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{c} x\right)}{8\sqrt{2} b^{5/4} c^{7/4}} \\
&= -\frac{(bB - Ac)x^{3/2}}{2bc(b + cx^2)} - \frac{(3bB + Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{5/4} c^{7/4}} + \frac{(3bB + Ac) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{5/4} c^{7/4}} +
\end{aligned}$$

**Mathematica [C]** time = 0.18, size = 95, normalized size = 0.36

$$\frac{2x^{3/2}(Ac - bB) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^2c} + \frac{B\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right) + \tanh^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)\right)}{\sqrt[4]{-b}c^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (B\*(ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b\*c^(1/4)\*Sqrt[x])/(-b)^(5/4)]))/((-b)^(1/4)\*c^(7/4)) + (2\*(-(b\*B) + A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -(c\*x^2)/b])/(3\*b^2\*c)

**IntegrateAlgebraic [A]** time = 0.68, size = 160, normalized size = 0.61

$$-\frac{(Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}\right)}{4\sqrt{2} b^{5/4} c^{7/4}} - \frac{(Ac + 3bB) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2} b^{5/4} c^{7/4}} + \frac{x^{3/2}(Ac - bB)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] ((-(b\*B) + A\*c)\*x^(3/2))/(2\*b\*c\*(b + c\*x^2)) - ((3\*b\*B + A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(4\*Sqrt[2]\*b^(5/4)\*c^(7/4)) - ((3\*b\*B + A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)])/(4\*Sqrt[2]\*b^(5/4)\*c^(7/4))

**fricas [B]** time = 0.45, size = 912, normalized size = 3.49

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 
$$-1/8*(4*(B*b - A*c)*x^{(3/2)} + 4*(b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(1/4)} * \arctan(\sqrt{(729*B^6*b^6 + 1458*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 + 540*A^3*B^3*b^3*c^3 + 135*A^4*B^2*b^2*c^4 + 18*A^5*B*b*c^5 + A^6*c^6)*x - (81*B^4*b^7*c^3 + 108*A*B^3*b^6*c^4 + 54*A^2*B^2*b^5*c^5 + 12*A^3*B*b^4*c^6 + A^4*b^3*c^7)*\sqrt{-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7)}}) * b*c^2 * (-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(1/4)} - (27*B^3*b^4*c^2 + 27*A*B^2*b^3*c^3 + 9*A^2*B*b^2*c^4 + A^3*b*c^5)*\sqrt{x} * (-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(1/4)} / (81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4) - (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(1/4)} * \log(b^4*c^5 * (-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(3/4)} + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3)*\sqrt{x}) + (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(1/4)} * \log(-b^4*c^5 * (-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^{(3/4)} + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3)*\sqrt{x})) / (b*c^2*x^2 + b^2*c)$$

**giac** [A] time = 0.21, size = 273, normalized size = 1.05

$$\frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{2}} Bb + (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + 2\sqrt{c}}}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8 b^2 c^4} + \frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{2}} Bb + (bc^3)^{\frac{1}{2}} Ac \right) \arctan \left( -\frac{\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} - 2\sqrt{c}}}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8 b^2 c^4} - \frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{2}} Bb + (bc^3)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + x + \sqrt{\frac{b}{c}}} \right)}{16 b^2 c^4} + \frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{2}} Bb + (bc^3)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{\left(\frac{b}{c}\right)^{\frac{1}{2}} + x + \sqrt{\frac{b}{c}}} \right)}{16 b^2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 
$$-1/2*(B*b*x^{(3/2)} - A*c*x^{(3/2)})/((c*x^2 + b)*b*c) + 1/8*\sqrt{2}*(3*(b*c^3)^{(3/4)}*B*b + (b*c^3)^{(3/4)}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*\sqrt{x}))/ (b/c)^{(1/4)} / (b^2*c^4) + 1/8*\sqrt{2}*(3*(b*c^3)^{(3/4)}*B*b + (b*c^3)^{(3/4)}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*\sqrt{x}))/ (b/c)^{(1/4)} / (b^2*c^4) - 1/16*\sqrt{2}*(3*(b*c^3)^{(3/4)}*B*b + (b*c^3)^{(3/4)}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c}) / (b^2*c^4) + 1/16*\sqrt{2}*(3*(b*c^3)^{(3/4)}*B*b + (b*c^3)^{(3/4)}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c}) / (b^2*c^4)$$

**maple** [A] time = 0.06, size = 305, normalized size = 1.17

$$\frac{\sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} - 1 \right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} bc} + \frac{\sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} + 1 \right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} bc} + \frac{\sqrt{2} A \ln \left( \frac{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x + \sqrt{\frac{b}{c}}}}{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x + \sqrt{\frac{b}{c}}}} \right)}{16 \left(\frac{b}{c}\right)^{\frac{1}{4}} bc} + \frac{3\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} - 1 \right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} + \frac{3\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} + 1 \right)}{8 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} + \frac{3\sqrt{2} B \ln \left( \frac{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x + \sqrt{\frac{b}{c}}}}{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{x + \sqrt{\frac{b}{c}}}} \right)}{16 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^2} + \frac{(Ac - bB)x^{\frac{3}{2}}}{2(c^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 
$$1/2*(A*c - B*b)/b/c*x^{(3/2)}/(c*x^2 + b) + 1/8/b/c/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)} - 1) + 1/16/b/c/(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x - (b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)} + (b/c)^{(1/2)})/(x + (b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)} + (b/c)^{(1/2)})) + 1/8/b/c/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)} + 1) + 3/8/c^{(1/2)}/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)} - 1) + 3/16/c^2/(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x - (b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)} + (b/c)^{(1/2)})/(x + (b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)} + (b/c)^{(1/2)})) + 3/8/c^2/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)} + 1)$$

**maxima [A]** time = 3.11, size = 217, normalized size = 0.83

$$\frac{(Bb - Ac)x^{\frac{3}{2}}}{2(bc^2x^2 + b^2c)} + \frac{(3Bb + Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} \right)}{16bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-1/2*(B*b - A*c)*x^{3/2}/(b*c^2*x^2 + b^2*c) + 1/16*(3*B*b + A*c)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})})/(\sqrt{(\sqrt{b}*\sqrt{c})}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})})/(\sqrt{(\sqrt{b}*\sqrt{c})}) - \sqrt{2}*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*\sqrt{x} + \sqrt{b})/(b^{1/4}*c^{3/4}) + \sqrt{2}*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*\sqrt{x} + \sqrt{b})/(b^{1/4}*c^{3/4})/(b*c)$

**mupad [B]** time = 0.23, size = 91, normalized size = 0.35

$$\frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac + 3Bb)}{4(-b)^{5/4}c^{7/4}} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac + 3Bb)}{4(-b)^{5/4}c^{7/4}} + \frac{x^{3/2}(Ac - Bb)}{2bc(cx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $(\operatorname{atanh}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(Ac + 3*B*b))/(4*(-b)^{5/4}*c^{7/4}) - (\operatorname{atan}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(Ac + 3*B*b))/(4*(-b)^{5/4}*c^{7/4}) + (x^{3/2}*(Ac - B*b))/(2*b*c*(b + c*x^2))$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(9/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.201 \quad \int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=261

$$\frac{(3Ac + bB) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{8\sqrt{2} b^{7/4} c^{5/4}} + \frac{(3Ac + bB) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{8\sqrt{2} b^{7/4} c^{5/4}} - \frac{(3Ac + bB)}{4\sqrt{2} b^{7/4} c^{5/4}}$$

Rubi [A] time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {1584, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(3Ac + bB) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{8\sqrt{2} b^{7/4} c^{5/4}} + \frac{(3Ac + bB) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{8\sqrt{2} b^{7/4} c^{5/4}} - \frac{(3Ac + bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{7/4} c^{5/4}} + \frac{(3Ac + bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{4\sqrt{2} b^{7/4} c^{5/4}} - \frac{\sqrt{x}(bB - Ac)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] -((b\*B - A\*c)\*Sqrt[x])/(2\*b\*c\*(b + c\*x^2)) - ((b\*B + 3\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(7/4)\*c^(5/4)) + ((b\*B + 3\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(7/4)\*c^(5/4))) - ((b\*B + 3\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(8\*Sqrt[2]\*b^(7/4)\*c^(5/4)) + ((b\*B + 3\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(8\*Sqrt[2]\*b^(7/4)\*c^(5/4)))

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 457

Int[((e\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*b\*e\*n\*(p + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(e\*x)^(m\*(a + b\*x^n)^(p + 1)), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n\*(p + 1))]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{\sqrt{x} (b + cx^2)^2} dx \\
&= -\frac{(bB - Ac)\sqrt{x}}{2bc(b + cx^2)} + \frac{\left(\frac{bB}{2} + \frac{3Ac}{2}\right) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{2bc} \\
&= -\frac{(bB - Ac)\sqrt{x}}{2bc(b + cx^2)} + \frac{\left(\frac{bB}{2} + \frac{3Ac}{2}\right) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{bc} \\
&= -\frac{(bB - Ac)\sqrt{x}}{2bc(b + cx^2)} + \frac{(bB + 3Ac) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{4b^{3/2}c} + \frac{(bB + 3Ac) \text{Subst}\left(\int \frac{\sqrt{b}}{b+cx^4} dx, x, \sqrt{x}\right)}{4b^{3/2}c} \\
&= -\frac{(bB - Ac)\sqrt{x}}{2bc(b + cx^2)} + \frac{(bB + 3Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{8b^{3/2}c^{3/2}} + \frac{(bB + 3Ac) \text{Subst}\left(\int \frac{\sqrt{b}}{b+cx^4} dx, x, \sqrt{x}\right)}{4b^{3/2}c} \\
&= -\frac{(bB - Ac)\sqrt{x}}{2bc(b + cx^2)} - \frac{(bB + 3Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB + 3Ac) \log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} \\
&= -\frac{(bB - Ac)\sqrt{x}}{2bc(b + cx^2)} - \frac{(bB + 3Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB + 3Ac) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 203, normalized size = 0.78

$$\frac{(3Ac+bB)\left(8b^{3/4}\sqrt[4]{c}\sqrt{x}-3\sqrt{2}(b+cx^2)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)\right)\right)}{b^{7/4}\sqrt[4]{c}} - 32B\sqrt{x}$$


---


$$48c(b + cx^2)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] (-32\*B\*Sqrt[x] + ((b\*B + 3\*A\*c)\*(8\*b^(3/4)\*c^(1/4)\*Sqrt[x] - 3\*Sqrt[2]\*(b + c\*x^2)\*(2\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 2\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])))/(b^(7/4)\*c^(1/4)))/(48\*c\*(b + c\*x^2))

**IntegrateAlgebraic [A]** time = 0.63, size = 160, normalized size = 0.61

$$-\frac{(3Ac + bB) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(3Ac + bB) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{\sqrt{x}(Ac - bB)}{2bc(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] (((-b\*B) + A\*c)\*Sqrt[x])/(2\*b\*c\*(b + c\*x^2)) - ((b\*B + 3\*A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(4\*Sqrt[2]\*b^(7/4)\*c^(5/4)) + ((b\*B + 3\*A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(4\*Sqrt[2]\*b^(7/4)\*c^(5/4))

**fricas [B]** time = 0.44, size = 717, normalized size = 2.75

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} + 2\sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2c^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} - 2\sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2c^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^2c^2} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^2c^2} - \frac{Bb\sqrt{x} - Ac\sqrt{x}}{2(cx^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out]  $\frac{1}{8} * (4 * (b * c^2 * x^2 + b^2 * c) * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{1/4} * \arctan(\sqrt{(b^4 * c^2 * \sqrt{x} - (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))} + (B^2 * b^2 + 6 * A * B * b * c + 9 * A^2 * c^2) * x) * b^5 * c^4 * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{3/4} - (B * b^6 * c^4 + 3 * A * b^5 * c^5) * \sqrt{x} * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{3/4}) / (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) + (b * c^2 * x^2 + b^2 * c) * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{1/4} * \log(b^2 * c * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{1/4} + (B * b + 3 * A * c) * \sqrt{x}) - (b * c^2 * x^2 + b^2 * c) * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{1/4} * \log(-b^2 * c * (- (B^4 * b^4 + 12 * A * B^3 * b^3 * c + 54 * A^2 * B^2 * b^2 * c^2 + 108 * A^3 * B * b * c^3 + 81 * A^4 * c^4) / (b^7 * c^5))^{1/4} + (B * b + 3 * A * c) * \sqrt{x}) - 4 * (B * b - A * c) * \sqrt{x}) / (b * c^2 * x^2 + b^2 * c)$

**giac [A]** time = 0.21, size = 273, normalized size = 1.05

$$\frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} + 2\sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2c^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} - 2\sqrt{x}}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2c^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^2c^2} - \frac{\sqrt{2} \left( (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^2c^2} - \frac{Bb\sqrt{x} - Ac\sqrt{x}}{2(cx^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $\frac{1}{8} * \sqrt{2} * ((b * c^3)^{\frac{1}{4}} * B * b + 3 * (b * c^3)^{\frac{1}{4}} * A * c) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * (b/c)^{\frac{1}{4}} + 2 * \sqrt{x}) / (b/c)^{\frac{1}{4}}) / (b^2 * c^2) + 1/8 * \sqrt{2} * ((b * c^3)^{\frac{1}{4}} * B * b + 3 * (b * c^3)^{\frac{1}{4}} * A * c) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * (b/c)^{\frac{1}{4}} - 2 * \sqrt{x}) / (b/c)^{\frac{1}{4}}) / (b^2 * c^2) + 1/16 * \sqrt{2} * ((b * c^3)^{\frac{1}{4}} * B * b + 3 * (b * c^3)^{\frac{1}{4}} * A * c) * \log(\sqrt{2} * \sqrt{x} * (b/c)^{\frac{1}{4}} + x + \sqrt{b/c}) / (b^2 * c^2) - 1/16 * \sqrt{2} * ((b * c^3)^{\frac{1}{4}} * B * b + 3 * (b * c^3)^{\frac{1}{4}} * A * c) * \log(-\sqrt{2} * \sqrt{x} * \sqrt{2} * \sqrt{x} * (b/c)^{\frac{1}{4}} + x + \sqrt{b/c}) / (b^2 * c^2) - 1/2 * (B * b * \sqrt{x} - A * c * \sqrt{x}) / ((c * x^2 + b) * b * c)$

**maple [A]** time = 0.06, size = 305, normalized size = 1.17

$$\frac{3 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2} + \frac{3 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2} + \frac{3 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} A \ln \left( \frac{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{16b^2} + \frac{\left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8bc} + \frac{\left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8bc} + \frac{\left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \ln \left( \frac{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{16bc} + \frac{(Ac - bB) \sqrt{x}}{2(cx^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $\frac{1}{2} * (A * c - B * b) / b * c * x^{1/2} / (c * x^2 + b) + 3/16 / b^2 * (b/c)^{1/4} * 2^{1/2} * A * \ln((x + (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2}) / (x - (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2})) + 3/8 / b^2 * (b/c)^{1/4} * 2^{1/2} * A * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} + 1) + 3/8 / b^2 * (b/c)^{1/4} * 2^{1/2} * A * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} - 1) + 1/16 / b * c * (b/c)^{1/4} * 2^{1/2} * B * \ln((x + (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2}) / (x - (b/c)^{1/4} * 2^{1/2} * x^{1/2} + (b/c)^{1/2})) + 1/8 / b * c * (b/c)^{1/4} * 2^{1/2} * B * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} + 1) + 1/8 / b * c * (b/c)^{1/4} * 2^{1/2} * B * \arctan(2^{1/2} / (b/c)^{1/4} * x^{1/2} - 1)$



**maxima [A]** time = 3.11, size = 241, normalized size = 0.92

$$\frac{2\sqrt{2}(Bb+3Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(Bb+3Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(Bb+3Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{cx+\sqrt{b}}\right)}{\frac{3}{b^{\frac{1}{4}}c^{\frac{1}{4}}}} - \frac{\sqrt{2}(Bb+3Ac)\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{cx+\sqrt{b}}\right)}{\frac{3}{b^{\frac{1}{4}}c^{\frac{1}{4}}}}$$

$$-\frac{(Bb-Ac)\sqrt{x}}{2(bc^2x^2+b^2c)} + \frac{16bc}{16bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $-1/2*(B*b - A*c)*\text{sqrt}(x)/(b*c^2*x^2 + b^2*c) + 1/16*(2*\text{sqrt}(2)*(B*b + 3*A*c) * \arctan(1/2*\text{sqrt}(2)*( \text{sqrt}(2)*b^{(1/4)}*c^{(1/4)} + 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(\text{sqrt}(b)*\text{sqrt}(c)))/(\text{sqrt}(b)*\text{sqrt}(\text{sqrt}(b)*\text{sqrt}(c))) + 2*\text{sqrt}(2)*(B*b + 3*A*c)*\arctan(-1/2*\text{sqrt}(2)*( \text{sqrt}(2)*b^{(1/4)}*c^{(1/4)} - 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(\text{sqrt}(b)*\text{sqrt}(c)))/(\text{sqrt}(b)*\text{sqrt}(\text{sqrt}(b)*\text{sqrt}(c))) + \text{sqrt}(2)*(B*b + 3*A*c)*\log(\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/(b^{(3/4)}*c^{(1/4)}) - \text{sqrt}(2)*(B*b + 3*A*c)*\log(-\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/(b^{(3/4)}*c^{(1/4)})/(b*c)$

**mupad [B]** time = 0.33, size = 750, normalized size = 2.87

$$\text{atan}\left(\frac{\frac{\frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}} + \frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}}}{\frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}} + \frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}}}}{4(-b)^{7/4}c^{5/4}}\right) + \frac{\sqrt{x}(Ac-Bb)}{2bc(c^2x^2+b^2)} + \text{atan}\left(\frac{\frac{\frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}} + \frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}}}{\frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}} + \frac{\sqrt{c}\sqrt{b^2+4ABb^2+4A^2c^2}}{b^2} \frac{(3Ac+Bb)}{8(-b)^{7/4}c^{5/4}}}}{4(-b)^{7/4}c^{5/4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $(\text{atan}(\frac{((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 - ((3Ac+Bb)*(24A^2c^3+8Bbc^2))/(8*(-b)^{(7/4)}c^{(5/4)})) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)} + ((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 + ((3Ac+Bb)*(24A^2c^3+8Bbc^2))/(8*(-b)^{(7/4)}c^{(5/4)})) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) / (((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 - ((3Ac+Bb)*(24A^2c^3+8Bbc^2))/(8*(-b)^{(7/4)}c^{(5/4)})) / (8*(-b)^{(7/4)}c^{(5/4)}) - ((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 + ((3Ac+Bb)*(24A^2c^3+8Bbc^2))/(8*(-b)^{(7/4)}c^{(5/4)})) / (8*(-b)^{(7/4)}c^{(5/4)}) * (3Ac+Bb) * 1i}{(4*(-b)^{(7/4)}c^{(5/4)} + (\text{atan}(\frac{((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 - ((3Ac+Bb)*(24A^2c^3+8Bbc^2)) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) / (8*(-b)^{(7/4)}c^{(5/4)})) / (((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 + ((3Ac+Bb)*(24A^2c^3+8Bbc^2)) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) / (8*(-b)^{(7/4)}c^{(5/4)})) / (((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 - ((3Ac+Bb)*(24A^2c^3+8Bbc^2)) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) - ((3Ac+Bb)*((x^{(1/2)}*(9A^2c^3+B^2b^2c+6ABbc^2))/b^2 + ((3Ac+Bb)*(24A^2c^3+8Bbc^2)) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) * 1i}{(8*(-b)^{(7/4)}c^{(5/4)})) * (3Ac+Bb)}{4*(-b)^{(7/4)}c^{(5/4)} + (x^{(1/2)}*(Ac-Bb))/(2b*c*(b+c*x^2))$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.202 \quad \int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=284

$$\frac{(bB - 5Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{9/4} c^{3/4}} - \frac{(bB - 5Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{9/4} c^{3/4}} - \frac{(bB - 5Ac) \tan^{-1}}{4\sqrt{2} b^9}$$

**Rubi [A]** time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB - 5Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{9/4} c^{3/4}} - \frac{(bB - 5Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{9/4} c^{3/4}} - \frac{(bB - 5Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{2} b^{9/4} c^{3/4}} + \frac{(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{4\sqrt{2} b^{9/4} c^{3/4}} + \frac{bB - 5Ac}{2b^2 c \sqrt{x}} - \frac{bB - Ac}{2bc \sqrt{x} (b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (b\*B - 5\*A\*c)/(2\*b^2\*c\*Sqrt[x]) - (b\*B - A\*c)/(2\*b\*c\*Sqrt[x]\*(b + c\*x^2)) - ((b\*B - 5\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(9/4)\*c^(3/4)) + ((b\*B - 5\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(9/4)\*c^(3/4)) + ((b\*B - 5\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(9/4)\*c^(3/4)) - ((b\*B - 5\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(9/4)\*c^(3/4))

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^{3/2} (b + cx^2)^2} dx \\
&= -\frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} + \frac{\left(-\frac{bB}{2} + \frac{5Ac}{2}\right) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{2bc} \\
&= \frac{bB - 5Ac}{2b^2c\sqrt{x}} - \frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} + \frac{(bB - 5Ac) \int \frac{\sqrt{x}}{b+cx^2} dx}{4b^2} \\
&= \frac{bB - 5Ac}{2b^2c\sqrt{x}} - \frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} + \frac{(bB - 5Ac) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{2b^2} \\
&= \frac{bB - 5Ac}{2b^2c\sqrt{x}} - \frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} - \frac{(bB - 5Ac) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{4b^2\sqrt{c}} + \frac{(bB - 5Ac) S}{(bB - 5Ac) S} \\
&= \frac{bB - 5Ac}{2b^2c\sqrt{x}} - \frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} + \frac{(bB - 5Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}}{\sqrt{c}} \frac{\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{8b^2c} + \frac{(bB - 5Ac) S}{(bB - 5Ac) S} \\
&= \frac{bB - 5Ac}{2b^2c\sqrt{x}} - \frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} + \frac{(bB - 5Ac) \log\left(\sqrt{b} - \sqrt{2} \frac{\sqrt[4]{b}}{\sqrt{c}} \sqrt[4]{c} \sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2} b^{9/4} c^{3/4}} - \frac{(bB - 5Ac) S}{(bB - 5Ac) S} \\
&= \frac{bB - 5Ac}{2b^2c\sqrt{x}} - \frac{bB - Ac}{2bc\sqrt{x} (b + cx^2)} - \frac{(bB - 5Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{9/4} c^{3/4}} + \frac{(bB - 5Ac) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{9/4} c^{3/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.32, size = 117, normalized size = 0.41

$$\frac{2x^{3/2}(bB - Ac) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right) + 3A\left((-b)^{3/4} \sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{c} \sqrt{x}}{\sqrt[4]{-b}}\right) - (-b)^{3/4} \sqrt[4]{c} \tanh^{-1}\left(\frac{\sqrt[4]{c} \sqrt{x}}{\sqrt[4]{-b}}\right) - \frac{2b}{\sqrt{x}}\right)}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] (3\*A\*((-2\*b)/Sqrt[x] + (-b)^(3/4)\*c^(1/4)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)]) - (-b)^(3/4)\*c^(1/4)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + 2\*(b\*B - A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -((c\*x^2)/b)]/(3\*b^3)

**IntegrateAlgebraic [A]** time = 0.67, size = 167, normalized size = 0.59

$$-\frac{(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}\right)}{4\sqrt{2} b^{9/4} c^{3/4}} - \frac{(bB - 5Ac) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{4\sqrt{2} b^{9/4} c^{3/4}} + \frac{-4Ab - 5Acx^2 + bBx^2}{2b^2\sqrt{x} (b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out] (-4\*A\*b + b\*B\*x^2 - 5\*A\*c\*x^2)/(2\*b^2\*Sqrt[x]\*(b + c\*x^2)) - ((b\*B - 5\*A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(4\*Sqrt[2]\*b^(9/4)\*c^(3/4)) - ((b\*B - 5\*A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]/(Sqrt[b] + Sqrt[c]\*x))/(4\*Sqrt[2]\*b^(9/4)\*c^(3/4))

**fricas [B]** time = 0.46, size = 920, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{8} \cdot (4 \cdot (b^2 \cdot c \cdot x^3 + b^3 \cdot x) \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{1/4} \cdot \arctan(\sqrt{(B^6 \cdot b^6 - 30 \cdot A \cdot B^5 \cdot b^5 \cdot c + 375 \cdot A^2 \cdot B^4 \cdot b^4 \cdot c^2 - 2500 \cdot A^3 \cdot B^3 \cdot b^3 \cdot c^3 + 9375 \cdot A^4 \cdot B^2 \cdot b^2 \cdot c^4 - 18750 \cdot A^5 \cdot B \cdot b \cdot c^5 + 15625 \cdot A^6 \cdot c^6)} \cdot x - (B^4 \cdot b^9 \cdot c - 20 \cdot A \cdot B^3 \cdot b^8 \cdot c^2 + 150 \cdot A^2 \cdot B^2 \cdot b^7 \cdot c^3 - 500 \cdot A^3 \cdot B \cdot b^6 \cdot c^4 + 625 \cdot A^4 \cdot b^5 \cdot c^5) \cdot \sqrt{-(B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4)} / (b^9 \cdot c^3)) \cdot b^2 \cdot c \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{1/4} + (B^3 \cdot b^5 \cdot c - 15 \cdot A \cdot B^2 \cdot b^4 \cdot c^2 + 75 \cdot A^2 \cdot B \cdot b^3 \cdot c^3 - 125 \cdot A^3 \cdot b^2 \cdot c^4) \cdot \sqrt{x} \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{1/4}) / (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) - (b^2 \cdot c \cdot x^3 + b^3 \cdot x) \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{1/4} \cdot \log(b^7 \cdot c^2 \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{3/4} - (B^3 \cdot b^3 - 15 \cdot A \cdot B^2 \cdot b^2 \cdot c + 75 \cdot A^2 \cdot B \cdot b \cdot c^2 - 125 \cdot A^3 \cdot c^3) \cdot \sqrt{x}) + (b^2 \cdot c \cdot x^3 + b^3 \cdot x) \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{1/4} \cdot \log(-b^7 \cdot c^2 \cdot (- (B^4 \cdot b^4 - 20 \cdot A \cdot B^3 \cdot b^3 \cdot c + 150 \cdot A^2 \cdot B^2 \cdot b^2 \cdot c^2 - 500 \cdot A^3 \cdot B \cdot b \cdot c^3 + 625 \cdot A^4 \cdot c^4) / (b^9 \cdot c^3))^{3/4} - (B^3 \cdot b^3 - 15 \cdot A \cdot B^2 \cdot b^2 \cdot c + 75 \cdot A^2 \cdot B \cdot b \cdot c^2 - 125 \cdot A^3 \cdot c^3) \cdot \sqrt{x}) + 4 \cdot ((B \cdot b - 5 \cdot A \cdot c) \cdot x^2 - 4 \cdot A \cdot b) \cdot \sqrt{x} / (b^2 \cdot c \cdot x^3 + b^3 \cdot x)$$

**giac [A]** time = 0.21, size = 278, normalized size = 0.98

$$\frac{Bbx^2 - 5Axc^2 - 4Ab}{2(cx^2 + b\sqrt{c})b^2} + \frac{\sqrt{2} \left( (bc^3)^{\frac{3}{4}} Bb - 5(bc^3)^{\frac{3}{4}} Ac \right) \arctan\left(\frac{\sqrt{2} \left( \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{c} \right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{\frac{3}{2}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \left( (bc^3)^{\frac{3}{4}} Bb - 5(bc^3)^{\frac{3}{4}} Ac \right) \arctan\left(-\frac{\sqrt{2} \left( \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{c} \right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{\frac{3}{2}}c^{\frac{3}{4}}} - \frac{\sqrt{2} \left( (bc^3)^{\frac{3}{4}} Bb - 5(bc^3)^{\frac{3}{4}} Ac \right) \log\left(\sqrt{2} \sqrt{c} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{b}}\right)}{16b^{\frac{3}{2}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \left( (bc^3)^{\frac{3}{4}} Bb - 5(bc^3)^{\frac{3}{4}} Ac \right) \log\left(-\sqrt{2} \sqrt{c} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{b}}\right)}{16b^{\frac{3}{2}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 
$$\frac{1}{2} \cdot (B \cdot b \cdot x^2 - 5 \cdot A \cdot c \cdot x^2 - 4 \cdot A \cdot b) / ((c \cdot x^2 + b \cdot \sqrt{x}) \cdot b^2) + \frac{1}{8} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - 5 \cdot (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (b/c)^{\frac{1}{4}} + 2 \cdot \sqrt{x}) / (b/c)^{\frac{1}{4}}) / (b^3 \cdot c^3) + \frac{1}{8} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - 5 \cdot (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (b/c)^{\frac{1}{4}} - 2 \cdot \sqrt{x}) / (b/c)^{\frac{1}{4}}) / (b^3 \cdot c^3) - \frac{1}{16} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - 5 \cdot (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \log(\sqrt{2} \cdot \sqrt{x} \cdot (b/c)^{\frac{1}{4}} + x + \sqrt{b/c}) / (b^3 \cdot c^3) + \frac{1}{16} \cdot \sqrt{2} \cdot ((b \cdot c^3)^{\frac{3}{4}} \cdot B \cdot b - 5 \cdot (b \cdot c^3)^{\frac{3}{4}} \cdot A \cdot c) \cdot \log(-\sqrt{2} \cdot \sqrt{x} \cdot (b/c)^{\frac{1}{4}} + x + \sqrt{b/c}) / (b^3 \cdot c^3)$$

**maple [A]** time = 0.06, size = 323, normalized size = 1.14

$$\frac{Acx^{\frac{3}{2}}}{2(cx^2 + b)b^2} + \frac{Bx^{\frac{3}{2}}}{2(cx^2 + b)b} - \frac{5\sqrt{2}A \arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2} - \frac{5\sqrt{2}A \arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2} - \frac{5\sqrt{2}A \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2} + \frac{\sqrt{2}B \arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc} + \frac{\sqrt{2}B \arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc} + \frac{\sqrt{2}B \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{c}{b}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}bc} - \frac{2A}{b^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out] 
$$-1/2 \cdot b^2 \cdot x^{3/2} / (c \cdot x^2 + b) \cdot A \cdot c + 1/2 \cdot b \cdot x^{3/2} / (c \cdot x^2 + b) \cdot B - 5/16 \cdot b^2 / (b/c)^{1/4} \cdot 2^{1/2} \cdot A \cdot \ln((x - (b/c)^{1/4}) \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4}) \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) - 5/8 \cdot b^2 / (b/c)^{1/4} \cdot 2^{1/2} \cdot A \cdot \arctan(2^{1/2} / (b/c)^{1/4} \cdot x^{1/2} + 1) - 5/8 \cdot b^2 / (b/c)^{1/4} \cdot 2^{1/2} \cdot A \cdot \arctan(2^{1/2} / (b/c)^{1/4} \cdot x^{1/2} - 1) + 1/16 \cdot b \cdot c / (b/c)^{1/4} \cdot 2^{1/2} \cdot B \cdot \ln((x - (b/c)^{1/4}) \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4}) \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) - 1/16 \cdot b \cdot c / (b/c)^{1/4} \cdot 2^{1/2} \cdot B \cdot \ln((x + (b/c)^{1/4}) \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2}) / (x - (b/c)^{1/4}) \cdot 2^{1/2} \cdot x^{1/2} + (b/c)^{1/2})$$

$(1/2)+(b/c)^{(1/2)}/(x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+1/8/b/c/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+1/8/b/c/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)-2*A/b^2/x^{(1/2)}$

**maxima** [A] time = 3.04, size = 222, normalized size = 0.78

$$\frac{(Bb - 5Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \right)}{2\left(b^2cx^{\frac{5}{2}}+b^3\sqrt{x}\right)} + \frac{16b^2}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $1/2*((B*b - 5*A*c)*x^2 - 4*A*b)/(b^2*c*x^{(5/2)} + b^3*\sqrt{x}) + 1/16*(B*b - 5*A*c)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} + 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})})/(\sqrt{(\sqrt{b}*\sqrt{c})}*\sqrt{c}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} - 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})})/(\sqrt{(\sqrt{b}*\sqrt{c})}*\sqrt{c}) - \sqrt{2}*\log(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c}*x + \sqrt{b}))/b^{(1/4)}*c^{(3/4)} + \sqrt{2}*\log(-\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c}*x + \sqrt{b}))/b^{(1/4)}*c^{(3/4)})/b^2$

**mupad** [B] time = 0.23, size = 104, normalized size = 0.37

$$\frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(5Ac - Bb)}{4(-b)^{9/4}c^{3/4}} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(5Ac - Bb)}{4(-b)^{9/4}c^{3/4}} - \frac{\frac{2A}{b} + \frac{x^2(5Ac - Bb)}{2b^2}}{b\sqrt{x} + cx^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x)

[Out]  $(\operatorname{atanh}((c^{(1/4)}*x^{(1/2)})/(-b)^{(1/4)})*(5*A*c - B*b))/(4*(-b)^{(9/4)}*c^{(3/4)}) - (\operatorname{atan}((c^{(1/4)}*x^{(1/2)})/(-b)^{(1/4)})*(5*A*c - B*b))/(4*(-b)^{(9/4)}*c^{(3/4)}) - ((2*A)/b + (x^2*(5*A*c - B*b))/(2*b^2))/(b*x^{(1/2)} + c*x^{(5/2)})$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.203 \quad \int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=289

$$\frac{(3bB - 7Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{(3bB - 7Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{11/4} \sqrt[4]{c}} - \frac{(3bB - 7Ac)}{2bcx^{3/2}}$$

**Rubi [A]** time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3bB - 7Ac}{6b^2cx^{3/2}} - \frac{(3bB - 7Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{(3bB - 7Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{11/4} \sqrt[4]{c}} - \frac{(3bB - 7Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{(3bB - 7Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{4\sqrt{2} b^{11/4} \sqrt[4]{c}} - \frac{bB - Ac}{2bcx^{3/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (3\*b\*B - 7\*A\*c)/(6\*b^2\*c\*x^(3/2)) - (b\*B - A\*c)/(2\*b\*c\*x^(3/2)\*(b + c\*x^2)) - ((3\*b\*B - 7\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(11/4)\*c^(1/4)) + ((3\*b\*B - 7\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(11/4)\*c^(1/4)) - ((3\*b\*B - 7\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(11/4)\*c^(1/4)) + ((3\*b\*B - 7\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(11/4)\*c^(1/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^{3/2} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^{5/2} (b + cx^2)^2} dx \\
&= -\frac{bB - Ac}{2bcx^{3/2} (b + cx^2)} + \frac{\left(-\frac{3bB}{2} + \frac{7Ac}{2}\right) \int \frac{1}{x^{5/2}(b+cx^2)} dx}{2bc} \\
&= \frac{3bB - 7Ac}{6b^2cx^{3/2}} - \frac{bB - Ac}{2bcx^{3/2} (b + cx^2)} + \frac{(3bB - 7Ac) \int \frac{1}{\sqrt{x} (b+cx^2)} dx}{4b^2} \\
&= \frac{3bB - 7Ac}{6b^2cx^{3/2}} - \frac{bB - Ac}{2bcx^{3/2} (b + cx^2)} + \frac{(3bB - 7Ac) \text{Subst} \left( \int \frac{1}{b+cx^4} dx, x, \sqrt{x} \right)}{2b^2} \\
&= \frac{3bB - 7Ac}{6b^2cx^{3/2}} - \frac{bB - Ac}{2bcx^{3/2} (b + cx^2)} + \frac{(3bB - 7Ac) \text{Subst} \left( \int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x} \right)}{4b^{5/2}} + \frac{(3bB - 7Ac) \text{Subst} \left( \int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x} \right)}{8b^{5/2}\sqrt{c}} \\
&= \frac{3bB - 7Ac}{6b^2cx^{3/2}} - \frac{bB - Ac}{2bcx^{3/2} (b + cx^2)} - \frac{(3bB - 7Ac) \log \left( \sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{c} x \right)}{8\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{(3bB - 7Ac) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} \right)}{4\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{(3bB - 7Ac) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} \right)}{4\sqrt{2} b^{11/4} \sqrt[4]{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.58, size = 355, normalized size = 1.23

$$\frac{-\frac{24Ab^3c\sqrt{c}}{b+cx^2} - \frac{32Ab^3c}{2b^2c^2} + \frac{6\sqrt{2}(7Ac-3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}} + 21\sqrt{2}Ac^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right) - 21\sqrt{2}Ac^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right) + \frac{24b^{7/4}B\sqrt{c}}{b+cx^2} - \frac{9\sqrt{2}88\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}} + \frac{9\sqrt{2}88\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}}{48b^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out]  $\left(\frac{-32A*b^{3/4}}{x^{3/2}} + \frac{24*b^{7/4}*B*\text{Sqrt}[x]}{(b + c*x^2)} - \frac{24A*b^{3/4}*c*\text{Sqrt}[x]}{(b + c*x^2)} + \frac{6*\text{Sqrt}[2]*(-3*b*B + 7A*c)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*\text{Sqrt}[x])/b^{1/4}]}{c^{1/4}} + \frac{6*\text{Sqrt}[2]*(3*b*B - 7A*c)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*\text{Sqrt}[x])/b^{1/4}]}{c^{1/4}} - \frac{9*\text{Sqrt}[2]*b*B*\text{Log}[\text{Sqrt}[b] - \text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[c]*x]}{c^{1/4}} + \frac{21*\text{Sqrt}[2]*A*c^{3/4}*\text{Log}[\text{Sqrt}[b] - \text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[c]*x] + (9*\text{Sqrt}[2]*b*B*\text{Log}[\text{Sqrt}[b] + \text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[c]*x])}{c^{1/4}} - 21*\text{Sqrt}[2]*A*c^{3/4}*\text{Log}[\text{Sqrt}[b] + \text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[c]*x]\right)/(48*b^{11/4})$

**IntegrateAlgebraic [A]** time = 0.67, size = 170, normalized size = 0.59

$$-\frac{(3bB - 7Ac) \tan^{-1} \left( \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}} \right)}{4\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{(3bB - 7Ac) \tanh^{-1} \left( \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}+\sqrt{c}x} \right)}{4\sqrt{2} b^{11/4} \sqrt[4]{c}} + \frac{-4Ab - 7Acx^2 + 3bBx^2}{6b^2x^{3/2} (b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out]  $(-4*A*b + 3*b*B*x^2 - 7*A*c*x^2)/(6*b^2*x^{(3/2)}*(b + c*x^2)) - ((3*b*B - 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^{(1/4)}*c^{(1/4)}*Sqrt[x]])/(4*Sqrt[2]*b^{(11/4)}*c^{(1/4)}) + ((3*b*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^{(1/4)}*c^{(1/4)}*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]/(4*Sqrt[2]*b^{(11/4)}*c^{(1/4)})$

**fricas [B]** time = 0.45, size = 741, normalized size = 2.56



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out]  $-1/24*(12*(b^2*c*x^4 + b^3*x^2)*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(1/4)}*arctan((sqrt(b^6*sqrt(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c)) + (9*B^2*b^2 - 42*A*B*b*c + 49*A^2*c^2)*x)*b^8*c*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(3/4)} + (3*B*b^9*c - 7*A*b^8*c^2)*sqrt(x)*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(3/4)}/(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4) + 3*(b^2*c*x^4 + b^3*x^2)*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(1/4)}*log(b^3*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(1/4)} - (3*B*b - 7*A*c)*sqrt(x) - 3*(b^2*c*x^4 + b^3*x^2)*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(1/4)}*log(-b^3*(-81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c))^{(1/4)} - (3*B*b - 7*A*c)*sqrt(x) - 4*((3*B*b - 7*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)$

**giac [A]** time = 0.19, size = 283, normalized size = 0.98

$$\frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{4}} Bb - 7 (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} + 2 \sqrt{x} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^3c} + \frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{4}} Bb - 7 (bc^3)^{\frac{1}{4}} Ac \right) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \left( \frac{b}{c} \right)^{\frac{1}{4}} - 2 \sqrt{x} \right)}{2 \left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^3c} + \frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{4}} Bb - 7 (bc^3)^{\frac{1}{4}} Ac \right) \log \left( \sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^3c} - \frac{\sqrt{2} \left( 3 (bc^3)^{\frac{1}{4}} Bb - 7 (bc^3)^{\frac{1}{4}} Ac \right) \log \left( -\sqrt{2} \sqrt{x} \left( \frac{b}{c} \right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^3c} + \frac{Bb\sqrt{x} - Ac\sqrt{x}}{2(cx^2 + b)b^2} - \frac{2A}{3b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $1/8*sqrt(2)*(3*(b*c^3)^{(1/4)}*B*b - 7*(b*c^3)^{(1/4)}*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^{(1/4)} + 2*sqrt(x))/(b/c)^{(1/4)})/(b^3*c) + 1/8*sqrt(2)*(3*(b*c^3)^{(1/4)}*B*b - 7*(b*c^3)^{(1/4)}*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^{(1/4)} - 2*sqrt(x))/(b/c)^{(1/4)})/(b^3*c) + 1/16*sqrt(2)*(3*(b*c^3)^{(1/4)}*B*b - 7*(b*c^3)^{(1/4)}*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^{(1/4)} + x + sqrt(b/c))/(b^3*c) - 1/16*sqrt(2)*(3*(b*c^3)^{(1/4)}*B*b - 7*(b*c^3)^{(1/4)}*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^{(1/4)} + x + sqrt(b/c))/(b^3*c) + 1/2*(B*b*sqrt(x) - A*c*sqrt(x))/((c*x^2 + b)*b^2) - 2/3*A/(b^2*x^{(3/2)})$

**maple [A]** time = 0.06, size = 317, normalized size = 1.10

$$\frac{Ac\sqrt{x}}{2(cx^2 + b)b^2} + \frac{B\sqrt{x}}{2(cx^2 + b)b} - \frac{7 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} Ac \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^3} - \frac{7 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} Ac \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^3} - \frac{7 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} Ac \ln \left( \frac{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{16b^3} + \frac{3 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x} - 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2} + \frac{3 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{x} + 1}{\left( \frac{b}{c} \right)^{\frac{1}{4}}} \right)}{8b^2} + \frac{3 \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} B \ln \left( \frac{x + \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}}{x - \left( \frac{b}{c} \right)^{\frac{1}{4}} \sqrt{2} \sqrt{x} + \sqrt{\frac{b}{c}}} \right)}{16b^2} - \frac{2A}{3b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^2,x)

[Out]  $-1/2/b^2*x^{(1/2)}/(c*x^2+b)*A*c+1/2/b*x^{(1/2)}/(c*x^2+b)*B-7/16/b^3*(b/c)^{(1/4)}*2^{(1/2)}*A*ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})))*c-7/8/b^3*(b/c)^{(1/4)}*2^{(1/2)}*A*arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)*c-7/8/b^3*(b/c)^{(1/4)}*2^{(1/2)}*A*arctan(2^{(1/2)}/(b/$



```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

$$3.204 \quad \int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2} dx$$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{c}(5bB-9Ac)\log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}}{2bcx^{5/2}(b+cx^2)}$$

Rubi [A] time = 0.27, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{\sqrt[4]{c}(5bB-9Ac)\log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{13/4}} - \frac{bB-Ac}{2bcx^{5/2}(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2,x]

[Out] (5\*b\*B - 9\*A\*c)/(10\*b^2\*c\*x^(5/2)) - (5\*b\*B - 9\*A\*c)/(2\*b^3\*Sqrt[x]) - (b\*B - A\*c)/(2\*b\*c\*x^(5/2)\*(b + c\*x^2)) + (c^(1/4)\*(5\*b\*B - 9\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(13/4)) - (c^(1/4)\*(5\*b\*B - 9\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]/(4\*Sqrt[2]\*b^(13/4)) - (c^(1/4)\*(5\*b\*B - 9\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(13/4)) + (c^(1/4)\*(5\*b\*B - 9\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(13/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x} (A + Bx^2)}{(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^{7/2} (b + cx^2)^2} dx \\
&= -\frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} + \frac{\left(-\frac{5bB}{2} + \frac{9Ac}{2}\right) \int \frac{1}{x^{7/2}(b+cx^2)} dx}{2bc} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} + \frac{(5bB - 9Ac) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{4b^2} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{5bB - 9Ac}{2b^3\sqrt{x}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} - \frac{(c(5bB - 9Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{4b^3} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{5bB - 9Ac}{2b^3\sqrt{x}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} - \frac{(c(5bB - 9Ac)) \text{Subst} \left( \int \frac{x^2}{b+cx^4} dx, x, \sqrt{x} \right)}{2b^3} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{5bB - 9Ac}{2b^3\sqrt{x}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} + \frac{(\sqrt{c} (5bB - 9Ac)) \text{Subst} \left( \int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx \right)}{4b^3} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{5bB - 9Ac}{2b^3\sqrt{x}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} - \frac{(5bB - 9Ac) \text{Subst} \left( \int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}}{\sqrt{c}} \frac{\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx \right)}{8b^3} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{5bB - 9Ac}{2b^3\sqrt{x}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} - \frac{\sqrt[4]{c} (5bB - 9Ac) \log \left( \sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x \right)}{8\sqrt{2} b^{13/4}} \\
&= \frac{5bB - 9Ac}{10b^2cx^{5/2}} - \frac{5bB - 9Ac}{2b^3\sqrt{x}} - \frac{bB - Ac}{2bcx^{5/2} (b + cx^2)} + \frac{\sqrt[4]{c} (5bB - 9Ac) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} \right)}{4\sqrt{2} b^{13/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.64, size = 151, normalized size = 0.49

$$\frac{2cx^{3/2}(Ac - bB) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^4} + \frac{4Ac - 2bB}{b^3\sqrt{x}} - \frac{2A}{5b^2x^{5/2}} + \frac{\sqrt[4]{c}(bB - 2Ac) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{(-b)^{13/4}} + \frac{b\sqrt[4]{c}(bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out]  $(-2*A)/(5*b^2*x^{(5/2)}) + (-2*b*B + 4*A*c)/(b^3*Sqrt[x]) + (c^{(1/4)}*(b*B - 2*A*c)*ArcTan[(c^{(1/4)}*Sqrt[x])/(-b)^{(1/4)}])/(-b)^{(13/4)} + (b*c^{(1/4)}*(b*B - 2*A*c)*ArcTanh[(c^{(1/4)}*Sqrt[x])/(-b)^{(1/4)}])/(-b)^{(17/4)} + (2*c*(-(b*B) + A*c)*x^{(3/2)}*Hypergeometric2F1[3/4, 2, 7/4, -(c*x^2)/b])/ (3*b^4)$

**IntegrateAlgebraic [A]** time = 0.68, size = 200, normalized size = 0.65

$$\frac{(5bB\sqrt[4]{c} - 9Ac^{5/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}b^{13/4}} + \frac{(5bB\sqrt[4]{c} - 9Ac^{5/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{13/4}} + \frac{-4Ab^2 + 36Abcx^2 + 45Ac^2x^4 - 20b^2Bx^2 - 25bBcx^4}{10b^3x^{5/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^2, x]

[Out]  $(-4*A*b^2 - 20*b^2*B*x^2 + 36*A*b*c*x^2 - 25*b*B*c*x^4 + 45*A*c^2*x^4)/(10*b^3*x^{5/2}*(b + c*x^2)) + ((5*b*B*c^{1/4} - 9*A*c^{5/4})*\text{ArcTan}[(\text{Sqrt}[b] - \text{Sqrt}[c]*x)/(\text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x])])/(4*\text{Sqrt}[2]*b^{13/4}) + ((5*b*B*c^{1/4} - 9*A*c^{5/4})*\text{ArcTanh}[(\text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x])/(\text{Sqrt}[b] + \text{Sqrt}[c]*x)])/(4*\text{Sqrt}[2]*b^{13/4})$

**fricas [B]** time = 0.46, size = 974, normalized size = 3.14

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")`

[Out]  $-1/40*(20*(b^3*c*x^5 + b^4*x^3)*(-625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13})^{1/4}*\arctan(\sqrt{(15625*B^6*b^6*c^2 - 168750*A*B^5*b^5*c^3 + 759375*A^2*B^4*b^4*c^4 - 1822500*A^3*B^3*b^3*c^5 + 2460375*A^4*B^2*b^2*c^6 - 1771470*A^5*B*b*c^7 + 531441*A^6*c^8)*x - (625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)*\sqrt{-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13}})/b^{13})^{1/4} + (125*B^3*b^6*c - 675*A*B^2*b^5*c^2 + 1215*A^2*B*b^4*c^3 - 729*A^3*b^3*c^4)*\sqrt{x}*(-625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13})^{1/4} + (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*\sqrt{x} + 5*(b^3*c*x^5 + b^4*x^3)*(-625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13})^{1/4}*\log(b^{10}*(-625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13})^{3/4} - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*\sqrt{x} + 5*(b^3*c*x^5 + b^4*x^3)*(-625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13})^{1/4}*\log(-b^{10}*(-625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^{13})^{3/4} - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*\sqrt{x} + 4*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x^2)*\sqrt{x})/(b^3*c*x^5 + b^4*x^3)$

**giac [A]** time = 0.21, size = 303, normalized size = 0.98

$$\frac{\sqrt{2} \left( 5 (bc)^{\frac{1}{2}} Bb - 9 (bc)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{2(cx^2 + b)b^3} + \frac{\sqrt{2} \left( 5 (bc)^{\frac{1}{2}} Bb - 9 (bc)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8b^4c^2} + \frac{\sqrt{2} \left( 5 (bc)^{\frac{1}{2}} Bb - 9 (bc)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{\frac{b}{c}} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^4c^2} + \frac{\sqrt{2} \left( 5 (bc)^{\frac{1}{2}} Bb - 9 (bc)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{\frac{b}{c}} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}} \right)}{16b^4c^2} - \frac{2(5Bb^2 - 10Acx^2 + Ab)}{5b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")`

[Out]  $-1/2*(B*b*c*x^{3/2} - A*c^2*x^{3/2})/((c*x^2 + b)*b^3) - 1/8*\sqrt{2}*(5*(b*c^3)^{3/4}*B*b - 9*(b*c^3)^{3/4}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} + 2*\sqrt{x})/(b/c)^{1/4})/(b^4*c^2) - 1/8*\sqrt{2}*(5*(b*c^3)^{3/4}*B*b - 9*(b*c^3)^{3/4}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} - 2*\sqrt{x})/(b/c)^{1/4})/(b^4*c^2) + 1/16*\sqrt{2}*(5*(b*c^3)^{3/4}*B*b - 9*(b*c^3)^{3/4}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^4*c^2) - 1/16*\sqrt{2}*(5*(b*c^3)^{3/4}*B*b - 9*(b*c^3)^{3/4}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^4*c^2) - 2/5*(5*B*b*x^2 - 10*A*c*x^2 + A*b)/(b^3*x^{5/2})$

**maple [A]** time = 0.06, size = 339, normalized size = 1.09

$$\frac{Ac^2x^3}{2(cx^2 + b)b^3} - \frac{Bcx^3}{2(cx^2 + b)b^2} + \frac{9\sqrt{2} Ac \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3} + \frac{9\sqrt{2} Ac \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3} + \frac{9\sqrt{2} Ac \ln \left( \frac{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{\frac{b}{c}} + \sqrt{\frac{b}{c}}}{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{\frac{b}{c}} + \sqrt{\frac{b}{c}}} \right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3} - \frac{5\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2} - \frac{5\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{\frac{b}{c}} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} \right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2} - \frac{5\sqrt{2} B \ln \left( \frac{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{\frac{b}{c}} + \sqrt{\frac{b}{c}}}{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{\frac{b}{c}} + \sqrt{\frac{b}{c}}} \right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2} + \frac{4Ac}{b^3\sqrt{x}} - \frac{2B}{b^2\sqrt{x}} - \frac{2A}{5b^2x^{\frac{5}{2}}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x)`

[Out]  $\frac{1}{2} \frac{1}{b^3} c^2 x^{3/2} / (c x^2 + b) A - \frac{1}{2} \frac{1}{b^2} c x^{3/2} / (c x^2 + b) B + \frac{9}{16} \frac{1}{b^3} c / (b/c)^{1/4} 2^{1/2} A \ln((x - (b/c)^{1/4}) 2^{1/2} x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4}) 2^{1/2} x^{1/2} + (b/c)^{1/2}) + \frac{9}{8} \frac{1}{b^3} c / (b/c)^{1/4} 2^{1/2} A \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} + 1) + \frac{9}{8} \frac{1}{b^3} c / (b/c)^{1/4} 2^{1/2} A \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} - 1) - \frac{5}{16} \frac{1}{b^2} / (b/c)^{1/4} 2^{1/2} B \ln((x - (b/c)^{1/4}) 2^{1/2} x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4}) 2^{1/2} x^{1/2} + (b/c)^{1/2}) - \frac{5}{8} \frac{1}{b^2} / (b/c)^{1/4} 2^{1/2} B \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} + 1) - \frac{5}{8} \frac{1}{b^2} / (b/c)^{1/4} 2^{1/2} B \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} - 1) - \frac{2}{5} \frac{1}{b^2} A / x^{5/2} + \frac{4}{b^3} x^{1/2} A c - \frac{2}{b^2} x^{1/2} B$

**maxima** [A] time = 3.03, size = 250, normalized size = 0.81

$$\frac{5(5Bbc - 9Ac^2)x^4 + 4Ab^2 + 4(5Bb^2 - 9Abc)x^2}{10(b^3cx^2 + b^4x^2)} - \frac{(5Bbc - 9Ac^2) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{x}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{x}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")`

[Out]  $-\frac{1}{10} \frac{(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x^2)}{(b^3*c*x^{9/2} + b^4*x^{5/2})} - \frac{1}{16} \frac{(5*B*b*c - 9*A*c^2)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} + 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})})/\sqrt{(\sqrt{b}*\sqrt{c})} + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*c^{1/4} - 2*\sqrt{c}*\sqrt{x}))/\sqrt{(\sqrt{b}*\sqrt{c})})/\sqrt{(\sqrt{b}*\sqrt{c})} - \sqrt{2}*\log(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b}))/b^{3/4} + \sqrt{2}*\log(-\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x} + \sqrt{c}*x + \sqrt{b}))/b^{3/4}}{b^3}$

**mupad** [B] time = 0.25, size = 121, normalized size = 0.39

$$\frac{\frac{2x^2(9Ac-5Bb)}{5b^2} - \frac{2A}{5b} + \frac{cx^4(9Ac-5Bb)}{2b^3}}{bx^{5/2} + cx^{9/2}} + \frac{(-c)^{1/4} \operatorname{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(9Ac-5Bb)}{4b^{13/4}} - \frac{(-c)^{1/4} \operatorname{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(9Ac-5Bb)}{4b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)`

[Out]  $\frac{(2*x^2*(9*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (c*x^4*(9*A*c - 5*B*b))/(2*b^3)}{(b*x^{5/2} + c*x^{9/2})} + \frac{((-c)^{1/4}*\operatorname{atan}(((c)^{1/4})*x^{1/2}))/b^{1/4}}{(9*A*c - 5*B*b)} - \frac{((-c)^{1/4}*\operatorname{atanh}(((c)^{1/4})*x^{1/2}))/b^{1/4}}{(9*A*c - 5*B*b)}$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**2,x)`

[Out] Timed out

$$3.205 \quad \int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=310

$$\frac{c^{3/4}(7bB - 11Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{15/4}} - \frac{c^{3/4}(7bB - 11Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{15/4}} + \frac{c^{3/4}(7bB - 11Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{15/4}}$$

**Rubi [A]** time = 0.26, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1584, 457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}(7bB - 11Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{15/4}} - \frac{c^{3/4}(7bB - 11Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{8\sqrt{2} b^{15/4}} + \frac{c^{3/4}(7bB - 11Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{15/4}} - \frac{c^{3/4}(7bB - 11Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}}{\sqrt[4]{b}} + 1\right)}{4\sqrt{2} b^{15/4}} - \frac{7bB - 11Ac}{6b^3 x^{3/2}} - \frac{7bB - 11Ac}{14b^2 c x^{7/2}} - \frac{bB - Ac}{2bcx^{7/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)^2), x]

[Out] (7\*b\*B - 11\*A\*c)/(14\*b^2\*c\*x^(7/2)) - (7\*b\*B - 11\*A\*c)/(6\*b^3\*x^(3/2)) - (b\*B - A\*c)/(2\*b\*c\*x^(7/2)\*(b + c\*x^2)) + (c^(3/4)\*(7\*b\*B - 11\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*b^(15/4)) - (c^(3/4)\*(7\*b\*B - 11\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*b^(15/4)) + (c^(3/4)\*(7\*b\*B - 11\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(15/4)) - (c^(3/4)\*(7\*b\*B - 11\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(15/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{\sqrt{x} (bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^{9/2} (b + cx^2)^2} dx \\
&= -\frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} + \frac{\left(-\frac{7bB}{2} + \frac{11Ac}{2}\right) \int \frac{1}{x^{9/2}(b+cx^2)} dx}{2bc} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} + \frac{(7bB - 11Ac) \int \frac{1}{x^{5/2}(b+cx^2)} dx}{4b^2} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} - \frac{(c(7bB - 11Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{4b^3} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} - \frac{(c(7bB - 11Ac)) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x,\right)}{2b^3} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} - \frac{(c(7bB - 11Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx\right)}{4b^{7/2}} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} - \frac{(\sqrt{c} (7bB - 11Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{b}}{\sqrt{c}}}\right)}{8b^{7/2}} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} + \frac{c^{3/4}(7bB - 11Ac) \log(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c})}{8\sqrt{2} b^{15/4}} \\
&= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2} (b + cx^2)} + \frac{c^{3/4}(7bB - 11Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2} b^{15/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 385, normalized size = 1.24

$$\frac{168A^2\sqrt{c} + 448AB\sqrt{c} - 168A^2c - 42\sqrt{2}c^{3/4}(7bB - 11Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) + 42\sqrt{2}c^{3/4}(11Ac - 7bB) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}} + 1\right) - 231\sqrt{2}Ac^{3/4} \log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}) + 231\sqrt{2}Ac^{3/4} \log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}) - \frac{168B^2\sqrt{c}}{c^{3/4}} - \frac{224B^2c}{c^{3/4}} + 147\sqrt{2}Bc^{3/4} \log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}) - 147\sqrt{2}Bc^{3/4} \log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c})}{336b^{15/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)^2), x]

[Out] ((-96\*A\*b^(7/4))/x^(7/2) - (224\*b^(7/4)\*B)/x^(3/2) + (448\*A\*b^(3/4)\*c)/x^(3/2) - (168\*b^(7/4)\*B\*c\*Sqrt[x])/(b + c\*x^2) + (168\*A\*b^(3/4)\*c^2\*Sqrt[x])/(b + c\*x^2) + 42\*Sqrt[2]\*c^(3/4)\*(7\*b\*B - 11\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 42\*Sqrt[2]\*c^(3/4)\*(-7\*b\*B + 11\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 147\*Sqrt[2]\*b\*B\*c^(3/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 231\*Sqrt[2]\*A\*c^(7/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 147\*Sqrt[2]\*b\*B\*c^(3/4)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 231\*Sqrt[2]\*A\*c^(7/4)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(336\*b^(15/4))

**IntegrateAlgebraic [A]** time = 0.70, size = 200, normalized size = 0.65

$$\frac{(7bBc^{3/4} - 11Ac^{7/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}b^{15/4}} - \frac{(7bBc^{3/4} - 11Ac^{7/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{15/4}} + \frac{-12Ab^2 + 44Abcx^2 + 77Ac^2x^4 - 28b^2Bx^2 - 49bBcx^4}{42b^3x^{7/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)^2),x]

[Out]  $(-12*A*b^2 - 28*b^2*B*x^2 + 44*A*b*c*x^2 - 49*b*B*c*x^4 + 77*A*c^2*x^4)/(42*b^3*x^{7/2}*(b + c*x^2)) + ((7*b*B*c^{3/4} - 11*A*c^{7/4})*\text{ArcTan}[(\text{Sqrt}[b] - \text{Sqrt}[c]*x)/(\text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x])])/(4*\text{Sqrt}[2]*b^{15/4}) - ((7*b*B*c^{3/4} - 11*A*c^{7/4})*\text{ArcTanh}[(\text{Sqrt}[2]*b^{1/4}*c^{1/4}*\text{Sqrt}[x])/(\text{Sqrt}[b] + \text{Sqrt}[c]*x)])/(4*\text{Sqrt}[2]*b^{15/4})$

**fricas [B]** time = 0.45, size = 795, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^2/x^(1/2),x, algorithm="fricas")

[Out]  $1/168*(84*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{1/4}*\text{arctan}(\text{sqrt}(b^8*\text{sqrt}(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15}) + (49*B^2*b^2*c^2 - 154*A*B*b*c^3 + 121*A^2*c^4)*x)*b^{11}*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{3/4} + (7*B*b^{12}*c - 11*A*b^{11}*c^2)*\text{sqrt}(x)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{3/4})/(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7) + 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{1/4}*\log(b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{1/4} - (7*B*b*c - 11*A*c^2)*\text{sqrt}(x)) - 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{1/4}*\log(-b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^{15})^{1/4} - (7*B*b*c - 11*A*c^2)*\text{sqrt}(x)) - 4*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*B*b^2 - 11*A*b*c)*x^2)*\text{sqrt}(x))/(b^3*c*x^6 + b^4*x^4)$

**giac [A]** time = 0.25, size = 292, normalized size = 0.94

$$\frac{\sqrt{2} \left( \frac{7}{8} (bc)^{\frac{1}{2}} Bb - 11 (bc)^{\frac{1}{2}} Ac \right) \arctan\left(\frac{\sqrt{2} \sqrt{\frac{b}{c}} \sqrt{x}}{\frac{b}{c}}\right)}{8b^4} + \frac{\sqrt{2} \left( \frac{7}{8} (bc)^{\frac{1}{2}} Bb - 11 (bc)^{\frac{1}{2}} Ac \right) \arctan\left(\frac{\sqrt{2} \sqrt{\frac{b}{c}} \sqrt{x}}{\frac{b}{c}}\right)}{8b^4} + \frac{\sqrt{2} \left( \frac{7}{8} (bc)^{\frac{1}{2}} Bb - 11 (bc)^{\frac{1}{2}} Ac \right) \log\left(\sqrt{2} \sqrt{\frac{b}{c}} \sqrt{x} + \sqrt{\frac{b}{c}}\right)}{16b^4} + \frac{\sqrt{2} \left( \frac{7}{8} (bc)^{\frac{1}{2}} Bb - 11 (bc)^{\frac{1}{2}} Ac \right) \log\left(-\sqrt{2} \sqrt{\frac{b}{c}} \sqrt{x} + \sqrt{\frac{b}{c}}\right)}{16b^4} + \frac{Bbc\sqrt{x} - Ac^2\sqrt{x}}{2(x^2+b)b^3} + \frac{2(7Bb^2 - 14Ac^2 + 3Ab)}{21b^5x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^2/x^(1/2),x, algorithm="giac")

[Out]  $-1/8*\text{sqrt}(2)*(7*(b*c^3)^{1/4}*B*b - 11*(b*c^3)^{1/4}*A*c)*\text{arctan}(1/2*\text{sqrt}(2)*( \text{sqrt}(2)*(b/c)^{1/4} + 2*\text{sqrt}(x))/(b/c)^{1/4})/b^4 - 1/8*\text{sqrt}(2)*(7*(b*c^3)^{1/4}*B*b - 11*(b*c^3)^{1/4}*A*c)*\text{arctan}(-1/2*\text{sqrt}(2)*( \text{sqrt}(2)*(b/c)^{1/4} - 2*\text{sqrt}(x))/(b/c)^{1/4})/b^4 - 1/16*\text{sqrt}(2)*(7*(b*c^3)^{1/4}*B*b - 11*(b*c^3)^{1/4}*A*c)*\log(\text{sqrt}(2)*\text{sqrt}(x)*(b/c)^{1/4} + x + \text{sqrt}(b/c))/b^4 + 1/16*\text{sqrt}(2)*(7*(b*c^3)^{1/4}*B*b - 11*(b*c^3)^{1/4}*A*c)*\log(-\text{sqrt}(2)*\text{sqrt}(x)*(b/c)^{1/4} + x + \text{sqrt}(b/c))/b^4 - 1/2*(B*b*c*\text{sqrt}(x) - A*c^2*\text{sqrt}(x))/((c*x^2 + b)*b^3) - 2/21*(7*B*b*x^2 - 14*A*c*x^2 + 3*A*b)/(b^3*x^{7/2})$

**maple [A]** time = 0.06, size = 348, normalized size = 1.12

$$\frac{A c^2 \sqrt{x}}{2(c x^2+b) b^3} - \frac{B c \sqrt{x}}{2(c x^2+b) b^3} + \frac{11 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 b^4} + \frac{11 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 b^4} + \frac{11 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A c^2 \ln\left(\frac{1+\sqrt{2} \sqrt{x} \sqrt{\frac{b}{c}}}{1-\sqrt{2} \sqrt{x} \sqrt{\frac{b}{c}}}\right)}{16 b^4} + \frac{7 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 b^3} - \frac{7 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c \arctan\left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8 b^3} + \frac{7 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B c \ln\left(\frac{1+\sqrt{2} \sqrt{x} \sqrt{\frac{b}{c}}}{1-\sqrt{2} \sqrt{x} \sqrt{\frac{b}{c}}}\right)}{16 b^3} + \frac{4 A c}{3 b^3 x^{\frac{3}{2}}} - \frac{2 B}{3 b^3 x^{\frac{3}{2}}} - \frac{2 A}{7 b^2 x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(c\*x^4+b\*x^2)^2/x^(1/2),x)

[Out] 1/2/b^3\*c^2\*x^(1/2)/(c\*x^2+b)\*A-1/2/b^2\*c\*x^(1/2)/(c\*x^2+b)\*B+11/8/b^4\*c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)+11/8/b^4\*c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)+11/16/b^4\*c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-7/8/b^3\*c\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-7/8/b^3\*c\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-7/16/b^3\*c\*(b/c)^(1/4)\*2^(1/2)\*B\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-2/7\*A/b^2/x^(7/2)+4/3/b^3/x^(3/2)\*A\*c-2/3/b^2/x^(3/2)\*B

maxima [A] time = 3.12, size = 286, normalized size = 0.92

$$\frac{7(7Bbc-11A^2)x^4+12Ab^2+4(7Bb^2-11Abc)x^2}{42\left(b^3cx^{\frac{11}{2}}+b^4x^{\frac{7}{2}}\right)} - \frac{2\sqrt{2}(7Bbc-11A^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\sqrt{c}\sqrt{b}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(7Bbc-11A^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\sqrt{c}\sqrt{b}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(7Bbc-11A^2)\log\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{16b^3} - \frac{\sqrt{2}(7Bbc-11A^2)\log\left(-\sqrt{2}b^{\frac{1}{4}}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^2/x^(1/2),x, algorithm="maxima")

[Out] -1/42\*(7\*(7\*B\*b\*c - 11\*A\*c^2)\*x^4 + 12\*A\*b^2 + 4\*(7\*B\*b^2 - 11\*A\*b\*c)\*x^2)/(b^3\*c\*x^(11/2) + b^4\*x^(7/2)) - 1/16\*(2\*sqrt(2)\*(7\*B\*b\*c - 11\*A\*c^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(b)\*sqrt(sqrt(b)\*sqrt(c))) + 2\*sqrt(2)\*(7\*B\*b\*c - 11\*A\*c^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(b)\*sqrt(sqrt(b)\*sqrt(c))) + sqrt(2)\*(7\*B\*b\*c - 11\*A\*c^2)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4)) - sqrt(2)\*(7\*B\*b\*c - 11\*A\*c^2)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4))/b^3

mupad [B] time = 0.40, size = 595, normalized size = 1.92

$$\frac{2\sqrt{2}(7Bbc-11A^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\sqrt{c}\sqrt{b}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4b^3cx^{\frac{11}{2}}+4b^4x^{\frac{7}{2}}} + \frac{2\sqrt{2}(7Bbc-11A^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\sqrt{c}\sqrt{b}\right)}{2\sqrt{b}\sqrt{c}}\right)}{4b^3cx^{\frac{11}{2}}+4b^4x^{\frac{7}{2}}} + \frac{\sqrt{2}(7Bbc-11A^2)\log\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{4b^3} - \frac{\sqrt{2}(7Bbc-11A^2)\log\left(-\sqrt{2}b^{\frac{1}{4}}\sqrt{c}+\sqrt{c}x+\sqrt{b}\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^(1/2)\*(b\*x^2 + c\*x^4)^2),x)

[Out] ((2\*x^2\*(11\*A\*c - 7\*B\*b))/(21\*b^2) - (2\*A)/(7\*b) + (c\*x^4\*(11\*A\*c - 7\*B\*b))/(6\*b^3))/(b\*x^(7/2) + c\*x^(11/2)) + ((-c)^(3/4)\*atan((((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(x^(1/2)\*(3872\*A^2\*b^9\*c^7 + 1568\*B^2\*b^11\*c^5 - 4928\*A\*B\*b^10\*c^6) - ((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(2816\*A\*b^13\*c^5 - 1792\*B\*b^14\*c^4)\*1i)/(8\*b^(15/4)))))/(8\*b^(15/4)) + ((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(x^(1/2)\*(3872\*A^2\*b^9\*c^7 + 1568\*B^2\*b^11\*c^5 - 4928\*A\*B\*b^10\*c^6) + ((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(2816\*A\*b^13\*c^5 - 1792\*B\*b^14\*c^4)\*1i)/(8\*b^(15/4)))))/(8\*b^(15/4)))/(((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(x^(1/2)\*(3872\*A^2\*b^9\*c^7 + 1568\*B^2\*b^11\*c^5 - 4928\*A\*B\*b^10\*c^6) - ((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(2816\*A\*b^13\*c^5 - 1792\*B\*b^14\*c^4)\*1i)/(8\*b^(15/4))))\*1i)/(8\*b^(15/4)) - ((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(x^(1/2)\*(3872\*A^2\*b^9\*c^7 + 1568\*B^2\*b^11\*c^5 - 4928\*A\*B\*b^10\*c^6) + ((-c)^(3/4)\*(11\*A\*c - 7\*B\*b)\*(2816\*A\*b^13\*c^5 - 1792\*B\*b^14\*c^4)\*1i)/(8\*b^(15/4))))\*(11\*A\*c - 7\*B\*b))/(4\*b^(15/4)) - ((-c)^(3/4)\*atan((A^3\*c^8\*x^(1/2)\*1331i - B^3\*b^3\*c^5\*x^(1/2)\*343i - A^2\*B\*b\*c^7\*x^(1/2)\*2541i + A\*B^2\*b^2\*c^6\*x^(1/2)\*1617i)/(b^(1/4)\*(-c)^(19/4)\*(c\*(c\*(1331\*A^3\*c - 2541\*A^2\*B\*b) + 1617\*A\*B^2\*b^2) - 343\*B^3\*b^3)))\*(11\*A\*c - 7\*B\*b)\*1i)/(4\*b^(15/4))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

$$3.206 \quad \int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=332

$$\frac{c^{5/4}(9bB - 13Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{17/4}} - \frac{c^{5/4}(9bB - 13Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{17/4}} - \frac{c^{5/4}(9bB - 13Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}}\right)}{4\sqrt{2} b^{17/4}} + \frac{c^{5/4}(9bB - 13Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}} + 1\right)}{4\sqrt{2} b^{17/4}} - \frac{9bB - 13Ac}{10b^3 c^{5/2}} + \frac{9bB - 13Ac}{18b^2 c^{3/2}} + \frac{c(9bB - 13Ac)}{2b^4 \sqrt{c}} - \frac{bB - Ac}{2bcx^2(b+cx^2)}$$

**Rubi [A]** time = 0.29, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(9bB - 13Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{17/4}} - \frac{c^{5/4}(9bB - 13Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{8\sqrt{2} b^{17/4}} - \frac{c^{5/4}(9bB - 13Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}}\right)}{4\sqrt{2} b^{17/4}} + \frac{c^{5/4}(9bB - 13Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}} + 1\right)}{4\sqrt{2} b^{17/4}} - \frac{9bB - 13Ac}{10b^3 c^{5/2}} + \frac{9bB - 13Ac}{18b^2 c^{3/2}} + \frac{c(9bB - 13Ac)}{2b^4 \sqrt{c}} - \frac{bB - Ac}{2bcx^2(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)^2), x]

[Out] (9\*b\*B - 13\*A\*c)/(18\*b^2\*c\*x^(9/2)) - (9\*b\*B - 13\*A\*c)/(10\*b^3\*x^(5/2)) + (c\*(9\*b\*B - 13\*A\*c))/(2\*b^4\*Sqrt[x]) - (b\*B - A\*c)/(2\*b\*c\*x^(9/2)\*(b + c\*x^2)) - (c^(5/4)\*(9\*b\*B - 13\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*b^(17/4)) + (c^(5/4)\*(9\*b\*B - 13\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(4\*Sqrt[2]\*b^(17/4)) + (c^(5/4)\*(9\*b\*B - 13\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(17/4)) - (c^(5/4)\*(9\*b\*B - 13\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(8\*Sqrt[2]\*b^(17/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457



```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

#### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^{3/2}(bx^2 + cx^4)^2} dx &= \int \frac{A + Bx^2}{x^{11/2}(b + cx^2)^2} dx \\
&= -\frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} + \frac{\left(-\frac{9bB}{2} + \frac{13Ac}{2}\right) \int \frac{1}{x^{11/2}(b+cx^2)} dx}{2bc} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} + \frac{(9bB - 13Ac) \int \frac{1}{x^{7/2}(b+cx^2)} dx}{4b^2} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} - \frac{(c(9bB - 13Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{4b^3} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} + \frac{(c^2(9bB - 13Ac)) \int \frac{1}{x^{1/2}(b+cx^2)} dx}{4b^4} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} + \frac{(c^2(9bB - 13Ac)) \operatorname{Sqrt}[x]}{4b^4} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} - \frac{(c^{3/2}(9bB - 13Ac)) \operatorname{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{b+cx^2}}\right]}{4b^4} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} - \frac{(c^{3/2}(9bB - 13Ac)) \operatorname{Sqrt}[x]}{4b^4} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} + \frac{(c(9bB - 13Ac)) \operatorname{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{b+cx^2}}\right]}{4b^4} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} + \frac{c^{5/4}(9bB - 13Ac) \operatorname{Log}\left[\frac{\sqrt{x}}{\sqrt{b+cx^2}}\right]}{4b^4} \\
&= \frac{9bB - 13Ac}{18b^2cx^{9/2}} - \frac{9bB - 13Ac}{10b^3x^{5/2}} + \frac{c(9bB - 13Ac)}{2b^4\sqrt{x}} - \frac{bB - Ac}{2bcx^{9/2}(b + cx^2)} - \frac{c^{5/4}(9bB - 13Ac) \operatorname{Tan}\left[\frac{\sqrt{x}}{\sqrt{b+cx^2}}\right]}{4\sqrt{2}b^{17/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.59, size = 176, normalized size = 0.53

$$\frac{2c^2x^{3/2}(bB - Ac) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^5} + \frac{2c(2bB - 3Ac)}{b^4\sqrt{x}} - \frac{2(bB - 2Ac)}{5b^3x^{5/2}} - \frac{2A}{9b^2x^{9/2}} + \frac{c^{5/4}(2bB - 3Ac) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}} + \frac{c^{5/4}(3Ac - 2bB) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)^2), x]

[Out] (-2\*A)/(9\*b^2\*x^(9/2)) - (2\*(b\*B - 2\*A\*c))/(5\*b^3\*x^(5/2)) + (2\*c\*(2\*b\*B - 3\*A\*c))/(b^4\*Sqrt[x]) + (c^(5/4)\*(2\*b\*B - 3\*A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) + (c^(5/4)\*(-2\*b\*B + 3\*A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) + (2\*c^2\*(b\*B - A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -(c\*x^2)/b])/ (3\*b^5)

**IntegrateAlgebraic [A]** time = 0.73, size = 224, normalized size = 0.67

$$\frac{(9bBc^{5/4} - 13Ac^{9/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{cx}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{4\sqrt{2}b^{17/4}} - \frac{(9bBc^{5/4} - 13Ac^{9/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{cx}}\right)}{4\sqrt{2}b^{17/4}} + \frac{-20Ab^3 + 52Ab^2cx^2 - 468Abc^2x^4 - 585Ac^3x^6 - 36b^3Bx^2 + 324b^2Bcx^4 + 405bBc^2x^6}{90b^4x^{9/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)^2), x]

[Out]  $(-20*Ab^3 - 36*b^3B*x^2 + 52*Ab^2*c*x^2 + 324*b^2*B*c*x^4 - 468*Ab*c^2*x^4 + 405*b*B*c^2*x^6 - 585*A*c^3*x^6)/(90*b^4*x^{9/2}*(b + c*x^2)) - ((9*b*B*c^{5/4} - 13*A*c^{9/4})*\text{ArcTan}[(\sqrt{b} - \sqrt{c}*x)/(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x})])/(4*\sqrt{2}*b^{17/4}) - ((9*b*B*c^{5/4} - 13*A*c^{9/4})*\text{ArcTanh}[(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x})/(\sqrt{b} + \sqrt{c}*x)])/(4*\sqrt{2}*b^{17/4})$

**fricas** [B] time = 0.48, size = 1024, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(3/2)/(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out]  $1/360*(180*(b^4*c*x^7 + b^5*x^5)*(-6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17})^{1/4}*\arctan(\sqrt{(531441*B^6*b^6*c^8 - 4605822*A*B^5*b^5*c^9 + 16632135*A^2*B^4*b^4*c^{10} - 32032260*A^3*B^3*b^3*c^{11} + 34701615*A^4*B^2*b^2*c^{12} - 20049822*A^5*B*b*c^{13} + 4826809*A^6*c^{14})*x - (6561*B^4*b^{13}*c^5 - 37908*A*B^3*b^{12}*c^6 + 82134*A^2*B^2*b^{11}*c^7 - 79092*A^3*B*b^{10}*c^8 + 28561*A^4*b^9*c^9)*\sqrt{-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17}})/b^{17})^{1/4} + (729*B^3*b^7*c^4 - 3159*A*B^2*b^6*c^5 + 4563*A^2*B*b^5*c^6 - 2197*A^3*b^4*c^7)*\sqrt{x}*(-6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17})^{1/4} / (6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9) - 45*(b^4*c*x^7 + b^5*x^5)*(-6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17})^{1/4} * \log(b^{13}*(-6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17})^{3/4} - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*\sqrt{x} + 45*(b^4*c*x^7 + b^5*x^5)*(-6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17})^{1/4} * \log(-b^{13}*(-6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^{17})^{3/4} - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*\sqrt{x} + 4*(45*(9*B*b^2*c - 13*A*b*c^2)*x^6 + 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*Ab^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)*\sqrt{x})/(b^4*c*x^7 + b^5*x^5)$

**giac** [A] time = 0.21, size = 328, normalized size = 0.99

$$\frac{\sqrt{2} \left( (bc)^{\frac{1}{2}} b^2 - 13 (bc)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( (bc)^{\frac{1}{2}} b^2 - 13 (bc)^{\frac{1}{2}} Ac \right)}{2 \left( (bc)^{\frac{1}{2}} \right)} \right)}{8 \sqrt{bc}} + \frac{\sqrt{2} \left( (bc)^{\frac{1}{2}} b^2 - 13 (bc)^{\frac{1}{2}} Ac \right) \arctan \left( \frac{\sqrt{2} \left( (bc)^{\frac{1}{2}} b^2 - 13 (bc)^{\frac{1}{2}} Ac \right)}{2 \left( (bc)^{\frac{1}{2}} \right)} \right)}{8 \sqrt{bc}} - \frac{\sqrt{2} \left( (bc)^{\frac{1}{2}} b^2 - 13 (bc)^{\frac{1}{2}} Ac \right) \log \left( \sqrt{2} \sqrt{\left( \frac{b}{c} \right)^{\frac{1}{2}} + x + \sqrt{c}} \right)}{16 \sqrt{bc}} + \frac{\sqrt{2} \left( (bc)^{\frac{1}{2}} b^2 - 13 (bc)^{\frac{1}{2}} Ac \right) \log \left( -\sqrt{2} \sqrt{\left( \frac{b}{c} \right)^{\frac{1}{2}} + x + \sqrt{c}} \right)}{16 \sqrt{bc}} + \frac{2 \left( 90 B b c^4 - 135 A c^2 x^4 - 9 B b^2 x^2 + 18 A b c x^2 - 5 A b^2 \right)}{2 (c^2 + b)^{5/4}} + \frac{2 \left( 90 B b c^4 - 135 A c^2 x^4 - 9 B b^2 x^2 + 18 A b c x^2 - 5 A b^2 \right)}{45 b^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(3/2)/(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out]  $1/8*\sqrt{2}*(9*(b*c^3)^{3/4}*B*b - 13*(b*c^3)^{3/4}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} + 2*\sqrt{x})/(b/c)^{1/4})/(b^5*c) + 1/8*\sqrt{2}*(9*(b*c^3)^{3/4}*B*b - 13*(b*c^3)^{3/4}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} - 2*\sqrt{x})/(b/c)^{1/4})/(b^5*c) - 1/16*\sqrt{2}*(9*(b*c^3)^{3/4}*B*b - 13*(b*c^3)^{3/4}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^5*c) + 1/16*\sqrt{2}*(9*(b*c^3)^{3/4}*B*b - 13*(b*c^3)^{3/4}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^5*c) + 1/2*(B*b*c^2*x^{3/2} - A*c^3*x^{3/2})/((c*x^2 + b)*b^4) + 2/45*(90*B*b*c*x^4 - 135*A*c^2*x^4 - 9*B*b^2*x^2 + 18*A*b*c*x^2 - 5*A*b^2)/(b^4*x^{9/2})$

**maple [A]** time = 0.07, size = 372, normalized size = 1.12

$$\frac{A c^2 x^{\frac{3}{2}}}{2(c x^2 + b)^{\frac{5}{2}}} + \frac{B c^2 x^{\frac{3}{2}}}{2(c x^2 + b)^{\frac{5}{2}}} - \frac{13\sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}} b^{\frac{5}{2}}} - \frac{13\sqrt{2} A c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}} b^{\frac{5}{2}}} - \frac{13\sqrt{2} A c^2 \ln\left(\frac{\sqrt{\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}}{\sqrt{\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}-\sqrt{c}}}\right)}{16\left(\frac{c}{b}\right)^{\frac{1}{4}} b^{\frac{5}{2}}} + \frac{9\sqrt{2} B c \arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}} b^{\frac{5}{2}}} + \frac{9\sqrt{2} B c \arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{c}{b}\right)^{\frac{1}{4}} b^{\frac{5}{2}}} + \frac{9\sqrt{2} B c \ln\left(\frac{\sqrt{\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}}{\sqrt{\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}-\sqrt{c}}}\right)}{16\left(\frac{c}{b}\right)^{\frac{1}{4}} b^{\frac{5}{2}}} - \frac{6Ac^2}{b^4\sqrt{c}} + \frac{4Bc}{b^3\sqrt{c}} + \frac{4Ac}{5b^3x^{\frac{3}{2}}} - \frac{2B}{5b^2x^{\frac{3}{2}}} - \frac{2A}{9b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/x^(3/2)/(c\*x^4+b\*x^2)^2,x)

[Out]  $-\frac{1}{2} \frac{b^4 c^3 x^{3/2}}{(c x^2 + b)^2} + \frac{1}{2} \frac{b^4 c^2 x^{3/2}}{(c x^2 + b)^2} + \frac{1}{2} \frac{b^4 c^2 x^{3/2}}{(c x^2 + b)^2} - \frac{13}{8} \frac{b^4 c^2}{(b/c)^{1/4}} \frac{2^{1/2} A \ln((x - (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}))}{(b/c)^{1/4}} - \frac{13}{8} \frac{b^4 c^2}{(b/c)^{1/4}} \frac{2^{1/2} A \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} + 1)}{(b/c)^{1/4}} - \frac{13}{8} \frac{b^4 c^2}{(b/c)^{1/4}} \frac{2^{1/2} A \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} - 1)}{(b/c)^{1/4}} + \frac{9}{16} \frac{b^4 c^2}{(b/c)^{1/4}} \frac{2^{1/2} B \ln((x - (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}) / (x + (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}))}{(b/c)^{1/4}} + \frac{9}{8} \frac{b^4 c^2}{(b/c)^{1/4}} \frac{2^{1/2} B \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} + 1)}{(b/c)^{1/4}} + \frac{9}{8} \frac{b^4 c^2}{(b/c)^{1/4}} \frac{2^{1/2} B \arctan(2^{1/2} / (b/c)^{1/4} x^{1/2} - 1)}{(b/c)^{1/4}} - \frac{2}{9} \frac{A}{b^2 x^{9/2}} + \frac{4}{5} \frac{B}{b^3 x^{5/2}} + \frac{A c^2}{5 b^4 x^{9/2}} - \frac{2 B c}{5 b^4 x^{5/2}} - \frac{6 A c^2}{b^4 \sqrt{c}} + \frac{4 B c}{b^3 \sqrt{c}} + \frac{4 A c}{5 b^3 x^{3/2}} - \frac{2 B}{5 b^2 x^{3/2}} - \frac{2 A}{9 b^2 x^{3/2}}$

**maxima [A]** time = 3.01, size = 276, normalized size = 0.83

$$\frac{45(9 B b c^2 - 13 A c^3) x^6 + 36(9 B b^2 c - 13 A b c^2) x^4 - 20 A b^3 - 4(9 B b^3 - 13 A b^2 c) x^2}{90(b^4 c x^2 + b^5 x^2)^{\frac{5}{2}}} + \frac{(9 B b c^2 - 13 A c^3) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}\sqrt{c}+2\sqrt{c}\sqrt{c}}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}\sqrt{c}-2\sqrt{c}\sqrt{c}}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} - \frac{\sqrt{2} \log\left(\frac{\sqrt{2}b^{\frac{1}{4}}\sqrt{c}+\sqrt{c}x+\sqrt{b}}{b^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}c^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(\frac{-\sqrt{2}b^{\frac{1}{4}}\sqrt{c}+\sqrt{c}x+\sqrt{b}}{b^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{16 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/x^(3/2)/(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out]  $\frac{1}{90} (45(9 B b^2 c^2 - 13 A b c^3) x^6 + 36(9 B b^2 c - 13 A b^2 c^2) x^4 - 20 A b^3 - 4(9 B b^3 - 13 A b^2 c) x^2) / (b^4 c x^2 + b^5 x^2)^{\frac{5}{2}} + \frac{1}{16} (9 B b^2 c^2 - 13 A b c^3) (2 \sqrt{2} \arctan(1/2 \sqrt{2} (\sqrt{2} b^{1/4} c^{1/4} + 2 \sqrt{c} \sqrt{c})) / (\sqrt{2} \sqrt{b} \sqrt{c}) + 2 \sqrt{2} \arctan(-1/2 \sqrt{2} (\sqrt{2} b^{1/4} c^{1/4} - 2 \sqrt{c} \sqrt{c})) / (\sqrt{2} \sqrt{b} \sqrt{c}) - \sqrt{2} \log(\sqrt{2} b^{1/4} c^{1/4} \sqrt{c} x + \sqrt{c} x + \sqrt{b}) / (b^{1/4} c^{3/4}) + \sqrt{2} \log(-\sqrt{2} b^{1/4} c^{1/4} \sqrt{c} x + \sqrt{c} x + \sqrt{b}) / (b^{1/4} c^{3/4})) / b^4$

**mupad [B]** time = 0.20, size = 142, normalized size = 0.43

$$\frac{(-c)^{5/4} \operatorname{atan}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (13 A c - 9 B b)}{4 b^{17/4}} - \frac{2 A}{9 b} - \frac{2 x^2 (13 A c - 9 B b)}{45 b^2} + \frac{c^2 x^6 (13 A c - 9 B b)}{2 b^4} + \frac{2 c x^4 (13 A c - 9 B b)}{5 b^3} - \frac{(-c)^{5/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (13 A c - 9 B b)}{4 b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(x^(3/2)\*(b\*x^2 + c\*x^4)^2),x)

[Out]  $((-c)^{5/4} \operatorname{atan}\left(\frac{(-c)^{1/4} x^{1/2}}{b^{1/4}}\right) (13 A c - 9 B b)) / (4 b^{17/4}) - ((2 A) / (9 b) - (2 x^2 (13 A c - 9 B b)) / (45 b^2) + (c^2 x^6 (13 A c - 9 B b)) / (2 b^4) + (2 c x^4 (13 A c - 9 B b)) / (5 b^3)) / (b x^{9/2} + c x^{13/2}) - ((-c)^{5/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} x^{1/2}}{b^{1/4}}\right) (13 A c - 9 B b)) / (4 b^{17/4})$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/x\*\*(3/2)/(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Timed out

$$3.207 \quad \int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=343

$$\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}} + \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}$$

Rubi [A] time = 0.28, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{-\frac{9\sqrt[4]{b}(13bB-5Ac)}{16b^2(b+cx^2)} + \frac{9\sqrt[4]{b}(13bB-5Ac)}{80bc^3} - \frac{9\sqrt[4]{b}(13bB-5Ac)}{16c^4} - \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}} + \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}} - \frac{9\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2}c^{17/4}} + \frac{9\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}+1\right)}{32\sqrt{2}c^{17/4}} - \frac{x^{13/2}(bB-5Ac)}{4bc(b+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(23/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (-9\*(13\*b\*B - 5\*A\*c)\*Sqrt[x])/(16\*c^4) + (9\*(13\*b\*B - 5\*A\*c)\*x^(5/2))/(80\*b\*c^3) - ((b\*B - A\*c)\*x^(13/2))/(4\*b\*c\*(b + c\*x^2)^2) - ((13\*b\*B - 5\*A\*c)\*x^(9/2))/(16\*b\*c^2\*(b + c\*x^2)) - (9\*b^(1/4)\*(13\*b\*B - 5\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*c^(17/4)) + (9\*b^(1/4)\*(13\*b\*B - 5\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*c^(17/4)) - (9\*b^(1/4)\*(13\*b\*B - 5\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*c^(17/4)) + (9\*b^(1/4)\*(13\*b\*B - 5\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*c^(17/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 288

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*n\*(p+1)), x] - Dist[(c^n\*(m-n+1))/(b\*n\*(p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I LtQ[(m+n\*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_.)*((c_) + (d_.)*(x_)^(n
_.)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{23/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^{11/2} (A + Bx^2)}{(b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} + \frac{\left(\frac{13bB}{2} - \frac{5Ac}{2}\right) \int \frac{x^{11/2}}{(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \frac{(9(13bB - 5Ac)) \int \frac{x^{7/2}}{b+cx^2} dx}{32bc^2} \\
&= \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} - \frac{(9(13bB - 5Ac)) \int \frac{x^{3/2}}{b+cx^2} dx}{32c^3} \\
&= -\frac{9(13bB - 5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \dots \\
&= -\frac{9(13bB - 5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \dots \\
&= -\frac{9(13bB - 5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \dots \\
&= -\frac{9(13bB - 5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \dots \\
&= -\frac{9(13bB - 5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \dots \\
&= -\frac{9(13bB - 5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB - 5Ac)x^{5/2}}{80bc^3} - \frac{(bB - Ac)x^{13/2}}{4bc(b + cx^2)^2} - \frac{(13bB - 5Ac)x^{9/2}}{16bc^2(b + cx^2)} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.63, size = 435, normalized size = 1.27

$$\frac{160b^{13/2}\sqrt{c} - 90\sqrt{2}\sqrt{(13bB - 5Ac)}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}}\right) + 90\sqrt{2}\sqrt{(13bB - 5Ac)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+cx^2}} + 1\right) + \frac{90bc^{13/2}\sqrt{x}}{16c^4} + 225\sqrt{2}A\sqrt{c}\log\left(-\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b+cx^2}\right) - 225\sqrt{2}A\sqrt{c}\log\left(\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b+cx^2}\right) + 1280A^{5/4}\sqrt{c} - 585\sqrt{2}B^{5/4}\log\left(-\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b+cx^2}\right) + 585\sqrt{2}B^{5/4}\log\left(\sqrt{2}\sqrt{c}\sqrt{x} + \sqrt{b+cx^2}\right) + \frac{160b^{13/2}\sqrt{c}}{16c^4} - \frac{160b^{13/2}\sqrt{c}}{16c^4} - 38400B\sqrt{c} + 256B^{5/4}\sqrt{c}}{640c^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(23/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (-3840\*b\*B\*c^(1/4)\*Sqrt[x] + 1280\*A\*c^(5/4)\*Sqrt[x] + 256\*B\*c^(5/4)\*x^(5/2) + (160\*b^3\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2)^2 - (160\*A\*b^2\*c^(5/4)\*Sqrt[x])/(b + c\*x^2)^2 - (1000\*b^2\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2) + (680\*A\*b\*c^(5/4)\*Sqrt[x])/(b + c\*x^2) - 90\*Sqrt[2]\*b^(1/4)\*(13\*b\*B - 5\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 90\*Sqrt[2]\*b^(1/4)\*(13\*b\*B - 5\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 585\*Sqrt[2]\*b^(5/4)\*B\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 225\*Sqrt[2]\*A\*b^(1/4)\*c\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 585\*Sqrt[2]\*b^(5/4)\*B\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 225\*Sqrt[2]\*A\*b^(1/4)\*c\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(640\*c^(17/4))

**IntegrateAlgebraic [A]** time = 0.90, size = 282, normalized size = 0.82

$$\left(\frac{45A\sqrt{b}}{32\sqrt{2}c^{13/4}} - \frac{117b^{5/4}B}{32\sqrt{2}c^{17/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x}}\right) + \frac{x^{5/2}(405Abc^2 - 1053b^2Bc) + \sqrt{x}(225Ab^2c - 585b^3B) + x^{9/2}(160Ac^3 - 416bBc^2) + 32Bc^3x^{13/2}}{80c^4(b+cx^2)^2} - \frac{45A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{13/4}} + \frac{117b^{5/4}B\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{17/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(23/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((-585\*b^3\*B + 225\*A\*b^2\*c)\*Sqrt[x] + (-1053\*b^2\*B\*c + 405\*A\*b\*c^2)\*x^(5/2) + (-416\*b\*B\*c^2 + 160\*A\*c^3)\*x^(9/2) + 32\*B\*c^3\*x^(13/2))/(80\*c^4\*(b + c\*x^2)^2) + (((-117\*b^(5/4)\*B)/(32\*Sqrt[2]\*c^(17/4)) + (45\*A\*b^(1/4))/(32\*Sqrt[2]\*c^(13/4)))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])] + (117\*b^(5/4)\*B\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)))/(32\*Sqrt[2]\*c^(17/4)) - (45\*A\*b^(1/4)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)))/(32\*Sqrt[2]\*c^(13/4))

**fricas [B]** time = 0.45, size = 817, normalized size = 2.38



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(23/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] -1/320\*(180\*(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4)\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(1/4)\*arctan((sqrt(c^8\*sqrt(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17) + (169\*B^2\*b^2 - 130\*A\*B\*b\*c + 25\*A^2\*c^2)\*x)\*c^13\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(3/4) + (13\*B\*b\*c^13 - 5\*A\*c^14)\*sqrt(x)\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(3/4))/(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)) + 45\*(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4)\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(1/4)\*log(9\*c^4\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(1/4) - 9\*(13\*B\*b - 5\*A\*c)\*sqrt(x)) - 45\*(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4)\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(1/4)\*log(-9\*c^4\*(-(28561\*B^4\*b^5 - 43940\*A\*B^3\*b^4\*c + 25350\*A^2\*B^2\*b^3\*c^2 - 6500\*A^3\*B\*b^2\*c^3 + 625\*A^4\*b\*c^4)/c^17)^(1/4) - 9\*(13\*B\*b - 5\*A\*c)\*sqrt(x)) - 4\*(32\*B\*c^3\*x^6 - 32\*(13\*B\*b\*c^2 - 5\*A\*c^3)\*x^4 - 585\*B\*b^3 + 225\*A\*b^2\*c - 81\*(13\*B\*b^2\*c - 5\*A\*b\*c^2)\*x^2)\*sqrt(x))/(c^6\*x^4 + 2\*b\*c^5\*x^2 + b^2\*c^4)

**giac [A]** time = 0.23, size = 321, normalized size = 0.94

$$\frac{9\sqrt{2}\left(13\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - 5\left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} - \sqrt{c}}}{\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}}}\right)}{64c^5} + \frac{9\sqrt{2}\left(13\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - 5\left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} - \sqrt{c}}}{\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}}}\right)}{64c^5} + \frac{9\sqrt{2}\left(13\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - 5\left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} + \sqrt{c}}\right)}{128c^5} + \frac{9\sqrt{2}\left(13\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - 5\left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}} + \sqrt{c}}\right)}{128c^5} + \frac{25Bb^2c^2 - 17Ab^2c^2 + 21Bb^2c^2 - 13Ab^2c^2}{16(c^2 + b)^2c^4} + \frac{2(13Bb^2c^2 - 15Bb^2c^2 + 5A^2c^2)}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(23/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 9/64\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/c^5 + 9/64\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/c^5 + 9/128\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 9/128\*sqrt(2)\*(13\*(b\*c^3)^(1/4)\*B\*b - 5\*(b\*c^3)^(1/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 1/16\*(25\*B\*b^2\*c\*x^(5/2) - 17\*A\*b\*c



$$\frac{2x^{5/2} + 21Bb^3\sqrt{x} - 13Ab^2c\sqrt{x}}{(cx^2 + b)^2c^4} + \frac{2}{5} \frac{(Bc^{12}x^{5/2} - 15Bb^3c^{11}\sqrt{x} + 5A^2c^{12}\sqrt{x})}{c^{15}}$$

**maple [A]** time = 0.08, size = 381, normalized size = 1.11

$$\frac{17Ab^3}{16(c^2x^2+b)^2c^3} - \frac{25Bb^3}{16(c^2x^2+b)^2c^3} + \frac{13A^2\sqrt{c}}{16(c^2x^2+b)^2c^3} - \frac{21Bb^3\sqrt{c}}{16(c^2x^2+b)^2c^3} - \frac{2Bb^3}{5c^3} - \frac{45\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{c}\sqrt{x}}{b}\right)}{64c^3} - \frac{45\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{c}\sqrt{x}}{b}+1\right)}{64c^3} - \frac{45\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}A\ln\left(\frac{+\sqrt{2}\sqrt{c}\sqrt{x}}{+\sqrt{2}\sqrt{c}\sqrt{x}}\right)}{128c^3} + \frac{117\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}Bb\arctan\left(\frac{\sqrt{c}\sqrt{x}}{b}\right)}{64c^3} + \frac{117\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}Bb\arctan\left(\frac{\sqrt{c}\sqrt{x}}{b}+1\right)}{64c^3} + \frac{117\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}Bb\ln\left(\frac{+\sqrt{2}\sqrt{c}\sqrt{x}}{+\sqrt{2}\sqrt{c}\sqrt{x}}\right)}{128c^3} + \frac{2A\sqrt{c}}{c^3} - \frac{6Bb\sqrt{c}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(23/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out]  $\frac{2}{5} \frac{Bx^5}{c^3} + \frac{2}{c^3} \frac{Ax^{1/2}}{c^3} - \frac{6}{c^4} \frac{bBx^{1/2}}{c^3} + \frac{17}{16} \frac{b^2}{c^2} \frac{1}{(cx^2+b)^2} x^{5/2} + \frac{A}{c^3} - \frac{25}{16} \frac{b^2}{c^3} \frac{1}{(cx^2+b)^2} x^{5/2} + \frac{13}{16} \frac{b^2}{c^3} \frac{1}{(cx^2+b)^2} \frac{A}{c^3} x^{1/2} - \frac{21}{16} \frac{b^3}{c^4} \frac{1}{(cx^2+b)^2} \frac{Bx^{1/2}}{c^3} - \frac{45}{64} \frac{b^3}{c^3} \left(\frac{b}{c}\right)^{1/4} 2^{1/2} \frac{A}{c^3} \arctan\left(2^{1/2} \frac{1}{(b/c)^{1/4}} x^{1/2} + 1\right) - \frac{45}{64} \frac{b^3}{c^3} \left(\frac{b}{c}\right)^{1/4} 2^{1/2} \frac{A}{c^3} \arctan\left(2^{1/2} \frac{1}{(b/c)^{1/4}} x^{1/2} - 1\right) - \frac{45}{128} \frac{b^3}{c^3} \left(\frac{b}{c}\right)^{1/4} 2^{1/2} \frac{A}{c^3} \ln\left(\frac{x + (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}}{x - (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}}\right) + \frac{117}{64} \frac{b^3}{c^4} \left(\frac{b}{c}\right)^{1/4} 2^{1/2} \frac{B}{c^3} \arctan\left(2^{1/2} \frac{1}{(b/c)^{1/4}} x^{1/2} + 1\right) + \frac{117}{64} \frac{b^3}{c^4} \left(\frac{b}{c}\right)^{1/4} 2^{1/2} \frac{B}{c^3} \arctan\left(2^{1/2} \frac{1}{(b/c)^{1/4}} x^{1/2} - 1\right) + \frac{117}{128} \frac{b^3}{c^4} \left(\frac{b}{c}\right)^{1/4} 2^{1/2} \frac{B}{c^3} \ln\left(\frac{x + (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}}{x - (b/c)^{1/4} 2^{1/2} x^{1/2} + (b/c)^{1/2}}\right)$

**maxima [A]** time = 3.08, size = 306, normalized size = 0.89

$$\frac{(25Bb^2c - 17Ab^2)x^{\frac{5}{2}} + (21Bb^3 - 13Ab^2c)\sqrt{x}}{16(c^5x^4 + 2bc^5x^2 + b^2c^4)} + \frac{9 \left( \frac{2\sqrt{2}(13Bb - 5Ac) \arctan\left(\frac{\sqrt{c}\sqrt{\frac{1}{4}x^2 + 2\sqrt{c}x}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(13Bb - 5Ac) \arctan\left(\frac{\sqrt{c}\sqrt{\frac{1}{4}x^2 + 2\sqrt{c}x}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(13Bb - 5Ac) \ln\left(\frac{\sqrt{2}b^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}\sqrt{c}} - \frac{\sqrt{2}(13Bb - 5Ac) \ln\left(\frac{\sqrt{2}b^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}\sqrt{c}} \right)}{128c^4} + \frac{2(Bc^{\frac{5}{2}} - 5(3Bb - Ac)\sqrt{x})}{5c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(23/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $-\frac{1}{16} \frac{(25Bb^2c - 17Ab^2c^2)x^{5/2} + (21Bb^3 - 13Ab^2c)\sqrt{x}}{(c^6x^4 + 2b^2c^5x^2 + b^2c^4)} + \frac{9}{128} \frac{2\sqrt{2}(13Bb - 5Ac) \arctan\left(\frac{1}{2}\sqrt{2} \frac{1}{(b/c)^{1/4}} \sqrt{c} \sqrt{x} + \sqrt{c}\sqrt{x}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(13Bb - 5Ac) \arctan\left(\frac{1}{2}\sqrt{2} \frac{1}{(b/c)^{1/4}} \sqrt{c} \sqrt{x} - \sqrt{c}\sqrt{x}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(13Bb - 5Ac) \ln\left(\frac{\sqrt{2}b^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}\sqrt{c}} - \frac{\sqrt{2}(13Bb - 5Ac) \ln\left(\frac{\sqrt{2}b^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}\sqrt{c}} + \frac{2(Bc^{\frac{5}{2}} - 5(3Bb - Ac)\sqrt{x})}{5c^4}$

**mupad [B]** time = 0.25, size = 865, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(23/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out]  $\frac{x^{5/2} \left( \frac{17Ab^2c}{16} - \frac{25Bb^2c}{16} \right) - x^{1/2} \left( \frac{21Bb^3}{16} - \frac{13Ab^2c}{16} \right)}{(b^2c^4 + c^6x^4 + 2b^2c^5x^2)} + \frac{x^{1/2} \left( \frac{2A}{c^3} - \frac{6Bb}{c^4} \right) + \frac{2Bx^{5/2}}{5c^3} + \frac{(-b)^{1/4} \operatorname{atan}\left(\frac{(-b)^{1/4} \left( \frac{81x^{1/2}}{2} \left( \frac{169B^2b^4 + 25A^2b^2c^2 - 130ABb^3c \right)}{64c^5} - \frac{81(-b)^{1/4} \left( \frac{5Ac - 13Bb \right) \left( \frac{13Bb^3 - 5Ab^2c}{64c^{21/4}} \right) \left( \frac{5Ac - 13Bb}{64c^{17/4}} \right) + \frac{9i}{64c^{17/4}} \right)}{(-b)^{1/4} \left( \frac{81x^{1/2}}{2} \left( \frac{169B^2b^4 + 25A^2b^2c^2 - 130ABb^3c \right)}{64c^5} + \frac{81(-b)^{1/4} \left( \frac{5Ac - 13Bb \right) \left( \frac{13Bb^3 - 5Ab^2c}{64c^{21/4}} \right) \left( \frac{5Ac - 13Bb}{64c^{17/4}} \right) + \frac{9i}{64c^{17/4}} \right)} \right)}{(-b)^{1/4} \left( \frac{81x^{1/2}}{2} \left( \frac{169B^2b^4 + 25A^2b^2c^2 - 130ABb^3c \right)}{64c^5} - \frac{81(-b)^{1/4} \left( \frac{5Ac - 13Bb \right) \left( \frac{13Bb^3 - 5Ab^2c}{64c^{21/4}} \right) \left( \frac{5Ac - 13Bb}{64c^{17/4}} \right) + \frac{9i}{64c^{17/4}} \right)} \right)}{(-b)^{1/4} \left( \frac{81x^{1/2}}{2} \left( \frac{169B^2b^4 + 25A^2b^2c^2 - 130ABb^3c \right)}{64c^5} + \frac{81(-b)^{1/4} \left( \frac{5Ac - 13Bb \right) \left( \frac{13Bb^3 - 5Ab^2c}{64c^{21/4}} \right) \left( \frac{5Ac - 13Bb}{64c^{17/4}} \right) + \frac{9i}{64c^{17/4}} \right)} \right)}{(-b)^{1/4} \left( \frac{81x^{1/2}}{2} \left( \frac{169B^2b^4 + 25A^2b^2c^2 - 130ABb^3c \right)}{64c^5} - \frac{81(-b)^{1/4} \left( \frac{5Ac - 13Bb \right) \left( \frac{13Bb^3 - 5Ab^2c}{64c^{21/4}} \right) \left( \frac{5Ac - 13Bb}{64c^{17/4}} \right) + \frac{9i}{64c^{17/4}} \right)} \right)}{(-b)^{1/4} \left( \frac{81x^{1/2}}{2} \left( \frac{169B^2b^4 + 25A^2b^2c^2 - 130ABb^3c \right)}{64c^5} + \frac{81(-b)^{1/4} \left( \frac{5Ac - 13Bb \right) \left( \frac{13Bb^3 - 5Ab^2c}{64c^{21/4}} \right) \left( \frac{5Ac - 13Bb}{64c^{17/4}} \right) + \frac{9i}{64c^{17/4}} \right)} \right)}$

```

*B*b^3 - 5*A*b^2*c))/(64*c^(21/4))*((5*A*c - 13*B*b))/(64*c^(17/4))))*(5*A*
c - 13*B*b)*9i)/(32*c^(17/4)) + (9*(-b)^(1/4)*atan(((9*(-b)^(1/4))*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)^(1/4)*
(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4))*((5*A*c - 13*B*b
))/(64*c^(17/4)) + (9*(-b)^(1/4))*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
- 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*
b^2*c)*81i)/(64*c^(21/4))*((5*A*c - 13*B*b))/(64*c^(17/4)))/(((-b)^(1/4))*((
81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)
^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4))*((5*A*c -
13*B*b)*9i)/(64*c^(17/4)) - ((-b)^(1/4))*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2
*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^
3 - 5*A*b^2*c)*81i)/(64*c^(21/4))*((5*A*c - 13*B*b)*9i)/(64*c^(17/4))))*(5*
A*c - 13*B*b))/(32*c^(17/4))

```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

$$3.208 \quad \int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=322

$$\frac{7(11bB - 3Ac) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} \sqrt[4]{b} c^{15/4}} + \frac{7(11bB - 3Ac) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} \sqrt[4]{b} c^{15/4}} + \frac{7(11bB - 3Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}}\right)}{32\sqrt{2} \sqrt[4]{b} c^{15/4}} - \frac{7(11bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}} + 1\right)}{32\sqrt{2} \sqrt[4]{b} c^{15/4}} - \frac{x^{11/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

**Rubi [A]** time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{7/2}(11bB - 3Ac)}{16bc^2(b + cx^2)} + \frac{7x^{3/2}(11bB - 3Ac)}{48bc^2} - \frac{7(11bB - 3Ac) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} \sqrt[4]{b} c^{15/4}} + \frac{7(11bB - 3Ac) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} \sqrt[4]{b} c^{15/4}} + \frac{7(11bB - 3Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}}\right)}{32\sqrt{2} \sqrt[4]{b} c^{15/4}} - \frac{7(11bB - 3Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{b}} + 1\right)}{32\sqrt{2} \sqrt[4]{b} c^{15/4}} - \frac{x^{11/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(21/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (7\*(11\*b\*B - 3\*A\*c)\*x^(3/2))/(48\*b\*c^3) - ((b\*B - A\*c)\*x^(11/2))/(4\*b\*c\*(b + c\*x^2)^2) - ((11\*b\*B - 3\*A\*c)\*x^(7/2))/(16\*b\*c^2\*(b + c\*x^2)) + (7\*(11\*b\*B - 3\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(1/4)\*c^(15/4)) - (7\*(11\*b\*B - 3\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(1/4)\*c^(15/4)) - (7\*(11\*b\*B - 3\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(1/4)\*c^(15/4)) + (7\*(11\*b\*B - 3\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(1/4)\*c^(15/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 288

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(c^n\*(m - n + 1))/(b\*n\*(p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n\*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_) + (b_.)*(x_)^(q_.))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{21/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^{9/2} (A + Bx^2)}{(b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} + \frac{\left(\frac{11bB}{2} - \frac{3Ac}{2}\right) \int \frac{x^{9/2}}{(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} + \frac{(7(11bB - 3Ac)) \int \frac{x^{5/2}}{b+cx^2} dx}{32bc^2} \\
&= \frac{7(11bB - 3Ac)x^{3/2}}{48bc^3} - \frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} - \frac{(7(11bB - 3Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{32c^3} \\
&= \frac{7(11bB - 3Ac)x^{3/2}}{48bc^3} - \frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} - \frac{(7(11bB - 3Ac)) \text{Subst}\left(\int \frac{\sqrt{x}}{b+cx^2} dx\right)}{16c^3} \\
&= \frac{7(11bB - 3Ac)x^{3/2}}{48bc^3} - \frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} + \frac{(7(11bB - 3Ac)) \text{Subst}\left(\int \frac{\sqrt{x}}{b+cx^2} dx\right)}{32c^{7/2}} \\
&= \frac{7(11bB - 3Ac)x^{3/2}}{48bc^3} - \frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} - \frac{(7(11bB - 3Ac)) \text{Subst}\left(\int \frac{\sqrt{x}}{b+cx^2} dx\right)}{64c^{11/4}} \\
&= \frac{7(11bB - 3Ac)x^{3/2}}{48bc^3} - \frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} - \frac{7(11bB - 3Ac) \log(\sqrt{b} - \sqrt{cx})}{64\sqrt{2} \sqrt[4]{b} c^{15/4}} \\
&= \frac{7(11bB - 3Ac)x^{3/2}}{48bc^3} - \frac{(bB - Ac)x^{11/2}}{4bc(b + cx^2)^2} - \frac{(11bB - 3Ac)x^{7/2}}{16bc^2(b + cx^2)} + \frac{7(11bB - 3Ac) \tan^{-1}\left(1 - \frac{\sqrt{b} - \sqrt{cx}}{\sqrt{2} \sqrt[4]{b} c^{15/4}}\right)}{32\sqrt{2} \sqrt[4]{b} c^{15/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.47, size = 176, normalized size = 0.55

$$\frac{2c^{3/4}x^{3/2}(3bB-2Ac)_2F_1\left(\frac{3}{4}, 2, \frac{7}{4}; -\frac{cx^2}{b}\right)}{b} + \frac{2c^{3/4}x^{3/2}(Ac-bB)_2F_1\left(\frac{3}{4}, 3, \frac{7}{4}; -\frac{cx^2}{b}\right)}{b} + \frac{(3Ac-9bB) \tan^{-1}\left(\frac{\sqrt[4]{c} \sqrt{x}}{\sqrt[4]{-b}}\right)}{\sqrt[4]{-b}} + \frac{(9bB-3Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c} \sqrt{x}}{\sqrt[4]{-b}}\right)}{\sqrt[4]{-b}} + 2Bc^{3/4}x^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(21/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (2\*B\*c^(3/4)\*x^(3/2) + ((-9\*b\*B + 3\*A\*c)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/(-b)^(1/4) + ((9\*b\*B - 3\*A\*c)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)])/(-b)^(1/4) + (2\*c^(3/4)\*(3\*b\*B - 2\*A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -(c\*x^2)/b])/b + (2\*c^(3/4)\*(-b\*B) + A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 3, 7/4, -(c\*x^2)/b])/b)/(3\*c^(15/4))

**IntegrateAlgebraic [A]** time = 1.03, size = 257, normalized size = 0.80

$$\left(\frac{77b^{3/4}B}{32\sqrt{2}c^{15/4}} - \frac{21A}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}\right) \tan^{-1}\left(\frac{\sqrt{b} - \sqrt{cx}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + \frac{x^{3/2}(77b^2B - 21Abc) + x^{7/2}(121bBc - 33Ac^2) + 32Bc^2x^{11/2}}{48c^3(b + cx^2)^2} - \frac{21A \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{cx}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{77b^{3/4}B \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{cx}}\right)}{32\sqrt{2}c^{15/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(21/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((77\*b^2\*B - 21\*A\*b\*c)\*x^(3/2) + (121\*b\*B\*c - 33\*A\*c^2)\*x^(7/2) + 32\*B\*c^2\*x^(11/2))/(48\*c^3\*(b + c\*x^2)^2) + ((77\*b^(3/4)\*B)/(32\*Sqrt[2]\*c^(15/4)) - (21\*A)/(32\*Sqrt[2]\*b^(1/4)\*c^(11/4)))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])] + (77\*b^(3/4)\*B\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)])/(32\*Sqrt[2]\*c^(15/4)) - (21\*A\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)])/(32\*Sqrt[2]\*b^(1/4)\*c^(11/4))

**fricas [B]** time = 0.47, size = 993, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(21/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] -1/192\*(84\*(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(1/4)\*arctan((sqrt((1771561\*B^6\*b^6 - 2898918\*A\*B^5\*b^5\*c + 1976535\*A^2\*B^4\*b^4\*c^2 - 718740\*A^3\*B^3\*b^3\*c^3 + 147015\*A^4\*B^2\*b^2\*c^4 - 16038\*A^5\*B\*b\*c^5 + 729\*A^6\*c^6)\*x - (14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/b\*c^15)))/c^4\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(1/4) + (1331\*B^3\*b^3\*c^4 - 1089\*A\*B^2\*b^2\*c^5 + 297\*A^2\*B\*b\*c^6 - 27\*A^3\*c^7)\*sqrt(x)\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(1/4))/(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4) - 21\*(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(1/4)\*log(343\*b\*c^11\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(3/4) - 343\*(1331\*B^3\*b^3 - 1089\*A\*B^2\*b^2\*c + 297\*A^2\*B\*b\*c^2 - 27\*A^3\*c^3)\*sqrt(x)) + 21\*(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(1/4)\*log(-343\*b\*c^11\*(-(14641\*B^4\*b^4 - 15972\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 1188\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b\*c^15))^(3/4) - 343\*(1331\*B^3\*b^3 - 1089\*A\*B^2\*b^2\*c + 297\*A^2\*B\*b\*c^2 - 27\*A^3\*c^3)\*sqrt(x)) - 4\*(32\*B\*c^2\*x^5 + 11\*(11\*B\*b\*c - 3\*A\*c^2)\*x^3 + 7\*(11\*B\*b^2 - 3\*A\*b\*c)\*x)\*sqrt(x))/(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)

**giac [A]** time = 0.23, size = 304, normalized size = 0.94

$$\frac{2Bx^{\frac{3}{2}} - 19Bbcx^{\frac{7}{2}} - 11Ac^2x^{\frac{11}{2}} + 15Bb^2c^2 - 7Abcx^{\frac{15}{2}}}{16(c^2 + b)^2c^3} + \frac{7\sqrt{2}(11(bc)^{\frac{3}{2}}Bb - 3(bc)^{\frac{3}{2}}Ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{x}}{2\sqrt{\frac{b}{c}}}\right)}{64bc^6} + \frac{7\sqrt{2}(11(bc)^{\frac{3}{2}}Bb - 3(bc)^{\frac{3}{2}}Ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{x}}{2\sqrt{\frac{b}{c}}}\right)}{64bc^6} + \frac{7\sqrt{2}(11(bc)^{\frac{3}{2}}Bb - 3(bc)^{\frac{3}{2}}Ac)\log\left(\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{x} + \sqrt{\frac{b}{c}}\right)}{128bc^6} + \frac{7\sqrt{2}(11(bc)^{\frac{3}{2}}Bb - 3(bc)^{\frac{3}{2}}Ac)\log\left(-\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{x} + \sqrt{\frac{b}{c}}\right)}{128bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(21/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 2/3\*B\*x^(3/2)/c^3 + 1/16\*(19\*B\*b\*c\*x^(7/2) - 11\*A\*c^2\*x^(7/2) + 15\*B\*b^2\*x^(3/2) - 7\*A\*b\*c\*x^(3/2))/((c\*x^2 + b)^2\*c^3) - 7/64\*sqrt(2)\*(11\*(b\*c^3)^(3/4)\*B\*b - 3\*(b\*c^3)^(3/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/(b\*c^6) - 7/64\*sqrt(2)\*(11\*(b\*c^3)^(3/4)\*B\*b - 3\*(b\*c^3)^(3/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/(b\*c^6) + 7/128\*sqrt(2)\*(11\*(b\*c^3)^(3/4)\*B\*b - 3\*(b\*c^3)^(3/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b\*c^6) - 7/128\*sqrt(2)\*(11\*(b\*c^3)^(3/4)\*B\*b - 3\*(b\*c^3)^(3/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b\*c^6)

**maple [A]** time = 0.07, size = 357, normalized size = 1.11

$$\frac{11Ax^2}{16(c^2+b)^2c} + \frac{19Bbx^2}{16(c^2+b)^2c^2} - \frac{7Abx^2}{16(c^2+b)^2c^2} + \frac{15Bb^2x^2}{16(c^2+b)^2c^3} + \frac{2Bx^3}{3c^3} + \frac{21\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3} + \frac{21\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3} + \frac{21\sqrt{2}A\ln\left(\frac{z-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{b}{c}}}{z+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3} - \frac{77\sqrt{2}Bb\arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^4} - \frac{77\sqrt{2}Bb\arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^4} - \frac{77\sqrt{2}Bb\ln\left(\frac{z-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{b}{c}}}{z+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(21/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 2/3\*B\*x^(3/2)/c^3-11/16/c/(c\*x^2+b)^2\*A\*x^(7/2)+19/16/c^2/(c\*x^2+b)^2\*B\*x^(7/2)\*b-7/16/c^2/(c\*x^2+b)^2\*x^(3/2)\*A\*b+15/16/c^3/(c\*x^2+b)^2\*x^(3/2)\*B\*b^2+21/128/c^3/(b/c)^(1/4)\*2^(1/2)\*A\*ln((x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-77/128/c^4/(b/c)^(1/4)\*2^(1/2)\*b\*B\*ln((x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-77/64/c^4/(b/c)^(1/4)\*2^(1/2)\*b\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-77/64/c^4/(b/c)^(1/4)\*2^(1/2)\*b\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)

**maxima [A]** time = 3.10, size = 256, normalized size = 0.80

$$\frac{(19Bbc-11Ac^2)x^2 + (15Bb^2-7Abc)x^3 + 2Bx^3}{16(c^5x^4+2bc^4x^2+b^2c^3)} + \frac{2Bx^3}{3c^3} - \frac{7(11Bb-3Ac)}{128c^3} \left( \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2b^4c^4+2\sqrt{c}\sqrt{c}}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2b^4c^4-2\sqrt{c}\sqrt{c}}}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}} - \frac{\sqrt{2}\log\left(\frac{\sqrt{2b^4c^4}\sqrt{x}+\sqrt{c}x+\sqrt{b}}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2b^4c^4}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(21/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 1/16\*((19\*B\*b\*c - 11\*A\*c^2)\*x^(7/2) + (15\*B\*b^2 - 7\*A\*b\*c)\*x^(3/2))/(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3) + 2/3\*B\*x^(3/2)/c^3 - 7/128\*(11\*B\*b - 3\*A\*c)\*(2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(sqrt(b)\*sqrt(c))\*sqrt(c) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(sqrt(b)\*sqrt(c))\*sqrt(c) - sqrt(2)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(1/4)\*c^(3/4)) + sqrt(2)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(1/4)\*c^(3/4))/c^3

**mupad [B]** time = 0.26, size = 138, normalized size = 0.43

$$\frac{x^{3/2} \left( \frac{15Bb^2}{16} - \frac{7Abc}{16} \right) - x^{7/2} \left( \frac{11Ac^2}{16} - \frac{19Bbc}{16} \right)}{b^2c^3 + 2bc^4x^2 + c^5x^4} + \frac{2Bx^{3/2}}{3c^3} + \frac{7\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(3Ac-11Bb)}{32(-b)^{1/4}c^{15/4}} + \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}1i}{(-b)^{1/4}}\right)(3Ac-11Bb)7i}{32(-b)^{1/4}c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(21/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] (x^(3/2)\*((15\*B\*b^2)/16 - (7\*A\*b\*c)/16) - x^(7/2)\*((11\*A\*c^2)/16 - (19\*B\*b\*c)/16))/(b^2\*c^3 + c^5\*x^4 + 2\*b\*c^4\*x^2) + (2\*B\*x^(3/2))/(3\*c^3) + (7\*atan((c^(1/4)\*x^(1/2))/(-b)^(1/4))\*(3\*A\*c - 11\*B\*b))/(32\*(-b)^(1/4)\*c^(15/4)) + (atan((c^(1/4)\*x^(1/2)\*1i)/(-b)^(1/4))\*(3\*A\*c - 11\*B\*b)\*7i)/(32\*(-b)^(1/4)\*c^(15/4))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(21/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.209 \quad \int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=322

$$\frac{5(9bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{3/4} c^{13/4}} - \frac{5(9bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{3/4} c^{13/4}} + \frac{5(9bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x}\right)}{32\sqrt{2} b^{3/4} c^{13/4}}$$

**Rubi [A]** time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5(9bB - Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{3/4} c^{13/4}} - \frac{5(9bB - Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{3/4} c^{13/4}} + \frac{5(9bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x}\right)}{32\sqrt{2} b^{3/4} c^{13/4}} - \frac{5(9bB - Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c} x} + 1\right)}{32\sqrt{2} b^{3/4} c^{13/4}} - \frac{x^{5/2}(9bB - Ac)}{16bc^2(b + cx^2)} + \frac{5\sqrt{x}(9bB - Ac)}{16bc^3} - \frac{x^{9/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(19/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (5\*(9\*b\*B - A\*c)\*Sqrt[x])/(16\*b\*c^3) - ((b\*B - A\*c)\*x^(9/2))/(4\*b\*c\*(b + c\*x^2)^2) - ((9\*b\*B - A\*c)\*x^(5/2))/(16\*b\*c^2\*(b + c\*x^2)) + (5\*(9\*b\*B - A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(3/4)\*c^(13/4)) - (5\*(9\*b\*B - A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(3/4)\*c^(13/4)) + (5\*(9\*b\*B - A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(3/4)\*c^(13/4)) - (5\*(9\*b\*B - A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(3/4)\*c^(13/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 288

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(c^n\*(m - n + 1))/(b\*n\*(p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n\*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]



Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^{19/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^{7/2} (A + Bx^2)}{(b + cx^2)^3} dx \\
 &= -\frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} + \frac{\left(\frac{9bB}{2} - \frac{Ac}{2}\right) \int \frac{x^{7/2}}{(b+cx^2)^2} dx}{4bc} \\
 &= -\frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} + \frac{(5(9bB - Ac)) \int \frac{x^{3/2}}{b+cx^2} dx}{32bc^2} \\
 &= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} - \frac{(5(9bB - Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{32c^3} \\
 &= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} - \frac{(5(9bB - Ac)) \text{Subst}\left(\int \frac{1}{b+cx^4} dx\right)}{16c^3} \\
 &= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} - \frac{(5(9bB - Ac)) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx\right)}{32\sqrt{b}c^3} \\
 &= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} - \frac{(5(9bB - Ac)) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right)}{64\sqrt{b}c^{7/2}} \\
 &= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} + \frac{5(9bB - Ac) \log(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c})}{64\sqrt{2}b^{3/4}c^{13/4}} \\
 &= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2(b + cx^2)} + \frac{5(9bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}}
 \end{aligned}$$

**Mathematica [A]** time = 0.63, size = 403, normalized size = 1.25

$$\frac{10\sqrt{2}(9bB - Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) - 10\sqrt{2}(9bB - Ac) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) + 5\sqrt{2}Ac \log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) + 5\sqrt{2}Ac \log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) + \frac{32Ab^{3/4}\sqrt{c}}{(b+cx)^2} - \frac{72Ac^{3/4}\sqrt{b}}{b+cx^2} - \frac{32b^2\sqrt{c}\sqrt{x}}{(b+cx)^2} + \frac{136b^2\sqrt{c}\sqrt{x}}{b+cx^2} + 45\sqrt{2}\sqrt[4]{b}B \log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) - 45\sqrt{2}\sqrt[4]{b}B \log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) + 256B\sqrt[4]{c}\sqrt{x}}{128c^{13/4}}$$

Antiderivative was successfully verified.

```

[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3, x]

[Out] (256*B*c^(1/4)*Sqrt[x] - (32*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (32*A*b*c^(5/4)*Sqrt[x])/(b + c*x^2)^2 + (136*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2) - (72*A*c^(5/4)*Sqrt[x])/(b + c*x^2) + (10*Sqrt[2]*(9*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(b^(3/4)) - (10*Sqrt[2]*(9*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(b^(3/4)) + 45*Sqrt[2]*b^(1/4)*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - (5*Sqrt[2]*A*c*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - 45*Sqrt[2]*b^(1/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + (5*Sqrt[2]*A*c*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(128*c^(13/4))
    
```

**IntegrateAlgebraic [A]** time = 0.73, size = 257, normalized size = 0.80

$$\left(\frac{45\sqrt[4]{b}B}{32\sqrt{2}c^{13/4}} - \frac{5A}{32\sqrt{2}b^{3/4}c^{9/4}}\right) \tan^{-1}\left(\frac{\sqrt{b} - \sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + \frac{5A \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2}b^{3/4}c^{9/4}} + \frac{\sqrt{x}(45b^2B - 5Abc) + x^{5/2}(81bBc - 9Ac^2) + 32Bc^2x^{9/2}}{16c^3(b + cx^2)^2} - \frac{45\sqrt[4]{b}B \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2}c^{13/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(19/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((45\*b^2\*B - 5\*A\*b\*c)\*Sqrt[x] + (81\*b\*B\*c - 9\*A\*c^2)\*x^(5/2) + 32\*B\*c^2\*x^(9/2))/(16\*c^3\*(b + c\*x^2)^2) + ((45\*b^(1/4)\*B)/(32\*Sqrt[2]\*c^(13/4)) - (5\*A)/(32\*Sqrt[2]\*b^(3/4)\*c^(9/4)))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])] - (45\*b^(1/4)\*B\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*c^(13/4)) + (5\*A\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*b^(3/4)\*c^(9/4))

**fricas** [B] time = 0.44, size = 793, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/64\*(20\*(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(1/4)\*arctan((sqrt(b^2\*c^6\*sqrt(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13)) + (81\*B^2\*b^2 - 18\*A\*B\*b\*c + A^2\*c^2)\*x)\*b^2\*c^10\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(3/4) + (9\*B\*b^3\*c^10 - A\*b^2\*c^11)\*sqrt(x)\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(3/4))/(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)) + 5\*(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(1/4)\*log(5\*b\*c^3\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(1/4) - 5\*(9\*B\*b - A\*c)\*sqrt(x)) - 5\*(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(1/4)\*log(-5\*b\*c^3\*(-(6561\*B^4\*b^4 - 2916\*A\*B^3\*b^3\*c + 486\*A^2\*B^2\*b^2\*c^2 - 36\*A^3\*B\*b\*c^3 + A^4\*c^4)/(b^3\*c^13))^(1/4) - 5\*(9\*B\*b - A\*c)\*sqrt(x)) + 4\*(32\*B\*c^2\*x^4 + 45\*B\*b^2 - 5\*A\*b\*c + 9\*(9\*B\*b\*c - A\*c^2)\*x^2)\*sqrt(x))/(c^5\*x^4 + 2\*b\*c^4\*x^2 + b^2\*c^3)

**giac** [A] time = 0.20, size = 304, normalized size = 0.94

$$\frac{2B\sqrt{x}}{c^3} - \frac{5\sqrt{2}\left(9\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - \left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}}}{z\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^4} - \frac{5\sqrt{2}\left(9\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - \left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}}}{z\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^4} - \frac{5\sqrt{2}\left(9\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - \left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{x}}\right)}{128bc^4} + \frac{5\sqrt{2}\left(9\left(\frac{b}{c}\right)^{\frac{1}{4}}Bb - \left(\frac{b}{c}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{x}}\right)}{128bc^4} + \frac{17Bbcx^{\frac{3}{2}} - 9Ac^2x^{\frac{3}{2}} + 13Bb^2\sqrt{x} - 5Abc\sqrt{x}}{16(c^2+b)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 2\*B\*sqrt(x)/c^3 - 5/64\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - (b\*c^3)^(1/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/(b\*c^4) - 5/64\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - (b\*c^3)^(1/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/(b\*c^4) - 5/128\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - (b\*c^3)^(1/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b\*c^4) + 5/128\*sqrt(2)\*(9\*(b\*c^3)^(1/4)\*B\*b - (b\*c^3)^(1/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b\*c^4) + 1/16\*(17\*B\*b\*c\*x^(5/2) - 9\*A\*c^2\*x^(5/2) + 13\*B\*b^2\*sqrt(x) - 5\*A\*b\*c\*sqrt(x))/(c\*x^2 + b)^2\*c^3

**maple** [A] time = 0.07, size = 363, normalized size = 1.13

$$\frac{9Ax^{\frac{3}{2}}}{16(c^2+b)^2c} + \frac{17Bbx^{\frac{3}{2}}}{16(c^2+b)^2c^2} - \frac{5Ab\sqrt{x}}{16(c^2+b)^2c^2} + \frac{13Bb^2\sqrt{x}}{16(c^2+b)^2c^2} + \frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^2} + \frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^2} + \frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\ln\left(\frac{1+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x+\sqrt{x}}}{1+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x-\sqrt{x}}}\right)}{128bc^2} - \frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64c^3} - \frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64c^3} - \frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\ln\left(\frac{1+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x+\sqrt{x}}}{1+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x-\sqrt{x}}}\right)}{128c^3} + \frac{2B\sqrt{x}}{c^3}$$



$$\frac{b^2 - 18ABb^2c}{64c^3} - \frac{((45Bb^2 - 5Ab^2c)(Ac - 9Bb)5i)}{64(-b)^{3/4}c^{13/4}} - \frac{((45Bb^2 - 5Ab^2c)(Ac - 9Bb)5i)}{64(-b)^{3/4}c^{13/4}} - \frac{(Ac - 9Bb)((25x^{1/2})(A^2c^2 + 81B^2b^2 - 18ABb^2c))}{64c^3} + \frac{((45Bb^2 - 5Ab^2c)(Ac - 9Bb)5i)}{64(-b)^{3/4}c^{13/4}} + \frac{((45Bb^2 - 5Ab^2c)(Ac - 9Bb)5i)}{64(-b)^{3/4}c^{13/4}} + \frac{(Ac - 9Bb)}{32(-b)^{3/4}c^{13/4}}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(19/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.210 \quad \int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=293

$$\frac{3(Ac + 7bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{5/4} c^{11/4}} - \frac{3(Ac + 7bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{5/4} c^{11/4}} - \frac{3(Ac + 7bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2} b^{5/4} c^{11/4}} + \frac{3(Ac + 7bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2} b^{5/4} c^{11/4}} - \frac{x^{3/2}(Ac + 7bB)}{16bc^2(b + cx^2)} - \frac{x^{7/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

**Rubi [A]** time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, number of rules used = 0.385, Rules used = {1584, 457, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3(Ac + 7bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{5/4} c^{11/4}} - \frac{3(Ac + 7bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{5/4} c^{11/4}} - \frac{3(Ac + 7bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2} b^{5/4} c^{11/4}} + \frac{3(Ac + 7bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2} b^{5/4} c^{11/4}} - \frac{x^{3/2}(Ac + 7bB)}{16bc^2(b + cx^2)} - \frac{x^{7/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -((b\*B - A\*c)\*x^(7/2))/(4\*b\*c\*(b + c\*x^2)^2) - ((7\*b\*B + A\*c)\*x^(3/2))/(16\*b\*c^2\*(b + c\*x^2)) - (3\*(7\*b\*B + A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(5/4)\*c^(11/4)) + (3\*(7\*b\*B + A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(5/4)\*c^(11/4)) + (3\*(7\*b\*B + A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(5/4)\*c^(11/4)) - (3\*(7\*b\*B + A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(5/4)\*c^(11/4))

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 288

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*n\*(p+1)), x] - Dist[(c^n\*n\*(m-n+1))/(b\*n\*(p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[m+n\*(p+1)+1/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a+(b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := -Simp[(b\*c - a\*d)\*(e\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{17/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^{5/2} (A + Bx^2)}{(b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)x^{7/2}}{4bc(b + cx^2)^2} + \frac{\left(\frac{7bB}{2} + \frac{Ac}{2}\right) \int \frac{x^{5/2}}{(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)x^{7/2}}{4bc(b + cx^2)^2} - \frac{(7bB + Ac)x^{3/2}}{16bc^2(b + cx^2)} + \frac{(3(7bB + Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{32bc^2} \\
&= -\frac{(bB - Ac)x^{7/2}}{4bc(b + cx^2)^2} - \frac{(7bB + Ac)x^{3/2}}{16bc^2(b + cx^2)} + \frac{(3(7bB + Ac)) \operatorname{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{16bc^2} \\
&= -\frac{(bB - Ac)x^{7/2}}{4bc(b + cx^2)^2} - \frac{(7bB + Ac)x^{3/2}}{16bc^2(b + cx^2)} - \frac{(3(7bB + Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{b} - \sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{32bc^{5/2}} + \frac{(3(7bB + Ac)) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{64bc^3} \\
&= -\frac{(bB - Ac)x^{7/2}}{4bc(b + cx^2)^2} - \frac{(7bB + Ac)x^{3/2}}{16bc^2(b + cx^2)} + \frac{3(7bB + Ac) \log(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{c}x)}{64\sqrt{2} b^{5/4} c^{11/4}} - \frac{3(7bB + Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{5/4} c^{11/4}} + \frac{3(7bB + Ac)}{32\sqrt{2} b^{5/4} c^{11/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.32, size = 137, normalized size = 0.47

$$\frac{2c^{3/4}x^{3/2}(Ac - 2bB) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right) + 2c^{3/4}x^{3/2}(bB - Ac) {}_2F_1\left(\frac{3}{4}, 3; \frac{7}{4}; -\frac{cx^2}{b}\right) + 3(-b)^{7/4}B \left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right) + \tanh^{-1}\left(\frac{b^{4/5}\sqrt{x}}{(-b)^{5/4}}\right)\right)}{3b^2c^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (3\*(-b)^(7/4)\*B\*(ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b\*c^(1/4)\*Sqrt[x])/(-b)^(5/4)]) + 2\*c^(3/4)\*(-2\*b\*B + A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -((c\*x^2)/b)] + 2\*c^(3/4)\*(b\*B - A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 3, 7/4, -((c\*x^2)/b)])/(3\*b^2\*c^(11/4))

**IntegrateAlgebraic [A]** time = 0.66, size = 249, normalized size = 0.85

$$\left(-\frac{3A}{32\sqrt{2}b^{5/4}c^{7/4}} - \frac{21B}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}\right) \tan^{-1}\left(\frac{\sqrt{b} - \sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - \frac{3A \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2}b^{5/4}c^{7/4}} + \frac{x^{3/2}(-Abc - 7b^2B) + x^{7/2}(3Ac^2 - 11bBc)}{16bc^2(b + cx^2)^2} - \frac{21B \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] ((-7\*b^2\*B - A\*b\*c)\*x^(3/2) + (-11\*b\*B\*c + 3\*A\*c^2)\*x^(7/2))/(16\*b\*c^2\*(b + c\*x^2)^2) + ((-21\*B)/(32\*Sqrt[2]\*b^(1/4)\*c^(11/4)) - (3\*A)/(32\*Sqrt[2]\*b^(5/4)\*c^(7/4)))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]



]]) - (21\*B\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*b^(1/4)\*c^(11/4)) - (3\*A\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*b^(5/4)\*c^(7/4))

**fricas [B]** time = 0.46, size = 990, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 
$$-1/64*(12*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4) * \arctan(\sqrt{(117649*B^6*b^6 + 100842*A*B^5*b^5*c + 36015*A^2*B^4*b^4*c^2 + 6860*A^3*B^3*b^3*c^3 + 735*A^4*B^2*b^2*c^4 + 42*A^5*B*b*c^5 + A^6*c^6)}*x - (2401*B^4*b^7*c^5 + 1372*A*B^3*b^6*c^6 + 294*A^2*B^2*b^5*c^7 + 28*A^3*B*b^4*c^8 + A^4*b^3*c^9)*\sqrt{-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)}) * b*c^3 * (-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4) - (343*B^3*b^4*c^3 + 147*A*B^2*b^3*c^4 + 21*A^2*B*b^2*c^5 + A^3*b*c^6)*\sqrt{x} * (-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4) / (2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4) - 3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4) * \log(27*b^4*c^8 * (-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*\sqrt{x}) + 3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4) * \log(-27*b^4*c^8 * (-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*\sqrt{x}) + 4*((11*B*b*c - 3*A*c^2)*x^3 + (7*B*b^2 + A*b*c)*x)*\sqrt{x} / (b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)$$

**giac [A]** time = 0.23, size = 293, normalized size = 1.00

$$\frac{11 B b c x^2 - 3 A c^2 x^2 + 7 B b^2 x^2 + A b c x^2}{16 (c x^2 + b)^2 b c^2} + \frac{3 \sqrt{2} \left( 7 (b c)^{\frac{3}{2}} B b + (b c)^{\frac{3}{2}} A c \right) \arctan \left( \frac{\sqrt{2} \sqrt{c} \sqrt{x}}{z} \right)}{64 b^2 c^3} + \frac{3 \sqrt{2} \left( 7 (b c)^{\frac{3}{2}} B b + (b c)^{\frac{3}{2}} A c \right) \arctan \left( \frac{\sqrt{2} \sqrt{c} \sqrt{x}}{z} \right)}{64 b^2 c^3} + \frac{3 \sqrt{2} \left( 7 (b c)^{\frac{3}{2}} B b + (b c)^{\frac{3}{2}} A c \right) \log \left( \sqrt{2} \sqrt{c} \sqrt{x} + x + \sqrt{c} \right)}{128 b^2 c^3} + \frac{3 \sqrt{2} \left( 7 (b c)^{\frac{3}{2}} B b + (b c)^{\frac{3}{2}} A c \right) \log \left( -\sqrt{2} \sqrt{c} \sqrt{x} + x + \sqrt{c} \right)}{128 b^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 
$$-1/16*(11*B*b*c*x^(7/2) - 3*A*c^2*x^(7/2) + 7*B*b^2*x^(3/2) + A*b*c*x^(3/2)) / ((c*x^2 + b)^2*b*c^2) + 3/64*\sqrt{2}*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^(1/4) + 2*\sqrt{x})/(b/c)^(1/4))/(b^2*c^5) + 3/64*\sqrt{2}*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^(1/4) - 2*\sqrt{x})/(b/c)^(1/4))/(b^2*c^5) - 3/128*\sqrt{2}*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^(1/4) + x + \sqrt{b/c})/(b^2*c^5) + 3/128*\sqrt{2}*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^(1/4) + x + \sqrt{b/c})/(b^2*c^5)$$

**maple [A]** time = 0.06, size = 325, normalized size = 1.11

$$\frac{3\sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{c} \sqrt{x}}{z} - 1 \right)}{64 \left( \frac{b}{c} \right)^{\frac{3}{4}} b c^2} + \frac{3\sqrt{2} A \arctan \left( \frac{\sqrt{2} \sqrt{c} \sqrt{x}}{z} + 1 \right)}{64 \left( \frac{b}{c} \right)^{\frac{3}{4}} b c^2} + \frac{3\sqrt{2} A \ln \left( \frac{x \left( \frac{b}{c} \right)^{\frac{3}{4}} \sqrt{2} \sqrt{c} \sqrt{x} + \sqrt{c}}{x \left( \frac{b}{c} \right)^{\frac{3}{4}} \sqrt{2} \sqrt{c} \sqrt{x} + \sqrt{c}} \right)}{128 \left( \frac{b}{c} \right)^{\frac{3}{4}} b c^2} + \frac{21\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{c} \sqrt{x}}{z} - 1 \right)}{64 \left( \frac{b}{c} \right)^{\frac{3}{4}} c^3} + \frac{21\sqrt{2} B \arctan \left( \frac{\sqrt{2} \sqrt{c} \sqrt{x}}{z} + 1 \right)}{64 \left( \frac{b}{c} \right)^{\frac{3}{4}} c^3} + \frac{21\sqrt{2} B \ln \left( \frac{x \left( \frac{b}{c} \right)^{\frac{3}{4}} \sqrt{2} \sqrt{c} \sqrt{x} + \sqrt{c}}{x \left( \frac{b}{c} \right)^{\frac{3}{4}} \sqrt{2} \sqrt{c} \sqrt{x} + \sqrt{c}} \right)}{128 \left( \frac{b}{c} \right)^{\frac{3}{4}} c^3} + \frac{(3Ac-11bB)\sqrt{2}}{16bc} - \frac{(Ac+7bB)\sqrt{2}}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out]  $2*(1/32*(3*A*c-11*B*b)/b/c*x^(7/2)-1/32*(A*c+7*B*b)/c^2*x^(3/2))/(c*x^2+b)^2+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*A*\arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*A*\arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/128/c^2/b/(b/c)^(1/4)*2^(1/2)*A*\ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^(1/2)*B*\arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)*B*\arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+21/128/c^3/(b/c)^(1/4)*2^(1/2)*B*\ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))$

**maxima** [A] time = 3.11, size = 251, normalized size = 0.86

$$\frac{(11Bbc - 3Ac^2)x^{\frac{7}{2}} + (7Bb^2 + Abc)x^{\frac{3}{2}}}{16(b^4x^4 + 2b^2c^3x^2 + b^3c^2)} + \frac{3(7Bb + Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{b}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{b}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{128bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $-1/16*((11*B*b*c - 3*A*c^2)*x^(7/2) + (7*B*b^2 + A*b*c)*x^(3/2))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2) + 3/128*(7*B*b + A*c)*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^(1/4)*c^(1/4) + 2*\sqrt{c}*\sqrt{x}))/\sqrt{c}*\sqrt{b}*\sqrt{c})/(\sqrt{c}*\sqrt{b}*\sqrt{c}*\sqrt{c}) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^(1/4)*c^(1/4) - 2*\sqrt{c}*\sqrt{x}))/\sqrt{c}*\sqrt{b}*\sqrt{c})/(\sqrt{c}*\sqrt{b}*\sqrt{c}*\sqrt{c}) - \sqrt{2}*\log(\sqrt{2}*b^(1/4)*c^(1/4)*\sqrt{x} + \sqrt{c}*x + \sqrt{b}))/b^(1/4)*c^(3/4) + \sqrt{2}*\log(-\sqrt{2}*b^(1/4)*c^(1/4)*\sqrt{x} + \sqrt{c}*x + \sqrt{b}))/b^(1/4)*c^(3/4))/b*c^2$

**mupad** [B] time = 0.17, size = 122, normalized size = 0.42

$$\frac{3 \operatorname{atanh}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (Ac + 7Bb)}{32(-b)^{5/4} c^{11/4}} - \frac{3 \operatorname{atan}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (Ac + 7Bb)}{32(-b)^{5/4} c^{11/4}} - \frac{x^{3/2} (Ac + 7Bb)}{16c^2} - \frac{x^{7/2} (3Ac - 11Bb)}{16bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(17/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out]  $(3*\operatorname{atanh}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(A*c + 7*B*b))/(32*(-b)^{5/4}*c^{11/4}) - (3*\operatorname{atan}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(A*c + 7*B*b))/(32*(-b)^{5/4}*c^{11/4}) - ((x^{3/2}*(A*c + 7*B*b))/(16*c^2) - (x^{7/2}*(3*A*c - 11*B*b))/(16*b*c))/b^2 + c^2*x^4 + 2*b*c*x^2$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(17/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.211 \quad \int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=298

$$\frac{(3Ac + 5bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{7/4} c^{9/4}} + \frac{(3Ac + 5bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{7/4} c^{9/4}} - \frac{(3Ac + 5bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{7/4} c^{9/4}} + \frac{(3Ac + 5bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{32\sqrt{2} b^{7/4} c^{9/4}} - \frac{\sqrt{x}(3Ac + 5bB)}{16bc^2(b + cx^2)} - \frac{x^{5/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

**Rubi [A]** time = 0.24, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1584, 457, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(3Ac + 5bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{7/4} c^{9/4}} + \frac{(3Ac + 5bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{7/4} c^{9/4}} - \frac{(3Ac + 5bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{7/4} c^{9/4}} + \frac{(3Ac + 5bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{32\sqrt{2} b^{7/4} c^{9/4}} - \frac{\sqrt{x}(3Ac + 5bB)}{16bc^2(b + cx^2)} - \frac{x^{5/2}(bB - Ac)}{4bc(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -((b\*B - A\*c)\*x^(5/2))/(4\*b\*c\*(b + c\*x^2)^2) - ((5\*b\*B + 3\*A\*c)\*Sqrt[x])/((16\*b\*c^2\*(b + c\*x^2)) - ((5\*b\*B + 3\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(7/4)\*c^(9/4)) + ((5\*b\*B + 3\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(7/4)\*c^(9/4)) - ((5\*b\*B + 3\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(7/4)\*c^(9/4)) + ((5\*b\*B + 3\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(7/4)\*c^(9/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 288

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*n\*(p+1)), x] - Dist[(c^n\*(m-n+1))/(b\*n\*(p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !LtQ[(m+n\*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^(p), x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{15/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{x^{3/2} (A + Bx^2)}{(b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} + \frac{\left(\frac{5bB}{2} + \frac{3Ac}{2}\right) \int \frac{x^{3/2}}{(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} - \frac{(5bB + 3Ac)\sqrt{x}}{16bc^2(b + cx^2)} + \frac{(5bB + 3Ac) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{32bc^2} \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} - \frac{(5bB + 3Ac)\sqrt{x}}{16bc^2(b + cx^2)} + \frac{(5bB + 3Ac) \text{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{16bc^2} \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} - \frac{(5bB + 3Ac)\sqrt{x}}{16bc^2(b + cx^2)} + \frac{(5bB + 3Ac) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{32b^{3/2}c^2} + \dots \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} - \frac{(5bB + 3Ac)\sqrt{x}}{16bc^2(b + cx^2)} + \frac{(5bB + 3Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \sqrt{2} \frac{\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, \sqrt{x}\right)}{64b^{3/2}c^{5/2}} \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} - \frac{(5bB + 3Ac)\sqrt{x}}{16bc^2(b + cx^2)} - \frac{(5bB + 3Ac) \log\left(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2} b^{7/4} c^{9/4}} \\
&= -\frac{(bB - Ac)x^{5/2}}{4bc(b + cx^2)^2} - \frac{(5bB + 3Ac)\sqrt{x}}{16bc^2(b + cx^2)} - \frac{(5bB + 3Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2} b^{7/4} c^{9/4}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.71, size = 389, normalized size = 1.31

$$\frac{-\frac{2\sqrt{2}(3Ac+5bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/4}} + \frac{2\sqrt{2}(3Ac+5bB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}+1\right)}{b^{7/4}} - \frac{3\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}} + \frac{3\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{c}\sqrt{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}} + \frac{8A_2^{3/4}\sqrt{c}}{b^2+bcx^2} - \frac{32A_2^{3/4}\sqrt{c}}{(b+cx^2)^2} - \frac{5\sqrt{2}B\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}} + \frac{5\sqrt{2}B\log\left(\sqrt{2}\sqrt[4]{c}\sqrt{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}} - \frac{72B\sqrt[4]{c}\sqrt{c}}{b+cx^2} + \frac{32B\sqrt[4]{c}\sqrt{c}}{(b+cx^2)^2}}{128c^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((32\*b\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2)^2 - (32\*A\*c^(5/4)\*Sqrt[x])/(b + c\*x^2)^2 - (72\*B\*c^(1/4)\*Sqrt[x])/(b + c\*x^2) + (8\*A\*c^(5/4)\*Sqrt[x])/(b^2 + b\*c\*x^2) - (2\*Sqrt[2]\*(5\*b\*B + 3\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/b^(7/4) + (2\*Sqrt[2]\*(5\*b\*B + 3\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/b^(7/4) - (5\*Sqrt[2]\*B\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/b^(3/4) - (3\*Sqrt[2]\*A\*c\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/b^(7/4) + (5\*Sqrt[2]\*B\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/b^(3/4) + (3\*Sqrt[2]\*A\*c\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/b^(7/4))/(128\*c^(9/4))

**IntegrateAlgebraic [A]** time = 0.94, size = 191, normalized size = 0.64

$$\frac{(3Ac + 5bB) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}\right)}{32\sqrt{2} b^{7/4} c^{9/4}} + \frac{(3Ac + 5bB) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2} b^{7/4} c^{9/4}} + \frac{-3Abc\sqrt{x} + Ac^2x^{5/2} - 5b^2B\sqrt{x} - 9bBcx^{5/2}}{16bc^2(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(15/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (-5\*b^2\*B\*Sqrt[x] - 3\*A\*b\*c\*Sqrt[x] - 9\*b\*B\*c\*x^(5/2) + A\*c^2\*x^(5/2))/(16\*b\*c^2\*(b + c\*x^2)^2 - ((5\*b\*B + 3\*A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(32\*Sqrt[2]\*b^(7/4)\*c^(9/4)) + ((5\*b\*B + 3\*A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*b^(7/4)\*c^(9/4)))

**fricas** [B] time = 0.45, size = 806, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/64\*(4\*(b\*c^4\*x^4 + 2\*b^2\*c^3\*x^2 + b^3\*c^2)\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(1/4)\*arctan((sqrt(b^4\*c^4\*sqrt(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9)) + (25\*B^2\*b^2 + 30\*A\*B\*b\*c + 9\*A^2\*c^2)\*x)\*b^5\*c^7\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(3/4) - (5\*B\*b^6\*c^7 + 3\*A\*b^5\*c^8)\*sqrt(x)\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(3/4))/(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4) + (b\*c^4\*x^4 + 2\*b^2\*c^3\*x^2 + b^3\*c^2)\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(1/4)\*log(b^2\*c^2\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(1/4) + (5\*B\*b + 3\*A\*c)\*sqrt(x)) - (b\*c^4\*x^4 + 2\*b^2\*c^3\*x^2 + b^3\*c^2)\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(1/4)\*log(-b^2\*c^2\*(-(625\*B^4\*b^4 + 1500\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 540\*A^3\*B\*b\*c^3 + 81\*A^4\*c^4)/(b^7\*c^9))^(1/4) + (5\*B\*b + 3\*A\*c)\*sqrt(x)) - 4\*(5\*B\*b^2 + 3\*A\*b\*c + (9\*B\*b\*c - A\*c^2)\*x^2)\*sqrt(x))/(b\*c^4\*x^4 + 2\*b^2\*c^3\*x^2 + b^3\*c^2)

**giac** [A] time = 0.19, size = 298, normalized size = 1.00

$$\frac{\sqrt{5} (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \arctan\left(\frac{\sqrt{5} \sqrt{(bc^3)^{\frac{1}{4}} + 2\sqrt{5}}}{z(bc^3)^{\frac{1}{4}}}\right)}{64 b^2 c^3} + \frac{\sqrt{5} (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \arctan\left(\frac{\sqrt{5} \sqrt{(bc^3)^{\frac{1}{4}} - 2\sqrt{5}}}{z(bc^3)^{\frac{1}{4}}}\right)}{64 b^2 c^3} + \frac{\sqrt{5} (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \log\left(\sqrt{5} \sqrt{(bc^3)^{\frac{1}{4}} + x + \sqrt{5}}\right)}{128 b^2 c^3} - \frac{\sqrt{5} (bc^3)^{\frac{1}{4}} Bb + 3 (bc^3)^{\frac{1}{4}} Ac \log\left(-\sqrt{5} \sqrt{(bc^3)^{\frac{1}{4}} + x + \sqrt{5}}\right)}{128 b^2 c^3} - \frac{9 Bbcx^{\frac{3}{2}} - Ac^2x^{\frac{3}{2}} + 5 Bb^2\sqrt{x} + 3 Abc\sqrt{x}}{16 (cx^2 + b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 1/64\*sqrt(2)\*(5\*(b\*c^3)^(1/4)\*B\*b + 3\*(b\*c^3)^(1/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/(b^2\*c^3) + 1/64\*sqrt(2)\*(5\*(b\*c^3)^(1/4)\*B\*b + 3\*(b\*c^3)^(1/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/(b^2\*c^3) + 1/128\*sqrt(2)\*(5\*(b\*c^3)^(1/4)\*B\*b + 3\*(b\*c^3)^(1/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b^2\*c^3) - 1/128\*sqrt(2)\*(5\*(b\*c^3)^(1/4)\*B\*b + 3\*(b\*c^3)^(1/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b^2\*c^3) - 1/16\*(9\*B\*b\*c\*x^(5/2) - A\*c^2\*x^(5/2) + 5\*B\*b^2\*sqrt(x) + 3\*A\*b\*c\*sqrt(x))/((c\*x^2 + b)^2\*b\*c^2)

**maple** [A] time = 0.06, size = 334, normalized size = 1.12

$$\frac{3 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A \arctan\left(\frac{\sqrt{2} \sqrt{c} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 b^2 c} + \frac{3 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A \arctan\left(\frac{\sqrt{2} \sqrt{c} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 b^2 c} + \frac{3 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} A \ln\left(\frac{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{c}}{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{c}}\right)}{128 b^2 c} + \frac{5 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B \arctan\left(\frac{\sqrt{2} \sqrt{c} - 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 b c^2} + \frac{5 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B \arctan\left(\frac{\sqrt{2} \sqrt{c} + 1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 b c^2} + \frac{5 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} B \ln\left(\frac{x + \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{c}}{x - \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{c} + \sqrt{c}}\right)}{128 b c^2} + \frac{(Ac - 9Bb) \sqrt{c}^{\frac{3}{2}} - (3Ac + 5Bb) \sqrt{c}}{16 bc (cx^2 + b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(15/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)



sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(15/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out



$$3.212 \quad \int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=298

$$\frac{(5Ac + 3bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{9/4} c^{7/4}} - \frac{(5Ac + 3bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{9/4} c^{7/4}} - \frac{(5Ac + 3bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{9/4} c^{7/4}} + \frac{(5Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{32\sqrt{2} b^{9/4} c^{7/4}} + \frac{x^{3/2}(5Ac + 3bB)}{16b^2c(b+cx^2)} - \frac{x^{3/2}(bB - Ac)}{4bc(b+cx^2)^2}$$

**Rubi [A]** time = 0.23, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1584, 457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(5Ac + 3bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{9/4} c^{7/4}} - \frac{(5Ac + 3bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{9/4} c^{7/4}} - \frac{(5Ac + 3bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{9/4} c^{7/4}} + \frac{(5Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{32\sqrt{2} b^{9/4} c^{7/4}} + \frac{x^{3/2}(5Ac + 3bB)}{16b^2c(b+cx^2)} - \frac{x^{3/2}(bB - Ac)}{4bc(b+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -((b\*B - A\*c)\*x^(3/2))/(4\*b\*c\*(b + c\*x^2)^2) + ((3\*b\*B + 5\*A\*c)\*x^(3/2))/(16\*b^2\*c\*(b + c\*x^2)) - ((3\*b\*B + 5\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(9/4)\*c^(7/4)) + ((3\*b\*B + 5\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(9/4)\*c^(7/4)) + ((3\*b\*B + 5\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(9/4)\*c^(7/4)) - ((3\*b\*B + 5\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(9/4)\*c^(7/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 290

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*n\*(p+1)), x] + Dist[(m+n\*(p+1)+1)/(a\*n\*(p+1)), Int[(c\*x)^m\*(a+b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a+(b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*(e\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{\sqrt{x} (A + Bx^2)}{(b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{\left(\frac{3bB}{2} + \frac{5Ac}{2}\right) \int \frac{\sqrt{x}}{(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{(3bB + 5Ac)x^{3/2}}{16b^2c(b + cx^2)} + \frac{(3bB + 5Ac) \int \frac{\sqrt{x}}{b+cx^2} dx}{32b^2c} \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{(3bB + 5Ac)x^{3/2}}{16b^2c(b + cx^2)} + \frac{(3bB + 5Ac) \text{Subst}\left(\int \frac{x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{16b^2c} \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{(3bB + 5Ac)x^{3/2}}{16b^2c(b + cx^2)} - \frac{(3bB + 5Ac) \text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{32b^2c^{3/2}} + \dots \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{(3bB + 5Ac)x^{3/2}}{16b^2c(b + cx^2)} + \frac{(3bB + 5Ac) \text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{64b^2c^2} \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{(3bB + 5Ac)x^{3/2}}{16b^2c(b + cx^2)} + \frac{(3bB + 5Ac) \log\left(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2} b^{9/4} c^{7/4}} \\
&= -\frac{(bB - Ac)x^{3/2}}{4bc(b + cx^2)^2} + \frac{(3bB + 5Ac)x^{3/2}}{16b^2c(b + cx^2)} - \frac{(3bB + 5Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{9/4} c^{7/4}} + \frac{(3bB + 5Ac)}{16b^2c}
\end{aligned}$$

**Mathematica [C]** time = 0.09, size = 62, normalized size = 0.21

$$\frac{2x^{3/2} \left( (Ac - bB) {}_2F_1\left(\frac{3}{4}, 3; \frac{7}{4}; -\frac{cx^2}{b}\right) + bB {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right) \right)}{3b^3c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (2\*x^(3/2)\*(b\*B\*Hypergeometric2F1[3/4, 2, 7/4, -((c\*x^2)/b)] + (-b\*B) + A\*c)\*Hypergeometric2F1[3/4, 3, 7/4, -((c\*x^2)/b)])/(3\*b^3\*c)

**IntegrateAlgebraic [A]** time = 1.03, size = 182, normalized size = 0.61

$$\frac{(5Ac + 3bB) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}\right)}{32\sqrt{2} b^{9/4} c^{7/4}} - \frac{(5Ac + 3bB) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{32\sqrt{2} b^{9/4} c^{7/4}} - \frac{x^{3/2} (-9Abc - 5Ac^2x^2 + b^2B - 3bBcx^2)}{16b^2c(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(13/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -1/16\*(x^(3/2)\*(b^2\*B - 9\*A\*b\*c - 3\*b\*B\*c\*x^2 - 5\*A\*c^2\*x^2))/(b^2\*c\*(b + c\*x^2)^2) - ((3\*b\*B + 5\*A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(32\*Sqrt[2]\*b^(9/4)\*c^(7/4)) - ((3\*b\*B + 5\*A\*c)\*ArcTanh[(

Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]]/(Sqrt[b + Sqrt[c]\*x]]/(32\*Sqrt[2]\*b^(9/4)\*c^(7/4))

**fricas [B]** time = 0.46, size = 1005, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] -1/64\*(4\*(b^2\*c^3\*x^4 + 2\*b^3\*c^2\*x^2 + b^4\*c)\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(1/4)\*arctan((sqrt((729\*B^6\*b^6 + 7290\*A\*B^5\*b^5\*c + 30375\*A^2\*B^4\*b^4\*c^2 + 67500\*A^3\*B^3\*b^3\*c^3 + 84375\*A^4\*B^2\*b^2\*c^4 + 56250\*A^5\*B\*b\*c^5 + 15625\*A^6\*c^6))\*x - (81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7)))^2\*c^2\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(1/4) - (27\*B^3\*b^3\*c^3 + 135\*A\*B^2\*b^2\*c^2 + 225\*A^2\*B\*b\*c^2 + 125\*A^3\*c^3)\*sqrt(x))\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(1/4))/(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4) - (b^2\*c^3\*x^4 + 2\*b^3\*c^2\*x^2 + b^4\*c)\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(1/4)\*log(b^7\*c^5\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(3/4) + (27\*B^3\*b^3 + 135\*A\*B^2\*b^2\*c + 225\*A^2\*B\*b\*c^2 + 125\*A^3\*c^3)\*sqrt(x)) + (b^2\*c^3\*x^4 + 2\*b^3\*c^2\*x^2 + b^4\*c)\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(1/4)\*log(-b^7\*c^5\*(-(81\*B^4\*b^4 + 540\*A\*B^3\*b^3\*c + 1350\*A^2\*B^2\*b^2\*c^2 + 1500\*A^3\*B\*b\*c^3 + 625\*A^4\*c^4)/(b^9\*c^7))^(3/4) + (27\*B^3\*b^3 + 135\*A\*B^2\*b^2\*c + 225\*A^2\*B\*b\*c^2 + 125\*A^3\*c^3)\*sqrt(x)) - 4\*((3\*B\*b\*c + 5\*A\*c^2)\*x^3 - (B\*b^2 - 9\*A\*b\*c)\*x)\*sqrt(x))/(b^2\*c^3\*x^4 + 2\*b^3\*c^2\*x^2 + b^4\*c)

**giac [A]** time = 0.23, size = 298, normalized size = 1.00

$$\frac{3Bbcx^7 + 5Ac^2x^7 - Bb^2x^3 + 9Abcx^3}{16(c^2 + b)^{7/4}b^2c} + \frac{\sqrt{2}(3(hc^3)^{3/4}Bb + 5(hc^3)^{1/4}Ac) \arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{1/4} + 2\sqrt{x}}}{\left(\frac{b}{c}\right)^{1/4}}\right)}{64b^{5/4}c} + \frac{\sqrt{2}(3(hc^3)^{3/4}Bb + 5(hc^3)^{1/4}Ac) \arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{1/4} - 2\sqrt{x}}}{\left(\frac{b}{c}\right)^{1/4}}\right)}{64b^{5/4}c} + \frac{\sqrt{2}(3(hc^3)^{3/4}Bb + 5(hc^3)^{1/4}Ac) \log\left(\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{1/4} + x + \sqrt{x}}\right)}{128b^{5/4}c} + \frac{\sqrt{2}(3(hc^3)^{3/4}Bb + 5(hc^3)^{1/4}Ac) \log\left(-\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{1/4} + x + \sqrt{x}}\right)}{128b^{5/4}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 1/16\*(3\*B\*b\*c\*x^(7/2) + 5\*A\*c^2\*x^(7/2) - B\*b^2\*x^(3/2) + 9\*A\*b\*c\*x^(3/2))/(c\*x^2 + b)^2\*b^2\*c + 1/64\*sqrt(2)\*(3\*(b\*c^3)^(3/4)\*B\*b + 5\*(b\*c^3)^(3/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/(b^3\*c^4) + 1/64\*sqrt(2)\*(3\*(b\*c^3)^(3/4)\*B\*b + 5\*(b\*c^3)^(3/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/(b^3\*c^4) - 1/128\*sqrt(2)\*(3\*(b\*c^3)^(3/4)\*B\*b + 5\*(b\*c^3)^(3/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b^3\*c^4) + 1/128\*sqrt(2)\*(3\*(b\*c^3)^(3/4)\*B\*b + 5\*(b\*c^3)^(3/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b^3\*c^4)

**maple [A]** time = 0.07, size = 335, normalized size = 1.12

$$\frac{5\sqrt{2} A \arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{1/4}}\right)}{64\left(\frac{b}{c}\right)^{1/4}b^2c} + \frac{5\sqrt{2} A \arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{1/4}}\right)}{64\left(\frac{b}{c}\right)^{1/4}b^2c} + \frac{5\sqrt{2} A \ln\left(\frac{x-\left(\frac{b}{c}\right)^{1/4}\sqrt{2}\sqrt{x+\sqrt{x}}}{x+\left(\frac{b}{c}\right)^{1/4}\sqrt{2}\sqrt{x+\sqrt{x}}}\right)}{128\left(\frac{b}{c}\right)^{1/4}b^2c} + \frac{3\sqrt{2} B \arctan\left(\frac{\sqrt{2}\sqrt{x}-1}{\left(\frac{b}{c}\right)^{1/4}}\right)}{64\left(\frac{b}{c}\right)^{1/4}b^2c} + \frac{3\sqrt{2} B \arctan\left(\frac{\sqrt{2}\sqrt{x}+1}{\left(\frac{b}{c}\right)^{1/4}}\right)}{64\left(\frac{b}{c}\right)^{1/4}b^2c} + \frac{3\sqrt{2} B \ln\left(\frac{x-\left(\frac{b}{c}\right)^{1/4}\sqrt{2}\sqrt{x+\sqrt{x}}}{x+\left(\frac{b}{c}\right)^{1/4}\sqrt{2}\sqrt{x+\sqrt{x}}}\right)}{128\left(\frac{b}{c}\right)^{1/4}b^2c} + \frac{(5Ac+3B)x^2}{16b^2} + \frac{(9Ac-bB)x^2}{16bc} + \frac{1}{(cx^2+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^{13/2}*(B*x^2+A)/(c*x^4+b*x^2)^3,x)$

[Out]  $2*(1/32*(5*A*c+3*B*b)/b^2*x^{7/2}+1/32*(9*A*c-B*b)/b/c*x^{3/2})/(c*x^2+b)^2+5/128/b^2/c/(b/c)^{1/4}*2^{1/2}*A*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})/(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))+5/64/b^2/c/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)+5/64/b^2/c/(b/c)^{1/4}*2^{1/2}*A*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1)+3/128/b/c^2/(b/c)^{1/4}*2^{1/2}*B*\ln((x-(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2})/(x+(b/c)^{1/4}*2^{1/2}*x^{1/2}+(b/c)^{1/2}))+3/64/b/c^2/(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}-1)+3/64/b/c^2/(b/c)^{1/4}*2^{1/2}*B*\arctan(2^{1/2}/(b/c)^{1/4}*x^{1/2}+1)$

**maxima** [A] time = 3.01, size = 253, normalized size = 0.85

$$\frac{(3Bb + 5Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \right)}{16(b^2c^3x^4 + 2b^3c^2x^2 + b^4c)} + \frac{\quad}{128b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^{13/2}*(B*x^2+A)/(c*x^4+b*x^2)^3,x, \text{algorithm}="maxima")$

[Out]  $1/16*((3*B*b*c + 5*A*c^2)*x^{7/2} - (B*b^2 - 9*A*b*c)*x^{3/2})/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c) + 1/128*(3*B*b + 5*A*c)*(2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(sqrt(2)*b^{1/4}*c^{1/4} + 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(sqrt(b)*\text{sqrt}(c))*\text{sqrt}(c)) + 2*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*(sqrt(2)*b^{1/4}*c^{1/4} - 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(sqrt(b)*\text{sqrt}(c))*\text{sqrt}(c)) - \text{sqrt}(2)*\log(\text{sqrt}(2)*b^{1/4}*c^{1/4}*sqrt(x) + \text{sqrt}(c)*x + \text{sqrt}(b)))/(b^{1/4}*c^{3/4}) + \text{sqrt}(2)*\log(-\text{sqrt}(2)*b^{1/4}*c^{1/4}*sqrt(x) + \text{sqrt}(c)*x + \text{sqrt}(b)))/(b^{1/4}*c^{3/4}))/b^2*c$

**mupad** [B] time = 0.16, size = 124, normalized size = 0.42

$$\frac{x^{7/2}(5Ac+3Bb)}{16b^2} + \frac{x^{3/2}(9Ac-Bb)}{16bc} + \frac{\text{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(5Ac+3Bb)}{32(-b)^{9/4}c^{7/4}} - \frac{\text{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(5Ac+3Bb)}{32(-b)^{9/4}c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^{13/2}*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)$

[Out]  $((x^{7/2}*(5*A*c + 3*B*b))/(16*b^2) + (x^{3/2}*(9*A*c - B*b))/(16*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (\text{atan}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(5*A*c + 3*B*b))/(32*(-b)^{9/4}*c^{7/4}) - (\text{atanh}((c^{1/4}*x^{1/2})/(-b)^{1/4})*(5*A*c + 3*B*b))/(32*(-b)^{9/4}*c^{7/4})$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)$

[Out] Timed out

$$3.213 \quad \int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=293

$$\frac{3(7Ac + bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{11/4} c^{5/4}} + \frac{3(7Ac + bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{11/4} c^{5/4}} - \frac{3(7Ac + bB)}{32}$$

**Rubi [A]** time = 0.23, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1584, 457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3(7Ac + bB) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{11/4} c^{5/4}} + \frac{3(7Ac + bB) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{11/4} c^{5/4}} - \frac{3(7Ac + bB) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{11/4} c^{5/4}} + \frac{3(7Ac + bB) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} + 1\right)}{32\sqrt{2} b^{11/4} c^{5/4}} + \frac{\sqrt{x}(7Ac + bB)}{16b^2c(b + cx^2)} - \frac{\sqrt{x}(bB - Ac)}{4bc(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] -((b\*B - A\*c)\*Sqrt[x])/(4\*b\*c\*(b + c\*x^2)^2) + ((b\*B + 7\*A\*c)\*Sqrt[x])/(16\*b^2\*c\*(b + c\*x^2)) - (3\*(b\*B + 7\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(11/4)\*c^(5/4)) + (3\*(b\*B + 7\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(11/4)\*c^(5/4)) - (3\*(b\*B + 7\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(11/4)\*c^(5/4)) + (3\*(b\*B + 7\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(11/4)\*c^(5/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 290

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*n\*(p + 1)), x] + Dist[(m + n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(c\*x)^m\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 457

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{\sqrt{x} (b + cx^2)^3} dx \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{\left(\frac{bB}{2} + \frac{7Ac}{2}\right) \int \frac{1}{\sqrt{x}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{(bB + 7Ac)\sqrt{x}}{16b^2c(b + cx^2)} + \frac{(3(bB + 7Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{32b^2c} \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{(bB + 7Ac)\sqrt{x}}{16b^2c(b + cx^2)} + \frac{(3(bB + 7Ac)) \operatorname{Subst}\left(\int \frac{1}{b+cx^4} dx, x, \sqrt{x}\right)}{16b^2c} \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{(bB + 7Ac)\sqrt{x}}{16b^2c(b + cx^2)} + \frac{(3(bB + 7Ac)) \operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, \sqrt{x}\right)}{32b^{5/2}c} + \dots \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{(bB + 7Ac)\sqrt{x}}{16b^2c(b + cx^2)} + \frac{(3(bB + 7Ac)) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{c}} + x^2} dx, x, \sqrt{x}\right)}{64b^{5/2}c^{3/2}} + \dots \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{(bB + 7Ac)\sqrt{x}}{16b^2c(b + cx^2)} - \frac{3(bB + 7Ac) \log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}} + \dots \\
&= -\frac{(bB - Ac)\sqrt{x}}{4bc(b + cx^2)^2} + \frac{(bB + 7Ac)\sqrt{x}}{16b^2c(b + cx^2)} - \frac{3(bB + 7Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} + \frac{3(bB + 7Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.38, size = 230, normalized size = 0.78

$$\frac{(7Ac+bB)\left(7(b+cx^2)\left(8b^{3/4}\sqrt[4]{c}\sqrt{x}-3\sqrt{2}(b+cx^2)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)\right)\right)+32b^{7/4}\sqrt[4]{c}\sqrt{x}\right)}{b^{11/4}\sqrt[4]{c}} - 256B\sqrt{x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (-256\*B\*Sqrt[x] + ((b\*B + 7\*A\*c)\*(32\*b^(7/4)\*c^(1/4)\*Sqrt[x] + 7\*(b + c\*x^2)\*(8\*b^(3/4)\*c^(1/4)\*Sqrt[x] - 3\*Sqrt[2]\*(b + c\*x^2)\*(2\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 2\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)]) + Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])))/(b^(11/4)\*c^(1/4))/(896\*c\*(b + c\*x^2)^2)

**IntegrateAlgebraic [A]** time = 0.83, size = 181, normalized size = 0.62

$$-\frac{3(7Ac + bB) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} + \frac{3(7Ac + bB) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} - \frac{\sqrt{x}(-11Abc - 7Ac^2x^2 + 3b^2B - bBcx^2)}{16b^2c(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]



[Out]  $-1/16*(\text{Sqrt}[x]*(3*b^2*B - 11*A*b*c - b*B*c*x^2 - 7*A*c^2*x^2))/(b^2*c*(b + c*x^2)^2) - (3*(b*B + 7*A*c)*\text{ArcTan}[(\text{Sqrt}[b] - \text{Sqrt}[c]*x)/(\text{Sqrt}[2]*b^{(1/4)}*c^{(1/4)}*\text{Sqrt}[x]))/(32*\text{Sqrt}[2]*b^{(11/4)}*c^{(5/4)}) + (3*(b*B + 7*A*c)*\text{ArcTanh}[(\text{Sqrt}[2]*b^{(1/4)}*c^{(1/4)}*\text{Sqrt}[x])/(\text{Sqrt}[b] + \text{Sqrt}[c]*x)]/(32*\text{Sqrt}[2]*b^{(11/4)}*c^{(5/4)})$

**fricas [B]** time = 0.44, size = 793, normalized size = 2.71

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")`

[Out]  $1/64*(12*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(1/4)}*\arctan(\text{sqrt}(b^6*c^2*\text{sqrt}(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5)) + (B^2*b^2 + 14*A*B*b*c + 49*A^2*c^2)*x)*b^8*c^4*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(3/4)} - (B*b^9*c^4 + 7*A*b^8*c^5)*\text{sqrt}(x)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(3/4)})/(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4) + 3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(1/4)}*\log(3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(1/4)} + 3*(B*b + 7*A*c)*\text{sqrt}(x)) - 3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(1/4)}*\log(-3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^{11}*c^5))^{(1/4)} + 3*(B*b + 7*A*c)*\text{sqrt}(x)) - 4*(3*B*b^2 - 11*A*b*c - (B*b*c + 7*A*c^2)*x^2)*\text{sqrt}(x)/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)$

**giac [A]** time = 0.20, size = 293, normalized size = 1.00

$$\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{c}}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c^2} + \frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{c}}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c^2} + \frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{c}}\right)}{128b^3c^2} + \frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{c}}\right)}{128b^3c^2} + \frac{Bbcx^{\frac{3}{2}}+7Ac^3x^{\frac{3}{2}}-3Bb^2\sqrt{c}+11Abc\sqrt{c}}{16(c^2+b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")`

[Out]  $3/64*\text{sqrt}(2)*((b*c^3)^{(1/4)}*B*b + 7*(b*c^3)^{(1/4)}*A*c)*\arctan(1/2*\text{sqrt}(2))*(\text{sqrt}(2)*(b/c)^{(1/4)} + 2*\text{sqrt}(x))/(b/c)^{(1/4)}/(b^3*c^2) + 3/64*\text{sqrt}(2)*((b*c^3)^{(1/4)}*B*b + 7*(b*c^3)^{(1/4)}*A*c)*\arctan(-1/2*\text{sqrt}(2))*(\text{sqrt}(2)*(b/c)^{(1/4)} - 2*\text{sqrt}(x))/(b/c)^{(1/4)}/(b^3*c^2) + 3/128*\text{sqrt}(2)*((b*c^3)^{(1/4)}*B*b + 7*(b*c^3)^{(1/4)}*A*c)*\log(\text{sqrt}(2)*\text{sqrt}(x)*(b/c)^{(1/4)} + x + \text{sqrt}(b/c))/(b^3*c^2) - 3/128*\text{sqrt}(2)*((b*c^3)^{(1/4)}*B*b + 7*(b*c^3)^{(1/4)}*A*c)*\log(-\text{sqrt}(2)*\text{sqrt}(x)*(b/c)^{(1/4)} + x + \text{sqrt}(b/c))/(b^3*c^2) + 1/16*(B*b*c*x^{(5/2)} + 7*A*c^2*x^{(5/2)} - 3*B*b^2*\text{sqrt}(x) + 11*A*b*c*\text{sqrt}(x))/((c*x^2 + b)^2*b^2*c)$

**maple [A]** time = 0.06, size = 325, normalized size = 1.11

$$\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3} + \frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3} + \frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}\right)}{128b^3} + \frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{c}-1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^2c} + \frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{c}+1}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^2c} + \frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}B\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c}+\sqrt{c}}\right)}{128b^2c} + \frac{(7Ac+8B)c^{\frac{3}{2}} + (11Ac-3Bb)\sqrt{c}}{(cx^2+b)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)`

[Out]  $2*(1/32*(7*A*c+B*b)/b^2*x^{(5/2)}+1/32*(11*A*c-3*B*b)/b*c*x^{(1/2)})/(c*x^2+b)^2+21/64/b^3*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)+21/$

$128/b^3*(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+21/64/b^3*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+3/64/b^2/c*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)+3/128/b^2/c*(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+3/64/b^2/c*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)$

**maxima** [A] time = 3.05, size = 276, normalized size = 0.94

$$\frac{(Bbc + 7Ac^2)x^{\frac{5}{2}} - (3Bb^2 - 11Abc)\sqrt{x}}{16(b^2c^3x^4 + 2b^3c^2x^2 + b^4c)} + \frac{3 \left( \frac{2\sqrt{2}(Bb+7Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(Bb+7Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(Bb+7Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}(Bb+7Ac)\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{\frac{3}{b^{\frac{3}{4}}c^{\frac{1}{4}}}} \right)}{128b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out]  $1/16*((B*b*c + 7*A*c^2)*x^{(5/2)} - (3*B*b^2 - 11*A*b*c)*\text{sqrt}(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c) + 3/128*(2*\text{sqrt}(2)*(B*b + 7*A*c)*\arctan(1/2*\text{sqrt}(2)*(sqrt(2)*b^{(1/4)}*c^{(1/4)} + 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(b)*\text{sqrt}(sqrt(b)*\text{sqrt}(c))) + 2*\text{sqrt}(2)*(B*b + 7*A*c)*\arctan(-1/2*\text{sqrt}(2)*(sqrt(2)*b^{(1/4)}*c^{(1/4)} - 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(b)*\text{sqrt}(sqrt(b)*\text{sqrt}(c))) + \text{sqrt}(2)*(B*b + 7*A*c)*\log(\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/(b^{(3/4)}*c^{(1/4)}) - \text{sqrt}(2)*(B*b + 7*A*c)*\log(-\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/(b^{(3/4)}*c^{(1/4)})/(b^2*c)$

**mupad** [B] time = 0.39, size = 780, normalized size = 2.66

$$\frac{\text{atan}\left(\frac{\frac{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x}}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\sqrt{b^2c^3x^4+2b^3c^2x^2+b^4c}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{32(-b)^{\frac{11}{4}}c^{\frac{5}{4}}}\sqrt{b^2c^3x^4+2b^3c^2x^2+b^4c} + 3\text{atan}\left(\frac{\frac{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x}}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\sqrt{b^2c^3x^4+2b^3c^2x^2+b^4c}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{32(-b)^{\frac{11}{4}}c^{\frac{5}{4}}}\sqrt{b^2c^3x^4+2b^3c^2x^2+b^4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(11/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out]  $((x^{(5/2)}*(7*A*c + B*b))/(16*b^2) + (x^{(1/2)}*(11*A*c - 3*B*b))/(16*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (\text{atan}(((7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)))/(64*(-b)^{(15/4)}*c^{(5/4)})))*3i)/(64*(-b)^{(11/4)}*c^{(5/4)}) + ((7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) + (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)))/(64*(-b)^{(15/4)}*c^{(5/4)}))*3i)/(64*(-b)^{(11/4)}*c^{(5/4)})/((3*(7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)))/(64*(-b)^{(15/4)}*c^{(5/4)})))/((3*(7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)))/(64*(-b)^{(15/4)}*c^{(5/4)})))*7i)/(64*(-b)^{(11/4)}*c^{(5/4)}) + ((7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2))*9i)/(64*(-b)^{(15/4)}*c^{(5/4)})))/((3*(7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2))*9i)/(64*(-b)^{(15/4)}*c^{(5/4)})))*3i)/(64*(-b)^{(11/4)}*c^{(5/4)}) - ((7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2))*9i)/(64*(-b)^{(15/4)}*c^{(5/4)})))/((3*(7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2))*9i)/(64*(-b)^{(15/4)}*c^{(5/4)})))*3i)/(64*(-b)^{(11/4)}*c^{(5/4)}) - ((7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2))*9i)/(64*(-b)^{(15/4)}*c^{(5/4)})))/((3*(7*A*c + B*b)*((9*x^{(1/2)}*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2)))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2))*9i)/(64*(-b)^{(15/4)}*c^{(5/4)})))*3i)/(64*(-b)^{(11/4)}*c^{(5/4)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(11/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.214 \quad \int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=316

$$\frac{5(bB - 9Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{13/4} c^{3/4}} - \frac{5(bB - 9Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{13/4} c^{3/4}} - \frac{5(bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{32\sqrt{2} b^{13/4} c^{3/4}} + \frac{5(bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} - 1\right)}{32\sqrt{2} b^{13/4} c^{3/4}} - \frac{bB - 9Ac}{16b^2 c \sqrt{x} (b + cx^2)} + \frac{5(bB - 9Ac)}{16b^3 c \sqrt{x}} - \frac{bB - Ac}{4bc \sqrt{x} (b + cx^2)^2}$$

**Rubi [A]** time = 0.26, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5(bB - 9Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{13/4} c^{3/4}} - \frac{5(bB - 9Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c} x\right)}{64\sqrt{2} b^{13/4} c^{3/4}} - \frac{5(bB - 9Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2} b^{13/4} c^{3/4}} + \frac{5(bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{32\sqrt{2} b^{13/4} c^{3/4}} - \frac{bB - 9Ac}{16b^2 c \sqrt{x} (b + cx^2)} + \frac{5(bB - 9Ac)}{16b^3 c \sqrt{x}} - \frac{bB - Ac}{4bc \sqrt{x} (b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (5\*(b\*B - 9\*A\*c))/(16\*b^3\*c\*Sqrt[x]) - (b\*B - A\*c)/(4\*b\*c\*Sqrt[x]\*(b + c\*x^2)^2) - (b\*B - 9\*A\*c)/(16\*b^2\*c\*Sqrt[x]\*(b + c\*x^2)) - (5\*(b\*B - 9\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(13/4)\*c^(3/4)) + (5\*(b\*B - 9\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(13/4)\*c^(3/4)) + (5\*(b\*B - 9\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(13/4)\*c^(3/4)) - (5\*(b\*B - 9\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(13/4)\*c^(3/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 290

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*n\*(p + 1)), x] + Dist[(m + n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(c\*x)^m\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^{3/2} (b + cx^2)^3} dx \\
&= -\frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} + \frac{\left(-\frac{bB}{2} + \frac{9Ac}{2}\right) \int \frac{1}{x^{3/2}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} - \frac{(5(bB - 9Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{5(bB - 9Ac)}{16b^3c\sqrt{x}} - \frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} + \frac{(5(bB - 9Ac)) \int \frac{\sqrt{x}}{b+cx^2} dx}{32b^3} \\
&= \frac{5(bB - 9Ac)}{16b^3c\sqrt{x}} - \frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} + \frac{(5(bB - 9Ac)) \text{Subst} \left( \int \frac{x^2}{b+cx^4} dx \right)}{16b^3} \\
&= \frac{5(bB - 9Ac)}{16b^3c\sqrt{x}} - \frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} - \frac{(5(bB - 9Ac)) \text{Subst} \left( \int \frac{\sqrt{b}-\sqrt{c}}{b+cx^4} dx \right)}{32b^3\sqrt{c}} \\
&= \frac{5(bB - 9Ac)}{16b^3c\sqrt{x}} - \frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} + \frac{(5(bB - 9Ac)) \text{Subst} \left( \int \frac{\frac{\sqrt{b}-\sqrt{c}}{\sqrt{c}} - \frac{\sqrt{2}}{\sqrt{c}}}{\sqrt{c}} dx \right)}{64b^3c} \\
&= \frac{5(bB - 9Ac)}{16b^3c\sqrt{x}} - \frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} + \frac{5(bB - 9Ac) \log(\sqrt{b} - \sqrt{2} \sqrt[4]{b})}{64\sqrt{2} b^{13/4} c^{3/4}} \\
&= \frac{5(bB - 9Ac)}{16b^3c\sqrt{x}} - \frac{bB - Ac}{4bc\sqrt{x} (b + cx^2)^2} - \frac{bB - 9Ac}{16b^2c\sqrt{x} (b + cx^2)} - \frac{5(bB - 9Ac) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt[4]{b}} \right)}{32\sqrt{2} b^{13/4} c^{3/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.29, size = 147, normalized size = 0.47

$$\frac{2x^{3/2}(bB - Ac) {}_2F_1\left(\frac{3}{4}, 3; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^4} - \frac{2Acx^{3/2} {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^4} - \frac{2A}{b^3\sqrt{x}} + \frac{A\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{13/4}} + \frac{Ab\sqrt[4]{c} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (-2\*A)/(b^3\*Sqrt[x]) + (A\*c^(1/4)\*ArcTan[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)]/(-b)^(13/4) + (A\*b\*c^(1/4)\*ArcTanh[(c^(1/4)\*Sqrt[x])/(-b)^(1/4)]/(-b)^(17/4) - (2\*A\*c\*x^(3/2)\*Hypergeometric2F1[3/4, 2, 7/4, -((c\*x^2)/b)]/(3\*b^4) + (2\*(b\*B - A\*c)\*x^(3/2)\*Hypergeometric2F1[3/4, 3, 7/4, -((c\*x^2)/b)]/(3\*b^4))

**IntegrateAlgebraic [A]** time = 0.79, size = 190, normalized size = 0.60

$$-\frac{5(bB - 9Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{32\sqrt{2} b^{13/4} c^{3/4}} - \frac{5(bB - 9Ac) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2} b^{13/4} c^{3/4}} + \frac{-32Ab^2 - 81Abcx^2 - 45Ac^2x^4 + 9b^2Bx^2 + 5bBcx^4}{16b^3\sqrt{x} (b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(9/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] 
$$\frac{(-32*A*b^2 + 9*b^2*B*x^2 - 81*A*b*c*x^2 + 5*b*B*c*x^4 - 45*A*c^2*x^4)/(16*b^3*\sqrt{x}*(b + c*x^2)^2) - (5*(b*B - 9*A*c)*\text{ArcTan}[(\sqrt{b} - \sqrt{c})*x]/(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x}))}{(32*\sqrt{2}*b^{13/4}*c^{3/4})} - \frac{(5*(b*B - 9*A*c)*\text{ArcTanh}[(\sqrt{2}*b^{1/4}*c^{1/4}*\sqrt{x})/(\sqrt{b} + \sqrt{c}*x)])}{(32*\sqrt{2}*b^{13/4}*c^{3/4})}$$

**fricas** [B] time = 0.47, size = 988, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 
$$\frac{1}{64}*(20*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3))^{1/4}*\arctan(\frac{\sqrt{(B^6*b^6 - 54*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 - 14580*A^3*B^3*b^3*c^3 + 98415*A^4*B^2*b^2*c^4 - 354294*A^5*B*b*c^5 + 531441*A^6*c^6)*x - (B^4*b^{11}*c - 36*A*B^3*b^{10}*c^2 + 486*A^2*B^2*b^9*c^3 - 2916*A^3*B*b^8*c^4 + 6561*A^4*b^7*c^5)*\sqrt{-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3)}}{(b^{13}*c^3))^{1/4}} + (B^3*b^6*c - 27*A*B^2*b^5*c^2 + 243*A^2*B*b^4*c^3 - 729*A^3*b^3*c^4)*\sqrt{x}*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3))^{1/4})/(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4) - 5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3))^{1/4}*\log(125*b^{10}*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3))^{3/4} - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*\sqrt{x}) + 5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3))^{1/4}*\log(-125*b^{10}*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^{13}*c^3))^{3/4} - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*\sqrt{x}) + 4*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)*\sqrt{x})/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)$$

**giac** [A] time = 0.21, size = 300, normalized size = 0.95

$$\frac{-2A}{b^3\sqrt{c}} + \frac{5Bbcx^2 - 13Ac^2x^3 + 9Bb^2x^3 - 17Abcx^3}{16(c^2+b)^2b} + \frac{5\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb - 9(bc^2)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + \sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c^3} + \frac{5\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb - 9(bc^2)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - \sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c^3} + \frac{5\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb - 9(bc^2)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{\frac{b}{c}}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128b^3c^3} + \frac{5\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb - 9(bc^2)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{\frac{b}{c}}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 
$$-2*A/(b^3*\sqrt{x}) + 1/16*(5*B*b*c*x^{7/2} - 13*A*c^2*x^{7/2} + 9*B*b^2*x^{3/2} - 17*A*b*c*x^{3/2})/((c*x^2 + b)^2*b^3) + 5/64*\sqrt{2}*((bc^3)^{3/4}*B*b - 9*(bc^3)^{3/4}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} + 2*\sqrt{x})/(b/c)^{1/4})/(b^4*c^3) + 5/64*\sqrt{2}*((bc^3)^{3/4}*B*b - 9*(bc^3)^{3/4}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{1/4} - 2*\sqrt{x})/(b/c)^{1/4})/(b^4*c^3) - 5/128*\sqrt{2}*((bc^3)^{3/4}*B*b - 9*(bc^3)^{3/4}*A*c)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^4*c^3) + 5/128*\sqrt{2}*((bc^3)^{3/4}*B*b - 9*(bc^3)^{3/4}*A*c)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{1/4} + x + \sqrt{b/c})/(b^4*c^3)$$

**maple** [A] time = 0.07, size = 363, normalized size = 1.15

$$\frac{-13Ac^2x^2}{16(c^2+b)^2b^3} + \frac{5Bcx^2}{16(c^2+b)^2b^2} + \frac{17Acx^3}{16(c^2+b)^2b^2} + \frac{9Bx^3}{16(c^2+b)^2b} + \frac{45\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3} + \frac{45\sqrt{2}A\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3} + \frac{45\sqrt{2}A\ln\left(\frac{x\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c} + \sqrt{\frac{b}{c}}}{x\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c} + \sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3} + \frac{5\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3c} + \frac{5\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3c} + \frac{5\sqrt{2}B\ln\left(\frac{x\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c} + \sqrt{\frac{b}{c}}}{x\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{c} + \sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3c} - \frac{2A}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^{(9/2)}*(B*x^2+A)/(c*x^4+b*x^2)^3,x)$

[Out]  $-13/16/b^3/(c*x^2+b)^2*x^{(7/2)}*A*c^2+5/16/b^2/(c*x^2+b)^2*x^{(7/2)}*B*c-17/16/b^2/(c*x^2+b)^2*A*x^{(3/2)}*c+9/16/b/(c*x^2+b)^2*B*x^{(3/2)}-45/128/b^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))-45/64/b^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)-45/64/b^3/(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)+5/128/b^2/c/(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+5/64/b^2/c/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+5/64/b^2/c/(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)-2*A/b^3/x^{(1/2)}$

**maxima [A]** time = 3.07, size = 255, normalized size = 0.81

$$\frac{5(Bb-9Ac)}{16(b^3c^2x^2+2b^4cx^2+b^5\sqrt{x})} + \frac{5(Bb-9Ac) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2b^4c^4+2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2b^4c^4-2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2b^4c^4}\sqrt{x}+\sqrt{cx}+\sqrt{b}\right)}{\frac{1}{b^4c^4}} + \frac{\sqrt{2} \log\left(-\sqrt{2b^4c^4}\sqrt{x}+\sqrt{cx}+\sqrt{b}\right)}{\frac{1}{b^4c^4}} \right)}{128b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^{(9/2)}*(B*x^2+A)/(c*x^4+b*x^2)^3,x, \text{algorithm}="maxima")$

[Out]  $1/16*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)/(b^3*c^2*x^{(9/2)} + 2*b^4*c*x^{(5/2)} + b^5*\text{sqrt}(x)) + 5/128*(B*b - 9*A*c)*(2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(sqrt(2)*b^{(1/4)}*c^{(1/4)} + 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(sqrt(b)*\text{sqrt}(c))*\text{sqrt}(c)) + 2*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*(sqrt(2)*b^{(1/4)}*c^{(1/4)} - 2*\text{sqrt}(c)*\text{sqrt}(x))/\text{sqrt}(sqrt(b)*\text{sqrt}(c)))/(\text{sqrt}(sqrt(b)*\text{sqrt}(c))*\text{sqrt}(c)) - \text{sqrt}(2)*\log(\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/b^{(1/4)}*c^{(3/4)} + \text{sqrt}(2)*\log(-\text{sqrt}(2)*b^{(1/4)}*c^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(c)*x + \text{sqrt}(b))/b^{(1/4)}*c^{(3/4)})/b^3$

**mupad [B]** time = 0.26, size = 133, normalized size = 0.42

$$\frac{5 \operatorname{atan}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (9Ac - Bb)}{32(-b)^{13/4} c^{3/4}} - \frac{\frac{2A}{b} + \frac{9x^2(9Ac - Bb)}{16b^2} + \frac{5cx^4(9Ac - Bb)}{16b^3}}{b^2 \sqrt{x} + c^2 x^{9/2} + 2bcx^{5/2}} - \frac{5 \operatorname{atanh}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (9Ac - Bb)}{32(-b)^{13/4} c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^{(9/2)}*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)$

[Out]  $(5*\operatorname{atan}((c^{(1/4)}*x^{(1/2)})/(-b)^{(1/4)})*(9*A*c - B*b))/(32*(-b)^{(13/4)}*c^{(3/4)}) - ((2*A)/b + (9*x^2*(9*A*c - B*b))/(16*b^2) + (5*c*x^4*(9*A*c - B*b))/(16*b^3))/(b^2*x^{(1/2)} + c^2*x^{(9/2)} + 2*b*c*x^{(5/2)}) - (5*\operatorname{atanh}((c^{(1/4)}*x^{(1/2)})/(-b)^{(1/4)})*(9*A*c - B*b))/(32*(-b)^{(13/4)}*c^{(3/4)})$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^{(9/2)}*(B*x^2+A)/(c*x^4+b*x^2)^3,x)$

[Out] Timed out



$$3.215 \quad \int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=322

$$\frac{7(3bB - 11Ac) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} b^{15/4} \sqrt[4]{c}} + \frac{7(3bB - 11Ac) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} b^{15/4} \sqrt[4]{c}} - \frac{7(3bB - 11Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2} b^{15/4} \sqrt[4]{c}} + \frac{7(3bB - 11Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{32\sqrt{2} b^{15/4} \sqrt[4]{c}} - \frac{bB - Ac}{4bcx^2(b + cx^2)^2}$$

**Rubi [A]** time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3bB - 11Ac}{16b^2cx^2(b + cx^2)^2} + \frac{7(3bB - 11Ac)}{48b^2cx^2} - \frac{7(3bB - 11Ac) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} b^{15/4} \sqrt[4]{c}} + \frac{7(3bB - 11Ac) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2} b^{15/4} \sqrt[4]{c}} - \frac{7(3bB - 11Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2} b^{15/4} \sqrt[4]{c}} + \frac{7(3bB - 11Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{b}} + 1\right)}{32\sqrt{2} b^{15/4} \sqrt[4]{c}} - \frac{bB - Ac}{4bcx^2(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (7\*(3\*b\*B - 11\*A\*c))/(48\*b^3\*c\*x^(3/2)) - (b\*B - A\*c)/(4\*b\*c\*x^(3/2)\*(b + c\*x^2)^2) - (3\*b\*B - 11\*A\*c)/(16\*b^2\*c\*x^(3/2)\*(b + c\*x^2)) - (7\*(3\*b\*B - 11\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(15/4)\*c^(1/4)) + (7\*(3\*b\*B - 11\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(15/4)\*c^(1/4)) - (7\*(3\*b\*B - 11\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(15/4)\*c^(1/4)) + (7\*(3\*b\*B - 11\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(15/4)\*c^(1/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 290

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*n\*(p + 1)), x] + Dist[(m + n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(c\*x)^(m\*(a + b\*x^n)^(p + 1)), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n)^(p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^{5/2} (b + cx^2)^3} dx \\
&= -\frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} + \frac{\left(-\frac{3bB}{2} + \frac{11Ac}{2}\right) \int \frac{1}{x^{5/2}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} - \frac{(7(3bB - 11Ac)) \int \frac{1}{x^{5/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{7(3bB - 11Ac)}{48b^3cx^{3/2}} - \frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} + \frac{(7(3bB - 11Ac)) \int \frac{1}{\sqrt{x}(b+cx^2)} dx}{32b^3} \\
&= \frac{7(3bB - 11Ac)}{48b^3cx^{3/2}} - \frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} + \frac{(7(3bB - 11Ac)) \text{Subst}\left(\int \frac{1}{\sqrt{x}(b+cx^2)} dx\right)}{16b^3} \\
&= \frac{7(3bB - 11Ac)}{48b^3cx^{3/2}} - \frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} + \frac{(7(3bB - 11Ac)) \text{Subst}\left(\int \frac{1}{\sqrt{x}(b+cx^2)} dx\right)}{32b^3} \\
&= \frac{7(3bB - 11Ac)}{48b^3cx^{3/2}} - \frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} + \frac{(7(3bB - 11Ac)) \text{Subst}\left(\int \frac{1}{\sqrt{x}(b+cx^2)} dx\right)}{64b^3} \\
&= \frac{7(3bB - 11Ac)}{48b^3cx^{3/2}} - \frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} - \frac{7(3bB - 11Ac) \log(\sqrt{b} - \sqrt{cx})}{64\sqrt{2} b^{15/4}} \\
&= \frac{7(3bB - 11Ac)}{48b^3cx^{3/2}} - \frac{bB - Ac}{4bcx^{3/2} (b + cx^2)^2} - \frac{3bB - 11Ac}{16b^2cx^{3/2} (b + cx^2)} - \frac{7(3bB - 11Ac) \tan^{-1}\left(1 - \frac{\sqrt{b} - \sqrt{cx}}{\sqrt{2} b^{15/4}}\right)}{32\sqrt{2} b^{15/4} \sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.50, size = 400, normalized size = 1.24

$$\frac{-\frac{96A^{3/4}b\sqrt{c}}{(b+cx^2)^2} - \frac{360A^{3/4}c\sqrt{c}}{b+cx^2} - \frac{256A^{3/4}}{3c^2} + \frac{42\sqrt{2}(11Ac-3bB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{b}\right)}{c^2} + \frac{42\sqrt{2}(3bB-11Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{b}\right)}{c^2} + 231\sqrt{2}A^{3/4}\log(-\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) - 231\sqrt{2}A^{3/4}\log(\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx}) + \frac{96A^{3/4}B\sqrt{c}}{(b+cx^2)^2} + \frac{168A^{3/4}B\sqrt{c}}{b+cx^2} - \frac{63\sqrt{2}B\log(-\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx})}{c^2} + \frac{63\sqrt{2}B\log(\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{x} + \sqrt{b} + \sqrt{cx})}{c^2}}{384b^{15/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((-256\*A\*b^(3/4))/x^(3/2) + (96\*b^(11/4)\*B\*Sqrt[x])/(b + c\*x^2)^2 - (96\*A\*b^(7/4)\*c\*Sqrt[x])/(b + c\*x^2)^2 + (168\*b^(7/4)\*B\*Sqrt[x])/(b + c\*x^2) - (360\*A\*b^(3/4)\*c\*Sqrt[x])/(b + c\*x^2) + (42\*Sqrt[2]\*(-3\*b\*B + 11\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/c^(1/4) + (42\*Sqrt[2]\*(3\*b\*B - 11\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/c^(1/4) - (63\*Sqrt[2]\*b\*B\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/c^(1/4) + 231\*Sqrt[2]\*A\*c^(3/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + (63\*Sqrt[2]\*b\*B\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/c^(1/4) - 231\*Sqrt[2]\*A\*c^(3/4)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(384\*b^(15/4))

**IntegrateAlgebraic [A]** time = 0.64, size = 192, normalized size = 0.60

$$\frac{7(3bB - 11Ac) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB - 11Ac) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{-32Ab^2 - 121Abcx^2 - 77Ac^2x^4 + 33b^2Bx^2 + 21bBcx^4}{48b^3x^{3/2}(b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (-32\*A\*b^2 + 33\*b^2\*B\*x^2 - 121\*A\*b\*c\*x^2 + 21\*b\*B\*c\*x^4 - 77\*A\*c^2\*x^4)/(4\*8\*b^3\*x^(3/2)\*(b + c\*x^2)^2) - (7\*(3\*b\*B - 11\*A\*c)\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(32\*Sqrt[2]\*b^(15/4)\*c^(1/4)) + (7\*(3\*b\*B - 11\*A\*c)\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]/(Sqrt[b] + Sqrt[c]\*x)))/(32\*Sqrt[2]\*b^(15/4)\*c^(1/4))

**fricas [B]** time = 0.46, size = 809, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] -1/192\*(84\*(b^3\*c^2\*x^6 + 2\*b^4\*c\*x^4 + b^5\*x^2)\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(1/4)\*arctan((sqrt(b^8\*sqrt(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))) + (9\*B^2\*b^2 - 66\*A\*B\*b\*c + 121\*A^2\*c^2)\*x)\*b^11\*c\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(3/4) + (3\*B\*b^12\*c - 11\*A\*b^11\*c^2)\*sqrt(x)\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(3/4))/(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)) + 21\*(b^3\*c^2\*x^6 + 2\*b^4\*c\*x^4 + b^5\*x^2)\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(1/4)\*log(7\*b^4\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(1/4) - 7\*(3\*B\*b - 11\*A\*c)\*sqrt(x)) - 21\*(b^3\*c^2\*x^6 + 2\*b^4\*c\*x^4 + b^5\*x^2)\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(1/4)\*log(-7\*b^4\*(-(81\*B^4\*b^4 - 1188\*A\*B^3\*b^3\*c + 6534\*A^2\*B^2\*b^2\*c^2 - 15972\*A^3\*B\*b\*c^3 + 14641\*A^4\*c^4)/(b^15\*c))^(1/4) - 7\*(3\*B\*b - 11\*A\*c)\*sqrt(x)) - 4\*(7\*(3\*B\*b\*c - 11\*A\*c^2)\*x^4 - 32\*A\*b^2 + 11\*(3\*B\*b^2 - 11\*A\*b\*c)\*x^2)\*sqrt(x))/(b^3\*c^2\*x^6 + 2\*b^4\*c\*x^4 + b^5\*x^2)

**giac [A]** time = 0.20, size = 304, normalized size = 0.94

$$\frac{7\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}\arctan\left(\frac{\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}}{z\sqrt[4]{b}}\right)}{64\sqrt[4]{c}} + \frac{7\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}\arctan\left(\frac{\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}}{z\sqrt[4]{b}}\right)}{64\sqrt[4]{c}} + \frac{7\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}\log\left(\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}\sqrt[4]{z^2+x+\sqrt{c}}\right)}{128\sqrt[4]{c}} + \frac{7\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}\log\left(-\sqrt{2}\sqrt[4]{(bc)^2Bb-11(bc)^2Ac}\sqrt[4]{z^2+x+\sqrt{c}}\right)}{128\sqrt[4]{c}} + \frac{2A}{3b^3x^2} + \frac{7Bbcx^3-15A^2x^2+11Bb^2\sqrt{c}-19Abc\sqrt{c}}{16(x^2+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 7/64\*sqrt(2)\*(3\*(b\*c^3)^(1/4)\*B\*b - 11\*(b\*c^3)^(1/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/(b^4\*c) + 7/64\*sqrt(2)\*(3\*(b\*c^3)^(1/4)\*B\*b - 11\*(b\*c^3)^(1/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/(b^4\*c) + 7/128\*sqrt(2)\*(3\*(b\*c^3)^(1/4)\*B\*b - 11\*(b\*c^3)^(1/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b^4\*c) - 7/128\*sqrt(2)\*(3\*(b\*c^3)^(1/4)\*B\*b - 11\*(b\*c^3)^(1/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/(b^4\*c) - 2/3\*A/(b^3\*x^(3/2)) + 1/16\*(7\*B\*b\*c\*x^(5/2) - 15\*A\*c^2\*x^(5/2) + 11\*B\*b^2\*sqrt(x) - 19\*A\*b\*c\*sqrt(x))/((c\*x^2 + b)^2\*b^3)

**maple [A]** time = 0.07, size = 357, normalized size = 1.11

$$-\frac{15Ac^2}{16(c^2+b)^3} - \frac{7Bc}{16(c^2+b)^2} - \frac{19Ac\sqrt{c}}{16(c^2+b)^2} + \frac{11B\sqrt{c}}{16(c^2+b)b} - \frac{77\left(\frac{c}{b}\right)^{\frac{1}{2}}\sqrt{2}A\arctan\left(\frac{\sqrt{c}\sqrt{c}-1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{64b^4} - \frac{77\left(\frac{c}{b}\right)^{\frac{1}{2}}\sqrt{2}A\arctan\left(\frac{\sqrt{c}\sqrt{c}+1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{64b^4} - \frac{77\left(\frac{c}{b}\right)^{\frac{1}{2}}\sqrt{2}A\ln\left(\frac{+\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{c}\sqrt{c}+\sqrt{c}}{-\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{c}\sqrt{c}+\sqrt{c}}\right)}{128b^4} + \frac{21\left(\frac{c}{b}\right)^{\frac{1}{2}}\sqrt{2}B\arctan\left(\frac{\sqrt{c}\sqrt{c}-1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{64b^3} + \frac{21\left(\frac{c}{b}\right)^{\frac{1}{2}}\sqrt{2}B\arctan\left(\frac{\sqrt{c}\sqrt{c}+1}{\left(\frac{c}{b}\right)^{\frac{1}{4}}}\right)}{64b^3} + \frac{21\left(\frac{c}{b}\right)^{\frac{1}{2}}\sqrt{2}B\ln\left(\frac{+\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{c}\sqrt{c}+\sqrt{c}}{-\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{c}\sqrt{c}+\sqrt{c}}\right)}{128b^3} - \frac{2A}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] -15/16/b^3/(c\*x^2+b)^2\*x^(5/2)\*A\*c^2+7/16/b^2/(c\*x^2+b)^2\*x^(5/2)\*B\*c-19/16/b^2/(c\*x^2+b)^2\*A\*x^(1/2)\*c+11/16/b/(c\*x^2+b)^2\*B\*x^(1/2)-77/64/b^4\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)\*c-77/128/b^4\*(b/c)^(1/4)\*2^(1/2)\*A\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))\*c-77/64/b^4\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)\*c+21/64/b^3\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)+21/128/b^3\*(b/c)^(1/4)\*2^(1/2)\*B\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))+21/64/b^3\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-2/3\*A/b^3/x^(3/2)

**maxima [A]** time = 3.09, size = 285, normalized size = 0.89

$$\frac{7(3Bbc-11Ac^2)x^4-32Ab^2+11(3Bb^2-11Abc)x^2}{48(b^3c^2x^2+2b^4cx^2+b^5x^2)} + \frac{2\sqrt{2}(3Bb-11Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{b^4c^2+2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{2\sqrt{2}(3Bb-11Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{b^4c^2-2\sqrt{c}}\right)}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{b}\sqrt{c}} + \frac{\sqrt{2}(3Bb-11Ac)\log\left(\sqrt{2b^4c^2}\sqrt{x+\sqrt{c}x+\sqrt{b}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}(3Bb-11Ac)\log\left(-\sqrt{2b^4c^2}\sqrt{x+\sqrt{c}x+\sqrt{b}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] 1/48\*(7\*(3\*B\*b\*c - 11\*A\*c^2)\*x^4 - 32\*A\*b^2 + 11\*(3\*B\*b^2 - 11\*A\*b\*c)\*x^2)/(b^3\*c^2\*x^(11/2) + 2\*b^4\*c\*x^(7/2) + b^5\*x^(3/2)) + 7/128\*(2\*sqrt(2)\*(3\*B\*b - 11\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(b)\*sqrt(sqrt(b)\*sqrt(c)) + 2\*sqrt(2)\*(3\*B\*b - 11\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(b)\*sqrt(sqrt(b)\*sqrt(c)) + sqrt(2)\*(3\*B\*b - 11\*A\*c)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4)) - sqrt(2)\*(3\*B\*b - 11\*A\*c)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4))/b^3

**mupad [B]** time = 0.45, size = 888, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(7/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] - ((2\*A)/(3\*b) + (11\*x^2\*(11\*A\*c - 3\*B\*b))/(48\*b^2) + (7\*c\*x^4\*(11\*A\*c - 3\*B\*b))/(48\*b^3))/(b^2\*x^(3/2) + c^2\*x^(11/2) + 2\*b\*c\*x^(7/2)) - (atan((((11\*A\*c - 3\*B\*b)\*(x^(1/2)\*(97140736\*A^2\*b^9\*c^5 + 7225344\*B^2\*b^11\*c^3 - 52985856\*A\*B\*b^10\*c^4) - (7\*(11\*A\*c - 3\*B\*b)\*(80740352\*A\*b^13\*c^4 - 22020096\*B\*b^14\*c^3))/(64\*(-b)^(15/4)\*c^(1/4))))\*7i)/(64\*(-b)^(15/4)\*c^(1/4)) + ((11\*A\*c - 3\*B\*b)\*(x^(1/2)\*(97140736\*A^2\*b^9\*c^5 + 7225344\*B^2\*b^11\*c^3 - 52985856\*A\*B\*b^10\*c^4) + (7\*(11\*A\*c - 3\*B\*b)\*(80740352\*A\*b^13\*c^4 - 22020096\*B\*b^14\*c^3))/(64\*(-b)^(15/4)\*c^(1/4))))\*7i)/(64\*(-b)^(15/4)\*c^(1/4)))/((7\*(11\*A\*c - 3\*B\*b)\*(x^(1/2)\*(97140736\*A^2\*b^9\*c^5 + 7225344\*B^2\*b^11\*c^3 - 52985856\*A\*B\*b^10\*c^4) - (7\*(11\*A\*c - 3\*B\*b)\*(80740352\*A\*b^13\*c^4 - 22020096\*B\*b^14\*c^3))/(64\*(-b)^(15/4)\*c^(1/4))))/(64\*(-b)^(15/4)\*c^(1/4)) - (7\*(11\*A\*c - 3\*B\*b)\*(x^(1/2)\*(97140736\*A^2\*b^9\*c^5 + 7225344\*B^2\*b^11\*c^3 - 52985856\*A\*B\*b^10\*c^4) + (7\*(11\*A\*c - 3\*B\*b)\*(80740352\*A\*b^13\*c^4 - 22020096\*B\*b^14\*c^3))/(64\*(-b)^(15/4)\*c^(1/4))))/(64\*(-b)^(15/4)\*c^(1/4)))\*((11\*A\*c - 3\*B\*b)\*7i)/(3

$$\begin{aligned}
& 2*(-b)^{(15/4)}*c^{(1/4)} - (7*\operatorname{atan}(((7*(11*A*c - 3*B*b))*(x^{(1/2)}*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^{11}*c^3 - 52985856*A*B*b^{10}*c^4) - ((11*A*c - 3*B*b)*(80740352*A*b^{13}*c^4 - 22020096*B*b^{14}*c^3)*7i)/(64*(-b)^{(15/4)}*c^{(1/4)})))/(64*(-b)^{(15/4)}*c^{(1/4)}) + (7*(11*A*c - 3*B*b)*(x^{(1/2)}*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^{11}*c^3 - 52985856*A*B*b^{10}*c^4) + ((11*A*c - 3*B*b)*(80740352*A*b^{13}*c^4 - 22020096*B*b^{14}*c^3)*7i)/(64*(-b)^{(15/4)}*c^{(1/4)})))/(64*(-b)^{(15/4)}*c^{(1/4)})))/((11*A*c - 3*B*b)*(x^{(1/2)}*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^{11}*c^3 - 52985856*A*B*b^{10}*c^4) - ((11*A*c - 3*B*b)*(80740352*A*b^{13}*c^4 - 22020096*B*b^{14}*c^3)*7i)/(64*(-b)^{(15/4)}*c^{(1/4)})))*7i)/(64*(-b)^{(15/4)}*c^{(1/4)}) - ((11*A*c - 3*B*b)*(x^{(1/2)}*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^{11}*c^3 - 52985856*A*B*b^{10}*c^4) + ((11*A*c - 3*B*b)*(80740352*A*b^{13}*c^4 - 22020096*B*b^{14}*c^3)*7i)/(64*(-b)^{(15/4)}*c^{(1/4)})))*7i)/(64*(-b)^{(15/4)}*c^{(1/4)})))*7i)/(64*(-b)^{(15/4)}*c^{(1/4)})))/(32*(-b)^{(15/4)}*c^{(1/4)})
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.216 \quad \int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

Optimal. Leaf size=343

$$\frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}$$

**Rubi [A]** time = 0.28, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{-\frac{5bB-13Ac}{16a^2cx^2(b+cx^2)} + \frac{9(5bB-13Ac)}{80b^3cx^2} - \frac{9(5bB-13Ac)}{16b^4\sqrt{c}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}+1\right)}{32\sqrt{2}b^{17/4}} - \frac{bB-Ac}{4bcx^2(b+cx^2)^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (9\*(5\*b\*B - 13\*A\*c))/(80\*b^3\*c\*x^(5/2)) - (9\*(5\*b\*B - 13\*A\*c))/(16\*b^4\*sqrt[x]) - (b\*B - A\*c)/(4\*b\*c\*x^(5/2)\*(b + c\*x^2)^2) - (5\*b\*B - 13\*A\*c)/(16\*b^2\*c\*x^(5/2)\*(b + c\*x^2)) + (9\*c^(1/4)\*(5\*b\*B - 13\*A\*c)\*ArcTan[1 - (sqrt[2]\*c^(1/4)\*sqrt[x])/b^(1/4)])/(32\*sqrt[2]\*b^(17/4)) - (9\*c^(1/4)\*(5\*b\*B - 13\*A\*c)\*ArcTan[1 + (sqrt[2]\*c^(1/4)\*sqrt[x])/b^(1/4)])/(32\*sqrt[2]\*b^(17/4)) - (9\*c^(1/4)\*(5\*b\*B - 13\*A\*c)\*Log[sqrt[b] - sqrt[2]\*b^(1/4)\*c^(1/4)\*sqrt[x] + sqrt[c]\*x])/(64\*sqrt[2]\*b^(17/4)) + (9\*c^(1/4)\*(5\*b\*B - 13\*A\*c)\*Log[sqrt[b] + sqrt[2]\*b^(1/4)\*c^(1/4)\*sqrt[x] + sqrt[c]\*x])/(64\*sqrt[2]\*b^(17/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 290

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*n\*(p+1)), x] + Dist[(m+n\*(p+1)+1)/(a\*n\*(p+1)), Int[(c\*x)^m\*(a+b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1)/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_) + (b_.)*(x_)^(q_.))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps



$$\begin{aligned}
\int \frac{x^{5/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^{7/2} (b + cx^2)^3} dx \\
&= -\frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} + \frac{\left(-\frac{5bB}{2} + \frac{13Ac}{2}\right) \int \frac{1}{x^{7/2}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} - \frac{(9(5bB - 13Ac)) \int \frac{1}{x^{7/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} + \frac{(9(5bB - 13Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB - 13Ac)}{16b^4\sqrt{x}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} - \frac{(9c(5bB - 13Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB - 13Ac)}{16b^4\sqrt{x}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} - \frac{(9c(5bB - 13Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB - 13Ac)}{16b^4\sqrt{x}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} + \frac{(9\sqrt{c} (5bB - 13Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB - 13Ac)}{16b^4\sqrt{x}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} - \frac{(9(5bB - 13Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB - 13Ac)}{16b^4\sqrt{x}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} + \frac{9\sqrt{c} (5bB - 13Ac) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{9(5bB - 13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB - 13Ac)}{16b^4\sqrt{x}} - \frac{bB - Ac}{4bcx^{5/2} (b + cx^2)^2} - \frac{5bB - 13Ac}{16b^2cx^{5/2} (b + cx^2)} + \frac{9\sqrt{c} (5bB - 13Ac) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3}
\end{aligned}$$

**Mathematica [C]** time = 0.52, size = 189, normalized size = 0.55

$$-\frac{2cx^{3/2}(bB-2Ac) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^5} + \frac{2cx^{3/2}(Ac-bB) {}_2F_1\left(\frac{3}{4}, 3; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^5} + \frac{6Ac-2bB}{b^4\sqrt{x}} - \frac{2A}{5b^3x^{5/2}} + \frac{\sqrt[4]{c}(3Ac-bB) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}} + \frac{\sqrt[4]{c}(bB-3Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out]  $(-2*A)/(5*b^3*x^{5/2}) + (-2*b*B + 6*A*c)/(b^4*\text{Sqrt}[x]) + (c^{1/4})*(-(b*B) + 3*A*c)*\text{ArcTan}[(c^{1/4}*\text{Sqrt}[x])/(-b)^{1/4}]/(-b)^{17/4} + (c^{1/4}*(b*B - 3*A*c))*\text{ArcTanh}[(c^{1/4}*\text{Sqrt}[x])/(-b)^{1/4}]/(-b)^{17/4} - (2*c*(b*B - 2*A*c))*x^{3/2}*\text{Hypergeometric2F1}[3/4, 2, 7/4, -((c*x^2)/b)]/(3*b^5) + (2*c*(-(b*B) + A*c))*x^{3/2}*\text{Hypergeometric2F1}[3/4, 3, 7/4, -((c*x^2)/b)]/(3*b^5)$

**IntegrateAlgebraic [A]** time = 0.66, size = 224, normalized size = 0.65

$$\frac{9(5bB\sqrt[4]{c} - 13Ac^{5/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{32\sqrt{2}b^{17/4}} + \frac{9(5bB\sqrt[4]{c} - 13Ac^{5/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{17/4}} + \frac{-32Ab^3 + 416Ab^2cx^2 + 1053Abc^2x^4 + 585Ac^3x^6 - 160b^3Bx^2 - 405b^2Bcx^4 - 225bBc^2x^6}{80b^4x^{5/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] (-32*A*b^3 - 160*b^3*B*x^2 + 416*A*b^2*c*x^2 - 405*b^2*B*c*x^4 + 1053*A*b*c^2*x^4 - 225*b*B*c^2*x^6 + 585*A*c^3*x^6)/(80*b^4*x^(5/2)*(b + c*x^2)^2) + (9*(5*b*B*c^(1/4) - 13*A*c^(5/4))*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]])/(32*Sqrt[2]*b^(17/4)) + (9*(5*b*B*c^(1/4) - 13*A*c^(5/4))*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]/(32*Sqrt[2]*b^(17/4))
fricas [B] time = 0.46, size = 1043, normalized size = 3.04
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] -1/320*(180*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*arctan((sqrt((15625*B^6*b^6*c^2 - 243750*A*B^5*b^5*c^3 + 1584375*A^2*B^4*b^4*c^4 - 5492500*A^3*B^3*b^3*c^5 + 10710375*A^4*B^2*b^2*c^6 - 11138790*A^5*B*b*c^7 + 4826809*A^6*c^8)*x - (625*B^4*b^13*c - 6500*A*B^3*b^12*c^2 + 25350*A^2*B^2*b^11*c^3 - 43940*A^3*B*b^10*c^4 + 28561*A^4*b^9*c^5)*sqrt(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)))*b^4*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4) + (125*B^3*b^7*c - 975*A*B^2*b^6*c^2 + 2535*A^2*B*b^5*c^3 - 2197*A^3*b^4*c^4)*sqrt(x)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4))/(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5) - 45*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*log(729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 45*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*log(-729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 4*(45*(5*B*b*c^2 - 13*A*c^3)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32*A*b^3 + 32*(5*B*b^3 - 13*A*b^2*c)*x^2)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

```
giac [A] time = 0.22, size = 326, normalized size = 0.95
```

$$\frac{9\sqrt{2}\left(\frac{5}{4}b^2c^2 - 13\frac{5}{4}b^2c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{b^2c^2 + c^2}}{z}\right) + 9\sqrt{2}\left(\frac{5}{4}b^2c^2 - 13\frac{5}{4}b^2c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{b^2c^2 + c^2}}{z}\right)}{64b^2c^2} + \frac{9\sqrt{2}\left(\frac{5}{4}b^2c^2 - 13\frac{5}{4}b^2c^2\right)\log\left(\sqrt{2}\sqrt{b^2c^2 + c^2} + z + \sqrt{\frac{c}{b}}\right) + 9\sqrt{2}\left(\frac{5}{4}b^2c^2 - 13\frac{5}{4}b^2c^2\right)\log\left(-\sqrt{2}\sqrt{b^2c^2 + c^2} + z + \sqrt{\frac{c}{b}}\right)}{128b^2c^2} + \frac{1388b^2c^2 - 21A^2b^2c^2 + 17Bb^2c^2 - 25Ab^2c^2}{16(c^2 + b)^4} + \frac{2(588b^2 - 15Ac^2 + Ab)}{5b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] -9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) - 9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) + 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 1/16*(13*B*b*c^2*x^(7/2) - 21*A*c^3*x^(7/2) + 17*B*b^2*c*x^(3/2) - 25*A*b*c^2*x^(3/2))/((c*x^2 + b)^2*b^4) - 2/5*(5*B*b*x^2 - 15*A*c*x^2 + A*b)/(b^4*x^(5/2))
```

**maple [A]** time = 0.07, size = 381, normalized size = 1.11

$$\frac{21A c^3 b^2}{16(c^2 x^2 + b)^2 b^4} - \frac{13B c^2 x^2}{16(c^2 x^2 + b)^2 b^3} + \frac{25A c^2 x^2}{16(c^2 x^2 + b)^2 b^3} - \frac{17B c x^2}{16(c^2 x^2 + b)^2 b^2} + \frac{117\sqrt{2} A c \arctan\left(\frac{\sqrt{2}\sqrt{c}}{(b)^{\frac{1}{4}}}\right)}{64(b)^{\frac{1}{4}} b^4} + \frac{117\sqrt{2} A c \arctan\left(\frac{\sqrt{2}\sqrt{c} + 1}{(b)^{\frac{1}{4}}}\right)}{64(b)^{\frac{1}{4}} b^4} + \frac{117\sqrt{2} A c \ln\left(\frac{(-b)^{\frac{1}{4}} \sqrt{c} \sqrt{c} + \sqrt{c}}{(+b)^{\frac{1}{4}} \sqrt{c} \sqrt{c} + \sqrt{c}}\right)}{128(b)^{\frac{1}{4}} b^4} - \frac{45\sqrt{2} B \arctan\left(\frac{\sqrt{2}\sqrt{c}}{(b)^{\frac{1}{4}}}\right)}{64(b)^{\frac{1}{4}} b^3} - \frac{45\sqrt{2} B \arctan\left(\frac{\sqrt{2}\sqrt{c} + 1}{(b)^{\frac{1}{4}}}\right)}{64(b)^{\frac{1}{4}} b^3} - \frac{45\sqrt{2} B \ln\left(\frac{(-b)^{\frac{1}{4}} \sqrt{c} \sqrt{c} + \sqrt{c}}{(+b)^{\frac{1}{4}} \sqrt{c} \sqrt{c} + \sqrt{c}}\right)}{128(b)^{\frac{1}{4}} b^3} + \frac{6Ac}{b^4 \sqrt{c}} - \frac{2B}{b^3 \sqrt{c}} - \frac{2A}{5b^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 21/16/b^4\*c^3/(c\*x^2+b)^2\*x^(7/2)\*A-13/16/b^3\*c^2/(c\*x^2+b)^2\*x^(7/2)\*B+25/16/b^3\*c^2/(c\*x^2+b)^2\*A\*x^(3/2)-17/16/b^2\*c/(c\*x^2+b)^2\*B\*x^(3/2)+117/128/b^4\*c/(b/c)^(1/4)\*2^(1/2)\*A\*ln((x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))+117/64/b^4\*c/(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)+117/64/b^4\*c/(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-45/128/b^3/(b/c)^(1/4)\*2^(1/2)\*B\*ln((x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-2/5\*A/b^3/x^(5/2)+6/b^4/x^(1/2)\*A\*c-2/b^3/x^(1/2)\*B

**maxima [A]** time = 3.04, size = 285, normalized size = 0.83

$$\frac{45(5Bbc^2 - 13Ac^3)x^6 + 81(5Bb^2c - 13Abc^2)x^4 + 32Ab^3 + 32(5Bb^3 - 13Ab^2c)x^2}{80(b^4c^2x^2 + 2b^5cx + b^6x^2)} - \frac{9(5Bbc - 13Ac^2) \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{c}(\sqrt{b}i^{\frac{1}{4}} + 2\sqrt{c}\sqrt{c})}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{c}(\sqrt{2}i^{\frac{1}{4}} + 2\sqrt{c}\sqrt{c})}{2\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}\sqrt{c}} - \frac{\sqrt{2} \log\left(\frac{\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}}{b^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}c^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(\frac{-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{c} + \sqrt{c}x + \sqrt{b}}{b^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}c^{\frac{1}{4}}} \right)}{128b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/80\*(45\*(5\*B\*b\*c^2 - 13\*A\*c^3)\*x^6 + 81\*(5\*B\*b^2\*c - 13\*A\*b\*c^2)\*x^4 + 32\*A\*b^3 + 32\*(5\*B\*b^3 - 13\*A\*b^2\*c)\*x^2)/(b^4\*c^2\*x^(13/2) + 2\*b^5\*c\*x^(9/2) + b^6\*x^(5/2)) - 9/128\*(5\*B\*b\*c - 13\*A\*c^2)\*(2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(sqrt(b)\*sqrt(c))\*sqrt(c) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/sqrt(sqrt(b)\*sqrt(c))\*sqrt(c) - sqrt(2)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(1/4)\*c^(3/4)) + sqrt(2)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(1/4)\*c^(3/4))/b^4

**mupad [B]** time = 0.19, size = 152, normalized size = 0.44

$$\frac{\frac{2x^2(13Ac-5Bb)}{5b^2} - \frac{2A}{5b} + \frac{9c^2x^6(13Ac-5Bb)}{16b^4} + \frac{81cx^4(13Ac-5Bb)}{80b^3}}{b^2x^{5/2} + c^2x^{13/2} + 2bcx^{9/2}} + \frac{9(-c)^{1/4} \operatorname{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(13Ac-5Bb)}{32b^{17/4}} - \frac{9(-c)^{1/4} \operatorname{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(13Ac-5Bb)}{32b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(5/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] ((2\*x^2\*(13\*A\*c - 5\*B\*b))/(5\*b^2) - (2\*A)/(5\*b) + (9\*c^2\*x^6\*(13\*A\*c - 5\*B\*b))/(16\*b^4) + (81\*c\*x^4\*(13\*A\*c - 5\*B\*b))/(80\*b^3))/(b^2\*x^(5/2) + c^2\*x^(13/2) + 2\*b\*c\*x^(9/2)) + (9\*(-c)^(1/4)\*atan((-c)^(1/4)\*x^(1/2)/b^(1/4))\*(13\*A\*c - 5\*B\*b)/(32\*b^(17/4)) - (9\*(-c)^(1/4)\*atanh((-c)^(1/4)\*x^(1/2)/b^(1/4))\*(13\*A\*c - 5\*B\*b)/(32\*b^(17/4))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.217 \quad \int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=343

$$\frac{11c^{3/4}(7bB - 15Ac) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2}b^{19/4}} - \frac{11c^{3/4}(7bB - 15Ac) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2}b^{19/4}} + \dots$$

**Rubi [A]** time = 0.28, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11c^{3/4}(7bB - 15Ac) \log(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2}b^{19/4}} - \frac{11c^{3/4}(7bB - 15Ac) \log(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x)}{64\sqrt{2}b^{19/4}} + \frac{11c^{3/4}(7bB - 15Ac) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}}\right)}{32\sqrt{2}b^{19/4}} - \frac{11c^{3/4}(7bB - 15Ac) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}} + 1\right)}{32\sqrt{2}b^{19/4}} - \frac{7bB - 15Ac}{16b^2c^{7/2}(b + cx^2)} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} + \frac{11(7bB - 15Ac)}{112b^2cx^{7/2}} - \frac{bB - Ac}{4bcx^{7/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (11\*(7\*b\*B - 15\*A\*c))/(112\*b^3\*c\*x^(7/2)) - (11\*(7\*b\*B - 15\*A\*c))/(48\*b^4\*x^(3/2)) - (b\*B - A\*c)/(4\*b\*c\*x^(7/2)\*(b + c\*x^2)^2) - (7\*b\*B - 15\*A\*c)/(16\*b^2\*c\*x^(7/2)\*(b + c\*x^2)) + (11\*c^(3/4)\*(7\*b\*B - 15\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(19/4)) - (11\*c^(3/4)\*(7\*b\*B - 15\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(19/4)) + (11\*c^(3/4)\*(7\*b\*B - 15\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(64\*Sqrt[2]\*b^(19/4)) - (11\*c^(3/4)\*(7\*b\*B - 15\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x]/(64\*Sqrt[2]\*b^(19/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 290

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := -Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*n\*(p + 1)), x] + Dist[(m + n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(c\*x)^m\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 325

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^{9/2} (b + cx^2)^3} dx \\
&= -\frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} + \frac{\left(-\frac{7bB}{2} + \frac{15Ac}{2}\right) \int \frac{1}{x^{9/2}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} - \frac{(11(7bB - 15Ac)) \int \frac{1}{x^{9/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} + \frac{(11(7bB - 15Ac)) \int \frac{1}{x^{5/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} - \frac{(11c(7bB - 15Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} - \frac{(11c(7bB - 15Ac)) \int \frac{1}{x^{1/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} - \frac{(11c(7bB - 15Ac)) \int \frac{1}{x^{1/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} - \frac{(11\sqrt{c} (7bB - 15Ac)) \int \frac{1}{x^{1/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} - \frac{(11c^{3/4} (7bB - 15Ac)) \int \frac{1}{x^{1/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} + \frac{11c^{3/4} (7bB - 15Ac) \sqrt{c} \operatorname{ArcTan}\left[\frac{\sqrt{c} \sqrt{bx^2 + cx^4}}{b + cx^2}\right]}{32b^2c} \\
&= \frac{11(7bB - 15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB - 15Ac)}{48b^4x^{3/2}} - \frac{bB - Ac}{4bcx^{7/2} (b + cx^2)^2} - \frac{7bB - 15Ac}{16b^2cx^{7/2} (b + cx^2)} + \frac{11c^{3/4} (7bB - 15Ac) \sqrt{c} \operatorname{ArcTan}\left[\frac{\sqrt{c} \sqrt{bx^2 + cx^4}}{b + cx^2}\right]}{32b^2c}
\end{aligned}$$

**Mathematica [A]** time = 0.57, size = 433, normalized size = 1.26

$$\frac{672\sqrt{c}^3\sqrt{c} + 384\sqrt{c}^3\sqrt{c} + 320\sqrt{c}^3\sqrt{c} - 288\sqrt{c}^3\sqrt{c} + 462\sqrt{c}^{11/2}(7bB - 15Ac)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{bx^2 + cx^4}}{b + cx^2}\right) + 462\sqrt{c}^{11/2}(7bB - 15Ac)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{bx^2 + cx^4}}{b + cx^2}\right) - 3465\sqrt{c}^{11/2}\log\left(\frac{\sqrt{c}\sqrt{bx^2 + cx^4}}{b + cx^2}\right) + 3465\sqrt{c}^{11/2}\log\left(\frac{\sqrt{c}\sqrt{bx^2 + cx^4}}{b + cx^2}\right) - \frac{672\sqrt{c}^3\sqrt{c} + 384\sqrt{c}^3\sqrt{c} + 320\sqrt{c}^3\sqrt{c} - 288\sqrt{c}^3\sqrt{c}}{32b^2c} + 1617\sqrt{c}^{11/2}\log\left(\frac{\sqrt{c}\sqrt{bx^2 + cx^4}}{b + cx^2}\right) - 1617\sqrt{c}^{11/2}\log\left(\frac{\sqrt{c}\sqrt{bx^2 + cx^4}}{b + cx^2}\right)}{388800c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] ((-768\*A\*b^(7/4))/x^(7/2) - (1792\*b^(7/4)\*B)/x^(3/2) + (5376\*A\*b^(3/4)\*c)/x^(3/2) - (672\*b^(11/4)\*B\*c\*Sqrt[x])/(b + c\*x^2)^2 + (672\*A\*b^(7/4)\*c^2\*Sqrt[x])/(b + c\*x^2)^2 - (2520\*b^(7/4)\*B\*c\*Sqrt[x])/(b + c\*x^2) + (3864\*A\*b^(3/4)\*c^2\*Sqrt[x])/(b + c\*x^2) + 462\*Sqrt[2]\*c^(3/4)\*(7\*b\*B - 15\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 462\*Sqrt[2]\*c^(3/4)\*(-7\*b\*B + 15\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 1617\*Sqrt[2]\*b\*B\*c^(3/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 3465\*Sqrt[2]\*A\*c^(7/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 1617\*Sqrt[2]\*b\*B\*c^(3/4)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]

] \* x] + 3465 \* Sqrt[2] \* A \* c^(7/4) \* Log[Sqrt[b] + Sqrt[2] \* b^(1/4) \* c^(1/4) \* Sqrt[x] + Sqrt[c] \* x]) / (2688 \* b^(19/4))

**IntegrateAlgebraic [A]** time = 0.73, size = 224, normalized size = 0.65

$$\frac{11(7bBc^{3/4} - 15Ac^{7/4}) \tan^{-1}\left(\frac{\sqrt{b} - \sqrt{cx}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 11(7bBc^{3/4} - 15Ac^{7/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{cx}}\right) + \frac{-96Ab^3 + 480Ab^2cx^2 + 1815Abc^2x^4 + 1155Ac^3x^6 - 224b^3Bx^2 - 847b^2Bcx^4 - 539bBc^2x^6}{336b^4x^{7/2}(b + cx)^2}}{32\sqrt{2}b^{19/4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out] (-96\*A\*b^3 - 224\*b^3\*B\*x^2 + 480\*A\*b^2\*c\*x^2 - 847\*b^2\*B\*c\*x^4 + 1815\*A\*b\*c^2\*x^4 - 539\*b\*B\*c^2\*x^6 + 1155\*A\*c^3\*x^6)/(336\*b^4\*x^(7/2)\*(b + c\*x^2)^2) + (11\*(7\*b\*B\*c^(3/4) - 15\*A\*c^(7/4))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(32\*Sqrt[2]\*b^(19/4)) - (11\*(7\*b\*B\*c^(3/4) - 15\*A\*c^(7/4))\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*b^(19/4))

**fricas [B]** time = 0.46, size = 864, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 1/1344\*(924\*(b^4\*c^2\*x^8 + 2\*b^5\*c\*x^6 + b^6\*x^4)\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(1/4)\*arctan((sqrt(b^10\*sqrt(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19) + (49\*B^2\*b^2\*c^2 - 210\*A\*B\*b\*c^3 + 225\*A^2\*c^4)\*x)\*b^14\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(3/4) + (7\*B\*b^15\*c - 15\*A\*b^14\*c^2)\*sqrt(x)\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(3/4))/(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)) + 231\*(b^4\*c^2\*x^8 + 2\*b^5\*c\*x^6 + b^6\*x^4)\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(1/4)\*log(11\*b^5\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(1/4) - 11\*(7\*B\*b\*c - 15\*A\*c^2)\*sqrt(x)) - 231\*(b^4\*c^2\*x^8 + 2\*b^5\*c\*x^6 + b^6\*x^4)\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(1/4)\*log(-11\*b^5\*(-(2401\*B^4\*b^4\*c^3 - 20580\*A\*B^3\*b^3\*c^4 + 66150\*A^2\*B^2\*b^2\*c^5 - 94500\*A^3\*B\*b\*c^6 + 50625\*A^4\*c^7)/b^19)^(1/4) - 11\*(7\*B\*b\*c - 15\*A\*c^2)\*sqrt(x)) - 4\*(77\*(7\*B\*b\*c^2 - 15\*A\*c^3)\*x^6 + 121\*(7\*B\*b^2\*c - 15\*A\*b\*c^2)\*x^4 + 96\*A\*b^3 + 32\*(7\*B\*b^3 - 15\*A\*b^2\*c)\*x^2)\*sqrt(x))/(b^4\*c^2\*x^8 + 2\*b^5\*c\*x^6 + b^6\*x^4)

**giac [A]** time = 0.22, size = 315, normalized size = 0.92

$$\frac{11\sqrt{2}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{x}}{z(b)^2}\right)\arctan\left(\frac{\sqrt{b}\sqrt{c}\sqrt{x}}{z(b)^2}\right) - 11\sqrt{2}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{x}}{z(b)^2}\right)\arctan\left(\frac{\sqrt{b}\sqrt{c}\sqrt{x}}{z(b)^2}\right) + 11\sqrt{2}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{x}}{z(b)^2}\right)\log\left(\sqrt{2}\sqrt{c}\left(\frac{z(b)^2}{z(b)^2}\right)^{1/2} + \sqrt{c}\right) - 11\sqrt{2}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{x}}{z(b)^2}\right)\log\left(-\sqrt{2}\sqrt{c}\left(\frac{z(b)^2}{z(b)^2}\right)^{1/2} + \sqrt{c}\right) + \frac{15Bb^{3/2} - 23A^{3/2} + 19Bb^2\sqrt{c} - 27Abc\sqrt{c} - 2\sqrt{B}b^{3/2} - 21Ac^2 + 3Ab}{16(c^2 + b)^{19/4}}}{64b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] -11/64\*sqrt(2)\*(7\*(b\*c^3)^(1/4)\*B\*b - 15\*(b\*c^3)^(1/4)\*A\*c)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) + 2\*sqrt(x))/(b/c)^(1/4))/b^5 - 11/64\*sqrt(2)\*(7\*(b\*c^3)^(1/4)\*B\*b - 15\*(b\*c^3)^(1/4)\*A\*c)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(b/c)^(1/4) - 2\*sqrt(x))/(b/c)^(1/4))/b^5 - 11/128\*sqrt(2)\*(7\*(b\*c^3)^(1/4)\*B\*b

- 15\*(b\*c^3)^(1/4)\*A\*c)\*log(sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/b^5 + 11/128\*sqrt(2)\*(7\*(b\*c^3)^(1/4)\*B\*b - 15\*(b\*c^3)^(1/4)\*A\*c)\*log(-sqrt(2)\*sqrt(x)\*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 1/16\*(15\*B\*b\*c^2\*x^(5/2) - 23\*A\*c^3\*x^(5/2) + 19\*B\*b^2\*c\*sqrt(x) - 27\*A\*b\*c^2\*sqrt(x))/((c\*x^2 + b)^2\*b^4) - 2/21\*(7\*B\*b\*x^2 - 21\*A\*c\*x^2 + 3\*A\*b)/(b^4\*x^(7/2))

**maple [A]** time = 0.07, size = 390, normalized size = 1.14

$$\frac{23A^2c^3}{16(c^2x^2+b)^4} - \frac{15Bc^2c^3}{16(c^2x^2+b)^4} + \frac{27A^2c^2\sqrt{c}}{16(c^2x^2+b)^4} - \frac{19Bc\sqrt{c}}{16(c^2x^2+b)^4} + \frac{165\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}A^2c^2\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}-1\right)}{64b^5} + \frac{165\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}A^2c^2\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}+1\right)}{64b^5} + \frac{165\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}A^2c^2\ln\left(\frac{-(\frac{c}{b})^{\frac{1}{4}}\sqrt{c}\sqrt{x}+\sqrt{c}}{+(\frac{c}{b})^{\frac{1}{4}}\sqrt{c}\sqrt{x}+\sqrt{c}}\right)}{128b^5} - \frac{77\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}Bc\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}-1\right)}{64b^4} - \frac{77\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}Bc\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}+1\right)}{64b^4} - \frac{77\left(\frac{c}{b}\right)^{\frac{1}{4}}\sqrt{2}Bc\ln\left(\frac{-(\frac{c}{b})^{\frac{1}{4}}\sqrt{c}\sqrt{x}+\sqrt{c}}{+(\frac{c}{b})^{\frac{1}{4}}\sqrt{c}\sqrt{x}+\sqrt{c}}\right)}{128b^4} + \frac{2Ac}{b^4} - \frac{2B}{3b^4} - \frac{2A}{7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x)

[Out] 23/16/b^4\*c^3/(c\*x^2+b)^2\*x^(5/2)\*A-15/16/b^3\*c^2/(c\*x^2+b)^2\*x^(5/2)\*B+27/16/b^3\*c^2/(c\*x^2+b)^2\*A\*x^(1/2)-19/16/b^2\*c/(c\*x^2+b)^2\*B\*x^(1/2)+165/64/b^5\*c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)+165/128/b^5\*c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))+165/64/b^5\*c^2\*(b/c)^(1/4)\*2^(1/2)\*A\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-77/64/b^4\*c\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)-1)-77/128/b^4\*c\*(b/c)^(1/4)\*2^(1/2)\*B\*ln((x+(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)\*2^(1/2)\*x^(1/2)+(b/c)^(1/2)))-77/64/b^4\*c\*(b/c)^(1/4)\*2^(1/2)\*B\*arctan(2^(1/2)/(b/c)^(1/4)\*x^(1/2)+1)-2/7\*A/b^3/x^(7/2)+2/b^4/x^(3/2)\*A\*c-2/3/b^3/x^(3/2)\*B

**maxima [A]** time = 3.15, size = 321, normalized size = 0.94

$$\frac{77(7Bbc^2-15Ac^3)x^6+121(7Bb^2c-15Abc^2)x^4+96Ab^3+32(7Bb^3-15Ab^2c)x^2}{336(b^4c^2x^2+2b^3cx+2b^4)} + \frac{2\sqrt{2}(7Bbc-15Ac^2)\arctan\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{c}}}{2\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}}\right)}{\sqrt{c}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}} + \frac{2\sqrt{2}(7Bbc-15Ac^2)\arctan\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x-\sqrt{c}}}{2\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}}\right)}{\sqrt{c}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}} + \frac{\sqrt{2}(7Bbc-15Ac^2)\log\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{c}}+\sqrt{c}}{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x-\sqrt{c}}+\sqrt{c}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}(7Bbc-15Ac^2)\log\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{c}}-\sqrt{c}}{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x-\sqrt{c}}+\sqrt{c}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(B\*x^2+A)/(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] -1/336\*(77\*(7\*B\*b\*c^2 - 15\*A\*c^3)\*x^6 + 121\*(7\*B\*b^2\*c - 15\*A\*b\*c^2)\*x^4 + 96\*A\*b^3 + 32\*(7\*B\*b^3 - 15\*A\*b^2\*c)\*x^2)/(b^4\*c^2\*x^(15/2) + 2\*b^5\*c\*x^(11/2) + b^6\*x^(7/2)) - 11/128\*(2\*sqrt(2)\*(7\*B\*b\*c - 15\*A\*c^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) + 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/(sqrt(b)\*sqrt(sqrt(b)\*sqrt(c))) + 2\*sqrt(2)\*(7\*B\*b\*c - 15\*A\*c^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*c^(1/4) - 2\*sqrt(c)\*sqrt(x))/sqrt(sqrt(b)\*sqrt(c)))/(sqrt(b)\*sqrt(sqrt(b)\*sqrt(c))) + sqrt(2)\*(7\*B\*b\*c - 15\*A\*c^2)\*log(sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4)) - sqrt(2)\*(7\*B\*b\*c - 15\*A\*c^2)\*log(-sqrt(2)\*b^(1/4)\*c^(1/4)\*sqrt(x) + sqrt(c)\*x + sqrt(b))/(b^(3/4)\*c^(1/4))/b^4

**mapad [B]** time = 0.45, size = 626, normalized size = 1.83

$$\frac{11(-c)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{c}}}{2\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}}\right)}{\sqrt{c}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}} + \frac{11(-c)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x-\sqrt{c}}}{2\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}}\right)}{\sqrt{c}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}}} + \frac{11(-c)^{\frac{3}{4}}\log\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{c}}+\sqrt{c}}{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x-\sqrt{c}}+\sqrt{c}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{11(-c)^{\frac{3}{4}}\log\left(\frac{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{c}}-\sqrt{c}}{\sqrt{2}\sqrt{b^{\frac{1}{4}}c^{\frac{1}{4}}x-\sqrt{c}}+\sqrt{c}}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3/2)\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x)

[Out] ((2\*x^2\*(15\*A\*c - 7\*B\*b))/(21\*b^2) - (2\*A)/(7\*b) + (11\*c^2\*x^6\*(15\*A\*c - 7\*B\*b))/(48\*b^4) + (121\*c\*x^4\*(15\*A\*c - 7\*B\*b))/(336\*b^3))/(b^2\*x^(7/2) + c^2\*x^(15/2) + 2\*b\*c\*x^(11/2)) + (11\*(-c)^(3/4)\*atan(((11\*(-c)^(3/4)\*(15\*A\*c - 7\*B\*b)\*(x^(1/2)\*(446054400\*A^2\*b^12\*c^7 + 97140736\*B^2\*b^14\*c^5 - 416317440\*A\*B\*b^13\*c^6) - ((-c)^(3/4)\*(15\*A\*c - 7\*B\*b)\*(173015040\*A\*b^17\*c^5 - 80740352\*B\*b^18\*c^4)\*11i)/(64\*b^(19/4)))/(64\*b^(19/4)) + (11\*(-c)^(3/4)\*(15\*A\*c - 7\*B\*b)\*(x^(1/2)\*(446054400\*A^2\*b^12\*c^7 + 97140736\*B^2\*b^14\*c^5 - 41631



$$7440*A*B*b^{13}*c^6) + ((-c)^{(3/4)}*(15*A*c - 7*B*b)*(173015040*A*b^{17}*c^5 - 80740352*B*b^{18}*c^4)*11i)/(64*b^{(19/4)})))/(64*b^{(19/4)})))/(((c)^{(3/4)}*(15*A*c - 7*B*b)*(x^{(1/2)}*(446054400*A^2*b^{12}*c^7 + 97140736*B^2*b^{14}*c^5 - 416317440*A*B*b^{13}*c^6) - ((-c)^{(3/4)}*(15*A*c - 7*B*b)*(173015040*A*b^{17}*c^5 - 80740352*B*b^{18}*c^4)*11i)/(64*b^{(19/4)})))*11i)/(64*b^{(19/4)}) - ((-c)^{(3/4)}*(15*A*c - 7*B*b)*(x^{(1/2)}*(446054400*A^2*b^{12}*c^7 + 97140736*B^2*b^{14}*c^5 - 416317440*A*B*b^{13}*c^6) + ((-c)^{(3/4)}*(15*A*c - 7*B*b)*(173015040*A*b^{17}*c^5 - 80740352*B*b^{18}*c^4)*11i)/(64*b^{(19/4)})))*11i)/(64*b^{(19/4)})))*(15*A*c - 7*B*b))/(32*b^{(19/4)}) - ((-c)^{(3/4)}*atan((A^3*c^8*x^{(1/2)}*3375i - B^3*b^3*c^5*x^{(1/2)}*343i - A^2*B*b*c^7*x^{(1/2)}*4725i + A*B^2*b^2*c^6*x^{(1/2)}*2205i)/(b^{(1/4)}*(-c)^{(19/4)}*(c*(c*(3375*A^3*c - 4725*A^2*B*b) + 2205*A*B^2*b^2) - 343*B^3*b^3)))*(15*A*c - 7*B*b)*11i)/(32*b^{(19/4)})$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*(B\*x\*\*2+A)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.218 \quad \int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=365

$$\frac{13c^{5/4}(9bB - 17Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{21/4}} - \frac{13c^{5/4}(9bB - 17Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{21/4}}$$

**Rubi [A]** time = 0.32, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13c^{5/4}(9bB - 17Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{21/4}} - \frac{13c^{5/4}(9bB - 17Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{64\sqrt{2} b^{21/4}} - \frac{13c^{5/4}(9bB - 17Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{x}}\right)}{32\sqrt{2} b^{21/4}} + \frac{13c^{5/4}(9bB - 17Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt[4]{c}}{\sqrt{x}} + 1\right)}{32\sqrt{2} b^{21/4}} - \frac{13(9bB - 17Ac)}{80b^{5/2}} - \frac{9bB - 17Ac}{16b^2 c^{3/2} (b + cx^2)} + \frac{13(9bB - 17Ac)}{144b^3 c^{3/2}} + \frac{13c(9bB - 17Ac)}{16b^5 \sqrt{c}} - \frac{bB - Ac}{4bc^{3/2} (b + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] (13\*(9\*b\*B - 17\*A\*c))/(144\*b^3\*c\*x^(9/2)) - (13\*(9\*b\*B - 17\*A\*c))/(80\*b^4\*x^(5/2)) + (13\*c\*(9\*b\*B - 17\*A\*c))/(16\*b^5\*Sqrt[x]) - (b\*B - A\*c)/(4\*b\*c\*x^(9/2)\*(b + c\*x^2)^2) - (9\*b\*B - 17\*A\*c)/(16\*b^2\*c\*x^(9/2)\*(b + c\*x^2)) - (13\*c^(5/4)\*(9\*b\*B - 17\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(21/4)) + (13\*c^(5/4)\*(9\*b\*B - 17\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(21/4)) + (13\*c^(5/4)\*(9\*b\*B - 17\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(21/4)) - (13\*c^(5/4)\*(9\*b\*B - 17\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(21/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 290

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*n\*(p+1)), x] + Dist[(m+n\*(p+1)+1)/(a\*n\*(p+1)), Int[(c\*x)^m\*(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x} (A + Bx^2)}{(bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^{11/2} (b + cx^2)^3} dx \\
&= -\frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} + \frac{\left(-\frac{9bB}{2} + \frac{17Ac}{2}\right) \int \frac{1}{x^{11/2}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} - \frac{(13(9bB - 17Ac)) \int \frac{1}{x^{11/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} + \frac{(13(9bB - 17Ac)) \int \frac{1}{x^{7/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} - \frac{(13c(9bB - 17Ac)) \int \frac{1}{x^{3/2}(b+cx^2)} dx}{32b^3} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)} \\
&= \frac{13(9bB - 17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB - 17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB - 17Ac)}{16b^5\sqrt{x}} - \frac{bB - Ac}{4bcx^{9/2} (b + cx^2)^2} - \frac{9bB - 17Ac}{16b^2cx^{9/2} (b + cx^2)}
\end{aligned}$$

**Mathematica [C]** time = 0.55, size = 216, normalized size = 0.59

$$\frac{2c^2x^{3/2}(2bB - 3Ac) {}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^6} + \frac{2c^2x^{3/2}(bB - Ac) {}_2F_1\left(\frac{3}{4}, 3; \frac{7}{4}; -\frac{cx^2}{b}\right)}{3b^6} + \frac{6c(bB - 2Ac)}{b^5\sqrt{x}} - \frac{2(bB - 3Ac)}{5b^4x^{5/2}} - \frac{2A}{9b^3x^{9/2}} - \frac{3c^{5/4}(bB - 2Ac) \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{-b}}\right)}{(-b)^{21/4}} + \frac{3c^{5/4}(bB - 2Ac) \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt{-b}}\right)}{(-b)^{21/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3, x]

[Out]  $(-2*A)/(9*b^3*x^{(9/2)}) - (2*(b*B - 3*A*c))/(5*b^4*x^{(5/2)}) + (6*c*(b*B - 2*A*c))/(b^5*\text{Sqrt}[x]) - (3*c^{(5/4)}*(b*B - 2*A*c)*\text{ArcTan}[(c^{(1/4)}*\text{Sqrt}[x])/(-b)^{(1/4)}])/(-b)^{(21/4)} + (3*c^{(5/4)}*(b*B - 2*A*c)*\text{ArcTanh}[(c^{(1/4)}*\text{Sqrt}[x])/(-b)^{(1/4)}])/(-b)^{(21/4)} + (2*c^2*(2*b*B - 3*A*c)*x^{(3/2)}*\text{Hypergeometric2F1}[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^6) + (2*c^2*(b*B - A*c)*x^{(3/2)}*\text{Hypergeometric2F1}[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^6)$

**IntegrateAlgebraic [A]** time = 0.70, size = 248, normalized size = 0.68

$$\frac{13(9bBc^{5/4} - 17Ac^{9/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{cx}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 13(9bBc^{5/4} - 17Ac^{9/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{cx}}\right) - 160Ab^4 + 544Ab^3cx^2 - 7072Ab^2c^2x^4 - 17901Abc^3x^6 - 9945Ac^4x^8 - 288b^4Bx^2 + 3744b^3Bcx^4 + 9477b^2Bc^2x^6 + 5265bBc^3x^8}{32\sqrt{2}b^{21/4}} - \frac{13(9bBc^{5/4} - 17Ac^{9/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{cx}}\right)}{32\sqrt{2}b^{21/4}} + \frac{-160Ab^4 + 544Ab^3cx^2 - 7072Ab^2c^2x^4 - 17901Abc^3x^6 - 9945Ac^4x^8 - 288b^4Bx^2 + 3744b^3Bcx^4 + 9477b^2Bc^2x^6 + 5265bBc^3x^8}{720b^5x^{9/2}(b+cx)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x]\*(A + B\*x^2))/(b\*x^2 + c\*x^4)^3,x]

[Out] 
$$\frac{(-160*Ab^4 - 288*b^4*B*x^2 + 544*Ab^3*c*x^2 + 3744*b^3*B*c*x^4 - 7072*Ab^2*c^2*x^4 + 9477*b^2*B*c^2*x^6 - 17901*Ab*c^3*x^6 + 5265*b*B*c^3*x^8 - 9945*Ac^4*x^8)/(720*b^5*x^{(9/2)}*(b + c*x^2)^2) - (13*(9*b*B*c^{(5/4)} - 17*Ac^{(9/4)})*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^{(1/4)}*c^{(1/4)}*Sqrt[x]])/(32*Sqrt[2]*b^{(21/4)}) - (13*(9*b*B*c^{(5/4)} - 17*Ac^{(9/4)})*ArcTanh[(Sqrt[2]*b^{(1/4)}*c^{(1/4)}*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]/(32*Sqrt[2]*b^{(21/4)})$$

**fricas [B]** time = 0.46, size = 1093, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*x^(1/2)/(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] 
$$\frac{1}{2880}*(2340*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(1/4)}*\arctan(\sqrt{(531441*B^6*b^6*c^8 - 6022998*Ab^5*b^5*c^9 + 28441935*A^2*B^4*b^4*c^{10} - 71631540*A^3*B^3*b^3*c^{11} + 101478015*A^4*B^2*b^2*c^{12} - 76672278*A^5*B*b*c^{13} + 24137569*A^6*c^{14})x - (6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b^12*c^8 + 83521*A^4*b^{11}*c^9)*\sqrt{-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21}}) + (729*B^3*b^8*c^4 - 4131*Ab^2*b^7*c^5 + 7803*A^2*B*b^6*c^6 - 4913*A^3*b^5*c^7)*\sqrt{x}*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(1/4)} + (6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(1/4)} + (729*B^3*b^8*c^4 - 4131*Ab^2*b^7*c^5 + 7803*A^2*B*b^6*c^6 - 4913*A^3*b^5*c^7)*\sqrt{x}*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(1/4)} + (729*B^3*b^8*c^4 - 4131*Ab^2*b^7*c^5 + 7803*A^2*B*b^6*c^6 - 4913*A^3*b^5*c^7)*\sqrt{x} + 585*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(1/4)}*\log(2197*b^{16}*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(3/4)} - 2197*(729*B^3*b^3*c^4 - 4131*Ab^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*\sqrt{x}) + 585*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(1/4)}*\log(-2197*b^{16}*(-(6561*B^4*b^4*c^5 - 49572*Ab^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^{21})^{(3/4)} - 2197*(729*B^3*b^3*c^4 - 4131*Ab^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*\sqrt{x}) + 4*(585*(9*B*b*c^3 - 17*Ac^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*Ab*c^3)*x^6 - 160*Ab^4 + 416*(9*B*b^3*c - 17*Ab^2*c^2)*x^4 - 32*(9*B*b^4 - 17*Ab^3*c)*x^2)*\sqrt{x})/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)$$

**giac [A]** time = 0.26, size = 351, normalized size = 0.96

$$\frac{13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{b+c}\right) - 13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{b+c}\right) - 13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b+c}\right) + 13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b+c}\right) - 2188b^2c^3 - 29Ac^2c^3 + 258b^2c^2c^3 - 33Ab^2c^3 + 2(135Bbc^4 - 270Ac^2c^4 - 988b^2c^2 + 27Abc^2 - 5Ab^2)}{64b^6c} - \frac{13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{b+c}\right) - 13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{b+c}\right) - 13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b+c}\right) + 13\sqrt{2}(b(c^2)^{3/4} - 17(b^2)^{3/4}Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b+c}\right) - 2188b^2c^3 - 29Ac^2c^3 + 258b^2c^2c^3 - 33Ab^2c^3 + 2(135Bbc^4 - 270Ac^2c^4 - 988b^2c^2 + 27Abc^2 - 5Ab^2)}{64b^6c} + \frac{-160Ab^4 + 544Ab^3cx^2 - 7072Ab^2c^2x^4 - 17901Abc^3x^6 - 9945Ac^4x^8 - 288b^4Bx^2 + 3744b^3Bcx^4 + 9477b^2Bc^2x^6 + 5265bBc^3x^8}{16(c^2 + b)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)\*x^(1/2)/(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] 
$$13/64*\sqrt{2}*(9*(b*c^3)^{(3/4)}*B*b - 17*(b*c^3)^{(3/4)}*A*c)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*\sqrt{x}))/b^{(1/4)}/(b^6*c) + 13/64*\sqrt{2}*(9*(b*c^3)^{(3/4)}*B*b - 17*(b*c^3)^{(3/4)}*A*c)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*\sqrt{x}))/b^{(1/4)}/(b^6*c) + 13/64*\sqrt{2}*(9*(b*c^3)^{(3/4)}*B*b - 17*(b*c^3)^{(3/4)}*A*c)*\log(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b+c}) - 13/64*\sqrt{2}*(9*(b*c^3)^{(3/4)}*B*b - 17*(b*c^3)^{(3/4)}*A*c)*\log(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b+c}) - 2188b^2c^3 - 29Ac^2c^3 + 258b^2c^2c^3 - 33Ab^2c^3 + 2(135Bbc^4 - 270Ac^2c^4 - 988b^2c^2 + 27Abc^2 - 5Ab^2)$$

$$\frac{1}{c} \sqrt[4]{\frac{x}{b/c}} - 2\sqrt{x} \sqrt[4]{\frac{b}{c}} / (b/c)^{1/4} / (b^6 c) - \frac{13}{128} \sqrt{2} (9(b^3 c)^{3/4} B^3 b - 17(b^3 c)^{3/4} A^3 c) \log(\sqrt{2} \sqrt{x} \sqrt[4]{\frac{b}{c}} + x + \sqrt{b/c}) / (b^6 c) + \frac{13}{128} \sqrt{2} (9(b^3 c)^{3/4} B^3 b - 17(b^3 c)^{3/4} A^3 c) \log(-\sqrt{2} \sqrt{x} \sqrt[4]{\frac{b}{c}} + x + \sqrt{b/c}) / (b^6 c) + \frac{1}{16} (21 B^3 b^3 c^3 x^{7/2} - 29 A^3 c^4 x^{7/2} + 25 B^3 b^2 c^2 x^{3/2} - 33 A^3 b^3 c^3 x^{3/2}) / ((c x^2 + b)^2 b^5) + \frac{2}{45} (135 B^3 b^3 c x^4 - 270 A^3 c^2 x^4 - 9 B^3 b^2 x^2 + 27 A^3 b^3 c x^2 - 5 A^3 b^2) / (b^5 x^{9/2})$$

**maple [A]** time = 0.07, size = 414, normalized size = 1.13

$$\frac{29A^3 c^3}{16(c^2 + b)^5} - \frac{21B^3 c^3}{16(c^2 + b)^4} + \frac{33A^3 c^3}{16(c^2 + b)^4} - \frac{25B^3 c^3}{16(c^2 + b)^4} + \frac{221\sqrt{2} A^2 c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) - 1}{64\left(\frac{b}{c}\right)^{3/4}} - \frac{221\sqrt{2} A^2 c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) + 1}{64\left(\frac{b}{c}\right)^{3/4}} - \frac{221\sqrt{2} A^2 c^2 \ln\left(\frac{(-\frac{b}{c})^{1/4} \sqrt{c} + \sqrt{c}}{(\frac{b}{c})^{1/4} \sqrt{c} + \sqrt{c}}\right)}{128\left(\frac{b}{c}\right)^{3/4}} + \frac{117\sqrt{2} Bc \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) - 1}{64\left(\frac{b}{c}\right)^{3/4}} - \frac{117\sqrt{2} Bc \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) + 1}{64\left(\frac{b}{c}\right)^{3/4}} - \frac{117\sqrt{2} Bc \ln\left(\frac{(-\frac{b}{c})^{1/4} \sqrt{c} + \sqrt{c}}{(\frac{b}{c})^{1/4} \sqrt{c} + \sqrt{c}}\right)}{128\left(\frac{b}{c}\right)^{3/4}} + \frac{12A^3 c^2}{b^5 \sqrt{c}} + \frac{6Bc}{b^4 \sqrt{c}} + \frac{6Ac}{5b^4} + \frac{2B}{5b^4} + \frac{2A}{9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x)`

[Out] 
$$-\frac{29}{16} \frac{1}{b^5 c^4} \frac{1}{(c x^2 + b)^2} A x^{7/2} + \frac{21}{16} \frac{1}{b^4 c^3} \frac{1}{(c x^2 + b)^2} B x^{7/2} - \frac{33}{16} \frac{1}{b^4 c^3} \frac{1}{(c x^2 + b)^2} x^{3/2} A + \frac{25}{16} \frac{1}{b^3 c^2} \frac{1}{(c x^2 + b)^2} x^{3/2} B - \frac{221}{128} \frac{1}{b^5 c^2} \frac{1}{(b/c)^{1/4}} x^{1/2} A \ln\left(\frac{x - (b/c)^{1/4} x^{1/2} + (b/c)^{1/2}}{x + (b/c)^{1/4} x^{1/2} + (b/c)^{1/2}}\right) - \frac{221}{64} \frac{1}{b^5 c^2} \frac{1}{(b/c)^{1/4}} x^{1/2} A \arctan\left(\frac{2^{1/2}}{(b/c)^{1/4} x^{1/2} + 1}\right) - \frac{221}{64} \frac{1}{b^5 c^2} \frac{1}{(b/c)^{1/4}} x^{1/2} A \arctan\left(\frac{2^{1/2}}{(b/c)^{1/4} x^{1/2} - 1}\right) + \frac{117}{128} \frac{1}{b^4 c} \frac{1}{(b/c)^{1/4}} x^{1/2} B \ln\left(\frac{x - (b/c)^{1/4} x^{1/2} + (b/c)^{1/2}}{x + (b/c)^{1/4} x^{1/2} + (b/c)^{1/2}}\right) + \frac{117}{64} \frac{1}{b^4 c} \frac{1}{(b/c)^{1/4}} x^{1/2} B \arctan\left(\frac{2^{1/2}}{(b/c)^{1/4} x^{1/2} + 1}\right) + \frac{117}{64} \frac{1}{b^4 c} \frac{1}{(b/c)^{1/4}} x^{1/2} B \arctan\left(\frac{2^{1/2}}{(b/c)^{1/4} x^{1/2} - 1}\right) - \frac{2}{9} \frac{A}{b^3} \frac{1}{x^{9/2}} + \frac{6}{5} \frac{1}{b^4} \frac{1}{x^{5/2}} A c - \frac{2}{5} \frac{1}{b^3} \frac{1}{x^{5/2}} B - 12 \frac{1}{b^5} \frac{1}{x^{1/2}} A + 6 \frac{1}{b^4} \frac{1}{x^{1/2}} B$$

**maxima [A]** time = 3.08, size = 311, normalized size = 0.85

$$\frac{13(9Bbc^2 - 17Ac^3)}{720(b^5 c^2 x^7 + 2b^6 c x^5 + b^7 x^3)} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) - 1}{\sqrt{b^5 c^2} \sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) + 1}{\sqrt{b^5 c^2} \sqrt{c}} - \frac{\sqrt{2} \log\left(\frac{(-\frac{b}{c})^{1/4} \sqrt{c} + \sqrt{c}}{(\frac{b}{c})^{1/4} \sqrt{c} + \sqrt{c}}\right)}{b^{3/4}} + \frac{\sqrt{2} \log\left(\frac{-\sqrt{2}\sqrt{c}}{b^{1/4}} \sqrt{c} + \sqrt{c} + \sqrt{b}\right)}{b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")`

[Out] 
$$\frac{1}{720} (585(9B^3 b^3 c^3 - 17A^3 c^4) x^8 + 1053(9B^3 b^2 c^2 - 17A^3 b^3 c^3) x^6 - 160A^3 b^4 + 416(9B^3 b^3 c - 17A^3 b^2 c^2) x^4 - 32(9B^3 b^4 - 17A^3 b^3 c) x^2) / (b^5 c^2 x^{17/2} + 2b^6 c x^{13/2} + b^7 x^{9/2}) + \frac{13}{128} (9B^3 b^3 c^2 - 17A^3 c^3) (2\sqrt{2} \arctan(1/2 \sqrt{2}) (\sqrt{2} b^{1/4} c^{1/4} + 2\sqrt{c} \sqrt{x}) / \sqrt{(\sqrt{b} \sqrt{c})} + 2\sqrt{2} \arctan(-1/2 \sqrt{2}) (\sqrt{2} b^{1/4} c^{1/4} - 2\sqrt{c} \sqrt{x}) / \sqrt{(\sqrt{b} \sqrt{c})}) / \sqrt{(\sqrt{b} \sqrt{c})} - \sqrt{2} \log(\sqrt{2} b^{1/4} c^{1/4} \sqrt{c} + \sqrt{c} + \sqrt{b}) / (b^{1/4} c^{3/4}) + \sqrt{2} \log(-\sqrt{2} b^{1/4} c^{1/4} \sqrt{c} + \sqrt{c} + \sqrt{b}) / (b^{1/4} c^{3/4})) / b^5$$

**mupad [B]** time = 0.29, size = 173, normalized size = 0.47

$$\frac{13(-c)^{5/4} \operatorname{atan}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (17Ac - 9Bb)}{32b^{21/4}} - \frac{2A}{9b} - \frac{2x^2(17Ac - 9Bb)}{45b^2} + \frac{117c^2 x^6(17Ac - 9Bb)}{80b^4} + \frac{13c^3 x^8(17Ac - 9Bb)}{16b^5} + \frac{26cx^4(17Ac - 9Bb)}{45b^3} - \frac{13(-c)^{5/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right) (17Ac - 9Bb)}{32b^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)`

[Out] 
$$(13(-c)^{5/4} \operatorname{atan}\left(\frac{(-c)^{1/4} x^{1/2}}{b^{1/4}}\right) (17Ac - 9Bb)) / (32b^{21/4}) - ((2A) / (9b) - (2x^2(17Ac - 9Bb)) / (45b^2) + (117c^2 x^6(17Ac - 9Bb)) / (80b^4) + (13c^3 x^8(17Ac - 9Bb)) / (16b^5) + (26cx^4(17Ac - 9Bb)) / (45b^3)) / (b^2 x^9/2 + c^2 x^{17/2} + 2bcx^{13/2})$$

$$\frac{b^4(17Ac - 9Bb)}{45b^3} \frac{1}{(b^2x^{9/2} + c^2x^{17/2} + 2bcx^{13/2})} - \frac{13(-c)^{5/4} \operatorname{atanh}\left(\frac{(-c)^{1/4}x^{1/2}}{b^{1/4}}\right)(17Ac - 9Bb)}{32b^{21/4}}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)\*x\*\*(1/2)/(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Timed out

$$3.219 \quad \int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3} dx$$

**Optimal.** Leaf size=365

$$\frac{15c^{7/4}(11bB - 19Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{64\sqrt{2} b^{23/4}} + \frac{15c^{7/4}(11bB - 19Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{64\sqrt{2} b^{23/4}}$$

**Rubi [A]** time = 0.32, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{15c^{7/4}(11bB - 19Ac) \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{64\sqrt{2} b^{23/4}} + \frac{15c^{7/4}(11bB - 19Ac) \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} \sqrt{x} + \sqrt{b} + \sqrt{cx}\right)}{64\sqrt{2} b^{23/4}} - \frac{15c^{7/4}(11bB - 19Ac) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}}{\sqrt[4]{b}}\right)}{32\sqrt{2} b^{23/4}} + \frac{15c^{7/4}(11bB - 19Ac) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}}{\sqrt[4]{b}} + 1\right)}{32\sqrt{2} b^{23/4}} + \frac{5c(11bB - 19Ac)}{16b^2 c^{3/2}} - \frac{15(11bB - 19Ac)}{112b^2 c^{3/2}} - \frac{11bB - 19Ac}{16b^2 c^{1/2}(b + cx^2)} + \frac{15(11bB - 19Ac)}{176b^2 c^{1/2}} - \frac{bB - Ac}{4cx^{1/2}(b + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)^3), x]

[Out] (15\*(11\*b\*B - 19\*A\*c))/(176\*b^3\*c\*x^(11/2)) - (15\*(11\*b\*B - 19\*A\*c))/(112\*b^4\*x^(7/2)) + (5\*c\*(11\*b\*B - 19\*A\*c))/(16\*b^5\*x^(3/2)) - (b\*B - A\*c)/(4\*b\*c\*x^(11/2)\*(b + c\*x^2)^2) - (11\*b\*B - 19\*A\*c)/(16\*b^2\*c\*x^(11/2)\*(b + c\*x^2)) - (15\*c^(7/4)\*(11\*b\*B - 19\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(23/4)) + (15\*c^(7/4)\*(11\*b\*B - 19\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)])/(32\*Sqrt[2]\*b^(23/4)) - (15\*c^(7/4)\*(11\*b\*B - 19\*A\*c)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(23/4)) + (15\*c^(7/4)\*(11\*b\*B - 19\*A\*c)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(64\*Sqrt[2]\*b^(23/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 290

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := -Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*n\*(p + 1)), x] + Dist[(m + n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(c\*x)^m\*(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 325

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]



Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
+ 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{\sqrt{x} (bx^2 + cx^4)^3} dx &= \int \frac{A + Bx^2}{x^{13/2} (b + cx^2)^3} dx \\
&= -\frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} + \frac{\left(-\frac{11bB}{2} + \frac{19Ac}{2}\right) \int \frac{1}{x^{13/2}(b+cx^2)^2} dx}{4bc} \\
&= -\frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{11bB - 19Ac}{16b^2cx^{11/2} (b + cx^2)} - \frac{(15(11bB - 19Ac)) \int \frac{1}{x^{13/2}(b+cx^2)} dx}{32b^2c} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{11bB - 19Ac}{16b^2cx^{11/2} (b + cx^2)} + \frac{(15(11bB - 19Ac)) \int \frac{1}{x}}{32b^3} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{11bB - 19Ac}{16b^2cx^{11/2} (b + cx^2)} - \frac{15 \ln|x|}{32b^3} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c} \\
&= \frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c}
\end{aligned}$$

**Mathematica [A]** time = 0.63, size = 467, normalized size = 1.28

$\frac{15(11bB - 19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB - 19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB - 19Ac)}{16b^5x^{3/2}} - \frac{bB - Ac}{4bcx^{11/2} (b + cx^2)^2} - \frac{15 \ln|x|}{16b^2c}$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)^3), x]

[Out] ((-1792\*A\*b^(11/4))/x^(11/2) - (2816\*b^(11/4)\*B)/x^(7/2) + (8448\*A\*b^(7/4)\*c)/x^(7/2) + (19712\*b^(7/4)\*B\*c)/x^(3/2) - (39424\*A\*b^(3/4)\*c^2)/x^(3/2) + (2464\*b^(11/4)\*B\*c^2\*Sqrt[x])/(b + c\*x^2)^2 - (2464\*A\*b^(7/4)\*c^3\*Sqrt[x])/(b + c\*x^2)^2 + (14168\*b^(7/4)\*B\*c^2\*Sqrt[x])/(b + c\*x^2) - (19096\*A\*b^(3/4)\*c^3\*Sqrt[x])/(b + c\*x^2) + 2310\*Sqrt[2]\*c^(7/4)\*(-11\*b\*B + 19\*A\*c)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] + 2310\*Sqrt[2]\*c^(7/4)\*(11\*b\*B - 19

\*A\*c)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*Sqrt[x])/b^(1/4)] - 12705\*Sqrt[2]\*b\*B\*c^(7/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 21945\*Sqrt[2]\*A\*c^(11/4)\*Log[Sqrt[b] - Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] + 12705\*Sqrt[2]\*b\*B\*c^(7/4)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - 21945\*Sqrt[2]\*A\*c^(11/4)\*Log[Sqrt[b] + Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(9856\*b^(23/4))

**IntegrateAlgebraic [A]** time = 0.69, size = 248, normalized size = 0.68

$$\frac{15(11bBc^{7/4} - 19Ac^{11/4}) \tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt{b}\sqrt{c}}\right) + 15(11bBc^{7/4} - 19Ac^{11/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}}{\sqrt{b}+\sqrt{c}x}\right) - 224Ab^4 + 608Ab^3cx^2 - 3040Ab^2c^2x^4 - 11495Abc^3x^6 - 7315Ac^4x^8 - 352b^4Bx^2 + 1760b^3Bcx^4 + 6655b^2Bc^2x^6 + 4235bBc^3x^8}{32\sqrt{2}b^{23/4}} + \frac{15(11bBc^{7/4} - 19Ac^{11/4}) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}}{\sqrt{b}+\sqrt{c}x}\right) - 224Ab^4 + 608Ab^3cx^2 - 3040Ab^2c^2x^4 - 11495Abc^3x^6 - 7315Ac^4x^8 - 352b^4Bx^2 + 1760b^3Bcx^4 + 6655b^2Bc^2x^6 + 4235bBc^3x^8}{1232b^5x^{11/2}(b+cx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x^2)/(Sqrt[x]\*(b\*x^2 + c\*x^4)^3), x]

[Out] (-224\*A\*b^4 - 352\*b^4\*B\*x^2 + 608\*A\*b^3\*c\*x^2 + 1760\*b^3\*B\*c\*x^4 - 3040\*A\*b^2\*c^2\*x^4 + 6655\*b^2\*B\*c^2\*x^6 - 11495\*A\*b\*c^3\*x^6 + 4235\*b\*B\*c^3\*x^8 - 7315\*A\*c^4\*x^8)/(1232\*b^5\*x^(11/2)\*(b + c\*x^2)^2) - (15\*(11\*b\*B\*c^(7/4) - 19\*A\*c^(11/4))\*ArcTan[(Sqrt[b] - Sqrt[c]\*x)/(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x]])/(32\*Sqrt[2]\*b^(23/4)) + (15\*(11\*b\*B\*c^(7/4) - 19\*A\*c^(11/4))\*ArcTanh[(Sqrt[2]\*b^(1/4)\*c^(1/4)\*Sqrt[x])/(Sqrt[b] + Sqrt[c]\*x)]/(32\*Sqrt[2]\*b^(23/4))

**fricas [B]** time = 0.44, size = 894, normalized size = 2.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^3/x^(1/2), x, algorithm="fricas")

[Out] -1/4928\*(4620\*(b^5\*c^2\*x^10 + 2\*b^6\*c\*x^8 + b^7\*x^6)\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(1/4)\*arctan((sqrt(b^12\*sqrt(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23) + (121\*B^2\*b^2\*c^4 - 418\*A\*B\*b\*c^5 + 361\*A^2\*c^6)\*x)\*b^17\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(3/4) + (11\*B\*b^18\*c^2 - 19\*A\*b^17\*c^3)\*sqrt(x)\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(3/4))/(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11) + 1155\*(b^5\*c^2\*x^10 + 2\*b^6\*c\*x^8 + b^7\*x^6)\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(1/4)\*log(15\*b^6\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(1/4) - 15\*(11\*B\*b\*c^2 - 19\*A\*c^3)\*sqrt(x)) - 1155\*(b^5\*c^2\*x^10 + 2\*b^6\*c\*x^8 + b^7\*x^6)\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(1/4)\*log(-15\*b^6\*(-(14641\*B^4\*b^4\*c^7 - 101156\*A\*B^3\*b^3\*c^8 + 262086\*A^2\*B^2\*b^2\*c^9 - 301796\*A^3\*B\*b\*c^10 + 130321\*A^4\*c^11)/b^23)^(1/4) - 15\*(11\*B\*b\*c^2 - 19\*A\*c^3)\*sqrt(x)) - 4\*(385\*(11\*B\*b\*c^3 - 19\*A\*c^4)\*x^8 + 605\*(11\*B\*b^2\*c^2 - 19\*A\*b\*c^3)\*x^6 - 224\*A\*b^4 + 160\*(11\*B\*b^3\*c - 19\*A\*b^2\*c^2)\*x^4 - 32\*(11\*B\*b^4 - 19\*A\*b^3\*c)\*x^2)\*sqrt(x))/(b^5\*c^2\*x^10 + 2\*b^6\*c\*x^8 + b^7\*x^6)

**giac [A]** time = 0.28, size = 351, normalized size = 0.96

$$\frac{15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\arctan\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right) + 15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\arctan\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right) - 224Ab^4 + 608Ab^3cx^2 - 3040Ab^2c^2x^4 - 11495Abc^3x^6 - 7315Ac^4x^8 - 352b^4Bx^2 + 1760b^3Bcx^4 + 6655b^2Bc^2x^6 + 4235bBc^3x^8}{64b^4} + \frac{15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\arctan\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right) - 224Ab^4 + 608Ab^3cx^2 - 3040Ab^2c^2x^4 - 11495Abc^3x^6 - 7315Ac^4x^8 - 352b^4Bx^2 + 1760b^3Bcx^4 + 6655b^2Bc^2x^6 + 4235bBc^3x^8}{64b^4} + \frac{15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\log\left(\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{1+x\sqrt{\frac{b}{c}}}\right) - 15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\log\left(-\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{1+x\sqrt{\frac{b}{c}}}\right) + 23Bb^2c^{\frac{1}{2}} - 31Ac^{\frac{1}{2}} + 27Bb^2c^{\frac{1}{2}} - 35Ab^2c^{\frac{1}{2}} - 2\sqrt{2}Bbc^{\frac{1}{2}} - 134Ac^{\frac{1}{2}} - 11Bb^2c^{\frac{1}{2}} + 33Ab^2c^{\frac{1}{2}} - 7Ab^2}{16(c^2+b)^2}}{128b^4} + \frac{15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\log\left(\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{1+x\sqrt{\frac{b}{c}}}\right) - 15\sqrt{2}\left((b^2)^{\frac{1}{2}}Bc-19(b^2)^{\frac{1}{2}}Ac\right)\log\left(-\sqrt{2}\sqrt{\frac{b}{c}}\sqrt{1+x\sqrt{\frac{b}{c}}}\right) + 23Bb^2c^{\frac{1}{2}} - 31Ac^{\frac{1}{2}} + 27Bb^2c^{\frac{1}{2}} - 35Ab^2c^{\frac{1}{2}} - 2\sqrt{2}Bbc^{\frac{1}{2}} - 134Ac^{\frac{1}{2}} - 11Bb^2c^{\frac{1}{2}} + 33Ab^2c^{\frac{1}{2}} - 7Ab^2}{16(c^2+b)^2}}{128b^4} + \frac{23Bb^2c^{\frac{1}{2}} - 31Ac^{\frac{1}{2}} + 27Bb^2c^{\frac{1}{2}} - 35Ab^2c^{\frac{1}{2}} - 2\sqrt{2}Bbc^{\frac{1}{2}} - 134Ac^{\frac{1}{2}} - 11Bb^2c^{\frac{1}{2}} + 33Ab^2c^{\frac{1}{2}} - 7Ab^2}{77b^2c^{\frac{1}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(c\*x^4+b\*x^2)^3/x^(1/2), x, algorithm="giac")

[Out]  $15/64*\sqrt{2}*(11*(b*c^3)^{(1/4)}*B*b*c - 19*(b*c^3)^{(1/4)}*A*c^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*\sqrt{x})/(b/c)^{(1/4)})/b^6 + 15/64*\sqrt{2}*(11*(b*c^3)^{(1/4)}*B*b*c - 19*(b*c^3)^{(1/4)}*A*c^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*\sqrt{x})/(b/c)^{(1/4)})/b^6 + 15/128*\sqrt{2}*(11*(b*c^3)^{(1/4)}*B*b*c - 19*(b*c^3)^{(1/4)}*A*c^2)*\log(\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c})/b^6 - 15/128*\sqrt{2}*(11*(b*c^3)^{(1/4)}*B*b*c - 19*(b*c^3)^{(1/4)}*A*c^2)*\log(-\sqrt{2}*\sqrt{x}*(b/c)^{(1/4)} + x + \sqrt{b/c})/b^6 + 1/16*(23*B*b*c^3*x^{(5/2)} - 31*A*c^4*x^{(5/2)} + 27*B*b^2*c^2*\sqrt{x} - 35*A*b*c^3*\sqrt{x})/((c*x^2 + b)^2*b^5) + 2/77*(77*B*b*c*x^4 - 154*A*c^2*x^4 - 11*B*b^2*x^2 + 33*A*b*c*x^2 - 7*A*b^2)/(b^5*x^{(11/2)})$

**maple [A]** time = 0.07, size = 420, normalized size = 1.15

$$\frac{3A^2c^2 + 23B^2c^2}{16(c^2 + b)^2} + \frac{35A^2c^2}{16(c^2 + b)^2} + \frac{27B^2c^2}{16(c^2 + b)^2} - \frac{285}{64b^6} \sqrt{2} A^2 c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) - \frac{285}{64b^6} \sqrt{2} A^2 c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b} + 1\right) - \frac{285}{128b^6} \sqrt{2} A^2 c^2 \ln\left(\frac{-(b/c)^{1/4} \sqrt{2}\sqrt{c} + \sqrt{c}}{(b/c)^{1/4}}\right) - \frac{165}{64b^6} \sqrt{2} B^2 c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right) - \frac{165}{64b^6} \sqrt{2} B^2 c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b} + 1\right) - \frac{165}{128b^6} \sqrt{2} B^2 c^2 \ln\left(\frac{-(b/c)^{1/4} \sqrt{2}\sqrt{c} + \sqrt{c}}{(b/c)^{1/4}}\right) - \frac{4A^2}{8b^5} + \frac{23c}{8b^5} - \frac{6Ac}{7b^5} - \frac{2B}{7b^5} - \frac{2A}{11b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2), x)`

[Out]  $-31/16/b^5*c^4/(c*x^2+b)^2*x^{(5/2)}*A+23/16/b^4*c^3/(c*x^2+b)^2*x^{(5/2)}*B-35/16/b^4*c^3/(c*x^2+b)^2*A*x^{(1/2)}+27/16/b^3*c^2/(c*x^2+b)^2*B*x^{(1/2)}-285/64/b^6*c^3*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)-285/128/b^6*c^3*(b/c)^{(1/4)}*2^{(1/2)}*A*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))-285/64/b^6*c^3*(b/c)^{(1/4)}*2^{(1/2)}*A*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)+165/64/b^5*c^2*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}-1)+165/128/b^5*c^2*(b/c)^{(1/4)}*2^{(1/2)}*B*\ln((x+(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)})/(x-(b/c)^{(1/4)}*2^{(1/2)}*x^{(1/2)}+(b/c)^{(1/2)}))+165/64/b^5*c^2*(b/c)^{(1/4)}*2^{(1/2)}*B*\arctan(2^{(1/2)}/(b/c)^{(1/4)}*x^{(1/2)}+1)-2/11*A/b^3/x^{(11/2)}+6/7/b^4/x^{(7/2)}*A*c-2/7/b^3/x^{(7/2)}*B-4*c^2/b^5/x^{(3/2)}*A+2*c/b^4/x^{(3/2)}*B$

**maxima [A]** time = 3.10, size = 353, normalized size = 0.97

$$\frac{15}{1232} \left( \frac{2\sqrt{2}(11Bb^2-19A^2)c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right)}{\sqrt{c}\sqrt{b}} + \frac{2\sqrt{2}(11Bb^2-19A^2)c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b} + 1\right)}{\sqrt{c}\sqrt{b}} + \frac{\sqrt{2}(11Bb^2-19A^2)c^2 \ln\left(\frac{-(b/c)^{1/4} \sqrt{2}\sqrt{c} + \sqrt{c}}{(b/c)^{1/4}}\right)}{b^{1/4}} + \frac{\sqrt{2}(11Bb^2-19A^2)c^2 \ln\left(\frac{-(b/c)^{1/4} \sqrt{2}\sqrt{c} + \sqrt{c}}{(b/c)^{1/4}}\right)}{b^{1/4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2), x, algorithm="maxima")`

[Out]  $1/1232*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c^2 - 19*A*b*c^3)*x^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32*(11*B*b^4 - 19*A*b^3*c)*x^2)/(b^5*c^2*x^{(19/2)} + 2*b^6*c*x^{(15/2)} + b^7*x^{(11/2)}) + 15/128*(2*\sqrt{2}*(11*B*b*c^2 - 19*A*c^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} + 2*\sqrt{c})*\sqrt{x})/\sqrt{b}*\sqrt{c})/(\sqrt{b}*\sqrt{c}) + 2*\sqrt{2}*(11*B*b*c^2 - 19*A*c^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{(1/4)}*c^{(1/4)} - 2*\sqrt{c})*\sqrt{x})/\sqrt{b}*\sqrt{c})/(\sqrt{b}*\sqrt{c}) + \sqrt{2}*(11*B*b*c^2 - 19*A*c^3)*\log(\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c})*x + \sqrt{b})/(b^{(3/4)}*c^{(1/4)}) - \sqrt{2}*(11*B*b*c^2 - 19*A*c^3)*\log(-\sqrt{2}*b^{(1/4)}*c^{(1/4)}*\sqrt{x} + \sqrt{c})*x + \sqrt{b})/(b^{(3/4)}*c^{(1/4)})/b^5$

**mupad [B]** time = 0.51, size = 639, normalized size = 1.75

$$\frac{15}{1232} \left( \frac{2\sqrt{2}(11Bb^2-19A^2)c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b}\right)}{\sqrt{c}\sqrt{b}} + \frac{2\sqrt{2}(11Bb^2-19A^2)c^2 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{b} + 1\right)}{\sqrt{c}\sqrt{b}} + \frac{\sqrt{2}(11Bb^2-19A^2)c^2 \ln\left(\frac{-(b/c)^{1/4} \sqrt{2}\sqrt{c} + \sqrt{c}}{(b/c)^{1/4}}\right)}{b^{1/4}} + \frac{\sqrt{2}(11Bb^2-19A^2)c^2 \ln\left(\frac{-(b/c)^{1/4} \sqrt{2}\sqrt{c} + \sqrt{c}}{(b/c)^{1/4}}\right)}{b^{1/4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^3), x)`

```
[Out] (15*(-c)^(7/4)*atan((6859*A^3*c^10*x^(1/2) - 1331*B^3*b^3*c^7*x^(1/2) - 119
13*A^2*B*b*c^9*x^(1/2) + 6897*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(
c*(c*(6859*A^3*c - 11913*A^2*B*b) + 6897*A*B^2*b^2) - 1331*B^3*b^3)))*(19*A
*c - 11*B*b))/(32*b^(23/4)) - ((2*A)/(11*b) - (2*x^2*(19*A*c - 11*B*b))/(77
*b^2) + (55*c^2*x^6*(19*A*c - 11*B*b))/(112*b^4) + (5*c^3*x^8*(19*A*c - 11*
B*b))/(16*b^5) + (10*c*x^4*(19*A*c - 11*B*b))/(77*b^3))/(b^2*x^(11/2) + c^2
*x^(19/2) + 2*b*c*x^(15/2)) - ((-c)^(7/4)*atan((((-c)^(7/4)*(19*A*c - 11*B*
b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 1540915200*
A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21*c^6 - 17
3015040*B*b^22*c^5))/(64*b^(23/4))))*15i)/(64*b^(23/4)) + ((-c)^(7/4)*(19*A*
c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 15
40915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21
*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))*15i)/(64*b^(23/4)))/((15*(-c)^(
7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b
^17*c^7 - 1540915200*A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(2988
44160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))))/(64*b^(23/4)) - (
15*(-c)^(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 4460544
00*B^2*b^17*c^7 - 1540915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*
b)*(298844160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))))/(64*b^(23
/4)))*(19*A*c - 11*B*b)*15i)/(32*b^(23/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

$$3.220 \quad \int x^m (A + Bx^2) (bx^2 + cx^4)^3 dx$$

**Optimal.** Leaf size=96

$$\frac{Ab^3x^{m+7}}{m+7} + \frac{b^2x^{m+9}(3Ac+bB)}{m+9} + \frac{c^2x^{m+13}(Ac+3bB)}{m+13} + \frac{3bcx^{m+11}(Ac+bB)}{m+11} + \frac{Bc^3x^{m+15}}{m+15}$$

**Rubi [A]** time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{b^2x^{m+9}(3Ac+bB)}{m+9} + \frac{Ab^3x^{m+7}}{m+7} + \frac{c^2x^{m+13}(Ac+3bB)}{m+13} + \frac{3bcx^{m+11}(Ac+bB)}{m+11} + \frac{Bc^3x^{m+15}}{m+15}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] (A\*b^3\*x^(7 + m))/(7 + m) + (b^2\*(b\*B + 3\*A\*c)\*x^(9 + m))/(9 + m) + (3\*b\*c\*(b\*B + A\*c)\*x^(11 + m))/(11 + m) + (c^2\*(3\*b\*B + A\*c)\*x^(13 + m))/(13 + m) + (B\*c^3\*x^(15 + m))/(15 + m)

#### Rule 448

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rubi steps

$$\begin{aligned} \int x^m (A + Bx^2) (bx^2 + cx^4)^3 dx &= \int x^{6+m} (A + Bx^2) (b + cx^2)^3 dx \\ &= \int (Ab^3x^{6+m} + b^2(bB + 3Ac)x^{8+m} + 3bc(bB + Ac)x^{10+m} + c^2(3bB + Ac)x^{12+m}) dx \\ &= \frac{Ab^3x^{7+m}}{7+m} + \frac{b^2(bB + 3Ac)x^{9+m}}{9+m} + \frac{3bc(bB + Ac)x^{11+m}}{11+m} + \frac{c^2(3bB + Ac)x^{13+m}}{13+m} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 89, normalized size = 0.93

$$x^{m+7} \left( \frac{Ab^3}{m+7} + \frac{b^2x^2(3Ac+bB)}{m+9} + \frac{c^2x^6(Ac+3bB)}{m+13} + \frac{3bcx^4(Ac+bB)}{m+11} + \frac{Bc^3x^8}{m+15} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] x^(7 + m)\*((A\*b^3)/(7 + m) + (b^2\*(b\*B + 3\*A\*c)\*x^2)/(9 + m) + (3\*b\*c\*(b\*B + A\*c)\*x^4)/(11 + m) + (c^2\*(3\*b\*B + A\*c)\*x^6)/(13 + m) + (B\*c^3\*x^8)/(15 + m))

IntegrateAlgebraic [F] time = 0.62, size = 0, normalized size = 0.00

$$\int x^m (A + Bx^2)(bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x]

[Out] Defer[IntegrateAlgebraic][x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3, x]

fricas [B] time = 0.41, size = 381, normalized size = 3.97

[[[a^m + 40\*b^m\*c + 3800\*b^m\*c^2 + 9009\*b^m\*c^3 + ((135\*c^2 + 4\*c^3)\*m^4 + 31185\*B\*b\*c^2 + 10395\*A\*c^3 + 42\*(3\*B\*b\*c^2 + A\*c^3)\*m^3 + 644\*(3\*B\*b\*c^2 + A\*c^3)\*m^2 + 4278\*(3\*B\*b\*c^2 + A\*c^3)\*m + 13285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^4 + 12285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^3 + 706\*(B\*b^2\*c + A\*b\*c^2)\*m^2 + 4884\*(B\*b^2\*c + A\*b\*c^2)\*m + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^4 + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^3 + 776\*(B\*b^3 + 3\*A\*b^2\*c)\*m^2 + 5666\*(B\*b^3 + 3\*A\*b^2\*c)\*m + 19305\*A\*b^3]\*x^m]/(m^5 + 55\*m^4 + 1190\*m^3 + 12650\*m^2 + 66009\*m + 135135)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="fricas")

[Out] ((B\*c^3\*m^4 + 40\*B\*c^3\*m^3 + 590\*B\*c^3\*m^2 + 3800\*B\*c^3\*m + 9009\*B\*c^3)\*x^15 + ((3\*B\*b\*c^2 + A\*c^3)\*m^4 + 31185\*B\*b\*c^2 + 10395\*A\*c^3 + 42\*(3\*B\*b\*c^2 + A\*c^3)\*m^3 + 644\*(3\*B\*b\*c^2 + A\*c^3)\*m^2 + 4278\*(3\*B\*b\*c^2 + A\*c^3)\*m)\*x^13 + 3\*((B\*b^2\*c + A\*b\*c^2)\*m^4 + 12285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^3 + 706\*(B\*b^2\*c + A\*b\*c^2)\*m^2 + 4884\*(B\*b^2\*c + A\*b\*c^2)\*m)\*x^11 + ((B\*b^3 + 3\*A\*b^2\*c)\*m^4 + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^3 + 776\*(B\*b^3 + 3\*A\*b^2\*c)\*m^2 + 5666\*(B\*b^3 + 3\*A\*b^2\*c)\*m)\*x^9 + (A\*b^3\*m^4 + 48\*A\*b^3\*m^3 + 854\*A\*b^3\*m^2 + 6672\*A\*b^3\*m + 19305\*A\*b^3)\*x^7)\*x^m/(m^5 + 55\*m^4 + 1190\*m^3 + 12650\*m^2 + 66009\*m + 135135)

giac [B] time = 0.22, size = 603, normalized size = 6.28

[[[a^m + 40\*b^m\*c + 3800\*b^m\*c^2 + 9009\*b^m\*c^3 + ((135\*c^2 + 4\*c^3)\*m^4 + 31185\*B\*b\*c^2 + 10395\*A\*c^3 + 42\*(3\*B\*b\*c^2 + A\*c^3)\*m^3 + 644\*(3\*B\*b\*c^2 + A\*c^3)\*m^2 + 4278\*(3\*B\*b\*c^2 + A\*c^3)\*m + 13285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^4 + 12285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^3 + 706\*(B\*b^2\*c + A\*b\*c^2)\*m^2 + 4884\*(B\*b^2\*c + A\*b\*c^2)\*m + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^4 + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^3 + 776\*(B\*b^3 + 3\*A\*b^2\*c)\*m^2 + 5666\*(B\*b^3 + 3\*A\*b^2\*c)\*m + 19305\*A\*b^3]\*x^m]/(m^5 + 55\*m^4 + 1190\*m^3 + 12650\*m^2 + 66009\*m + 135135)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="giac")

[Out] (B\*c^3\*m^4\*x^15\*x^m + 40\*B\*c^3\*m^3\*x^15\*x^m + 3\*B\*b\*c^2\*m^4\*x^13\*x^m + A\*c^3\*m^4\*x^13\*x^m + 590\*B\*c^3\*m^2\*x^15\*x^m + 126\*B\*b\*c^2\*m^3\*x^13\*x^m + 42\*A\*c^3\*m^3\*x^13\*x^m + 3800\*B\*c^3\*m\*x^15\*x^m + 3\*B\*b^2\*c\*m^4\*x^11\*x^m + 3\*A\*b\*c^2\*m^4\*x^11\*x^m + 1932\*B\*b\*c^2\*m^2\*x^13\*x^m + 644\*A\*c^3\*m^2\*x^13\*x^m + 9009\*B\*c^3\*x^15\*x^m + 132\*B\*b^2\*c\*m^3\*x^11\*x^m + 132\*A\*b\*c^2\*m^3\*x^11\*x^m + 12834\*B\*b\*c^2\*m\*x^13\*x^m + 4278\*A\*c^3\*m\*x^13\*x^m + B\*b^3\*m^4\*x^9\*x^m + 3\*A\*b^2\*c\*m^4\*x^9\*x^m + 2118\*B\*b^2\*c\*m^2\*x^11\*x^m + 2118\*A\*b\*c^2\*m^2\*x^11\*x^m + 31185\*B\*b\*c^2\*x^13\*x^m + 10395\*A\*c^3\*x^13\*x^m + 46\*B\*b^3\*m^3\*x^9\*x^m + 138\*A\*b^2\*c\*m^3\*x^9\*x^m + 14652\*B\*b^2\*c\*m\*x^11\*x^m + 14652\*A\*b\*c^2\*m\*x^11\*x^m + A\*b^3\*m^4\*x^7\*x^m + 776\*B\*b^3\*m^2\*x^9\*x^m + 2328\*A\*b^2\*c\*m^2\*x^9\*x^m + 36855\*B\*b^2\*c\*x^11\*x^m + 36855\*A\*b\*c^2\*x^11\*x^m + 48\*A\*b^3\*m^3\*x^7\*x^m + 5666\*B\*b^3\*m\*x^9\*x^m + 16998\*A\*b^2\*c\*m\*x^9\*x^m + 854\*A\*b^3\*m^2\*x^7\*x^m + 15015\*B\*b^3\*x^9\*x^m + 45045\*A\*b^2\*c\*x^9\*x^m + 6672\*A\*b^3\*m\*x^7\*x^m + 19305\*A\*b^3\*x^7\*x^m)/(m^5 + 55\*m^4 + 1190\*m^3 + 12650\*m^2 + 66009\*m + 135135)

maple [B] time = 0.05, size = 474, normalized size = 4.94

[[[a^m + 40\*b^m\*c + 3800\*b^m\*c^2 + 9009\*b^m\*c^3 + ((135\*c^2 + 4\*c^3)\*m^4 + 31185\*B\*b\*c^2 + 10395\*A\*c^3 + 42\*(3\*B\*b\*c^2 + A\*c^3)\*m^3 + 644\*(3\*B\*b\*c^2 + A\*c^3)\*m^2 + 4278\*(3\*B\*b\*c^2 + A\*c^3)\*m + 13285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^4 + 12285\*B\*b^2\*c + 12285\*A\*b\*c^2 + 44\*(B\*b^2\*c + A\*b\*c^2)\*m^3 + 706\*(B\*b^2\*c + A\*b\*c^2)\*m^2 + 4884\*(B\*b^2\*c + A\*b\*c^2)\*m + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^4 + 15015\*B\*b^3 + 45045\*A\*b^2\*c + 46\*(B\*b^3 + 3\*A\*b^2\*c)\*m^3 + 776\*(B\*b^3 + 3\*A\*b^2\*c)\*m^2 + 5666\*(B\*b^3 + 3\*A\*b^2\*c)\*m + 19305\*A\*b^3]\*x^m]/(m^5 + 55\*m^4 + 1190\*m^3 + 12650\*m^2 + 66009\*m + 135135)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x)

[Out] x^(m+7)\*(B\*c^3\*m^4\*x^8+40\*B\*c^3\*m^3\*x^8+A\*c^3\*m^4\*x^6+3\*B\*b\*c^2\*m^4\*x^6+590\*B\*c^3\*m^2\*x^8+42\*A\*c^3\*m^3\*x^6+126\*B\*b\*c^2\*m^3\*x^6+3800\*B\*c^3\*m\*x^8+3\*A\*b\*c^2\*m^4\*x^4+644\*A\*c^3\*m^2\*x^6+3\*B\*b^2\*c\*m^4\*x^4+1932\*B\*b\*c^2\*m^2\*x^6+9009\*B

$$\begin{aligned} & *c^3*x^8+132*A*b*c^2*m^3*x^4+4278*A*c^3*m*x^6+132*B*b^2*c*m^3*x^4+12834*B*b \\ & *c^2*m*x^6+3*A*b^2*c*m^4*x^2+2118*A*b*c^2*m^2*x^4+10395*A*c^3*x^6+B*b^3*m^4 \\ & *x^2+2118*B*b^2*c*m^2*x^4+31185*B*b*c^2*x^6+138*A*b^2*c*m^3*x^2+14652*A*b*c \\ & ^2*m*x^4+46*B*b^3*m^3*x^2+14652*B*b^2*c*m*x^4+A*b^3*m^4+2328*A*b^2*c*m^2*x^ \\ & 2+36855*A*b*c^2*x^4+776*B*b^3*m^2*x^2+36855*B*b^2*c*x^4+48*A*b^3*m^3+16998* \\ & A*b^2*c*m*x^2+5666*B*b^3*m*x^2+854*A*b^3*m^2+45045*A*b^2*c*x^2+15015*B*b^3* \\ & x^2+6672*A*b^3*m+19305*A*b^3)/(m+15)/(m+13)/(m+11)/(m+9)/(m+7) \end{aligned}$$

**maxima [A]** time = 1.37, size = 129, normalized size = 1.34

$$\frac{Bc^3x^{m+15}}{m+15} + \frac{3Bbc^2x^{m+13}}{m+13} + \frac{Ac^3x^{m+13}}{m+13} + \frac{3Bb^2cx^{m+11}}{m+11} + \frac{3Abc^2x^{m+11}}{m+11} + \frac{Bb^3x^{m+9}}{m+9} + \frac{3Ab^2cx^{m+9}}{m+9} + \frac{Ab^3x^{m+7}}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^3,x, algorithm="maxima")

[Out] B\*c^3\*x^(m + 15)/(m + 15) + 3\*B\*b\*c^2\*x^(m + 13)/(m + 13) + A\*c^3\*x^(m + 13)/(m + 13) + 3\*B\*b^2\*c\*x^(m + 11)/(m + 11) + 3\*A\*b\*c^2\*x^(m + 11)/(m + 11) + B\*b^3\*x^(m + 9)/(m + 9) + 3\*A\*b^2\*c\*x^(m + 9)/(m + 9) + A\*b^3\*x^(m + 7)/(m + 7)

**mupad [B]** time = 0.48, size = 291, normalized size = 3.03

$$\frac{A^3 x^m x^2 (m^4 + 48 m^3 + 854 m^2 + 6672 m + 19305)}{m^5 + 55 m^4 + 1190 m^3 + 12650 m^2 + 66009 m + 135135} + \frac{B^3 x^m x^3 (3 A c + B b) (m^4 + 46 m^3 + 776 m^2 + 5666 m + 15015)}{m^5 + 55 m^4 + 1190 m^3 + 12650 m^2 + 66009 m + 135135} + \frac{c^2 x^m x^{13} (A c + 3 B b) (m^4 + 42 m^3 + 644 m^2 + 4278 m + 10395)}{m^5 + 55 m^4 + 1190 m^3 + 12650 m^2 + 66009 m + 135135} + \frac{3 b c x^m x^{11} (A c + B b) (m^4 + 44 m^3 + 706 m^2 + 4884 m + 12285)}{m^5 + 55 m^4 + 1190 m^3 + 12650 m^2 + 66009 m + 135135}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^3,x)

[Out] (A\*b^3\*x^m\*x^7\*(6672\*m + 854\*m^2 + 48\*m^3 + m^4 + 19305))/(66009\*m + 12650\*m^2 + 1190\*m^3 + 55\*m^4 + m^5 + 135135) + (B\*c^3\*x^m\*x^15\*(3800\*m + 590\*m^2 + 40\*m^3 + m^4 + 9009))/(66009\*m + 12650\*m^2 + 1190\*m^3 + 55\*m^4 + m^5 + 135135) + (b^2\*x^m\*x^9\*(3\*A\*c + B\*b)\*(5666\*m + 776\*m^2 + 46\*m^3 + m^4 + 15015))/(66009\*m + 12650\*m^2 + 1190\*m^3 + 55\*m^4 + m^5 + 135135) + (c^2\*x^m\*x^13\*(A\*c + 3\*B\*b)\*(4278\*m + 644\*m^2 + 42\*m^3 + m^4 + 10395))/(66009\*m + 12650\*m^2 + 1190\*m^3 + 55\*m^4 + m^5 + 135135) + (3\*b\*c\*x^m\*x^11\*(A\*c + B\*b)\*(4884\*m + 706\*m^2 + 44\*m^3 + m^4 + 12285))/(66009\*m + 12650\*m^2 + 1190\*m^3 + 55\*m^4 + m^5 + 135135)

**sympy [A]** time = 9.05, size = 2077, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*3,x)

[Out] Piecewise((-A\*b\*\*3/(8\*x\*\*8) - A\*b\*\*2\*c/(2\*x\*\*6) - 3\*A\*b\*c\*\*2/(4\*x\*\*4) - A\*c\*\*3/(2\*x\*\*2) - B\*b\*\*3/(6\*x\*\*6) - 3\*B\*b\*\*2\*c/(4\*x\*\*4) - 3\*B\*b\*c\*\*2/(2\*x\*\*2) + B\*c\*\*3\*log(x), Eq(m, -15)), (-A\*b\*\*3/(6\*x\*\*6) - 3\*A\*b\*\*2\*c/(4\*x\*\*4) - 3\*A\*b\*c\*\*2/(2\*x\*\*2) + A\*c\*\*3\*log(x) - B\*b\*\*3/(4\*x\*\*4) - 3\*B\*b\*\*2\*c/(2\*x\*\*2) + 3\*B\*b\*c\*\*2\*log(x) + B\*c\*\*3\*x\*\*2/2, Eq(m, -13)), (-A\*b\*\*3/(4\*x\*\*4) - 3\*A\*b\*\*2\*c/(2\*x\*\*2) + 3\*A\*b\*c\*\*2\*log(x) + A\*c\*\*3\*x\*\*2/2 - B\*b\*\*3/(2\*x\*\*2) + 3\*B\*b\*\*2\*c\*log(x) + 3\*B\*b\*c\*\*2\*x\*\*2/2 + B\*c\*\*3\*x\*\*4/4, Eq(m, -11)), (-A\*b\*\*3/(2\*x\*\*2) + 3\*A\*b\*\*2\*c\*log(x) + 3\*A\*b\*c\*\*2\*x\*\*2/2 + A\*c\*\*3\*x\*\*4/4 + B\*b\*\*3\*log(x) + 3\*B\*b\*\*2\*c\*x\*\*2/2 + 3\*B\*b\*c\*\*2\*x\*\*4/4 + B\*c\*\*3\*x\*\*6/6, Eq(m, -9)), (A\*b\*\*3\*log(x) + 3\*A\*b\*\*2\*c\*x\*\*2/2 + 3\*A\*b\*c\*\*2\*x\*\*4/4 + A\*c\*\*3\*x\*\*6/6 + B\*b\*\*3\*x\*\*2/2 + 3\*B\*b\*\*2\*c\*x\*\*4/4 + B\*b\*c\*\*2\*x\*\*6/2 + B\*c\*\*3\*x\*\*8/8, Eq(m, -7)), (A\*b\*\*3\*m\*\*4\*x\*\*7\*x\*\*m/(m\*\*5 + 55\*m\*\*4 + 1190\*m\*\*3 + 12650\*m\*\*2 + 66009\*m + 135135) + 48\*A\*b\*\*3\*m\*\*3\*x\*\*7\*x\*\*m/(m\*\*5 + 55\*m\*\*4 + 1190\*m\*\*3 + 12650\*m\*\*2 + 66009\*m + 135135) + 854\*A\*b\*\*3\*m\*\*2\*x\*\*7\*x\*\*m/(m\*\*5 + 55\*m\*\*4 + 1190\*m\*\*3 + 12650\*m\*\*2 + 66009\*m + 135135) + 6672\*A\*b\*\*3\*m\*x\*\*7\*x\*\*m/(m\*\*5 + 55\*m\*\*4 + 1190\*m\*\*3 + 12650\*m\*\*2 + 66009\*m + 135135) + 19305\*A\*b\*\*3\*x\*\*7\*x\*\*m/(m



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**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b**2*c*m**
4*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
138*A*b**2*c*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 2328*A*b**2*c*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 +
12650*m**2 + 66009*m + 135135) + 16998*A*b**2*c*m*x**9*x**m/(m**5 + 55*m**4
+ 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 45045*A*b**2*c*x**9*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b*c**2*m**
4*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
132*A*b*c**2*m**3*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 2118*A*b*c**2*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3
+ 12650*m**2 + 66009*m + 135135) + 14652*A*b*c**2*m*x**11*x**m/(m**5 + 55*
m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 36855*A*b*c**2*x**11*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + A*c**3*m**
4*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
+ 42*A*c**3*m**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 644*A*c**3*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12
650*m**2 + 66009*m + 135135) + 4278*A*c**3*m*x**13*x**m/(m**5 + 55*m**4 + 1
190*m**3 + 12650*m**2 + 66009*m + 135135) + 10395*A*c**3*x**13*x**m/(m**5 +
55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*b**3*m**4*x**9*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 46*B*b**3
*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135
) + 776*B*b**3*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 5666*B*b**3*m*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 126
50*m**2 + 66009*m + 135135) + 15015*B*b**3*x**9*x**m/(m**5 + 55*m**4 + 1190
*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b**2*c*m**4*x**11*x**m/(m**5 +
55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 132*B*b**2*c*m**3*x
**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 21
18*B*b**2*c*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 14652*B*b**2*c*m*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1
2650*m**2 + 66009*m + 135135) + 36855*B*b**2*c*x**11*x**m/(m**5 + 55*m**4 +
1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b*c**2*m**4*x**13*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 126*B*b*c**2*m
**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
+ 1932*B*b*c**2*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 +
66009*m + 135135) + 12834*B*b*c**2*m*x**13*x**m/(m**5 + 55*m**4 + 1190*m**
3 + 12650*m**2 + 66009*m + 135135) + 31185*B*b*c**2*x**13*x**m/(m**5 + 55*m
**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*c**3*m**4*x**15*x**m/(
m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 40*B*c**3*m**
3*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
590*B*c**3*m**2*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 3800*B*c**3*m*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1265
0*m**2 + 66009*m + 135135) + 9009*B*c**3*x**15*x**m/(m**5 + 55*m**4 + 1190*
m**3 + 12650*m**2 + 66009*m + 135135), True))

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$$3.221 \quad \int x^m (A + Bx^2) (bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=71

$$\frac{Ab^2x^{m+5}}{m+5} + \frac{bx^{m+7}(2Ac+bB)}{m+7} + \frac{cx^{m+9}(Ac+2bB)}{m+9} + \frac{Bc^2x^{m+11}}{m+11}$$

**Rubi [A]** time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1584, 448}

$$\frac{Ab^2x^{m+5}}{m+5} + \frac{bx^{m+7}(2Ac+bB)}{m+7} + \frac{cx^{m+9}(Ac+2bB)}{m+9} + \frac{Bc^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] (A\*b^2\*x^(5 + m))/(5 + m) + (b\*(b\*B + 2\*A\*c)\*x^(7 + m))/(7 + m) + (c\*(2\*b\*B + A\*c)\*x^(9 + m))/(9 + m) + (B\*c^2\*x^(11 + m))/(11 + m)

Rule 448

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^m (A + Bx^2) (bx^2 + cx^4)^2 dx &= \int x^{4+m} (A + Bx^2) (b + cx^2)^2 dx \\ &= \int (Ab^2x^{4+m} + b(bB + 2Ac)x^{6+m} + c(2bB + Ac)x^{8+m} + Bc^2x^{10+m}) dx \\ &= \frac{Ab^2x^{5+m}}{5+m} + \frac{b(bB + 2Ac)x^{7+m}}{7+m} + \frac{c(2bB + Ac)x^{9+m}}{9+m} + \frac{Bc^2x^{11+m}}{11+m} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 66, normalized size = 0.93

$$x^{m+5} \left( \frac{Ab^2}{m+5} + \frac{cx^4(Ac+2bB)}{m+9} + \frac{bx^2(2Ac+bB)}{m+7} + \frac{Bc^2x^6}{m+11} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] x^(5 + m)\*((A\*b^2)/(5 + m) + (b\*(b\*B + 2\*A\*c)\*x^2)/(7 + m) + (c\*(2\*b\*B + A\*c)\*x^4)/(9 + m) + (B\*c^2\*x^6)/(11 + m))

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x]

[Out] Defer[IntegrateAlgebraic][x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2, x]

**fricas** [B] time = 0.41, size = 217, normalized size = 3.06

$$\frac{((B^2m^2 + 21Bc^2m^2 + 143B^2m + 315Bc^2)x^{11} + ((2Bbc + Ac^2)m^2 + 770Bbc + 385Ac^2 + 23(2Bbc + Ac^2)m^2 + 167(2Bbc + Ac^2)m)x^9 + ((Bb^2 + 2Abc)m^2 + 495Bb^2 + 990Abc + 25(Bb^2 + 2Abc)m^2 + 199(Bb^2 + 2Abc)m)x^7 + (Ab^2m^2 + 27Ab^2m + 239Ab^2m + 693Ab^2)x^5)m^3 + 32m^4 + 374m^2 + 1888m + 3465)}{m^4 + 32m^3 + 374m^2 + 1888m + 3465}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="fricas")

[Out] 
$$\frac{((B^2c^2m^3 + 21B^2c^2m^2 + 143B^2c^2m + 315B^2c^2)x^{11} + ((2B^2b^2c + Ac^2)m^3 + 770B^2b^2c + 385A^2c^2 + 23(2B^2b^2c + A^2c^2)m^2 + 167(2B^2b^2c + A^2c^2)m)x^9 + ((B^2b^2 + 2A^2b^2c)m^3 + 495B^2b^2 + 990A^2b^2c + 25(B^2b^2 + 2A^2b^2c)m^2 + 199(B^2b^2 + 2A^2b^2c)m)x^7 + (A^2b^2m^3 + 27A^2b^2m^2 + 239A^2b^2m + 693A^2b^2)x^5)x^m}{(m^4 + 32m^3 + 374m^2 + 1888m + 3465)}$$

**giac** [B] time = 0.19, size = 340, normalized size = 4.79

$$\frac{B^2c^2m^3 + 21B^2c^2m^2 + 143B^2c^2m + 315B^2c^2 + 2B^2b^2c^2m^3 + 770B^2b^2c^2m^2 + 385A^2c^2m^2 + 167(2B^2b^2c^2 + A^2c^2)m^2 + 199(B^2b^2 + 2A^2b^2c^2)m^2 + 495B^2b^2 + 990A^2b^2c^2 + 239A^2b^2m^2 + 693A^2b^2)x^m}{m^4 + 32m^3 + 374m^2 + 1888m + 3465}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="giac")

[Out] 
$$\frac{(B^2c^2m^3x^{11}x^m + 21B^2c^2m^2x^{11}x^m + 2B^2b^2c^2m^3x^9x^m + A^2c^2m^3x^9x^m + 143B^2c^2m^2x^{11}x^m + 46B^2b^2c^2m^2x^9x^m + 23A^2c^2m^2x^9x^m + 315B^2c^2m^2x^{11}x^m + B^2b^2m^3x^7x^m + 2A^2b^2c^2m^3x^7x^m + 334B^2b^2c^2m^2x^9x^m + 167A^2c^2m^2x^9x^m + 25B^2b^2m^2x^7x^m + 50A^2b^2c^2m^2x^7x^m + 770B^2b^2c^2x^9x^m + 385A^2c^2x^9x^m + A^2b^2m^3x^5x^m + 199B^2b^2m^2x^7x^m + 398A^2b^2c^2m^2x^7x^m + 27A^2b^2m^2x^5x^m + 495B^2b^2x^7x^m + 990A^2b^2c^2x^7x^m + 239A^2b^2m^2x^5x^m + 693A^2b^2x^5x^m)/(m^4 + 32m^3 + 374m^2 + 1888m + 3465)}$$

**maple** [B] time = 0.05, size = 262, normalized size = 3.69

$$\frac{(B^2c^2m^3 + 21B^2c^2m^2 + 143B^2c^2m + 315B^2c^2 + 2B^2b^2c^2m^3 + 770B^2b^2c^2m^2 + 385A^2c^2m^2 + 167(2B^2b^2c^2 + A^2c^2)m^2 + 199(B^2b^2 + 2A^2b^2c^2)m^2 + 495B^2b^2 + 990A^2b^2c^2 + 239A^2b^2m^2 + 693A^2b^2)x^{m+5}}{(m+11)(m+9)(m+7)(m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x)

[Out] 
$$x^{(m+5)} \cdot \frac{(B^2c^2m^3x^6 + 21B^2c^2m^2x^6 + A^2c^2m^3x^4 + 2B^2b^2c^2m^3x^4 + 143B^2c^2m^2x^6 + 23A^2c^2m^2x^4 + 46B^2b^2c^2m^2x^4 + 315B^2c^2m^2x^6 + 2A^2b^2c^2m^3x^2 + 167A^2c^2m^2x^4 + 50A^2b^2c^2m^2x^2 + 385A^2c^2x^4 + 25B^2b^2m^2x^2 + 770B^2b^2c^2x^4 + A^2b^2m^3x^2 + 199B^2b^2m^2x^2 + 27A^2b^2m^2x^2 + 990A^2b^2c^2x^2 + 495B^2b^2x^2 + 239A^2b^2m + 693A^2b^2)/(m+11)/(m+9)/(m+7)/(m+5)}$$

**maxima** [A] time = 1.37, size = 91, normalized size = 1.28

$$\frac{Bc^2x^{m+11}}{m+11} + \frac{2Bbcx^{m+9}}{m+9} + \frac{Ac^2x^{m+9}}{m+9} + \frac{Bb^2x^{m+7}}{m+7} + \frac{2Abcx^{m+7}}{m+7} + \frac{Ab^2x^{m+5}}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2)^2,x, algorithm="maxima")

[Out] 
$$B^2c^2x^{(m+11)}/(m+11) + 2B^2b^2c^2x^{(m+9)}/(m+9) + A^2c^2x^{(m+9)}/(m+9) + B^2b^2x^{(m+7)}/(m+7) + 2A^2b^2c^2x^{(m+7)}/(m+7) + A^2b^2x^{(m+5)}/(m+5)$$

**mupad [B]** time = 0.34, size = 179, normalized size = 2.52

$$x^m \left( \frac{A b^2 x^5 (m^3 + 27 m^2 + 239 m + 693)}{m^4 + 32 m^3 + 374 m^2 + 1888 m + 3465} + \frac{B c^2 x^{11} (m^3 + 21 m^2 + 143 m + 315)}{m^4 + 32 m^3 + 374 m^2 + 1888 m + 3465} + \frac{b x^7 (2 A c + B b) (m^3 + 25 m^2 + 199 m + 495)}{m^4 + 32 m^3 + 374 m^2 + 1888 m + 3465} + \frac{c x^9 (A c + 2 B b) (m^3 + 23 m^2 + 167 m + 385)}{m^4 + 32 m^3 + 374 m^2 + 1888 m + 3465} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4)^2,x)

[Out] x^m\*((A\*b^2\*x^5\*(239\*m + 27\*m^2 + m^3 + 693))/(1888\*m + 374\*m^2 + 32\*m^3 + m^4 + 3465) + (B\*c^2\*x^11\*(143\*m + 21\*m^2 + m^3 + 315))/(1888\*m + 374\*m^2 + 32\*m^3 + m^4 + 3465) + (b\*x^7\*(2\*A\*c + B\*b)\*(199\*m + 25\*m^2 + m^3 + 495))/(1888\*m + 374\*m^2 + 32\*m^3 + m^4 + 3465) + (c\*x^9\*(A\*c + 2\*B\*b)\*(167\*m + 23\*m^2 + m^3 + 385))/(1888\*m + 374\*m^2 + 32\*m^3 + m^4 + 3465))

**sympy [A]** time = 4.42, size = 1051, normalized size = 14.80



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*(B\*x\*\*2+A)\*(c\*x\*\*4+b\*x\*\*2)\*\*2,x)

[Out] Piecewise((-A\*b\*\*2/(6\*x\*\*6) - A\*b\*c/(2\*x\*\*4) - A\*c\*\*2/(2\*x\*\*2) - B\*b\*\*2/(4\*x\*\*4) - B\*b\*c/x\*\*2 + B\*c\*\*2\*log(x), Eq(m, -11)), (-A\*b\*\*2/(4\*x\*\*4) - A\*b\*c/x\*\*2 + A\*c\*\*2\*log(x) - B\*b\*\*2/(2\*x\*\*2) + 2\*B\*b\*c\*log(x) + B\*c\*\*2\*x\*\*2/2, Eq(m, -9)), (-A\*b\*\*2/(2\*x\*\*2) + 2\*A\*b\*c\*log(x) + A\*c\*\*2\*x\*\*2/2 + B\*b\*\*2\*log(x) + B\*b\*c\*x\*\*2 + B\*c\*\*2\*x\*\*4/4, Eq(m, -7)), (A\*b\*\*2\*log(x) + A\*b\*c\*x\*\*2 + A\*c\*\*2\*x\*\*4/4 + B\*b\*\*2\*x\*\*2/2 + B\*b\*c\*x\*\*4/2 + B\*c\*\*2\*x\*\*6/6, Eq(m, -5)), (A\*b\*\*2\*m\*\*3\*x\*\*5\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 27\*A\*b\*\*2\*m\*\*2\*x\*\*5\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 239\*A\*b\*\*2\*m\*x\*\*5\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 693\*A\*b\*\*2\*x\*\*5\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 2\*A\*b\*c\*m\*\*3\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 50\*A\*b\*c\*m\*\*2\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 398\*A\*b\*c\*m\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 990\*A\*b\*c\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + A\*c\*\*2\*m\*\*3\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 23\*A\*c\*\*2\*m\*\*2\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 167\*A\*c\*\*2\*m\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 385\*A\*c\*\*2\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + B\*b\*\*2\*m\*\*3\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 25\*B\*b\*\*2\*m\*\*2\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 199\*B\*b\*\*2\*m\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 495\*B\*b\*\*2\*x\*\*7\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 2\*B\*b\*c\*m\*\*3\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 46\*B\*b\*c\*m\*\*2\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 334\*B\*b\*c\*m\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 770\*B\*b\*c\*x\*\*9\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + B\*c\*\*2\*m\*\*3\*x\*\*11\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 21\*B\*c\*\*2\*m\*\*2\*x\*\*11\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 143\*B\*c\*\*2\*m\*x\*\*11\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465) + 315\*B\*c\*\*2\*x\*\*11\*x\*\*m/(m\*\*4 + 32\*m\*\*3 + 374\*m\*\*2 + 1888\*m + 3465), True))

$$3.222 \quad \int x^m (A + Bx^2) (bx^2 + cx^4) dx$$

**Optimal.** Leaf size=45

$$\frac{x^{m+5}(Ac + bB)}{m + 5} + \frac{Abx^{m+3}}{m + 3} + \frac{Bcx^{m+7}}{m + 7}$$

**Rubi [A]** time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1584, 448}

$$\frac{x^{m+5}(Ac + bB)}{m + 5} + \frac{Abx^{m+3}}{m + 3} + \frac{Bcx^{m+7}}{m + 7}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

[Out] (A\*b\*x^(3 + m))/(3 + m) + ((b\*B + A\*c)\*x^(5 + m))/(5 + m) + (B\*c\*x^(7 + m))/(7 + m)

**Rule 448**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(e\*x)^m\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rule 1584**

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

**Rubi steps**

$$\begin{aligned} \int x^m (A + Bx^2) (bx^2 + cx^4) dx &= \int x^{2+m} (A + Bx^2) (b + cx^2) dx \\ &= \int (Abx^{2+m} + (bB + Ac)x^{4+m} + Bcx^{6+m}) dx \\ &= \frac{Abx^{3+m}}{3 + m} + \frac{(bB + Ac)x^{5+m}}{5 + m} + \frac{Bcx^{7+m}}{7 + m} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 42, normalized size = 0.93

$$x^{m+3} \left( \frac{x^2(Ac + bB)}{m + 5} + \frac{Ab}{m + 3} + \frac{Bcx^4}{m + 7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

[Out] x^(3 + m)\*((A\*b)/(3 + m) + ((b\*B + A\*c)\*x^2)/(5 + m) + (B\*c\*x^4)/(7 + m))

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int x^m (A + Bx^2) (bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

[Out] Defer[IntegrateAlgebraic][x^m\*(A + B\*x^2)\*(b\*x^2 + c\*x^4), x]

**fricas** [B] time = 0.42, size = 94, normalized size = 2.09

$$\frac{((Bcm^2 + 8Bcm + 15Bc)x^7 + ((Bb + Ac)m^2 + 21Bb + 21Ac + 10(Bb + Ac)m)x^5 + (Abm^2 + 12Abm + 35Ab)x^3)x^m}{m^3 + 15m^2 + 71m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x, algorithm="fricas")

[Out] ((B\*c\*m^2 + 8\*B\*c\*m + 15\*B\*c)\*x^7 + ((B\*b + A\*c)\*m^2 + 21\*B\*b + 21\*A\*c + 10\*(B\*b + A\*c)\*m)\*x^5 + (A\*b\*m^2 + 12\*A\*b\*m + 35\*A\*b)\*x^3)\*x^m/(m^3 + 15\*m^2 + 71\*m + 105)

**giac** [B] time = 0.16, size = 149, normalized size = 3.31

$$\frac{Bcm^2x^7x^m + 8Bcmx^7x^m + Bbm^2x^5x^m + Acn^2x^5x^m + 15Bcx^7x^m + 10Bbm^2x^5x^m + 10Acn^2x^5x^m + Abm^2x^3x^m + 21Bbx^5x^m + 21Acx^5x^m + 12Abmx^3x^m + 35Abx^3x^m}{m^3 + 15m^2 + 71m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x, algorithm="giac")

[Out] (B\*c\*m^2\*x^7\*x^m + 8\*B\*c\*m\*x^7\*x^m + B\*b\*m^2\*x^5\*x^m + A\*c\*m^2\*x^5\*x^m + 15\*B\*c\*x^7\*x^m + 10\*B\*b\*m\*x^5\*x^m + 10\*A\*c\*m\*x^5\*x^m + A\*b\*m^2\*x^3\*x^m + 21\*B\*b\*x^5\*x^m + 21\*A\*c\*x^5\*x^m + 12\*A\*b\*m\*x^3\*x^m + 35\*A\*b\*x^3\*x^m)/(m^3 + 15\*m^2 + 71\*m + 105)

**maple** [B] time = 0.05, size = 110, normalized size = 2.44

$$\frac{(Bcm^2x^4 + 8Bcmx^4 + Ac m^2x^2 + Bbm^2x^2 + 15Bcx^4 + 10Ac m x^2 + 10Bbm x^2 + Ab m^2 + 21Ac x^2 + 21Bb x^2 + 12Abm + 35Ab)x^{m+3}}{(m+7)(m+5)(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x)

[Out] x^(m+3)\*(B\*c\*m^2\*x^4+8\*B\*c\*m\*x^4+A\*c\*m^2\*x^2+B\*b\*m^2\*x^2+15\*B\*c\*x^4+10\*A\*c\*m\*x^2+10\*B\*b\*m\*x^2+A\*b\*m^2+21\*A\*c\*x^2+21\*B\*b\*x^2+12\*A\*b\*m+35\*A\*b)/(m+7)/(m+5)/(m+3)

**maxima** [A] time = 1.30, size = 53, normalized size = 1.18

$$\frac{Bcx^{m+7}}{m+7} + \frac{Bbx^{m+5}}{m+5} + \frac{Acx^{m+5}}{m+5} + \frac{Abx^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(B\*x^2+A)\*(c\*x^4+b\*x^2), x, algorithm="maxima")

[Out] B\*c\*x^(m+7)/(m+7) + B\*b\*x^(m+5)/(m+5) + A\*c\*x^(m+5)/(m+5) + A\*b\*x^(m+3)/(m+3)

**mupad** [B] time = 0.25, size = 97, normalized size = 2.16

$$x^m \left( \frac{x^5 (Ac + Bb) (m^2 + 10m + 21)}{m^3 + 15m^2 + 71m + 105} + \frac{Abx^3 (m^2 + 12m + 35)}{m^3 + 15m^2 + 71m + 105} + \frac{Bcx^7 (m^2 + 8m + 15)}{m^3 + 15m^2 + 71m + 105} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^m(A + Bx^2)(bx^2 + cx^4), x)$

[Out]  $x^m((x^5(Ac + Bb)(10m + m^2 + 21))/(71m + 15m^2 + m^3 + 105) + (Abx^3(12m + m^2 + 35))/(71m + 15m^2 + m^3 + 105) + (Bcx^7(8m + m^2 + 15))/(71m + 15m^2 + m^3 + 105))$

sympy [A] time = 1.62, size = 415, normalized size = 9.22

$$\left\{ \begin{array}{ll} -\frac{Ab}{4x} - \frac{Ac}{2x^2} - \frac{Bb}{2x^2} + Bc \log(x) & \text{for } m = -7 \\ \frac{Ab}{2x} + Ac \log(x) + Bb \log(x) + \frac{Bcx^2}{2} & \text{for } m = -5 \\ Ab \log(x) + \frac{Acx^2}{2} + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} & \text{for } m = -3 \\ \frac{Abm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{12Abmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{35Abx^{m+105}}{m^3+15m^2+71m+105} + \frac{Acmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{10Acmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{21Acx^{m+105}}{m^3+15m^2+71m+105} + \frac{Bbmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{10Bbmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{21Bbx^{m+105}}{m^3+15m^2+71m+105} + \frac{Bcmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{8Bcmm^2x^{m+105}}{m^3+15m^2+71m+105} + \frac{15Bcx^{m+105}}{m^3+15m^2+71m+105} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^{**m}(B*x^{**2}+A)*(c*x^{**4}+b*x^{**2}), x)$

[Out]  $\text{Piecewise}((-A*b/(4*x^{**4}) - A*c/(2*x^{**2}) - B*b/(2*x^{**2}) + B*c*\log(x), \text{Eq}(m, -7)), (-A*b/(2*x^{**2}) + A*c*\log(x) + B*b*\log(x) + B*c*x^{**2}/2, \text{Eq}(m, -5)), (A*b*\log(x) + A*c*x^{**2}/2 + B*b*x^{**2}/2 + B*c*x^{**4}/4, \text{Eq}(m, -3)), (A*b*m^{**2}*x^{**3}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 12*A*b*m*x^{**3}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 35*A*b*x^{**3}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + A*c*m^{**2}*x^{**5}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 10*A*c*m*x^{**5}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 21*A*c*x^{**5}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + B*b*m^{**2}*x^{**5}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 10*B*b*m*x^{**5}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 21*B*b*x^{**5}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + B*c*m^{**2}*x^{**7}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 8*B*c*m*x^{**7}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105) + 15*B*c*x^{**7}*x^{**m}/(m^{**3} + 15*m^{**2} + 71*m + 105), \text{True}))$

$$3.223 \quad \int x^{-1+n-jp} (c + dx^n) (ax^j + bx^{j+n})^p dx$$

**Optimal.** Leaf size=95

$$\frac{dx^{n-j(p+1)} (ax^j + bx^{j+n})^{p+1}}{bn(p+2)} - \frac{x^{-j(p+1)}(ad - bc(p+2)) (ax^j + bx^{j+n})^{p+1}}{b^2n(p+1)(p+2)}$$

**Rubi [A]** time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {2039, 2014}

$$\frac{dx^{n-j(p+1)} (ax^j + bx^{j+n})^{p+1}}{bn(p+2)} - \frac{x^{-j(p+1)}(ad - bc(p+2)) (ax^j + bx^{j+n})^{p+1}}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n - j\*p)\*(c + d\*x^n)\*(a\*x^j + b\*x^(j + n))^p,x]

[Out] -(((a\*d - b\*c\*(2 + p))\*(a\*x^j + b\*x^(j + n))^(1 + p))/(b^2\*n\*(1 + p)\*(2 + p)\*x^(j\*(1 + p)))) + (d\*x^(n - j\*(1 + p))\*(a\*x^j + b\*x^(j + n))^(1 + p))/(b\*n\*(2 + p))

Rule 2014

Int[((c\_)\*(x\_))^(m\_)\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(c^(j - 1)\*(c\*x)^(m - j + 1)\*(a\*x^j + b\*x^n)^(p + 1))/(a\*(n - j)\*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n\*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2039

Int[((e\_)\*(x\_))^(m\_)\*((a\_)\*(x\_)^(j\_) + (b\_)\*(x\_)^(jn\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*e^(j - 1)\*(e\*x)^(m - j + 1)\*(a\*x^j + b\*x^(j + n))^(p + 1))/(b\*(m + n + p\*(j + n) + 1)), x] - Dist[(a\*d\*(m + j\*p + 1) - b\*c\*(m + n + p\*(j + n) + 1))/(b\*(m + n + p\*(j + n) + 1)), Int[(e\*x)^m\*(a\*x^j + b\*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b\*c - a\*d, 0] && NeQ[m + n + p\*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])

Rubi steps

$$\begin{aligned} \int x^{-1+n-jp} (c + dx^n) (ax^j + bx^{j+n})^p dx &= \frac{dx^{n-j(1+p)} (ax^j + bx^{j+n})^{1+p}}{bn(2+p)} - \left(-c + \frac{ad}{b(2+p)}\right) \int x^{-1+n-jp} (ax^j + bx^{j+n})^p \\ &= \frac{\left(c - \frac{ad}{b(2+p)}\right) x^{-j(1+p)} (ax^j + bx^{j+n})^{1+p}}{bn(1+p)} + \frac{dx^{n-j(1+p)} (ax^j + bx^{j+n})^{1+p}}{bn(2+p)} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 63, normalized size = 0.66

$$\frac{x^{-jp} (a + bx^n) (x^j (a + bx^n))^p (-ad + bc(p+2) + bd(p+1)x^n)}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n - j\*p)\*(c + d\*x^n)\*(a\*x^j + b\*x^(j + n))^p,x]



[Out]  $((a + b*x^n)*(x^j*(a + b*x^n))^p*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^n))/(b^2*n*(1 + p)*(2 + p)*x^{(j*p)})$

**IntegrateAlgebraic** [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{-1+n-jp} (c + dx^n) (ax^j + bx^{j+n})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x<sup>-1 + n - j\*p</sup>\*(c + d\*x<sup>n</sup>)\*(a\*x<sup>j</sup> + b\*x<sup>(j + n)</sup>)<sup>p</sup>, x]

[Out] Defer[IntegrateAlgebraic][x<sup>-1 + n - j\*p</sup>\*(c + d\*x<sup>n</sup>)\*(a\*x<sup>j</sup> + b\*x<sup>(j + n)</sup>)<sup>p</sup>, x]

**fricas** [A] time = 0.43, size = 140, normalized size = 1.47

$$\frac{((b^2 dp + b^2 d)xx^{-jp+n-1}x^{2n} + (2b^2c + (b^2c + abd)p)xx^{-jp+n-1}x^n + (abcp + 2abc - a^2d)xx^{-jp+n-1})\left(\frac{(bx^n+a)x^{j+n}}{x^n}\right)^p}{(b^2np^2 + 3b^2np + 2b^2n)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>-j\*p+n-1</sup>\*(c+d\*x<sup>n</sup>)\*(a\*x<sup>j</sup>+b\*x<sup>(j+n)</sup>)<sup>p</sup>, x, algorithm="fricas")

[Out]  $((b^2*d*p + b^2*d)*x*x^{(-j*p + n - 1)*x^{(2*n)} + (2*b^2*c + (b^2*c + a*b*d)*p)*x*x^{(-j*p + n - 1)*x^n} + (a*b*c*p + 2*a*b*c - a^2*d)*x*x^{(-j*p + n - 1)})*((b*x^n + a)*x^{(j + n)}/x^n)^p/((b^2*n*p^2 + 3*b^2*n*p + 2*b^2*n)*x^n)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)(bx^{j+n} + ax^j)^p x^{-jp+n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>-j\*p+n-1</sup>\*(c+d\*x<sup>n</sup>)\*(a\*x<sup>j</sup>+b\*x<sup>(j+n)</sup>)<sup>p</sup>, x, algorithm="giac")

[Out] integrate((d\*x<sup>n</sup> + c)\*(b\*x<sup>(j + n)</sup> + a\*x<sup>j</sup>)<sup>p</sup>\*x<sup>(-j\*p + n - 1)</sup>, x)

**maple** [F] time = 0.80, size = 0, normalized size = 0.00

$$\int (dx^n + c)x^{-jp+n-1} (ax^j + bx^{j+n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>-j\*p+n-1</sup>\*(d\*x<sup>n</sup>+c)\*(a\*x<sup>j</sup>+b\*x<sup>(j+n)</sup>)<sup>p</sup>, x)

[Out] int(x<sup>-j\*p+n-1</sup>\*(d\*x<sup>n</sup>+c)\*(a\*x<sup>j</sup>+b\*x<sup>(j+n)</sup>)<sup>p</sup>, x)

**maxima** [A] time = 2.04, size = 112, normalized size = 1.18

$$\frac{(bx^n + a)ce^{(-jp \log(x) + p \log(bx^n + a) + p \log(x^j))}}{bn(p + 1)} + \frac{(b^2(p + 1)x^{2n} + abpx^n - a^2)de^{(-jp \log(x) + p \log(bx^n + a) + p \log(x^j))}}{(p^2 + 3p + 2)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>-j\*p+n-1</sup>\*(c+d\*x<sup>n</sup>)\*(a\*x<sup>j</sup>+b\*x<sup>(j+n)</sup>)<sup>p</sup>, x, algorithm="maxima")

[Out]  $(b*x^n + a)*c*e^{(-j*p*\log(x) + p*\log(b*x^n + a) + p*\log(x^j))}/(b*n*(p + 1)) + (b^2*(p + 1)*x^{(2*n)} + a*b*p*x^n - a^2)*d*e^{(-j*p*\log(x) + p*\log(b*x^n + a) + p*\log(x^j))}/((p^2 + 3*p + 2)*b^2*n)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{n-jp-1} (ax^j + bx^{j+n})^p (c + dx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n - j\*p - 1)\*(a\*x^j + b\*x^(j + n))^p\*(c + d\*x^n), x)

[Out] int(x^(n - j\*p - 1)\*(a\*x^j + b\*x^(j + n))^p\*(c + d\*x^n), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(-j\*p+n-1)\*(c+d\*x\*\*n)\*(a\*x\*\*j+b\*x\*\*(j+n))\*\*p, x)

[Out] Timed out

$$3.224 \quad \int \frac{4+3x^4}{5x+2x^5} dx$$

Optimal. Leaf size=19

$$\frac{7}{40} \log(2x^4 + 5) + \frac{4 \log(x)}{5}$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1593, 446, 72}

$$\frac{7}{40} \log(2x^4 + 5) + \frac{4 \log(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(4 + 3\*x^4)/(5\*x + 2\*x^5), x]

[Out] (4\*Log[x])/5 + (7\*Log[5 + 2\*x^4])/40

Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{4 + 3x^4}{5x + 2x^5} dx &= \int \frac{4 + 3x^4}{x(5 + 2x^4)} dx \\ &= \frac{1}{4} \text{Subst} \left( \int \frac{4 + 3x}{x(5 + 2x)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left( \int \left( \frac{4}{5x} + \frac{7}{5(5 + 2x)} \right) dx, x, x^4 \right) \\ &= \frac{4 \log(x)}{5} + \frac{7}{40} \log(5 + 2x^4) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{7}{40} \log(2x^4 + 5) + \frac{4 \log(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3\*x^4)/(5\*x + 2\*x^5), x]

[Out]  $(4*\text{Log}[x])/5 + (7*\text{Log}[5 + 2*x^4])/40$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + 3x^4}{5x + 2x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + 3\*x^4)/(5\*x + 2\*x^5), x]

[Out] IntegrateAlgebraic[(4 + 3\*x^4)/(5\*x + 2\*x^5), x]

**fricas** [A] time = 0.39, size = 15, normalized size = 0.79

$$\frac{7}{40} \log(2x^4 + 5) + \frac{4}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^4+4)/(2\*x^5+5\*x), x, algorithm="fricas")

[Out] 7/40\*log(2\*x^4 + 5) + 4/5\*log(x)

**giac** [A] time = 0.16, size = 17, normalized size = 0.89

$$\frac{7}{40} \log(2x^4 + 5) + \frac{1}{5} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^4+4)/(2\*x^5+5\*x), x, algorithm="giac")

[Out] 7/40\*log(2\*x^4 + 5) + 1/5\*log(x^4)

**maple** [A] time = 0.05, size = 16, normalized size = 0.84

$$\frac{4 \ln(x)}{5} + \frac{7 \ln(2x^4 + 5)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^4+4)/(2\*x^5+5\*x), x)

[Out] 4/5\*ln(x)+7/40\*ln(2\*x^4+5)

**maxima** [A] time = 2.91, size = 15, normalized size = 0.79

$$\frac{7}{40} \log(2x^4 + 5) + \frac{4}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^4+4)/(2\*x^5+5\*x), x, algorithm="maxima")

[Out] 7/40\*log(2\*x^4 + 5) + 4/5\*log(x)

**mupad** [B] time = 0.17, size = 13, normalized size = 0.68

$$\frac{7 \ln\left(x^4 + \frac{5}{2}\right)}{40} + \frac{4 \ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 4)/(5*x + 2*x^5), x)
```

```
[Out] (7*log(x^4 + 5/2))/40 + (4*log(x))/5
```

**sympy [A]** time = 0.11, size = 17, normalized size = 0.89

$$\frac{4 \log(x)}{5} + \frac{7 \log(2x^4 + 5)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/(2*x**5+5*x), x)
```

```
[Out] 4*log(x)/5 + 7*log(2*x**4 + 5)/40
```

$$3.225 \quad \int \frac{1+x^6}{x-x^7} dx$$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{3} \log(1-x^6)$$

**Rubi [A]** time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1593, 446, 72}

$$\log(x) - \frac{1}{3} \log(1-x^6)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(x - x^7), x]

[Out] Log[x] - Log[1 - x^6]/3

#### Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
  x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.),
  x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
  *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
  b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
  ^((n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
  PosQ[q - p]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1+x^6}{x-x^7} dx &= \int \frac{1+x^6}{x(1-x^6)} dx \\ &= \frac{1}{6} \text{Subst} \left( \int \frac{1+x}{(1-x)x} dx, x, x^6 \right) \\ &= \frac{1}{6} \text{Subst} \left( \int \left( -\frac{2}{-1+x} + \frac{1}{x} \right) dx, x, x^6 \right) \\ &= \log(x) - \frac{1}{3} \log(1-x^6) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{3} \log(1-x^6)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^6)/(x - x^7), x]

[Out]  $\text{Log}[x] - \text{Log}[1 - x^6]/3$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{x-x^7} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(1 + x^6)/(x - x^7), x]`

[Out] `IntegrateAlgebraic[(1 + x^6)/(x - x^7), x]`

**fricas** [A] time = 0.39, size = 11, normalized size = 0.73

$$-\frac{1}{3} \log(x^6 - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6+1)/(-x^7+x), x, algorithm="fricas")`

[Out] `-1/3*log(x^6 - 1) + log(x)`

**giac** [A] time = 0.15, size = 16, normalized size = 1.07

$$\frac{1}{6} \log(x^6) - \frac{1}{3} \log(|x^6 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6+1)/(-x^7+x), x, algorithm="giac")`

[Out] `1/6*log(x^6) - 1/3*log(abs(x^6 - 1))`

**maple** [B] time = 0.05, size = 36, normalized size = 2.40

$$\ln(x) - \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} - \frac{\ln(x^2-x+1)}{3} - \frac{\ln(x^2+x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^6+1)/(-x^7+x), x)`

[Out] `-1/3*ln(x-1)-1/3*ln(x^2+x+1)-1/3*ln(x+1)+ln(x)-1/3*ln(x^2-x+1)`

**maxima** [B] time = 3.01, size = 35, normalized size = 2.33

$$-\frac{1}{3} \log(x^2 + x + 1) - \frac{1}{3} \log(x^2 - x + 1) - \frac{1}{3} \log(x + 1) - \frac{1}{3} \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6+1)/(-x^7+x), x, algorithm="maxima")`

[Out] `-1/3*log(x^2 + x + 1) - 1/3*log(x^2 - x + 1) - 1/3*log(x + 1) - 1/3*log(x - 1) + log(x)`

**mupad** [B] time = 0.15, size = 11, normalized size = 0.73

$$\ln(x) - \frac{\ln(x^6 - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x - x^7), x)
```

```
[Out] log(x) - log(x^6 - 1)/3
```

**sympy [A]** time = 0.12, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log(x^6 - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(-x**7+x), x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```



$$3.226 \quad \int \frac{8+5x^{10}}{2x-x^{11}} dx$$

Optimal. Leaf size=17

$$4 \log(x) - \frac{9}{10} \log(2 - x^{10})$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1593, 446, 72}

$$4 \log(x) - \frac{9}{10} \log(2 - x^{10})$$

Antiderivative was successfully verified.

[In] Int[(8 + 5\*x^10)/(2\*x - x^11), x]

[Out] 4\*Log[x] - (9\*Log[2 - x^10])/10

Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{8+5x^{10}}{2x-x^{11}} dx &= \int \frac{8+5x^{10}}{x(2-x^{10})} dx \\ &= \frac{1}{10} \text{Subst} \left( \int \frac{8+5x}{(2-x)x} dx, x, x^{10} \right) \\ &= \frac{1}{10} \text{Subst} \left( \int \left( -\frac{9}{-2+x} + \frac{4}{x} \right) dx, x, x^{10} \right) \\ &= 4 \log(x) - \frac{9}{10} \log(2 - x^{10}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$4 \log(x) - \frac{9}{10} \log(2 - x^{10})$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 5\*x^10)/(2\*x - x^11), x]

[Out]  $4*\text{Log}[x] - (9*\text{Log}[2 - x^{10}])/10$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8 + 5x^{10}}{2x - x^{11}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 5\*x^10)/(2\*x - x^11), x]

[Out] IntegrateAlgebraic[(8 + 5\*x^10)/(2\*x - x^11), x]

**fricas** [A] time = 0.39, size = 13, normalized size = 0.76

$$-\frac{9}{10} \log(x^{10} - 2) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^10+8)/(-x^11+2\*x), x, algorithm="fricas")

[Out] -9/10\*log(x^10 - 2) + 4\*log(x)

**giac** [A] time = 0.19, size = 16, normalized size = 0.94

$$\frac{2}{5} \log(x^{10}) - \frac{9}{10} \log(|x^{10} - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^10+8)/(-x^11+2\*x), x, algorithm="giac")

[Out] 2/5\*log(x^10) - 9/10\*log(abs(x^10 - 2))

**maple** [A] time = 0.05, size = 14, normalized size = 0.82

$$4 \ln(x) - \frac{9 \ln(x^{10} - 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^10+8)/(-x^11+2\*x), x)

[Out] -9/10\*ln(x^10-2)+4\*ln(x)

**maxima** [A] time = 2.94, size = 13, normalized size = 0.76

$$-\frac{9}{10} \log(x^{10} - 2) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^10+8)/(-x^11+2\*x), x, algorithm="maxima")

[Out] -9/10\*log(x^10 - 2) + 4\*log(x)

**mupad** [B] time = 0.14, size = 13, normalized size = 0.76

$$4 \ln(x) - \frac{9 \ln(x^{10} - 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^10 + 8)/(2\*x - x^11), x)

```
[Out] 4*log(x) - (9*log(x^10 - 2))/10
```

```
sympy [A] time = 0.15, size = 14, normalized size = 0.82
```

$$4 \log(x) - \frac{9 \log(x^{10} - 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**10+8)/(-x**11+2*x), x)
```

```
[Out] 4*log(x) - 9*log(x**10 - 2)/10
```

$$3.227 \quad \int \frac{-3+2x}{-x^2+x^3} dx$$

Optimal. Leaf size=16

$$-\frac{3}{x} - \log(1-x) + \log(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1593, 77}

$$-\frac{3}{x} - \log(1-x) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2\*x)/(-x^2 + x^3), x]

[Out] -3/x - Log[1 - x] + Log[x]

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{-3+2x}{-x^2+x^3} dx &= \int \frac{-3+2x}{(-1+x)x^2} dx \\ &= \int \left( \frac{1}{1-x} + \frac{3}{x^2} + \frac{1}{x} \right) dx \\ &= -\frac{3}{x} - \log(1-x) + \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$-\frac{3}{x} - \log(1-x) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 2\*x)/(-x^2 + x^3), x]

[Out] -3/x - Log[1 - x] + Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+2x}{-x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + 2\*x)/(-x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-3 + 2\*x)/(-x^2 + x^3), x]

**fricas** [A] time = 0.39, size = 18, normalized size = 1.12

$$-\frac{x \log(x-1) - x \log(x) + 3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2\*x)/(x^3-x^2), x, algorithm="fricas")

[Out] -(x\*log(x - 1) - x\*log(x) + 3)/x

**giac** [A] time = 0.18, size = 16, normalized size = 1.00

$$-\frac{3}{x} - \log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2\*x)/(x^3-x^2), x, algorithm="giac")

[Out] -3/x - log(abs(x - 1)) + log(abs(x))

**maple** [A] time = 0.05, size = 15, normalized size = 0.94

$$\ln(x) - \ln(x-1) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x-3)/(x^3-x^2), x)

[Out] -ln(x-1)+ln(x)-3/x

**maxima** [A] time = 1.62, size = 14, normalized size = 0.88

$$-\frac{3}{x} - \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2\*x)/(x^3-x^2), x, algorithm="maxima")

[Out] -3/x - log(x - 1) + log(x)

**mupad** [B] time = 0.13, size = 14, normalized size = 0.88

$$2 \operatorname{atanh}(2x-1) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x - 3)/(x^2 - x^3), x)

[Out] 2\*atanh(2\*x - 1) - 3/x

**sympy** [A] time = 0.10, size = 10, normalized size = 0.62

$$\log(x) - \log(x-1) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2\*x)/(x\*\*3-x\*\*2), x)

[Out] log(x) - log(x - 1) - 3/x

## 3.228

$$\int x^m (a + bx^n)^p (a(1 + m + q)x^q + b(1 + m + n(1 + p) + q)x^{n+q}) dx$$

**Optimal.** Leaf size=18

$$x^{m+q+1} (a + bx^n)^{p+1}$$

**Rubi [A]** time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {1584, 449}

$$x^{m+q+1} (a + bx^n)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^n)^p\*(a\*(1 + m + q)\*x^q + b\*(1 + m + n\*(1 + p) + q)\*x^(n + q)),x]

[Out] x^(1 + m + q)\*(a + b\*x^n)^(1 + p)

Rule 449

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1), 0] && NeQ[m, -1]

Rule 1584

Int[(u\_)\*(x\_)^(m\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :> Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\int x^m (a + bx^n)^p (a(1 + m + q)x^q + b(1 + m + n(1 + p) + q)x^{n+q}) dx = \int x^{m+q} (a + bx^n)^p (a(1 + m + q) + b(1 + m + n(1 + p) + q)x^n) dx = x^{1+m+q} (a + bx^n)^{1+p}$$

**Mathematica [C]** time = 0.22, size = 116, normalized size = 6.44

$$x^{m+q+1} (a + bx^n)^p \left( \frac{bx^n}{a} + 1 \right)^{-p} \left( \frac{bx^n(m + np + n + q + 1) {}_2F_1\left(-p, \frac{m+n+q+1}{n}; \frac{m+2n+q+1}{n}; -\frac{bx^n}{a}\right)}{m+n+q+1} + a {}_2F_1\left(-p, \frac{m+q+1}{n}; \frac{m+n+q+1}{n}; -\frac{bx^n}{a}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^n)^p\*(a\*(1 + m + q)\*x^q + b\*(1 + m + n\*(1 + p) + q)\*x^(n + q)),x]

[Out] (x^(1 + m + q)\*(a + b\*x^n)^p\*(a\*Hypergeometric2F1[-p, (1 + m + q)/n, (1 + m + n + q)/n, -(b\*x^n)/a]) + (b\*(1 + m + n + n\*p + q)\*x^n\*Hypergeometric2F1[-p, (1 + m + n + q)/n, (1 + m + 2\*n + q)/n, -(b\*x^n)/a]))/(1 + m + n + q))/(1 + (b\*x^n)/a)^p

**IntegrateAlgebraic [F]** time = 0.36, size = 0, normalized size = 0.00

$$\int x^m (a + bx^n)^p (a(1 + m + q)x^q + b(1 + m + n(1 + p) + q)x^{n+q}) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m\*(a + b\*x^n)^p\*(a\*(1 + m + q)\*x^q + b\*(1 + m + n\*(1 + p) + q)\*x^(n + q)),x]

[Out] Defer[IntegrateAlgebraic][x^m\*(a + b\*x^n)^p\*(a\*(1 + m + q)\*x^q + b\*(1 + m + n\*(1 + p) + q)\*x^(n + q)), x]

**fricas** [B] time = 0.43, size = 43, normalized size = 2.39

$$(bxx^m x^{n+q} + axx^m x^q) \left( \frac{bx^{n+q} + ax^q}{x^q} \right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(a+b\*x^n)^p\*(a\*(1+m+q)\*x^q+b\*(1+m+n\*(1+p)+q)\*x^(n+q)),x, algorithm="fricas")

[Out] (b\*x\*x^m\*x^(n + q) + a\*x\*x^m\*x^q)\*((b\*x^(n + q) + a\*x^q)/x^q)^p

**giac** [B] time = 0.37, size = 48, normalized size = 2.67

$$(bx^n + a)^p bxx^n e^{(m \log(x) + q \log(x))} + (bx^n + a)^p axe^{(m \log(x) + q \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(a+b\*x^n)^p\*(a\*(1+m+q)\*x^q+b\*(1+m+n\*(1+p)+q)\*x^(n+q)),x, algorithm="giac")

[Out] (b\*x^n + a)^p\*b\*x\*x^n\*e^(m\*log(x) + q\*log(x)) + (b\*x^n + a)^p\*a\*x\*e^(m\*log(x) + q\*log(x))

**maple** [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \left( (m + q + 1) a x^q + (m + (p + 1) n + q + 1) b x^{n+q} \right) x^m (b x^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(b\*x^n+a)^p\*(a\*(1+m+q)\*x^q+b\*(1+m+(p+1)\*n+q)\*x^(n+q)),x)

[Out] int(x^m\*(b\*x^n+a)^p\*(a\*(1+m+q)\*x^q+b\*(1+m+(p+1)\*n+q)\*x^(n+q)),x)

**maxima** [B] time = 2.15, size = 37, normalized size = 2.06

$$\left( axx^m + bxe^{(m \log(x) + n \log(x))} \right) e^{(p \log(bx^n + a) + q \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*(a+b\*x^n)^p\*(a\*(1+m+q)\*x^q+b\*(1+m+n\*(1+p)+q)\*x^(n+q)),x, algorithm="maxima")

[Out] (a\*x\*x^m + b\*x\*e^(m\*log(x) + n\*log(x)))\*e^(p\*log(b\*x^n + a) + q\*log(x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.06

$$\int x^m \left( a x^q (m + q + 1) + b x^{n+q} (m + q + n (p + 1) + 1) \right) (a + b x^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(a\*x^q\*(m + q + 1) + b\*x^(n + q)\*(m + q + n\*(p + 1) + 1))\*(a + b\*x^n)^p,x)

[Out]  $\text{int}(x^m(a*x^q*(m + q + 1) + b*x^{(n + q)*(m + q + n*(p + 1) + 1)}*(a + b*x^n)^p, x)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x**m*(a+b*x**n)**p*(a*(1+m+q)*x**q+b*(1+m+n*(1+p)+q)*x**(n+q)), x)$

[Out] Timed out



# Chapter 4

## Appendix

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## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica\_syntax\_CAS\_integration\_elementary\_version.zip

**Maple and Mupad format** Maple\_syntax\_CAS\_integration\_elementary\_version.zip

**Sympy format** SYMPY\_syntax\_CAS\_integration\_elementary\_version.zip

**Sage math format** SAGE\_syntax\_CAS\_integration\_elementary\_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```

(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3, ExpnType[expn[[1]]]],
              If[SpecialFunctionQ[Head[expn]],
                Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
                If[HypergeometricFunctionQ[Head[expn]],
                  Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
                  If[AppellFunctionQ[Head[expn]],
                    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
                    If[Head[expn]===RootSum,
                      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
                      If[Head[expn]===Integrate || Head[expn]===Int,
                        Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
                        9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ[func_] :=

```

```
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):

```

```

try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
    else:
        return False

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    )))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'^+^') or
type(expn,'*^')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function

```



```

def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

#### 4.2.4 SageMath grading function

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#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False

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else:
    return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):

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        return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
            return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

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    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```